Adjustment to the Single Point Forcing on the Z Grid ; Linear Theory

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Abstract: Geostrophic adjustment on the Z grid, for the single point forcing was analyzed. Various regimes of the ratio of Rossby deformation radius, (and grid distance), d were examined. Results were compared with the similar analysis for the E/B and C grids.

Keywords: Geostrophic adjustment ; Distribution of variables ; Source-sink ; Single point forcing

1 Introduction

Starting with Winingoff [1968] followed by many others, Arakawa and Lamb [1977] it was understood that the choice of the variables distribution i.e. the choice of the grid has the major influence in the simulation of the gravity-inertial waves propagation. Aside from gravity-inertial waves propagation it turns out that propagation of long waves, namely Rossby waves is also affected by the choice of the grid. Papers by Mesinger [1979], Wajsowicz [1986] and Gavrilov and Tosic [1999] among others have clearly demonstrated that point. The geostrophic adjustment problem in the extreme case of a single point forcing deserves special attention since, in a model, the so-called physics is usually active on exactly that, the smallest resolvable scale. To investigate that Arakawa [1972], Mesinger [1973] and Janjic and Mesinger [1989], hereafter JM, performed the so-called Source-Sink (S-S) experiments, wherein one simulates point disturbances of the stratified fluid by adding/removing water within the shallow water framework for different ranges of the ratio λ/d. For the equations and grid distributions for C and E grid, see Mesinger and Arakawa [1976]. Randall [1970] has analyzed gravity-inertial wave propagation with divergence and vorticity as the problem variables. This enables him to consider yet another grid distribution, which he calls the Z grid. In his paper he advocates the use of the Z grid since it has excellent properties regarding gravity-inertial waves propagation. Even in the region of $\lambda/d < 1$ where two other most often used grids, C which has very serious problems with phase and group speeds and B/E as he states have moderate problems, the Z grid behaves very well.

2 The Source-Sink experiments

The JM paper was concerned with the performance of the staggered C and semi-staggered E grids with respect to the single point forcing within the shallow water framework. In order to judge the successfulness of a grid in the process of the geostrophic adjustment, authors have proposed that the "true" solution is the one that would have been obtained when the forcing area is represented with very large number of grid points. In their paper that was accomplished with the sequence of three resolutions, progressively finer, of 250, 125 and 62.5 kilometers, spanning the same forcing area of 250 by 250 kilometers. Those resolutions resulted in representation with 1, 4 and 16 points of the forcing area, respectively. Regarding the λ/d ratio JM experiments where in the range of 4, for the single point forcing and 8, 16 and 32 for the higher resolutions. If one follows, for instance the height of the sink point (area) at the end of 24 hours integration, their simulations show quite reasonable indication of the existence and also of the value of the convergence point, the "true" value. During the whole convergence process, in comparison to the "true" solution, the C grid exhibits systematic overshooting while the E grid shows systematic undershooting. JM argue that this lack of good performance, for the C grid is the consequence of the wrong representation of the Coriolis force. For the E grid, they argue that the overestimation of the sink's height is the consequence of the grid separation, since E grid has two C grids as its sub-grids. Finally, in the case of the E grid additional influence comes from the modification term, which is introduced exactly to prevent separation between the two C sub-grids (Mesinger [1973], Janjic [1974] and Janjic [1979]). Qualitatively such behavior could be predicted from the ratio of the amplitude of the wave solution, corresponding to the geostrophic part, and the amplitude of the wave component of the initial disturbance. Formally the same could be obtained as the ratio of the amplitudes of the vorticity and divergence . We prefer this ratio as a meaningful parameter for the following reasons. The final height filed is the consequence of the adjustment process wherein part of the height disturbance disperses through gravity-inertial waves and part creates cyclonic/anticyclonic circulation that stays around the forcing areas. With stronger vorticity and smaller divergence the hight in the source area will be higher. therefore we expect a deeper depth of the sink region. Therefore the ratio of vorticity and divergence (ζ/δ) is the key in understandings and explanation of the results. But to get (simulate) this ratio right both,(ζ and δ, should be done right. That ratio, for the continuous case, is

$$\left|\frac{\zeta}{\delta}\right|^2_{Cnt} = \frac{1}{1+\frac{\lambda^2}{d^2}(k^2+l^2)} \tag{1}$$

The corresponding finite difference analogues for the three grids, considered in this paper, are

$$\left|\frac{\zeta}{\delta}\right|^2_{C} =$$

$$\frac{\cos^2(\frac{kd}{2})+\cos^2(\frac{ld}{2})}{\cos^2(\frac{kd}{2})+\cos^2(\frac{ld}{2})+\frac{4\lambda^2}{d^2}[\sin^2(\frac{kd}{2})+\sin^2(\frac{ld}{2})]} \tag{2}$$

$$\left|\frac{\zeta}{\delta}\right|^2_{E} = \frac{1}{1+2\frac{\lambda^2}{d^2}[\sin^2(kd/\sqrt{2})+\sin^2(ld/\sqrt{2})]} \tag{3}$$

$$\left|\frac{\zeta}{\delta}\right|^2_{Z} = \frac{1}{1+4\frac{\lambda^2}{d^2}[\sin^2(kd/2)+\sin^2(ld/2)]} \tag{4}$$

Since we are primarily interested in the single point forcing we will analyze the ratio ζ/δ for the shortest resolvable scales and how does it depend on the λ/d ratio. For the C and Z grids we have then $kd = ld = p$ while for the E grid we have $kd = ld = p$. With these values ratio ζ/δ is 0, 1 and $1/(1+8\lambda^2/d^2)$ respectively for C, E and Z grids. Note that only for the Z grid this ratio depends on ld ratio. With the corresponding values for the continuous case the following relations are valid $[\zeta/\delta]C < [\zeta/\delta]A < [\zeta/\delta]Z < [\zeta/\delta]E$. From the previous argumentation larger ζ/δ , compared to the continuous case means over sized hills in the source area and too deep wholes in the sink area. In the case of the E grid since $[\zeta/\delta]_A < [\zeta/\delta]_E$ we know immediately that the situation should be the opposite in comparison with the C grid. We have to note that with the E grid situation is more complicated due to the action of the modification term. Although, strictly speaking the modification is not a "part" of the E grid one should really consider it as the E grid's integral part since you always want to prevent sub-grid separation. In that case magnitude of the error depends on the parameters of the modification term. See JM for more detailed discussion of the influence of the modification term and its dependence on the time step. We will also come back to this point as we discuss our results for various λ/d ratios. Finally, from 4 we see that the Z grid should be on the E grid side but less erroneous.

Still these are only preliminary ideas and to complete the JM analysis of response of various grids, to the single point forcing, we have performed and analyzed the S-S experiment on the Z grid as well. In order to make full comparisons we have repeated the S-S experiments on the E and C grid covering wider range of values for the λ/d ratio. To check our code we have first repeated the JM experiment using the same values of all parameters as in their paper. But since the idea of this paper is to cover wider range in the λ/d ratio we have adopted a slightly different choice of parameters. Again we have the sequence, now of four progressively finer resolutions starting with 500 km and going down to 250, 125, 62.5 km. The values for λ/d ratio were 4 , 2, 1, 0.5 and 0.1. To get those values we had to choose different mean depths whose corresponding values are 4000, 1000, 200, 60 and 2.5 meters. To ensure validity of the linear approach and mutual scalability the strength of the forcing was chosen as 0.2 of the corresponding mean height. Finally we would like to point out a small difference between JM form of the continuity equation and ours. In their paper the continuity equation was written in the flux form, retaining its non-linearity, which enables them to have more accurate form of its finite difference analogue. We have linear form of the continuity equation on the E grid, though still following the ideas about cross-diagonal fluxes. This might worsen slightly the E grid results but since the analysis on the Z grid will be strictly linear we adopted the linear form for the continuity equation for all grids. In this manner we hoped to put all grids on as equal footing as possible. The core of our results is presented in Figure 1 and Figure 2.

Three panels in 1 and two panels in 2 are for five considered values of the λ/d ratio, for the single point forcing. In each panel, we have plotted difference between the mean depth of the sink point and the mean fluid height. The mean is over the number of the grid points that span the forcing area. On abscissa we present four resolutions in km and the corresponding λ/d ratio for that resolution, or more precisely for that d since the depth is constant. Again, the "true" value is the mean of the sink depth for all three grids and for the highest resolution. The first λ/d ratio shows the "canonical" positions of all three grids. The Z grid overestimates the sink depth compared to the "true" solution. The E grid also overestimates the sink depth but even more than the Z grid. The third grid, the C grid underestimates it.The absolute error for C, E and Z grid is -5.4, -10, and -8.9 meters,respectively. The corresponding relative errors are -20.9, 47 and 31.8. All three grids increase their accuracy substantially for the next resolution For the next λ/d ratio, $(\lambda/d = 2)$ we see that now, for the poorest resolution, the Z grid gives deeper sink than the E grid, while the C grid is again underestimating its depth but now is closest, of the three grids, to the "true" depth. This result, this sequence in the depths of the sink, is partially the consequence of the modification term for the E grid. Without it sequence would be the same as in the first case. For the higher resolutions, things are back to "normal" concerning relative deepness of the sink. The Z and the E grid are almost on top of each other. Again, the improvement is large with the next higher resolution. For the third case, $(\lambda/d = 1)$ the E grid goes on the other side, meaning that it now predicts shallower sink that the "true" one and is now in that sense together with the C grid. Looking at how close are depths to the true one the E grid is now leading the game with the C grid as second and the Z grid at the end. In the next panel, (λ/d=0.5) the order between C and E grid is reversed with C grid now leading with E being the second and the Z grid the third. For the last case $(\lambda/d = .1)$ the E grid does not recover even for the highest resolution. It is clear that single point forcing is very difficult problem for all grids but it is interesting that the Z and E grid show faster improvement with the increase of the resolution in comparison to the C grid (except for the E grid in

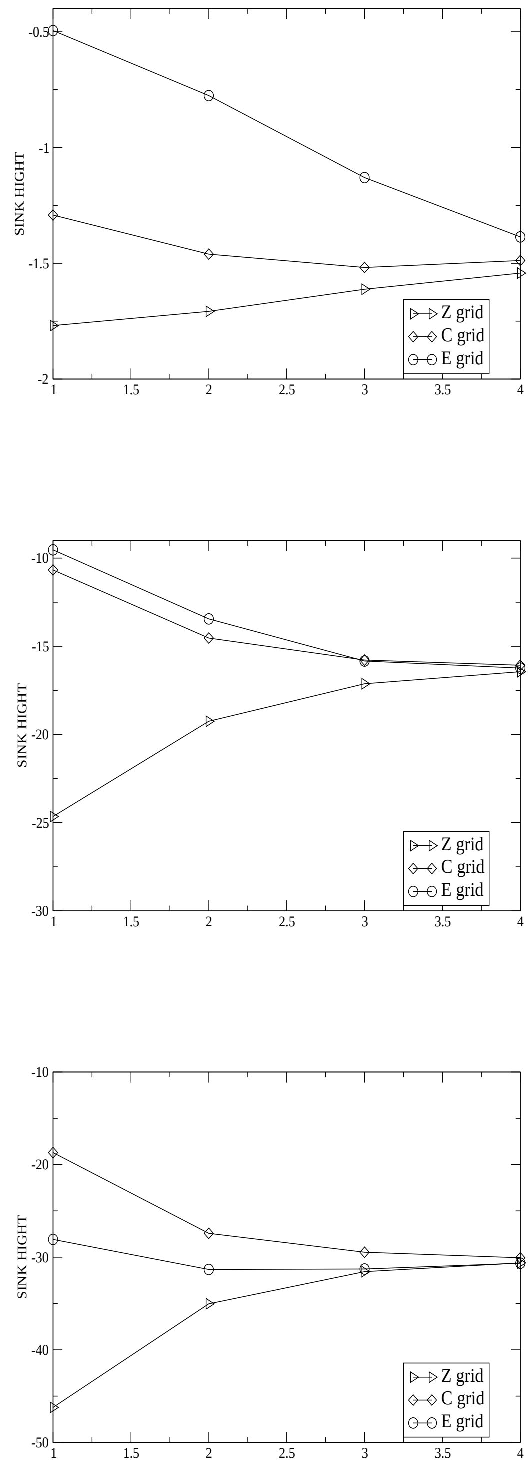

Figure 1: Ratio ζ/δ, top to bottom, for the values of λ/d 0.1, 0.5 and 1

the last, $\lambda/d = 0.1$, case)

3 Conclusions

The E grid, with the correction, has shown the best results for the $\lambda/d = 1$ case. For other values of λ/d the other two grids were better. Without the correction the E grid is always the worst, creating too deep lows. The C grid consistently overshoots while the Z grid undershoots. Finally the absolute error is similar for Z and C grids.

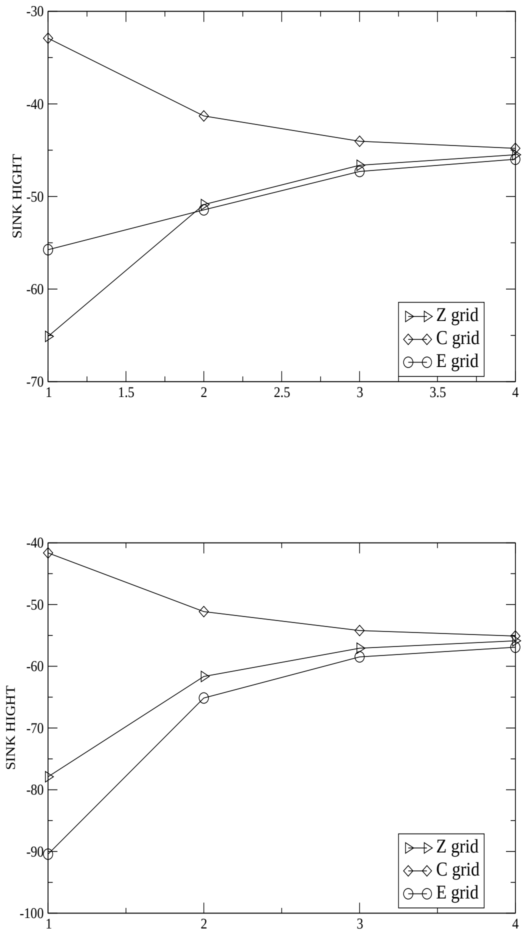

Figure 2: Ratio ζ/δ, top to bottom, for the values of λ/d 2 and 4

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