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Parameter Estimation for the Two-Parameter Weibull Distribution

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Master of Science

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ABSTRACT

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The Weibull distribution, an extreme value distribution, is frequently used to model survival, reliability, wind speed, and other data. One reason for this is its flexibility; it can mimic various distributions like the exponential or normal. The two-parameter Weibull has a shape (γ) and scale (β) parameter. Parameter estimation has been an ongoing search to find efficient, unbiased, and minimal variance estimators. Through data analysis and simulation studies, the following three methods of estimation will be discussed and compared: maximum likelihood estimation (MLE), method of moments estimation (MME), and median rank regression (MRR). The analysis of wind speed data from the TW Daniels Experimental Forest are used for this study to test the performance and flexibility of the Weibull distribution.

Keywords: weibull, extreme value distribution, parameter estimation, wind speed, TWDEF

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CHAPTER 1

INTRODUCTION

1.1 The Two-Parameter Weibull Distribution

There are many applications for the Weibull distribution in statistics. Although it was first identified by Fréchet in 1927, it is named after Waalobi Weibull and is a cousin to both the Fréchet and Gumbel distributions. Waalobi Weibull was the first to promote the usefulness of this distribution by modelling data sets from various disciplines (Murthy, Xie, and Jiang 2004). If $T \sim$ Weibull(γ,β) then its density function is defined as:

$$f(t|\gamma,\beta) = \frac{\gamma}{\beta^{\gamma}} t^{(\gamma-1)} \exp\left\{-\left(\frac{t}{\beta}\right)^{\gamma}\right\}, \text{for } \gamma > 0, \text{ and } \beta > 0.$$

In this density function, our γ represents the shape parameter, and our β represents the scale parameter. The distribution function can also be derived and is defined as:

$$F(t|\gamma,\beta) = 1 - \exp\left\{-\left(\frac{t}{\beta}\right)^{\gamma}\right\},$$

where, as noted before, γ and β are non-negative.

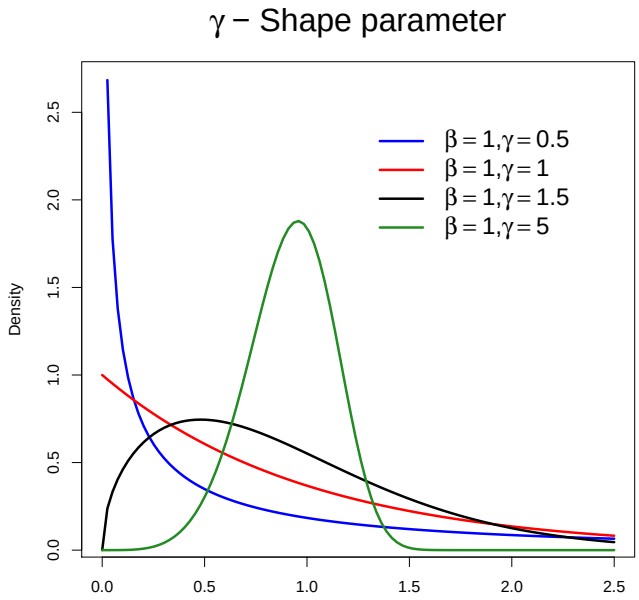


Figure 1.1: Examples of the Weibull density curve with various values of γ.

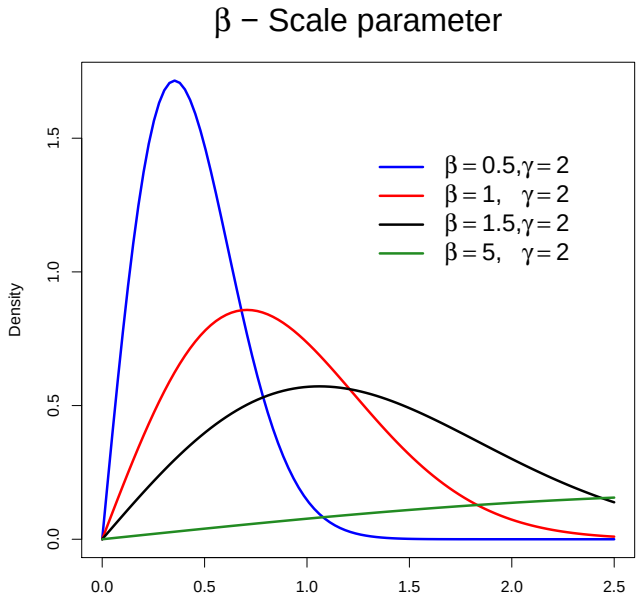


Figure 1.2: Examples of the Weibull density curve with various values of β.

1.2 Applications of the Weibull Distribution

Extreme value theory is a unique statistical discipline that develops "models for describing the unusual rather than the usual (Coles 2001)." Perhaps the simplest example of an extreme value distribution is the exponential distribution. The Weibull distribution is specifically used to model extreme value data. One example of this is the frequent use of the Weibull distribution to model failure time data (Murthy et al. 2004). Its use is also applicable in various situations including data containing product failure times to data containing survival times of cancer patients. The extreme values in these analyses would be the unusual longevity of the shelf life of a product or survival of affected patients. One reason it is widely used in reliability and life data analysis is due to its flexibility. It can mimic various distributions like the normal. Special cases of the Weibull include the exponential ($\gamma = 1$) and Rayleigh ($\gamma = 2$) distributions. Because of its uses in lifetime analysis, a more useful function is the probability that the lifetime exceeds any given time, (i.e. $\mathrm{P}(T > t)$). This is called the survival function or in the case of a product, the reliability. For the Weibull distribution, this is the following equation:

$$S(t) = 1 - P(T \leq t) = 1 - F(t|\gamma, \beta) = \exp\left\{-\left(\frac{t}{\beta}\right)^{\gamma}\right\}.$$

Another function of interest is the hazard function, which is often called the instantaneous failure rate. The hazard function is computed using both the reliability or survival function and the density function, $h(t) = \frac{f(t)}{S(t)}$. For the Weibull distribution, this is derived as follows:

$$h(t) = \frac{f(t)}{S(t)} = \frac{\frac{\gamma}{\beta^{\gamma}} t^{(\gamma-1)} \exp\left\{-\left(\frac{t}{\beta}\right)^{\gamma}\right\}}{\exp\left\{-\left(\frac{t}{\beta}\right)^{\gamma}\right\}} = \frac{\gamma}{\beta^{\gamma}} t^{(\gamma-1)}.$$

An interesting property of this equation is that this function is non-constant in comparison to the hazard function for the exponential distribution (i.e. $h(t) = \lambda$, for $f(t|\lambda) = \lambda e^{-\lambda t}$). This reveals an interesting property of the Weibull distribution. When the parameter $\gamma > 1$, it is said the "instantaneous failure rate," or hazard function, increases with time, if $\gamma = 1$ it remains constant, and if $\gamma < 1$ then the hazard function decreases over time (Collett 2009).

Often studies involving product failure times and patient survival times are censored for a given subset of subjects. The motivation to include these data in analyses, is that they do contain information about the reliability or survival times of the subjects. It would be unwise to simply throw out these data points. It would also be inappropriate to treat them the same as the subjects who each experienced their "events." An alternate form of the probability density function is suggested, as follows:

$$f(t_i|\gamma,\beta) = \left[\frac{\gamma}{\beta^\gamma} t_i^{(\gamma-1)} \exp\left\{-\left(\frac{t_i}{\beta}\right)^\gamma\right\}\right]^{\delta_i} \times \left[\exp\left\{-\left(\frac{t_i}{\beta}\right)^\gamma\right\}\right]^{(1-\delta_i)},$$

where γ and β are nonnegative and δ_i is the indicator function of the i^{th} subject's event. (i.e. $\delta_i = 1$ if the "event" has occurred.) This study will not address censored data.

Another discipline where the Weibull is applicable is in the analysis of wind speeds. According to Weisser (2003), wind energy, in comparison to other renewable sources of energy, such as solar or tidal energy, "has a more variable and diffuse energy flux." Because of this variability, the Weibull distribution is used to model the variation of wind speeds in order to maximize the benefit obtained from wind energy. This information can be used to optimize the design of wind energy conversion technologies (WECTs). In the past, the two-parameter Weibull distribution has been shown to effectively describe the variation of wind speed and is commonly used in modelling such data (Weisser 2003; Seguro and Lambert 2000; Celik 2003). Wind speed data is usually in time series format. It is reasonable to use the Weibull distribution to summarize the information contained in large sets of wind speed data into a couple parameter estimates. This project's focus will be the parameter estimation of wind speed data.

There are several functional and mechanical reasons for using the Weibull distribution in modeling wind speeds. In a study of wind speed durations above and below fixed speeds, Corotis (1979) determined that the Rayleigh distribution is reasonable in modeling wind speeds. In his previous research the following cumulative distribution function had been

developed to model run duration times (t):

$$\begin{aligned} F_1(t) &= 1 - \left(\frac{t}{t_0}\right)^{(1-b)} \qquad \text{for } t_0 \le t \le t_1, \\ F_2(t) &= 1 - A\exp\{-\lambda t\} \qquad \text{for } t_1 \le t \le \infty, \end{aligned}$$

where

$$\begin{aligned} t_0 &= \frac{1}{2}, \\ t_1 &= t_0 G^{\frac{1}{1-b}}, \\ A &= G\exp\{b-1\}, \text{ and} \\ \lambda &= \frac{b-1}{t_1}. \end{aligned}$$

G being approximated at 1/4 for run levels of practical interest. The remaining parameter b can be found using maximum likelihood estimation.

His research showed a strong correlation between the b parameter and seasonal mean wind speeds. He then presented a family of smoothed b-parameter curves, which indicated that the family of curves seemed to vary as a function of the run level to mean speed. The run level being the fixed wind speed. The Rayleigh distribution, a special case of the Weibull distribution (i.e. where $\gamma = 2$), is a one parameter distribution that implies a constant ratio of standard deviation to the mean. This implies a constant ratio of run level to the mean, where run level is considered as the number of standard deviations away from the mean. The reasonable fit of this distribution implies that the higher the run level is in comparison to the mean, the probability that a wind speed will continue to increase or that we will observe a "long duration above the run level" decreases. Now that the usefulness of the Rayleigh distribution has been established, the flexibility of the shape of the distribution is added by using the Weibull distribution because the skewness of a Weibull curve depends only on γ (Hennessey 1977).

Another functional reason for using the Weibull is because wind power (i.e. (wind speed)^3) is easily modeled by using a cubed transformation of the Weibull which is also distributed as a Weibull($\gamma/3$, β) (Hennessey 1977).

CHAPTER 2

PARAMETER ESTIMATION

2.1 Maximum Likelihood Estimator

The maximum likelihood estimator (MLE) is a well known estimator. It is defined by treating our parameters as unknown values and finding the joint density of all observations of a data set, which are assumed to be independent and identically distributed (iid). Once the likelihood function is defined, the maximum of that function is found. If the data points are all highly likely under specific parameter values, then their product will be the "most likely" outcome, or the maximum likelihood. This estimator is important in statistics because of is asymptotic unbiasedness and minimal variance.

Our MLE is computed by first assuming that $T_i \overset{iid}{\sim} \text{Weibull}(\gamma, \beta)$, with the probability given by its density function:

$$f(t_i|\gamma,\beta) = \frac{\gamma}{\beta^\gamma} t_i^{(\gamma-1)} \exp\left\{-\left(\frac{t_i}{\beta}\right)^\gamma\right\}, \text{for } \gamma > 0, \text{ and } \beta > 0.$$

The joint density of the likelihood is the product of the densities of each data point.

$$\begin{aligned} \mathrm{L}(\gamma,\beta|\mathbf{t}) &= \prod_{i=1}^{n} f(t_i|\gamma,\beta) \\ &= \prod_{i=1}^{n} \left[\frac{\gamma}{\beta^\gamma} t_i^{(\gamma-1)} \exp\left\{-\left(\frac{t_i}{\beta}\right)^\gamma\right\}\right] \\ &= \left(\frac{\gamma}{\beta^\gamma}\right)^n \prod_{i=1}^{n} t_i^{(\gamma-1)} \times \exp\left\{-\left(\frac{\sum t_i}{\beta}\right)^\gamma\right\}. \end{aligned}$$

Next, to make the math somewhat simpler, the natural log of our joint density is computed. This can be done while still preserving the true maximum of our likelihood function because

the log transformation is a monotonically increasing function.

$$\begin{aligned}
\ell(\gamma,\beta|\mathbf{t}) &= \log\left[\left(\frac{\gamma}{\beta^\gamma}\right)^n \prod_{i=1}^n t_i^{(\gamma-1)} \times \exp\left\{-\left(\frac{\sum t_i}{\beta}\right)^\gamma\right\}\right] \\
&= \log\left(\frac{\gamma}{\beta^\gamma}\right)^n + \log\left[\prod_{i=1}^n t_i^{(\gamma-1)}\right] - \left(\frac{\sum t_i}{\beta}\right)^\gamma \\
&= n\log\left(\frac{\gamma}{\beta^\gamma}\right) + (\gamma-1)\log\left[\prod_{i=1}^n t_i\right] - \sum_{i=1}^n \left(\frac{t_i}{\beta}\right)^\gamma \\
&= n\log\gamma - n\log\beta^\gamma + (\gamma-1)\sum_{i=1}^n \log(t_i) - \sum_{i=1}^n \left(\frac{t_i}{\beta}\right)^\gamma \\
&= n\log(\gamma) - \gamma n\log(\beta) + (\gamma-1)\sum_{i=1}^n \log(t_i) - \sum_{i=1}^n \left(\frac{t_i}{\beta}\right)^\gamma .
\end{aligned}$$

Once the likelihood function has been obtained, the function is optimized by negating it and finding its minimum. This is done iteratively using an optimization function in the *gsl* library in C and using the *nlm* function in R.

2.2 Method of Moments Estimator

Although in many cases the method of moments estimator (MME) is superseded by Fisher's MLE concerning asymptotic unbiasedness and minimal variance, the method of moments estimators can, in many cases and quite acurately, be derived by hand. The MME is defined by computing the sample moments,

$$\mu_k = \frac{1}{n}\sum_{i=1}^n t_i^k,$$

and setting them equal to the theoretical moments from the moment generating function, $M_T(t)$. The moment generating function for the Weibull is as follows:

$$M_K(t) = \beta^k \Gamma\left(1 + \frac{k}{\gamma}\right),$$

where k represents the k^{th} theoretical moment, and $\Gamma(\cdot)$ represents the gamma function, $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx$, where $\alpha > 0$ (Casella and Berger 2002).

Only the first two moments are needed to derive the parameter estimates for the Weibull. Using the equation, $\frac{\sigma^2}{\mu_1^2} = \frac{M_2(t)-[M_1(t)]^2}{[M_1(t)]^2}$, from Murthy, Xie, and Jiang (2004), we see that it can be simplified to the following equation:

$$\begin{aligned} \frac{\sigma^2}{\mu_1^2} &= \frac{M_2(t)-[M_1(t)]^2}{[M_1(t)]^2} \\ \Rightarrow \frac{\mu_2-\mu_1^2}{\mu_1^2} &= \frac{M_2(t)}{[M_1(t)]^2} - 1 \\ \Rightarrow \frac{\mu_2}{\mu_1^2} - 1 &= \frac{M_2(t)}{[M_1(t)]^2} - 1 \\ \Rightarrow \frac{\mu_2}{\mu_1^2} &= \frac{M_2(t)}{[M_1(t)]^2}. \end{aligned}$$

Next, taking the second sample moment divided by the square of the first sample moment, a function of the theoretical moments using our data is approximated.

$$\begin{aligned} \frac{\mu_2}{\mu_1^2} &= \frac{M_2(t)}{[M_1(t)]^2} \\ \Rightarrow \frac{\frac{1}{n}\sum_{i=1}^n t_i^2}{\left[\frac{1}{n}\sum_{i=1}^n t_i\right]^2} &\approx \frac{\beta^2\Gamma\left(1+\frac{2}{\hat{\gamma}}\right)}{\beta^2\Gamma^2\left(1+\frac{1}{\hat{\gamma}}\right)} \\ \Rightarrow \frac{n\sum_{i=1}^n t_i^2}{\left[\sum_{i=1}^n t_i\right]^2} &\approx \frac{\Gamma\left(1+\frac{2}{\hat{\gamma}}\right)}{\Gamma^2\left(1+\frac{1}{\hat{\gamma}}\right)}. \end{aligned}$$

This function solely depends on $\hat{\gamma}$ and therefore we can solve using root finding techniques. I used the bisection method (Jones, Maillardet, and Robinson 2009) because of its robustness and simplicity. It is, however, very slow in comparison to other root finding methods such as the Newton-Raphson method.

We begin by defining our function $f(x)$ as follows:

$$f(x) = \frac{\Gamma\left(1+\frac{2}{x}\right)}{\Gamma^2\left(1+\frac{1}{x}\right)} - \frac{\mu_2}{\mu_1^2} = 0,$$

knowing that μ_2/μ_1^2 is a constant given our data. The bisection method requires two initial points, x_1 and x_2, where $f(x_1)$ and $f(x_2)$ have opposite signs. The interval (x_1,x_2) is divided in half and the half where the end points evaluated in the function have opposite signs is

found. These values become the endpoints of our new interval and we divide it again. This process is repeated until the difference of two consecutive midpoints are within ϵ of each other, where $\epsilon > 0$. Once we have found our root, we set this equal to $\hat{\gamma}$. Finally, we can estimate β by the equation

$$\begin{aligned} \mu_1 = \bar{t} &= \hat{\beta}\Gamma\left(1 + \frac{1}{\hat{\gamma}}\right) \\ \Rightarrow \hat{\beta} &= \frac{\bar{t}}{\Gamma\left(1 + \frac{1}{\hat{\gamma}}\right)}, \end{aligned}$$

plugging in the value obtained in our root finding iteration for $\hat{\gamma}$.

2.3 Median Rank Regression Estimator

The last estimator is computed by using median rank regression. This method is the simplest of the three in this paper. Although it has neither the asymptotic properties of the MLE nor the accuracy of the MME estimator, it is quick, simple, and fairly accurate. This method was specifically used in the past as a method that could be done by hand. Thus, it is safe to assume that this method's accuracy will be mediocre in comparison to the previous methods.

The median rank regression estimator includes a simple algorithm. First, the distribution function for our ordered data is approximated. The data are first sorted in ascending order. The data are "ranked" by using the median rank approximation by solving the following equation for Z_i.

$$\sum_{k=i}^{n} \binom{n}{k} (Z_i)^k (1 - Z_i)^{n-k}. \tag{2.1}$$

Because this is computationally intensive we use Bernard's approximation,

$$F_T(t_i) \approx Z_i \approx \frac{i - 0.3}{n + 0.4},$$

which is an estimate to the solution for Equation 2.1, where i is the ascending rank of our data point and n is the total number of data in our data set (ReliaSoft Corportation 1996–2006). These estimates will be used later in setting up a regression model.

Finally, the distribution function is used to derive a linear relation between a transformation of the distribution function estimates and the log survival times.

$$\begin{aligned} 1 - F_T(t) &= \exp\left\{-\left(\frac{t}{\beta}\right)^{\gamma}\right\} \\ \log\left[1 - F_T(t)\right] &= -\left(\frac{t}{\beta}\right)^{\gamma} \\ \log\left[\frac{1}{S(t)}\right] &= \left(\frac{t}{\beta}\right)^{\gamma} \\ \log\left[\log\left[\frac{1}{S(t)}\right]\right] &= \gamma \log t - \gamma \log \beta \end{aligned}$$

Now we have a linear model and can use regression to find the parameter estimates $\dot{\gamma}$ and $\dot{\beta}$. Allowing $\mathbf{y}$ to equal the twice-logged inverse median-rank estimated survival function, and $\mathbf{x}$ to be equal to the log survival times,

$$\begin{aligned} \log\left[\log\left[\frac{1}{S(t)}\right]\right] &= \gamma \log t - \gamma \log \beta \\ \Rightarrow \mathbf{y} &= \gamma \mathbf{x} - \gamma \log \beta \\ \Rightarrow \mathbf{y} &= \psi_0 + \psi_1 \mathbf{x}, \end{aligned}$$

$$\begin{aligned} \text{where } \dot{\gamma} &= \psi_1, \text{ and} \\ \dot{\beta} &= \exp\left\{\frac{-\psi_0}{\dot{\gamma}}\right\}. \end{aligned}$$

Note that ψ_0 is the intercept of the least squares regression model and ψ_1 is the slope. This concept is illustrated in Figure 2.1 on the following page.

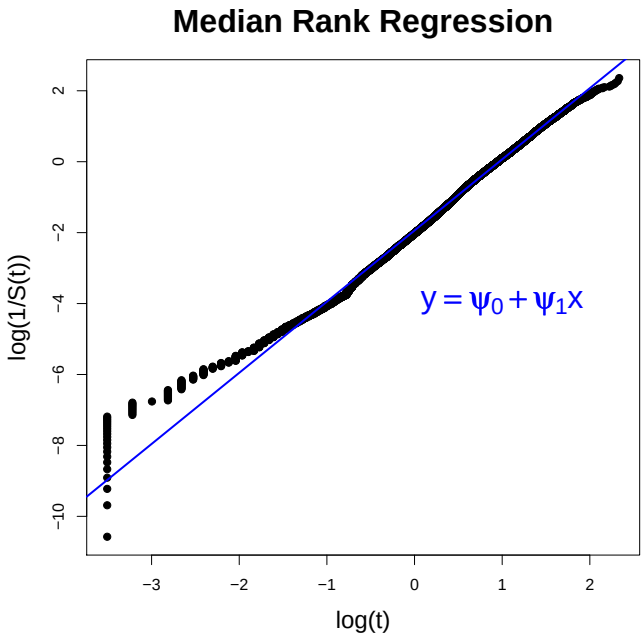


Figure 2.1: The linear relationship of the twice-logged inverse estimated survival times(Z_i) and logged survival times is graphically demonstrated.

CHAPTER 3

SIMULATION STUDIES

The first step in this simulation study will be to determine and justify parameter values and sample sizes in order to generate simulation data. For each combination of parameter values and sample sizes, 1000 data set are simulated. After the new data are generated, the parameter estimation methods discussed in chapter 2 will be used. Because the true parameter values of the different data sets are known, statistics measuring bias and variance of the estimated parameters can be computed using the 1000 repetitions. Once the bias, variance, and mean square error (defined and discussed later) are computed, the different methods can be compared in accuracy. Finally, data is then simulated under three different density functions to determine the accuracy of these parameterizations if misspecification is an issue.

3.1 Justification of Parameters and Sample Sizes

I chose the first parameters,γ and β, to be 2 and 2.5, respectively. I wanted to see how the estimators would handle values similar to the shape and scale parameters from the wind data. Next, I chose a second set of differing values. These values were 5 for γ and 90 for β. I conducted tests of accuracy for these values with small, moderate, and large ($n = 20, 100, 10000$) data sets in order to determine the accuracy of each method for various data sizes.

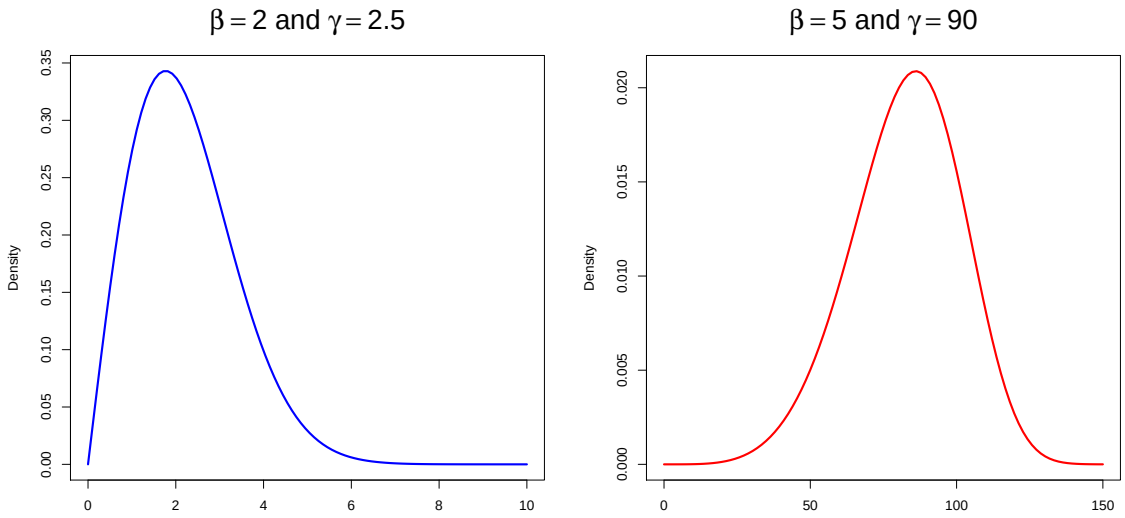


Figure 3.1: Weibull density curves for the parameters specified above. These parameters were selected for the simulation studies.

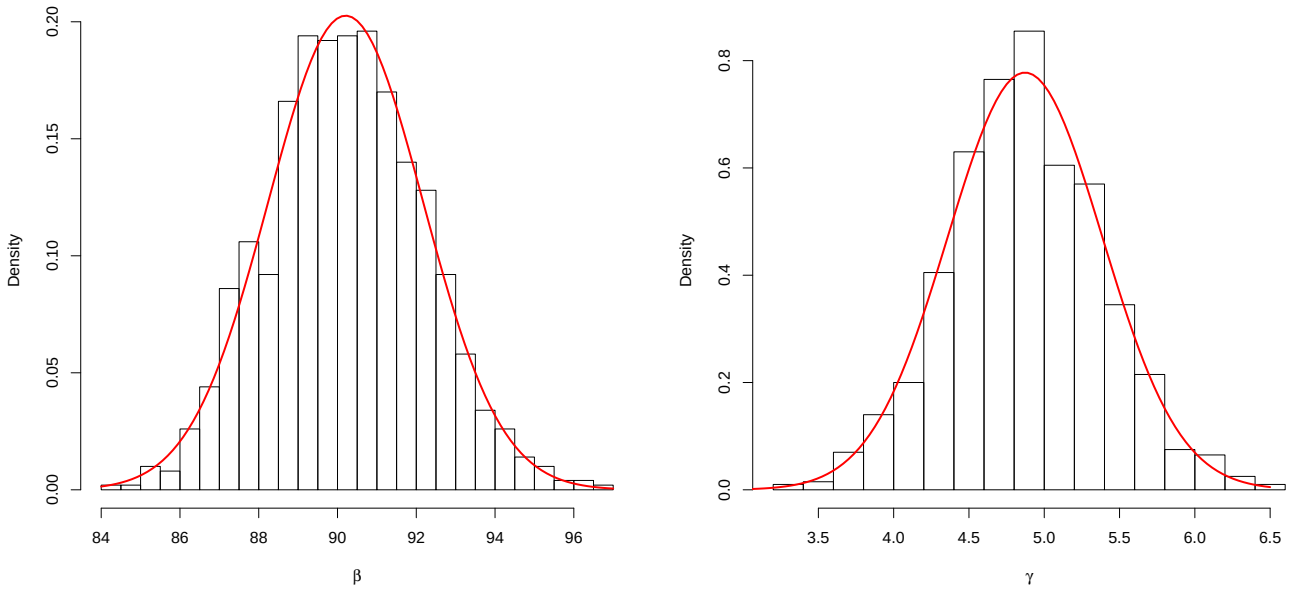


Figure 3.2: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the Regression Method, with $\gamma = 5$, $\beta = 90$, and $n = 100$.

After simulating the data sets and finding our parameter estimates, we assume that our parameter estimates are normally distributed. This assumption allows us to take the 1,000 repetitions of each parameter estimation and calculate the mean square error (MSE), or the bias squared plus the variance of the distribution of our parameter estimates. Graphs showing normality for all parameter estimates can be found in Appendix A.

3.2 Bias

Bias is defined as the difference between the true parameter value and the parameter estimate. Mathematically this is defined to be $E(\hat{\theta}) - \theta$, where $\hat{\theta}$ is our estimated parameter and θ is the true parameter value. This is computed by finding the average of each of the 1000 parameter estimates and subtracting the true value, which was used to generate the data.

$$\text{bias} = \frac{1}{n}\sum_{i=1}^{n} x_i - \text{truth}.$$

Below are the biases for each set of parameters and sample sizes.

	$\gamma = 5$			$\beta = 90$		
$n =$	20	100	10000	20	100	10000
MLE	0.38625	0.06758	0.00027	-0.18701	-0.06877	0.00302
MME	0.40456	0.06451	0.00067	-0.23079	-0.07345	0.00277
MRR	-0.16904	-0.12781	-0.00497	0.50473	0.20719	0.01181

Table 3.1: Biases for first set of parameters. Specifically for the MLE bias approaches zero as the sample size, n, gets large. Here all three methods demonstrate the tendency to asymptotic unbiasedness.

	$\gamma = 2$			$\beta = 2.5$		
$n =$	20	100	10000	20	100	10000
MLE	0.15450	0.02703	0.00011	-0.00192	-0.00267	0.00023
MME	0.14956	0.02705	0.00006	-0.00317	-0.00277	0.00022
MRR	-0.06761	-0.05112	-0.00199	0.04678	0.01666	0.00084

Table 3.2: Biases for the second set of parameters. Once again, the bias for all three methods approaches zero as n gets large.

Notice that the bias for the MLE estimates are all smaller than the biases for both MME and MRR estimates. Also, as n increases, the bias goes to zero for all estimators. This is a trait that was expected for the MLE, but which is also expressed in the other two methods. It appears that the MLE and MME converge the quickest to the true value.

3.3 Variance

Variance is defined to be the deviation about the mean. In this case, it is the sample variance for each of the parameters and sample sizes. This is computed by finding the sum of squared deviances of each data point from the data's average value and dividing by $n - 1$:

$$\text{variance} = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2\right].$$

The variances for the data simulation studies are shown in the tables below.

	$\gamma = 5$			$\beta = 90$		
$n =$	20	100	10000	20	100	10000
MLE	1.00863	0.16372	0.00160	19.16663	3.64719	0.03487
MME	1.19413	0.18421	0.00183	19.15801	3.65998	0.03489
MRR	1.22024	0.26308	0.00290	20.01155	3.87677	0.03652

Table 3.3: Variances for the first set of parameters. The variances' tendencies toward zero is expected as the sample size increases.

	$\gamma = 2$			$\beta = 2.5$		
$n =$	20	100	10000	20	100	10000
MLE	0.16138	0.02620	0.00026	0.09090	0.01758	0.00017
MME	0.15715	0.02673	0.00026	0.09106	0.01763	0.00017
MRR	0.19524	0.04209	0.00046	0.09732	0.01890	0.00018

Table 3.4: Variances for the second set of parameters.

In Tables 3.3 and 3.4, we see that the variance is smallest for the MLE estimates. However, note that the variances of the MME and MRR estimates are approximately the same as each of the variances for the MLE estimates.

3.4 Mean Square Error

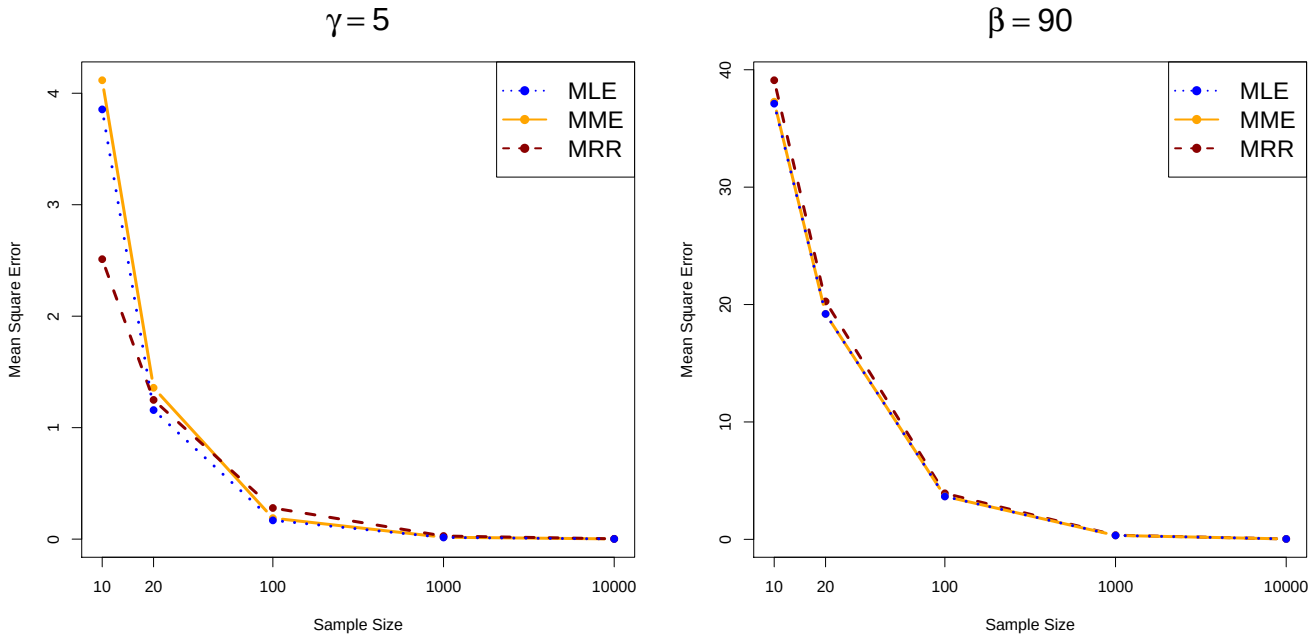
Once we have the measures for bias and variance we can compute the mean square error as defined below:

$$\text{MSE} = \text{bias}^2 + \text{variance}$$

Once again, Tables 3.5 and 3.6 below report the MSE for each set of parameters and sample sizes. Notice also Figure 3.2 which is a graphical representation of the measures of MSE for

each method. In all three cases for both parameters the MSE converges to zero, showing that as sample sizes increase, the accuracies of our methods also increase. Figure 3.3 tells a similar story for $\gamma = 2$ and $\beta = 2.5$.

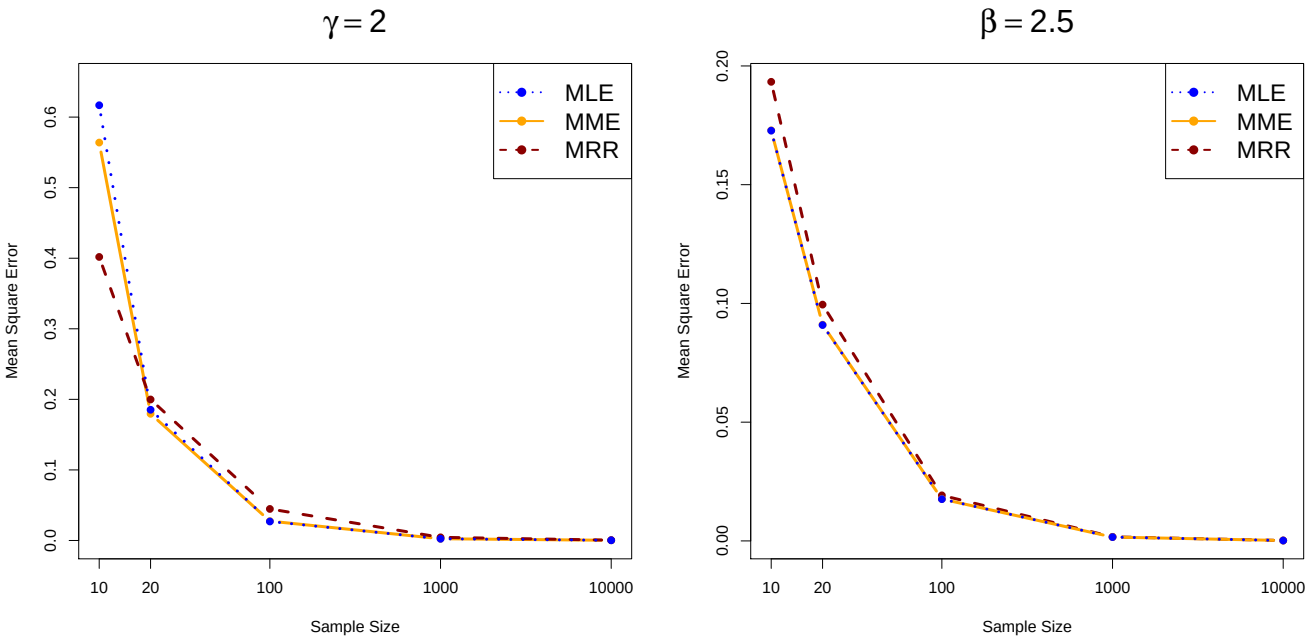
Figure 3.3: These two graphs show the mean square error for various sample sizes. The parameters $\gamma = 5$ and $\beta = 90$ are used for these estimations.



	$\gamma = 5$			$\beta = 90$		
$n =$	20	100	10000	20	100	10000
MLE	1.15782	0.16829	0.00160	19.20161	3.65192	0.03488
MME	1.35779	0.18837	0.00184	19.21128	3.66537	0.03489
MRR	1.24881	0.27942	0.00293	20.26631	3.91970	0.03666

Table 3.5: MSE for the first set of parameters.

Figure 3.4: These two graphs show the mean square error for various sample sizes. The parameters $\gamma = 2$ and $\beta = 2.5$ are used for these estimations.



	$\gamma = 2$			$\beta = 2.5$		
$n =$	20	100	10000	20	100	10000
MLE	0.18525	0.02693	0.00026	0.09090	0.01759	0.00017
MME	0.17952	0.02746	0.00026	0.09107	0.01764	0.00017
MRR	0.19981	0.04471	0.00047	0.09951	0.01918	0.00018

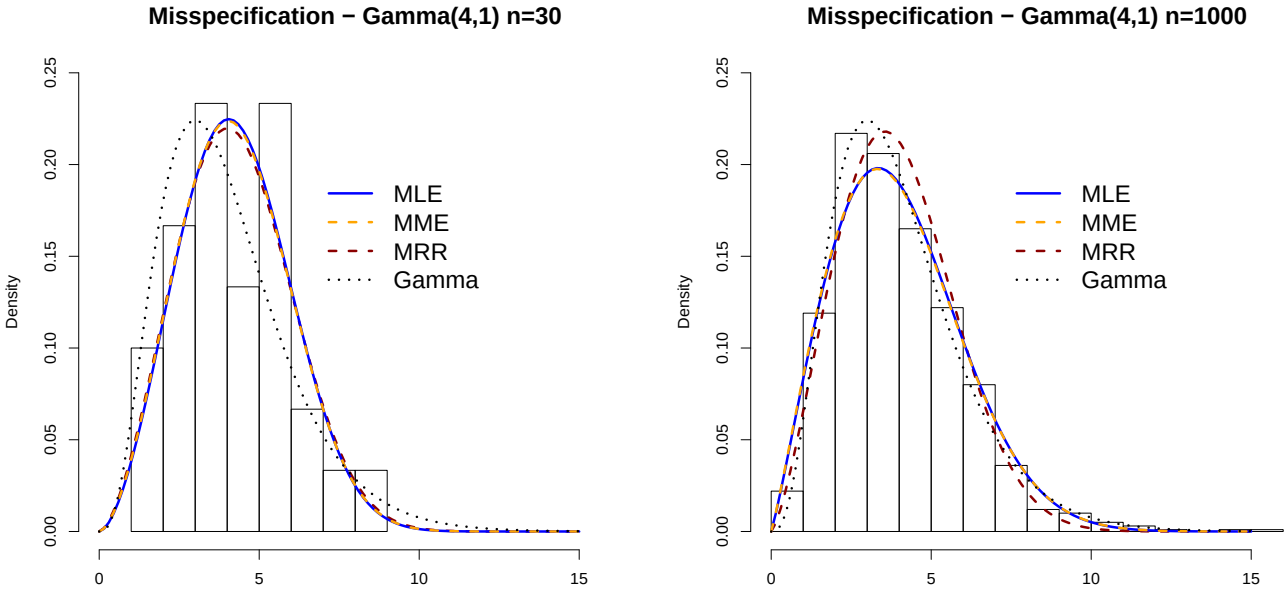
Table 3.6: MSE for second set of parameters.

Notice that the MLE had the smallest MSE for all four parameters. An interesting attribute of the median rank regression estimator is that for small data sets ($n = 10$) the MSE for γ is the smallest. However, we see again that the MLE is the best estimator, according to the MSE.

3.5 Distribution Misspecification

In general, the Weibull distribution is used in modeling a wide variety of data, including wind speed, patient survival, and product lifetime. There is the possibility that these data are not truly distributed as Weibull data. I simulated three different data sets which are distributed as gamma(4,1), normal(5,1), and zero-truncated normal(2,3). After simulating the data with sizes of n =30 and 1000, I used my three methods of parameter estimation for the Weibull to obtain estimates for the six data sets. To determine which parameter estimates fit the data best, I will use the Kolmgorov-Smirnov test in R. The KoImgorov-Smirnov test assesses whether or not data comes from a reference distribution. The null hypothesis is that the generated data is from the Weibull distribution, thus a p-value that is less than $\alpha = 0.05$ would suggest that the data did not come from the Weibull distribution. Figure 3.— below, you will find the histograms of the data with an overlaid Weibull curve with the estimated parameter estimates followed by the p-values for the Kolmgorov-Smirnov (KS) tests.

Figure 3.5: Data misspecification is a plausible error in modelling data. These plots show possible misspecification of the data.

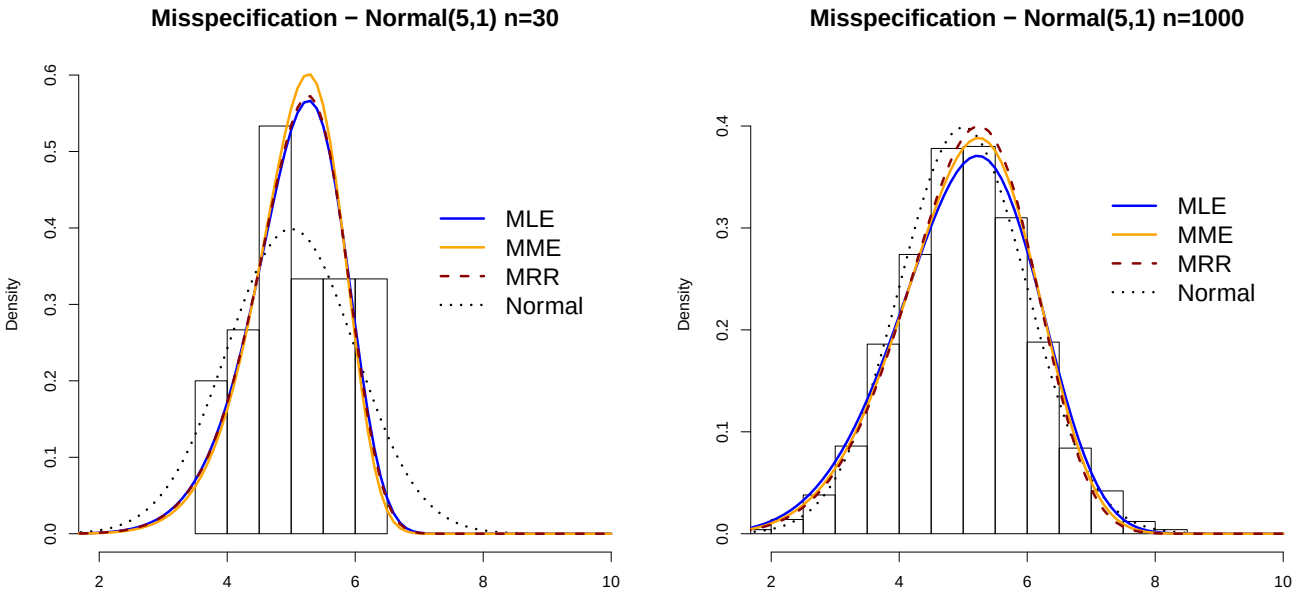


	$n = 30$	$n = 1000$
MLE	0.9739	0.2264
MME	0.9812	0.1800
MRR	0.9883	0.0634
Gamma	0.3442	0.9662

Table 3.7: p-values for the KS tests for the gamma misspecification. H_0: The data come from a Weibull distribution with corresponding parameter estimates.

Notice here that the MRR method performs best with small sample size and the MLE with large sample size. Because the p-values are not that different for the smaller sample data, I would conclude that the MLE is best for this misspecification.

Figure 3.6: Data misspecification for the normal.

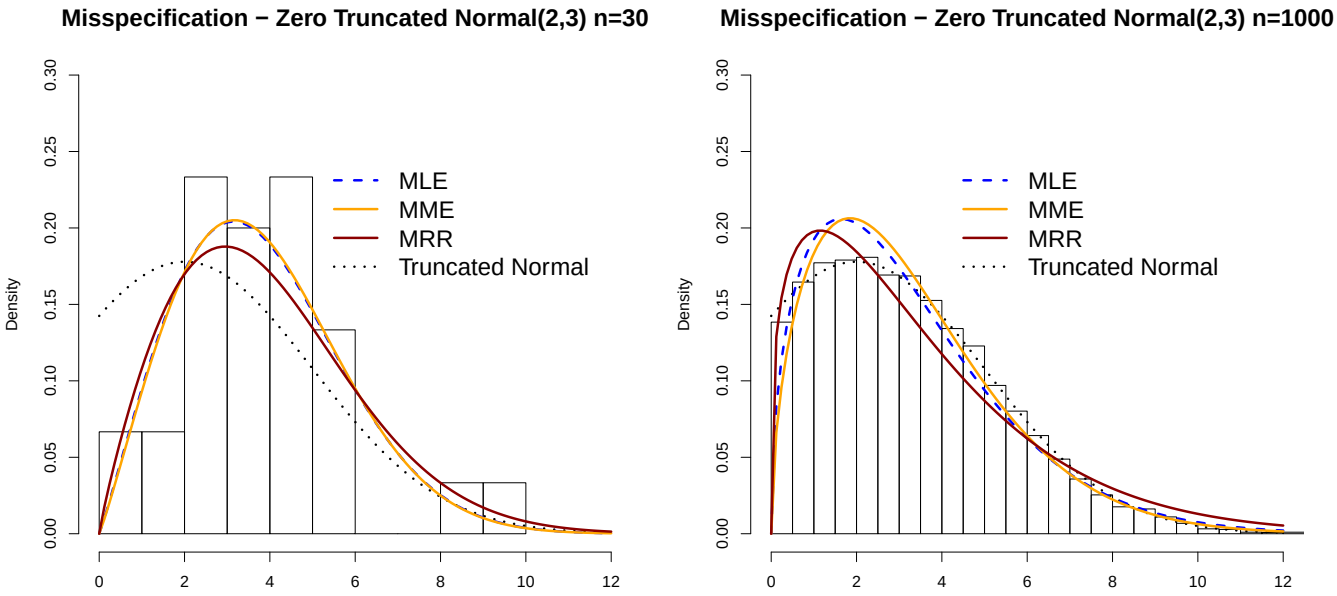


	$n = 30$	$n = 1000$
MLE	0.8484	0.2898
MME	0.7572	0.3330
MRR	0.8375	0.2402
Normal	0.7471	0.8634

Table 3.8: p-values for the KS tests for the normal misspecification. H_0: The data come from a Weibull distribution with corresponding parameter estimates.

Here we see that the best method of estimation for this type of misspecification is the MLE for small data sets and the MME for large sets.

Figure 3.7: Data misspecification for the zero-truncated normal.



	$n = 30$	$n = 1000$
MLE	0.7675	1.3389E-10
MME	0.7924	6.6811E-07
MRR	0.6688	0.0000
TruncNorm	0.0371	0.4912

Table 3.9: p-values for the KS tests for the truncated normal misspecification. H_0: The data come from a Weibull distribution with corresponding parameter estimates.

In this case we notice that our KS tests were significant in all the large sample size Weibull estimations, although the MME is best. I conclude that this is trivial because larger sample sizes that come from a truncated normal are not very likely to be misspecified as Weibull data. It is in small sample sizes that I would show more concern. The MME parameter estimates have the highest p-value with the MLE parameter estimate's p-value slightly lower.

In each case, assuming the Weibull is the underlying distribution of each of these data sets

is a reasonable misspecification. In general, the MLE estimates are the best fit for the misspecified data. This is a confirmation of the flexibility of the Weibull distribution as discussed in the introduction. While the fits above are not perfect, it is apparent that the Weibull is capable in parametrizing various types of data.

3.6 Summary

Overall, the maximum likelihood estimator was the best according to the mean square error. This is what we expected because of the asymptotic properties of the maximum likelihood estimator. Although the median rank regression method was worst according to the mean square error, because of its simplicity it is a very good estimator for finding the parameters. Both method of moments and maximum likelihood estimator required iterative algorithms, which was not necessary for the median rank regression method. Misspecification should raise no problems as seen in the previous section. The Weibull density's flexibility allows the modelling various types of data.

CHAPTER 4

DATA APPLICATION

4.1 Description of Data Set

In order to test our methods, I chose to use the T.W. Daniels Experimental Forest (TWDEF) dataset from the Department of Soil Physics at Utah State University (Utah State Universtity—Department of Soil Physics 2009–2010). The data contains the wind speed distributions for five seperate sites in the experimental forest. The set includes data from the following sites: aspen, conifer, grass, sage, and the summit. All non-missing data are the wind speeds, recorded every half hour at each site. Wind energy data are appropriately modelled using the Weibull distribution (Weisser 2003; Seguro and Lambert 2000; Celik 2003). The data set is large, and therefore we presume that the MLE method will likely be the most accurate as seen in our simulation studies.

4.2 Issues with the Wind Data

One of the main issues contained in this data is the fact that there are data equal to zero. Because the Weibull distribution is only for data that is greater than zero, this caused several problems for our parameter estimation, specifically for the MLE and MRR estimates. The solutions are as follows:

1. Add small values to each zero and find the estimates for the parameters using those values. In Figures 4.1, 4.2, and 4.3, these values are denoted in the key by "0.01, 0.0001, and 0.000001."

2. The data are separated into bins (i.e. just like they are separated before creating a histogram) and the estimates are found for the binned data. For my data, I rounded

each value up to the nearest hundredth, including zero up to one one-hundredth. I found this to be a common practice among wind energy analysts.

3. Delete the data valued at zero and find the estimates for our wind data. This last solution is shown to be equivalent to the parameter estimation for a mixture model of the point density at zero and the Weibull for the MLEs.

4.3 Maximum Likelihood Estimation for a Mixture Model

Here we find the MLE for the mixture model for a point density at zero and a Weibull density. The zero point density is as follows: $f(x) = I(x = 0)$. Also, let n be the total number of data and let m be the number of non-zero data points. Note that x_j below is the j^{th} non-zero wind speed measure.

$$\begin{aligned} g(x|\gamma,\beta,\pi) &= f(x)(1-\pi) + Weibull(\gamma,\beta)\pi \\ \Rightarrow \prod_{i=1}^{n}[g(x_i|\gamma,\beta)] &= \prod_{i=1}^{n}[f(x_i)(1-\pi) + Weibull(x_i|\gamma,\beta)\pi] \end{aligned}$$

When $x_i = 0$, $g(x_i|\gamma,\beta,\pi) = (1-\pi)$ and when $x_i \neq 0$, then $g(x_i|\gamma,\beta,\pi) = Weibull(x_i|\gamma,\beta)\pi$. Therefore we can write the equation as:

$$\prod_{i=1}^{n}[g(x_i|\gamma,\beta)] = (1-\pi)^{(n-m)}\pi^m \prod_{j=1}^{m} Weibull(x_j|\gamma,\beta)$$

Once this has been done, the MLE can be found for π by maximizing the log likelihood.

$$\begin{aligned}
\ell(\mathbf{x}|\gamma,\beta,\pi) &= \log\{(1-\pi)^{(n-m)}\pi^m \prod_{j=1}^{m} Weibull(x_j|\gamma,\beta)\} \\
&= (n-m)\log(1-\pi) + m\log\pi + \sum_{j=1}^{m} \log Weibull(x_j|\gamma,\beta) \\
\Rightarrow \frac{d\ell}{d\pi} &= \frac{(n-m)}{(1-\pi)}(-1) + \frac{m}{\pi} \\
\Rightarrow 0 &= \frac{(m-n)}{(1-\hat{\pi})} + \frac{m}{\hat{\pi}} \\
\Rightarrow 0 &= m\hat{\pi} - n\hat{\pi} + m - m\hat{\pi} \\
\Rightarrow 0 &= -n\hat{\pi} + m \Rightarrow \hat{\pi} = \frac{m}{n}
\end{aligned}$$

Notice that the same can be done with the parameters γ, and β. As you can see from $\ell(\mathbf{x}|\gamma,\beta,\pi)$, the MLEs for the remaining estimates will be the same as the MLE for the non-zero data. Therefore, the MLEs for the mixture model are the same as the estimates that are found in my method where we "delete" the zeros from the data.

4.4 Application Results

In the analyses of these data, we find that all three methods predict the following parameter estimates assuming the Weibull distribution.

Figure 4.1: Here is a histogram of the Grass Site data, with overlaid Weibull curves for the parameters chosen by our Maximum Likelihood Methods.

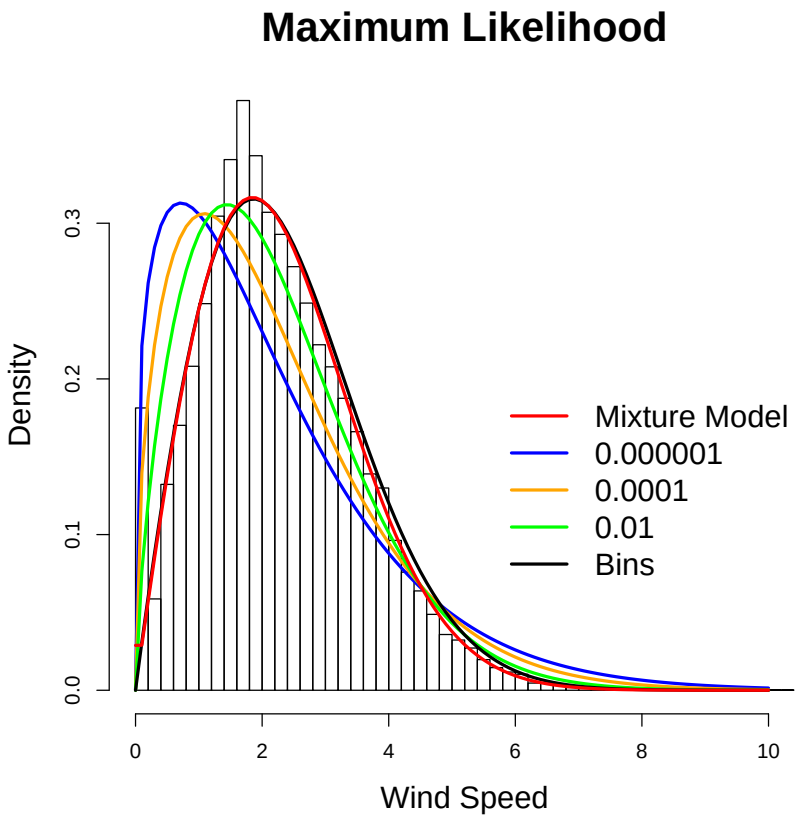


Table 4.1: MLEs for grass site data.

MLE method	γ	β
Mixture	1.99034	2.62306
Deleted	1.99034	2.62306
Binned	1.96030	2.68286
+ 0.000001	1.27802	2.35741
+ 0.0001	1.45597	2.41718
+ 0.01	1.67482	2.48765

Figure 4.2: Here is a histogram of the Grass Site data, with overlaid Weibull curves for the parameters chosen by the Method of Moments.

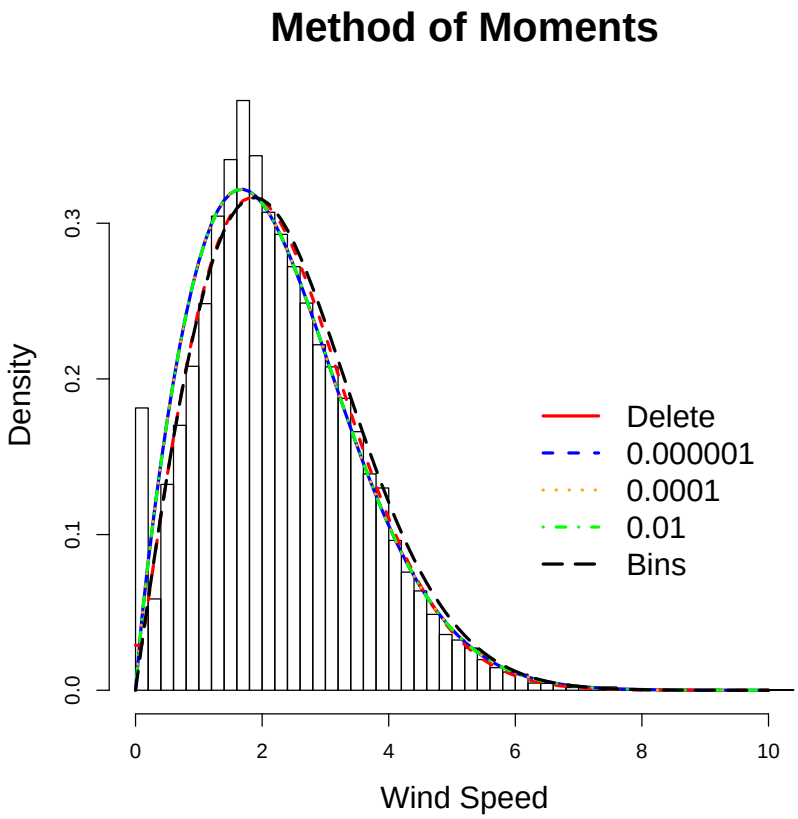


Table 4.2: MMEs for grass site data.

MME method	γ	β
Deleted	1.98974	2.62295
Binned	1.97344	2.68672
+ 0.000001	1.85277	2.54206
+ 0.0001	1.85279	2.54206
+ 0.01	1.85387	2.54244

Figure 4.3: Here is a histogram of the Grass Site data, with overlaid Weibull curves for the parameters chosen by our Median Rank Regression Method.

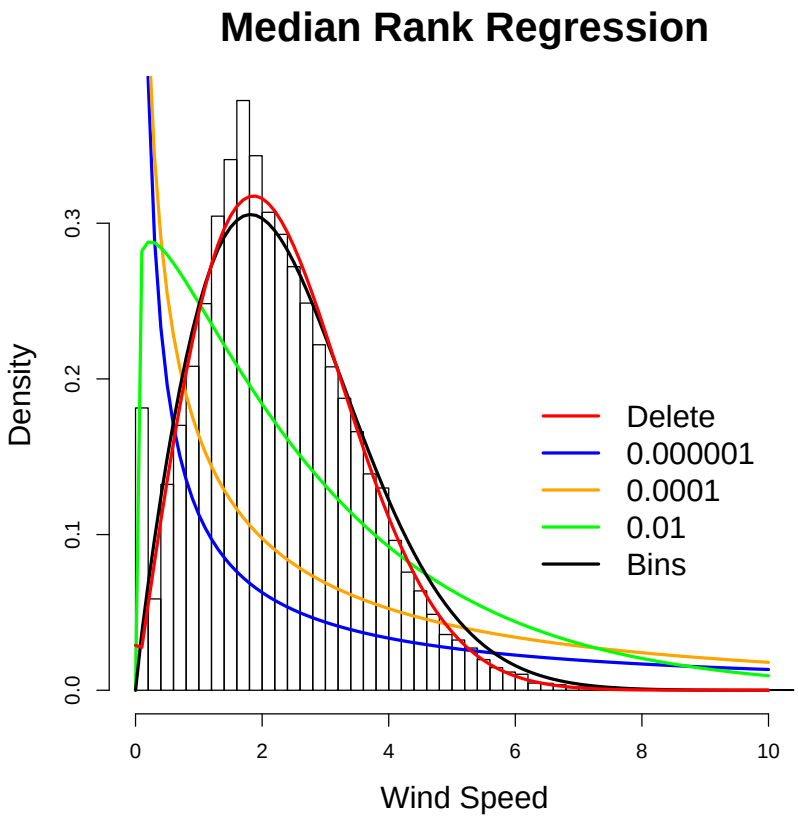


Table 4.3: MRR estimates for grass site data.

MRR method	γ	β
Deleted	2.00652	2.62908
Binned	1.88911	2.70797
+ 0.000001	0.36846	6.21900
+ 0.0001	0.57041	4.07870
+ 0.01	1.07205	2.90100

We can also see that all methods are similar enough to show that each method would be sufficient in determining our parameter estimates. Table 4.4 emphasizes this point because

the estimated parameter estimates are within 2 one-hundredths of each other. Graphs with overlaid Weibull curves and tables of parameter estimates for the remaining sites can be found in appendix A.

4.5 Summary

If we look at the resulting parameter estimates from each data set, we can see that the methods in which we replaced the zeros with small numbers gave too much weight to the small values in this data set. This was the case in both sets of the MLE and MRR estimates. The method of moments did not seem to change significantly between each of the methods. Also, looking at the binned data, I noticed that the density of the overlaid curve is slightly higher than the actual data because I am rounding the data up. Finally, for the MLE our mixture model proved to produce the exact estimates as our deleted data method, and the estimate for π was the ratio of the number of non-zero data points to the total number of data points. The values obtained by *nlm* were off by a few ten-millionths. If we look at what appear graphically to be the best estimates for each method, we see that they are all almost the same. This is shown in the table below. Note that the estimate for π in the MLE is compared to the ratio of the number of non-zero data points to the total number of data points in both the MOM and MRR methods.

Table 4.4: The parameter estimates for the MLE mixture and the MME and MRR deleted methods. The deleted methods mimic the mixture model. Notice how close these estimates are to each other.

	γ	β	π
MLE mixture	1.99034	2.62306	0.97118
MME deleted	1.98974	2.62295	0.97118
MRR deleted	2.00652	2.62908	0.97118

A graphical comparison showing each method would not tell us much because of how close the estimates are to each other; the three lines would overlay each other. From this we can see that no matter what the true parameters are, the estimates for each method are all going to have approximately the same accuracy. This is due to the fact that our data set is very large.

BIBLIOGRAPHY

Casella, G., and Berger, R. L. (2002), *Statistical Infrerence* (2nd ed.), Duxbury.

Celik, A. N. (2003), "A Statistical Analysis of Wind Power Density Based on the Weibull and Rayleigh Models at the Southern Region of Turkey," *Renewable Energy*, 29, 593–604, (http://www.elsvier.com/locate/renene).

Coles, S. (2001), *An Introduction to Statistical Modeling of Extreme Values*, Springer - Verlag London Limited.

Collett, D. (2009), *Modelling Survival Data in Medical Research* (2nd ed.), Chapman and Hall/CRC.

Corotis, R. B. (1979), "Statistical Models for Wind Characteristics at Potential Wind Energy Conversion Sites," *United States Department of Energy*, (Work performed under contract No. EY-76-S-06-2342).

Hennessey, Jr., J. P. (1977), "Some Aspects of Wind Power Statistics," *Journal of Applied Meteorology*, 16.

Jones, O., Maillardet, R., and Robinson, A. (2009), *Introduction to Scientific Programming and Simulation Using R*, Chapman and Hall/CRC.

Murthy, D. P., Xie, M., and Jiang, R. (2004), *Weibull Models*, John Wiley and Sons, Inc.

ReliaSoft Corportation (1996–2006), "Estimation of Weibull Parameters," (http://www.weibull.com/LifeDataWeb/).

Seguro, J., and Lambert, T. (2000), "Modern estimation of the parameters of the Weibull wind speed distribution for wind energy analysis," *Journal of Wind Engineering and Industrial Aerodynamics*, 85, 75–84.

Utah State Universtity—Department of Soil Physics (2009–2010), "TWDEF Site Data," (http://twdef.usu.edu/view.html).

Weisser, D. (2003), "A Wind Energy Analysis of Grenada: an Estimation Using the 'Weibull' Density Function," *Renewable Energy*, 28, 1803–1812, (http://www.elsvier.com/locate/renene).

Wu, S.-J. (2002), "Estimations of the Parameters of the Weibull Distribution with Progressively Censored Data," *Journal of The Japan Statistical Society*, 32, 155–163.

Zhang, L., Xie, M., and Tang, L. (2007), "A study of two estimation approaches for parameters of Weibull distribution based on WPP," *Reliability Engineering and System Safety*, 92, 360–368.

APPENDICES

APPENDIX A

ADDITIONAL GRAPHS

A.1 Graphs of Normality

Maximum Likelihood Method

Figure A.1: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the maximum likelihood method, with $\gamma = 5$, $\beta = 90$, and $n = 20$.

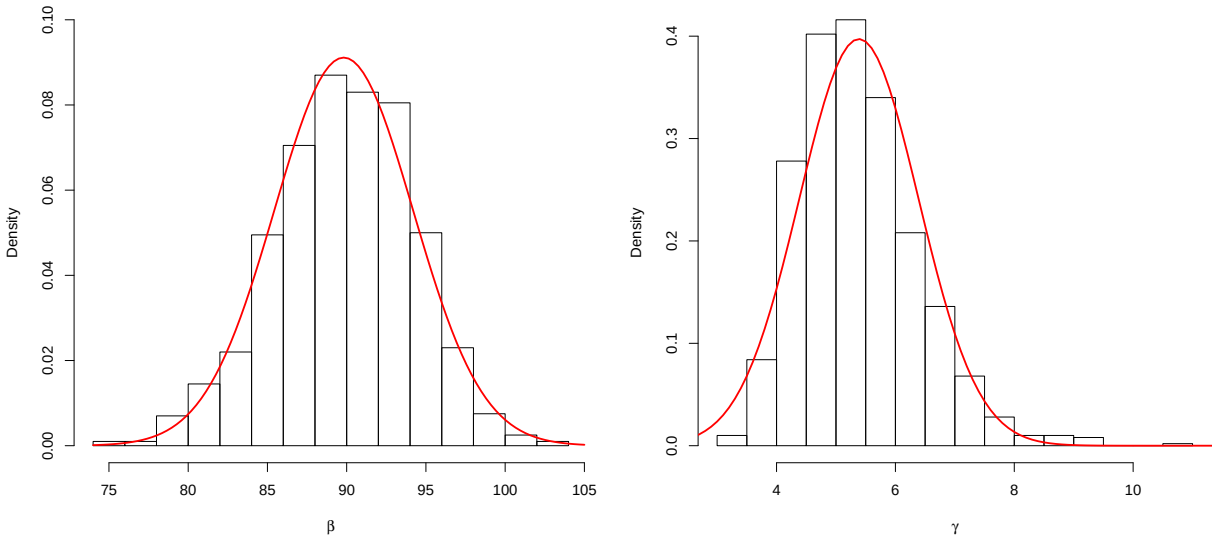


Figure A.2: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the maximum likelihood method, with $\gamma = 5$, $\beta = 90$, and $n = 100$.

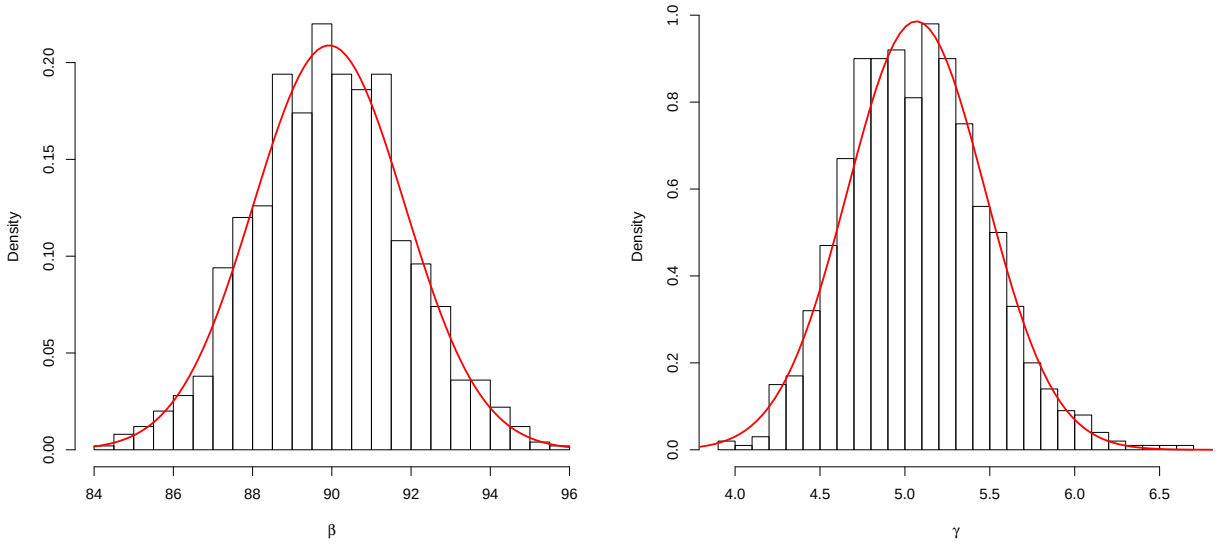


Figure A.3: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the maximum likelihood method, with $\gamma = 5$, $\beta = 90$, and $n = 10000$.

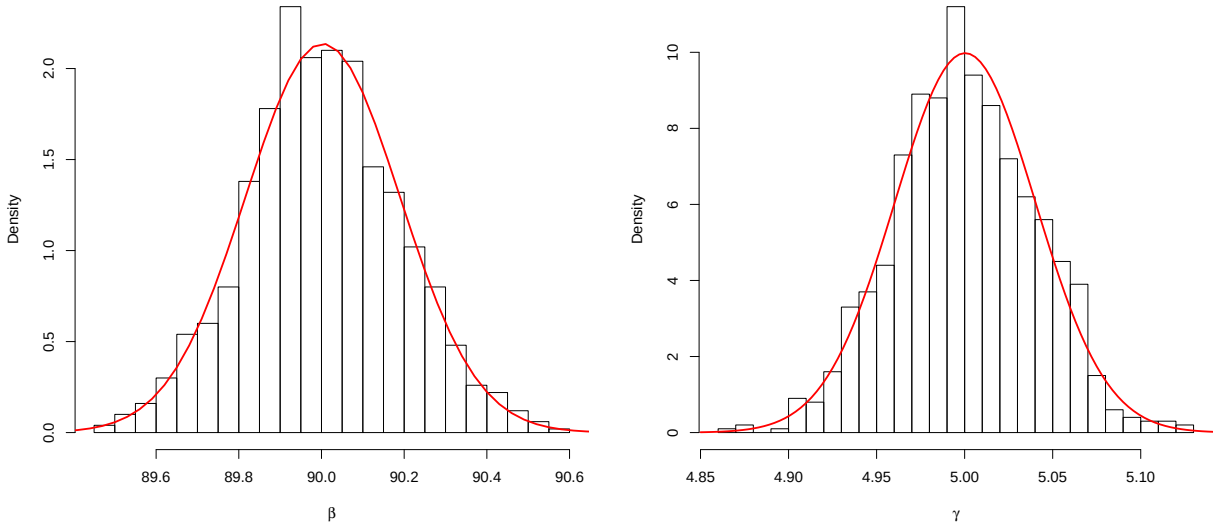


Figure A.4: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the maximum likelihood method, with $\gamma = 2$, $\beta = 2.5$, and $n = 20$.

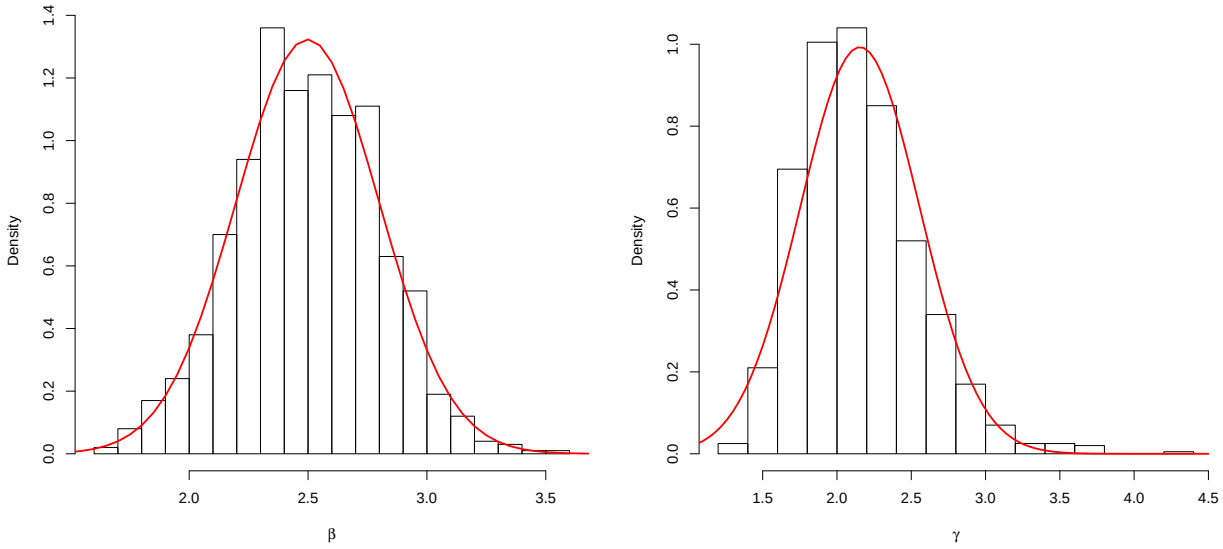


Figure A.5: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the maximum likelihood method, with $\gamma = 2$, $\beta = 2.5$, and $n = 100$.

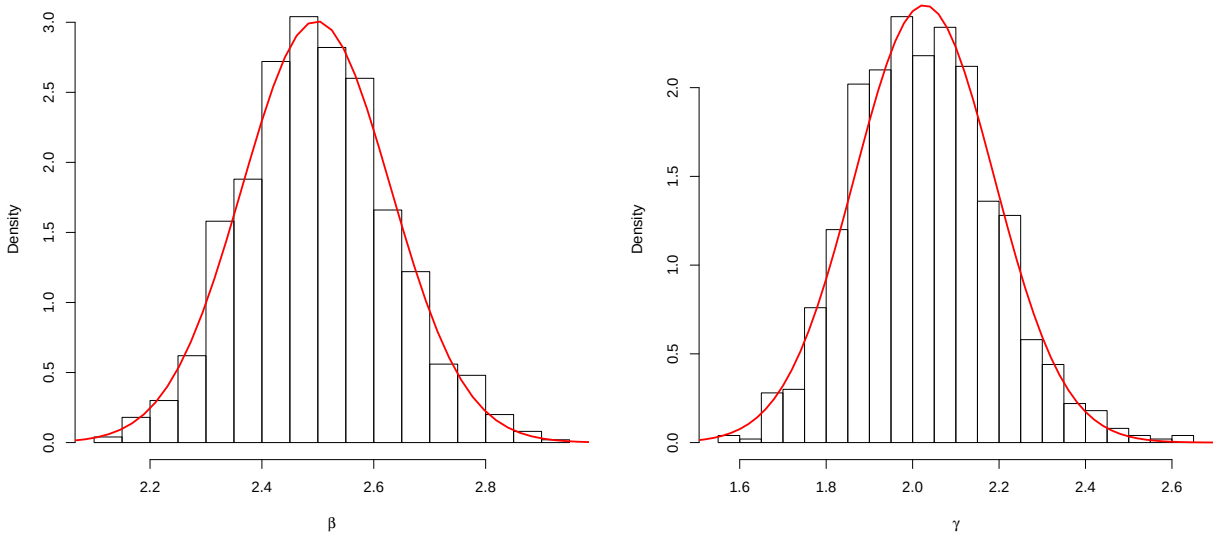
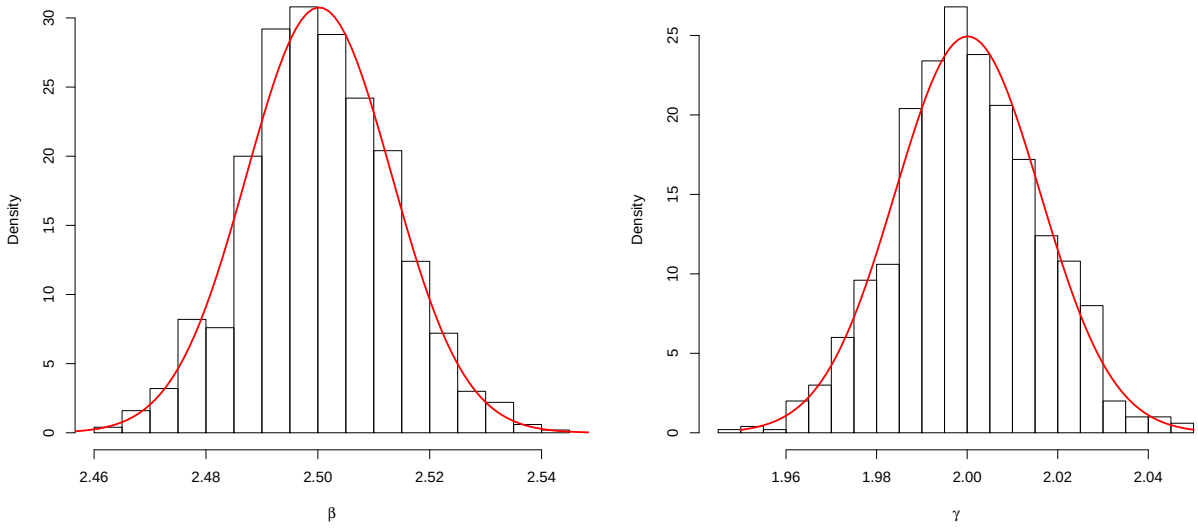


Figure A.6: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the maximum likelihood method, with $\gamma = 2$, $\beta = 2.5$, and $n = 10000$.



Method of Moments

Figure A.7: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the method of moments, with $\gamma = 5$, $\beta = 90$, and $n = 20$.

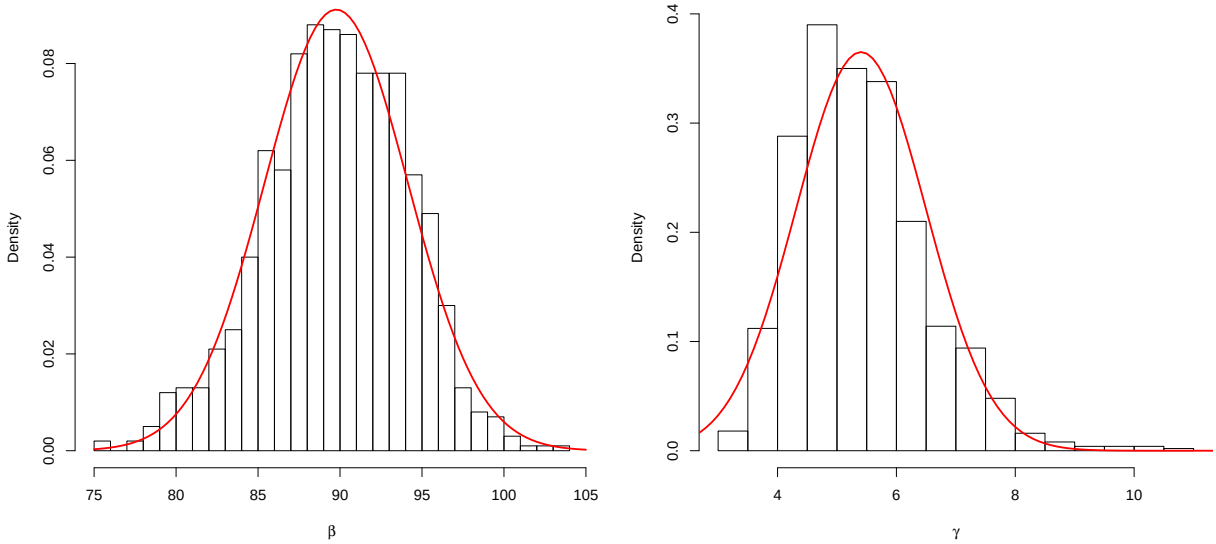


Figure A.8: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the method of moments, with $\gamma = 5$, $\beta = 90$, and $n = 100$.

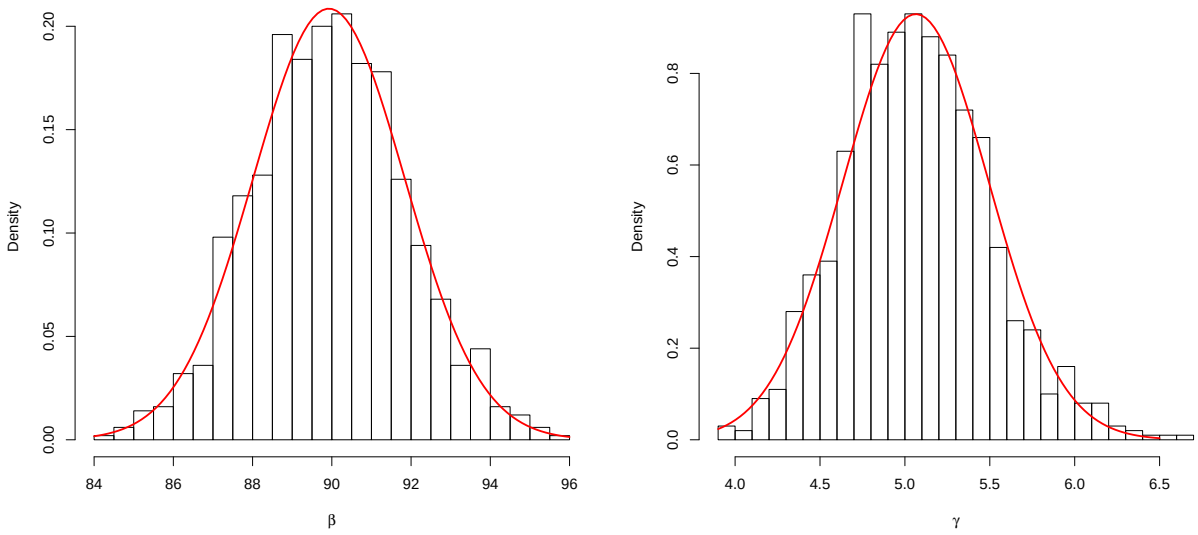


Figure A.9: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the method of moments, with $\gamma = 5$, $\beta = 90$, and $n = 10000$.

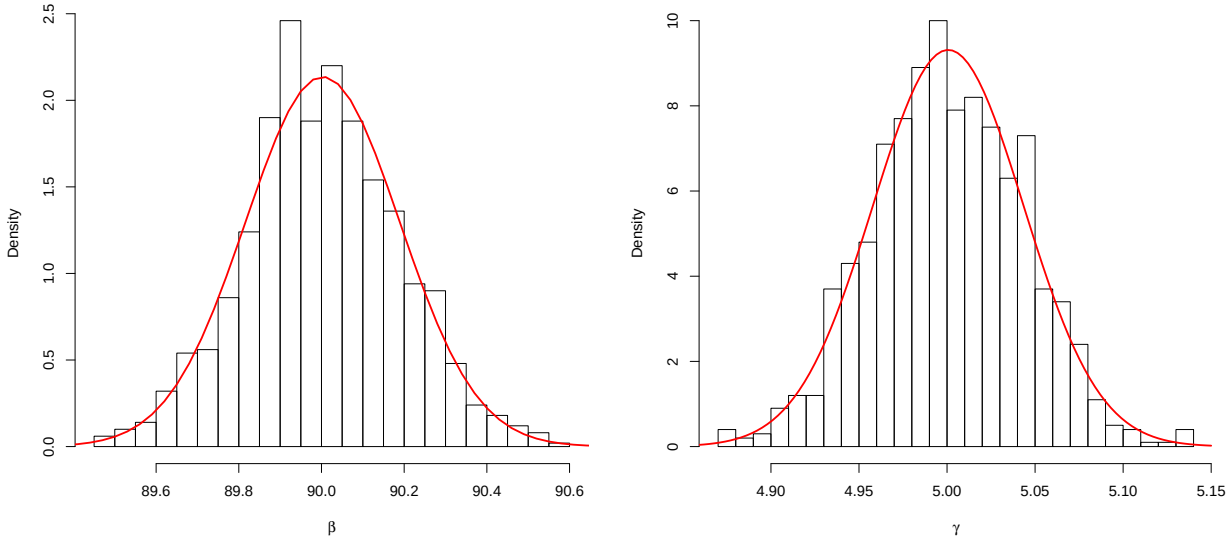


Figure A.10: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the method of moments, with $\gamma = 2$, $\beta = 2.5$, and $n = 20$.

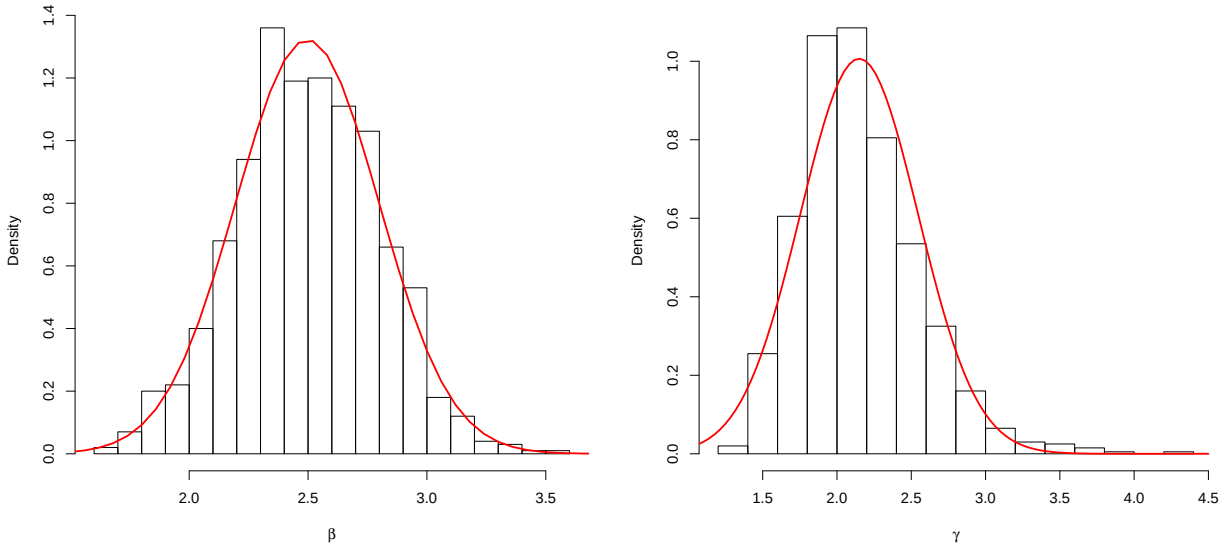


Figure A.11: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the method of moments, with $\gamma = 2$, $\beta = 2.5$, and $n = 100$.

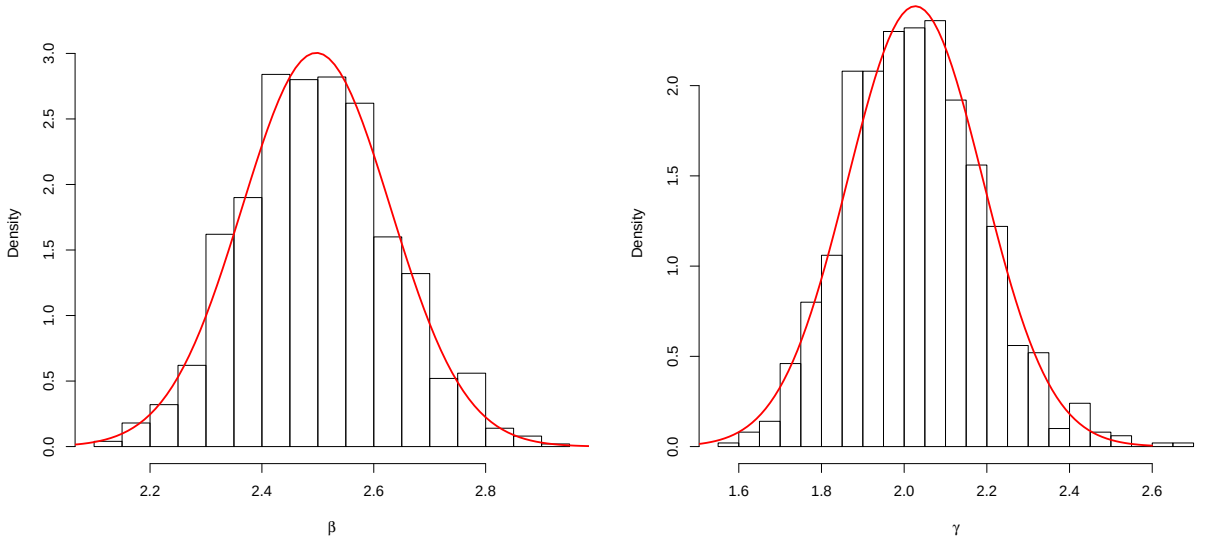
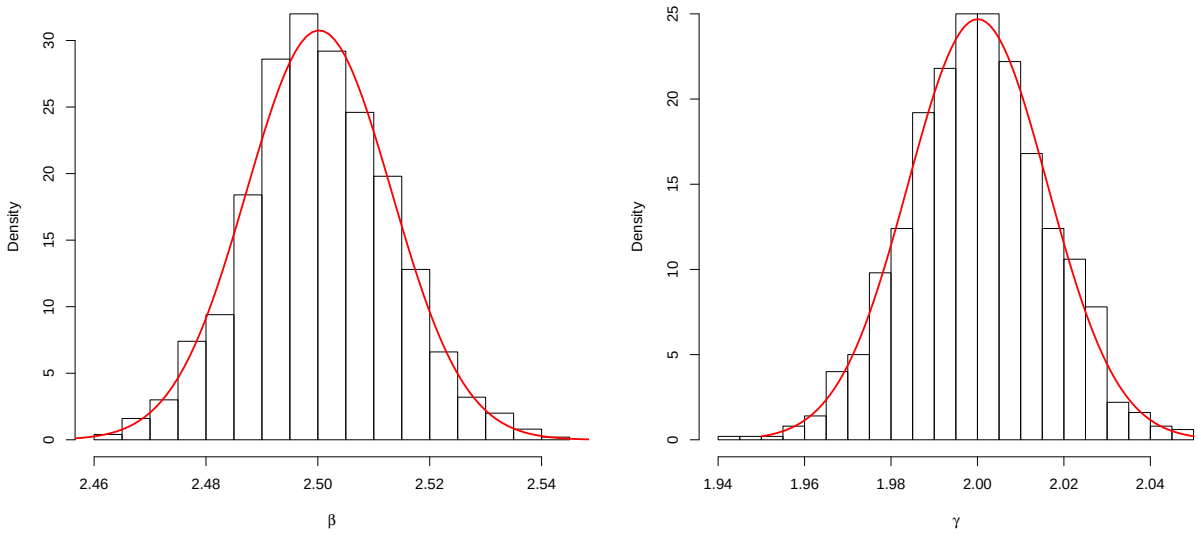


Figure A.12: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the method of moments, with $\gamma = 2$, $\beta = 2.5$, and $n = 10000$.



Median Rank Regression

Figure A.13: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the median rank regression method, with $\gamma = 5$, $\beta = 90$, and $n = 20$.

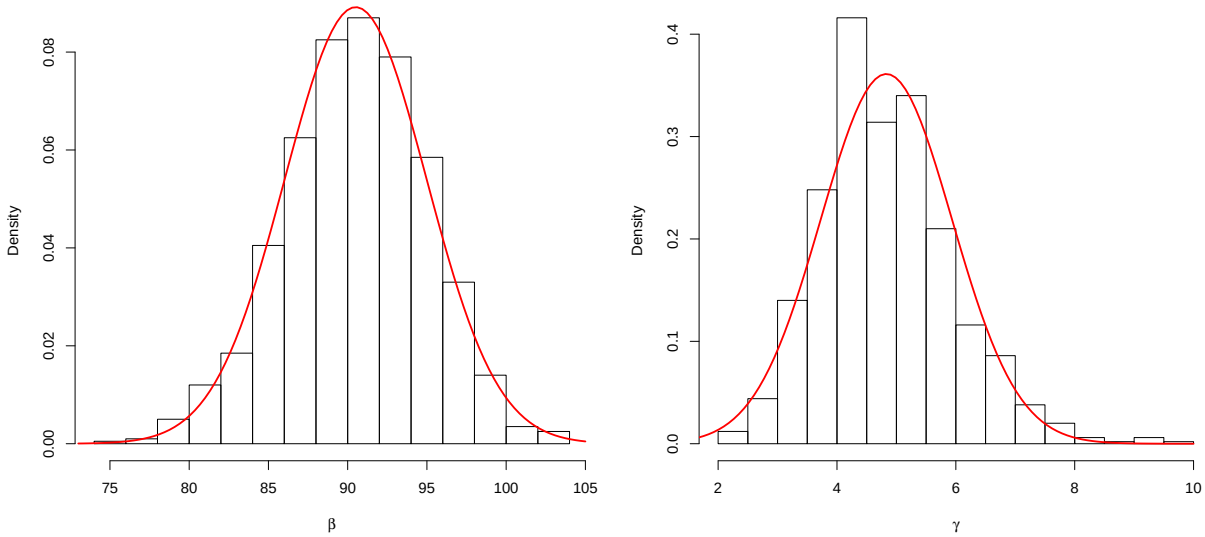


Figure A.14: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the median rank regression method, with $\gamma = 5$, $\beta = 90$, and $n = 100$.

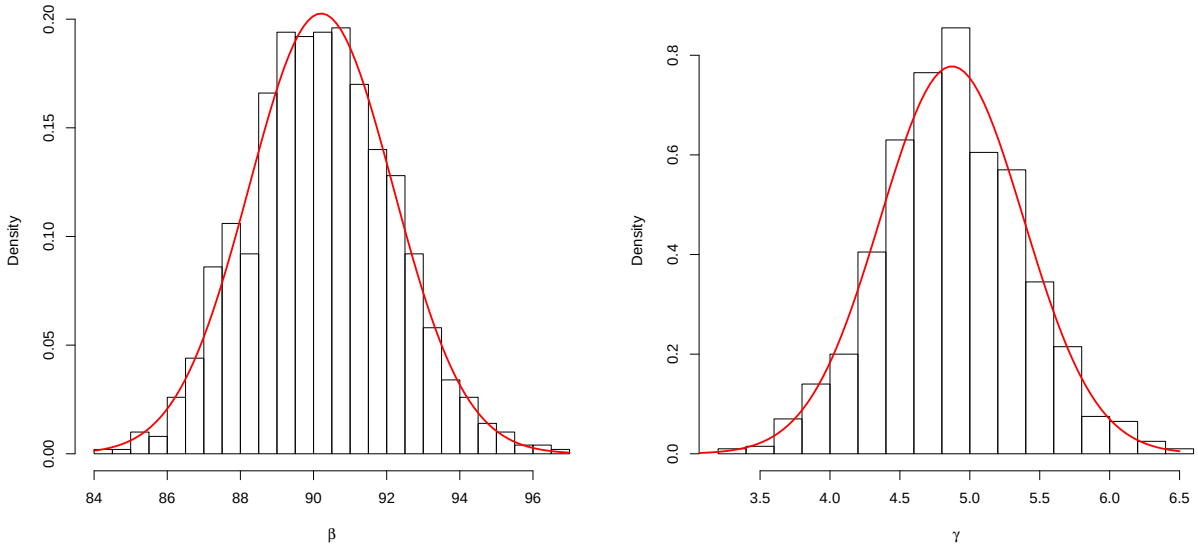


Figure A.15: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the median rank regression method, with $\gamma = 5$, $\beta = 90$, and $n = 10000$.

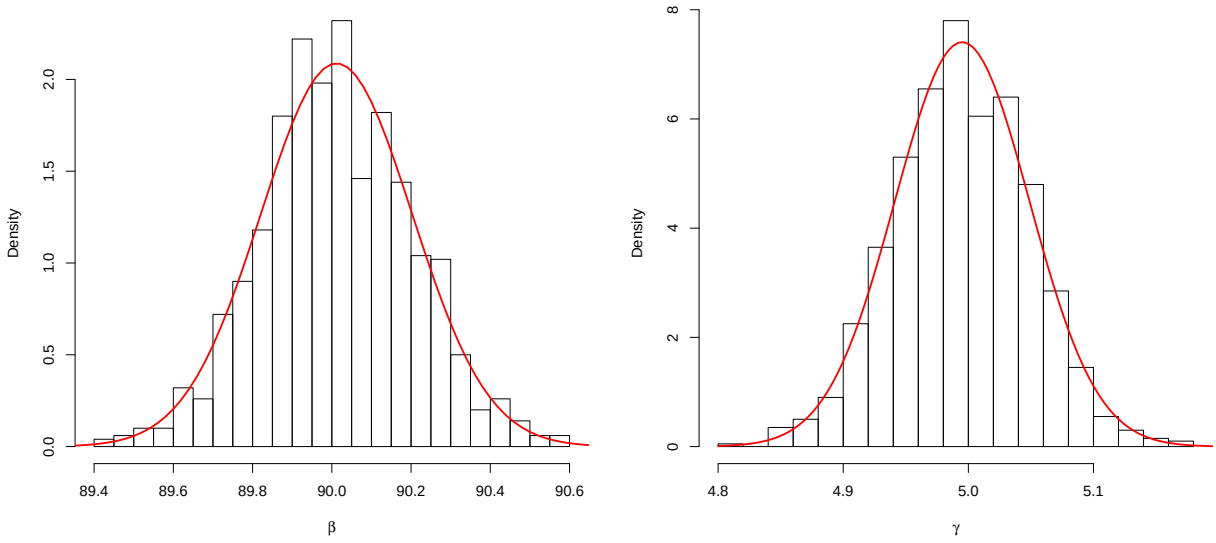


Figure A.16: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the median rank regression method, with $\gamma = 2$, $\beta = 2.5$, and $n = 20$.

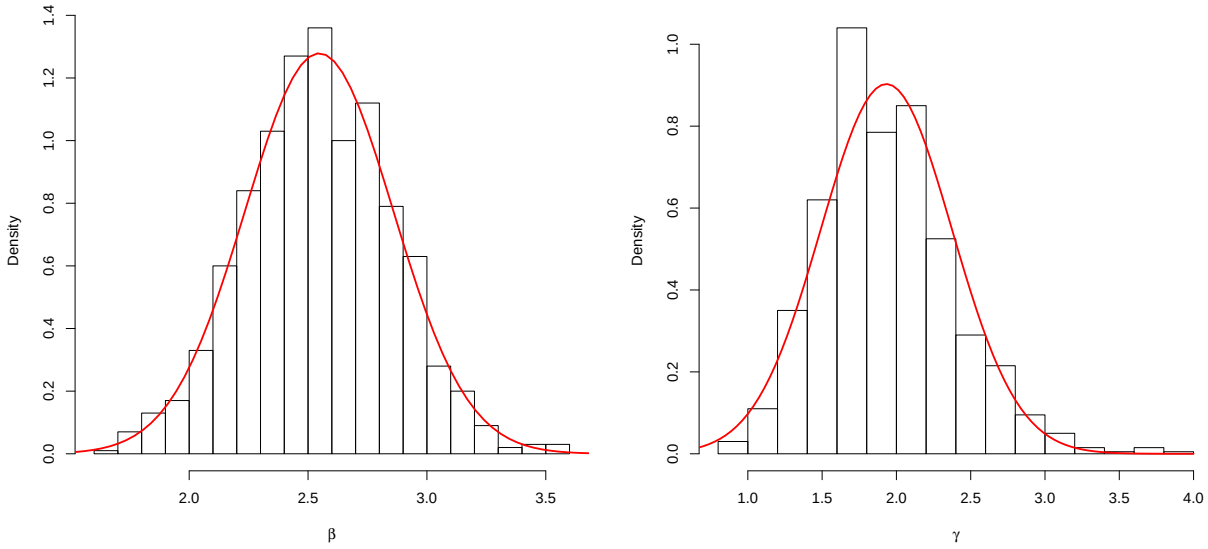


Figure A.17: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the median rank regression method, with $\gamma = 2$, $\beta = 2.5$, and $n = 100$.

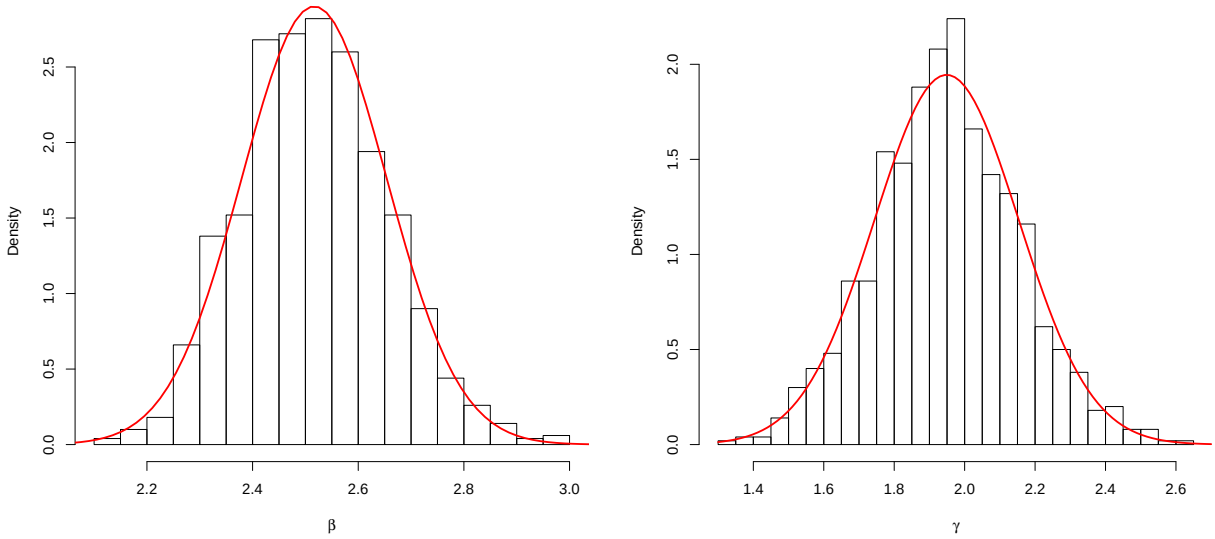
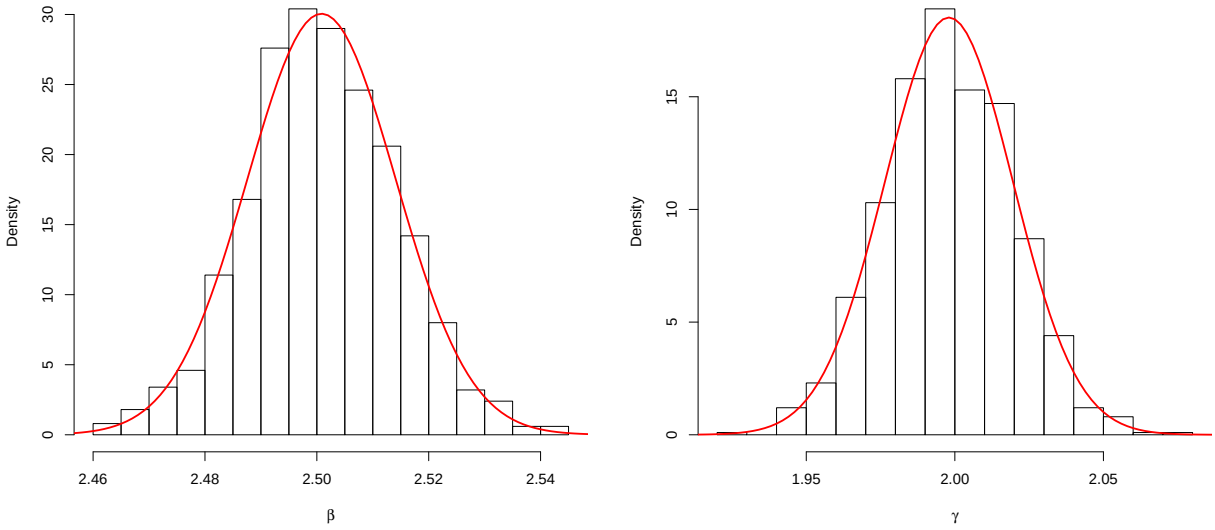


Figure A.18: **Normally Distributed Parameter Estimates:** Both are distributions of the parameters for the median rank regression method, with $\gamma = 2$, $\beta = 2.5$, and $n = 10000$.



MLE method	γ	β
Mixture	1.98968	1.31464
Deleted	1.98968	1.31464
Binned	1.94784	1.36516
Add 0.000001	0.77568	1.02308
Add 0.0001	1.00373	1.08067
Add 0.01	1.37274	1.15694

Table A.1: MLEs for the aspen site.

MME method	γ	β
Deleted	2.01276	1.31787
Binned	1.94667	1.36504
+ 0.000001	1.68236	1.20933
+ 0.0001	1.68241	1.20934
+ 0.01	1.68710	1.21042

Table A.2: MMEs for the aspen site.

MRR method	γ	β
Deleted	1.90188	1.33728
Binned	1.90098	1.37734
+ 0.000001	0.26718	3.02624
+ 0.0001	0.41547	1.98037
+ 0.01	0.84196	1.38628

Table A.3: MRR estimates for the aspen site.

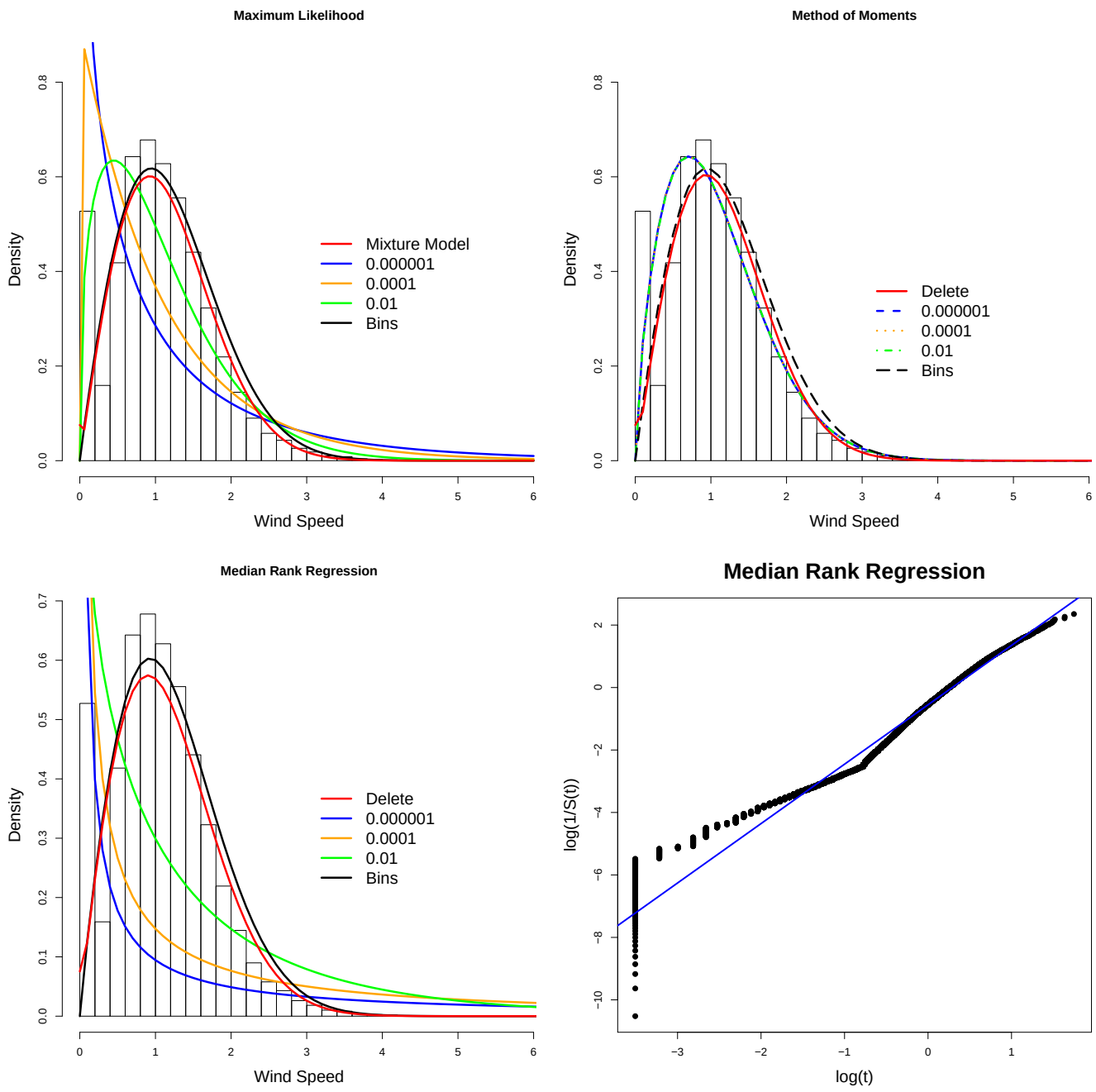


Figure A.19: Graphical summary of the parameter estimates for the aspen site. The method of moments appears to be most "robust" in estimation for all methods. In all methods, the deleted/mixture and binned data parameter estimates are most accurate.

MLE method	γ	β
Mixture	1.91104	0.71525
Deleted	1.91104	0.71525
Binned	1.08684	0.50213
+ 0.000001	0.25449	0.14215
+ 0.0001	0.37822	0.22965
+ 0.01	0.71221	0.38087

Table A.4: MLEs for the conifer site.

MME method	γ	β
Deleted	2.03695	0.72417
Binned	1.24820	0.52303
+ 0.000001	1.11050	0.46508
+ 0.0001	1.11067	0.46513
+ 0.01	1.12787	0.47051

Table A.5: MMEs for the conifer site.

MRR method	γ	β
Deleted	1.47805	0.75570
Binned	0.94479	0.52934
+ 0.000001	0.17376	0.26494
+ 0.0001	0.26972	0.32761
+ 0.01	0.57137	0.42676

Table A.6: MRR estimates for the conifer site.

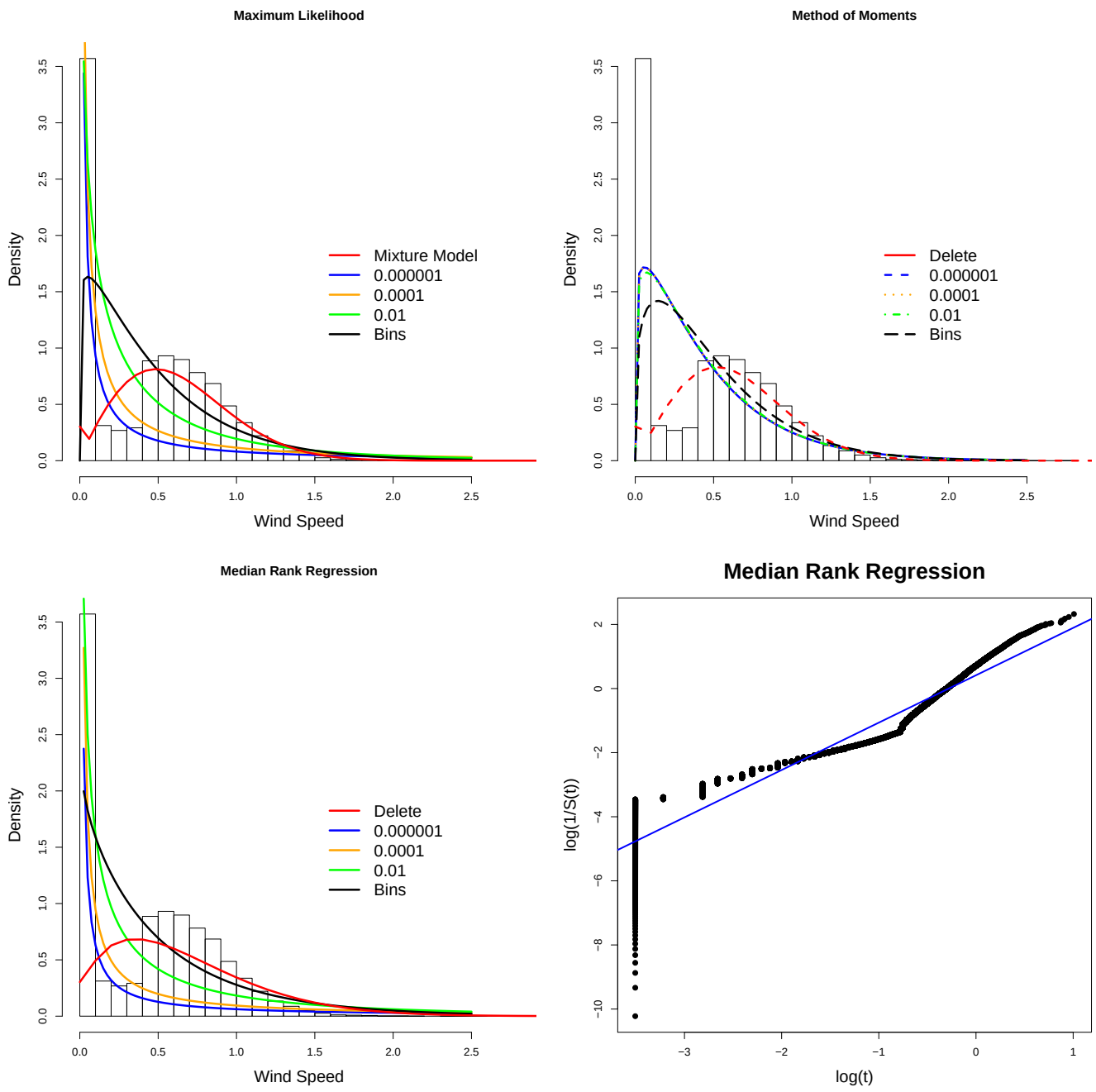


Figure A.20: Graphical summary of the parameter estimates for the conifer site. Notice the high proportion of values close to zero.

MLE method	γ	β
Mixture	1.80668	4.81714
Deleted	1.80668	4.81714
Binned	1.86742	4.98842
+ 0.000001	1.66072	4.70657
+ 0.0001	1.70347	4.73489
+ 0.01	1.74780	4.76414

Table A.7: MLEs for the summit.

MME method	γ	β
Deleted	1.78376	4.80543
Binned	1.84069	4.97420
+ 0.000001	1.76297	4.77663
+ 0.0001	1.76298	4.77663
+ 0.01	1.76307	4.77671

Table A.8: MMEs for the summit.

MRR method	γ	β
Deleted	1.87337	4.80686
Binned	1.98354	4.95250
+ 0.000001	0.75611	6.98805
+ 0.0001	1.07748	5.70484
+ 0.01	1.53393	4.98629

Table A.9: MRR estimates for the summit.

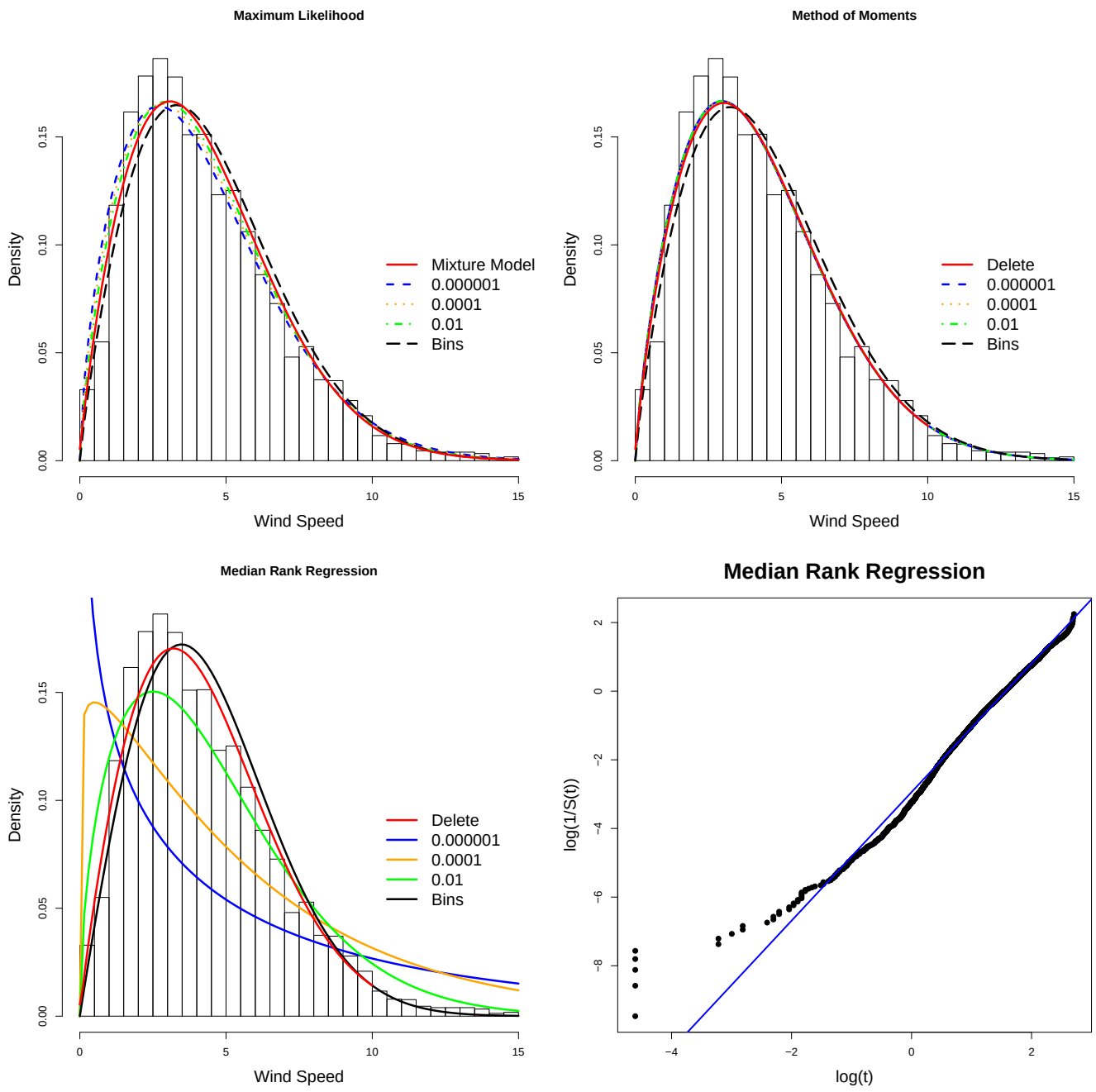


Figure A.21: Graphical summary for the parameter estimates for the summit site. There is evidence here that the MRR method is most sensitive to the zero data.

MLE method	γ	β
Mixture	1.44645	2.32531
Deleted	1.44645	2.32531
Binned	1.49738	2.40691
+ 0.000001	1.00917	2.05087
+ 0.0001	1.12541	2.11278
+ 0.01	1.26391	2.18411

Table A.10: MLEs for the sage site.

MME method	γ	β
Deleted	1.45800	2.33109
Binned	1.48168	2.40055
+ 0.000001	1.38578	2.24106
+ 0.0001	1.38579	2.24107
+ 0.01	1.38644	2.24158

Table A.11: MMEs for the summit.

MRR method	γ	β
Deleted	1.39711	2.34865
Binned	1.51890	2.40479
+ 0.000001	0.38197	4.50466
+ 0.0001	0.57915	3.11101
+ 0.01	0.97749	2.39434

Table A.12: MRR estimates for the sage site.

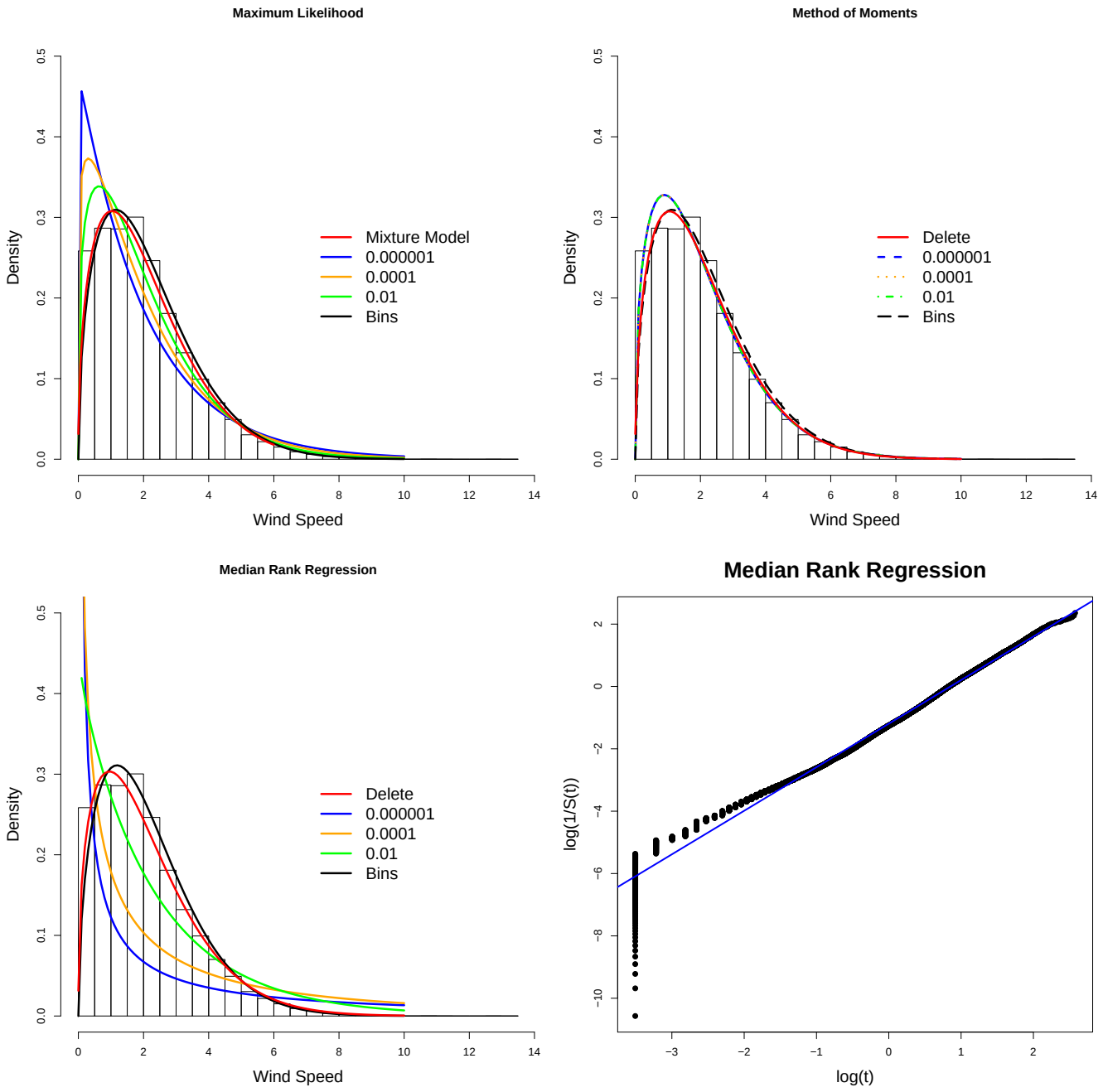


Figure A.22: Graphical summary of the parameter estimates for the sage site. Once again, the MME proves to be most robust when comparing each solution to remove the zeros from our data.

APPENDIX B

C-CODE

B.1 Code for Simulation Studies

```
1  /*******************
2  Mark Nielsen
3  C Project
4  (Adapted from code by Dr. David Engler)
5  ********************/
6
7
8  #include <stdio.h>
9  #include <gsl/gsl_rng.h>          // required for random number generation
10 #include <gsl/gsl_randist.h>      // required for data generation from distributions
11 #include <gsl/gsl_multimin.h>     // required for use of multimin (optimization)
12 //#include <gsl/gsl_cdf.h>          // required for CDF values
13 //#include <gsl/gsl_statistics.h>
14 //#include <gsl/gsl_sort.h>
15 #include <gsl/gsl_sf_gamma.h>
16
17 #include <math.h>
18
19 # define npop 10000     // number of data points
20 # define nparam 2      // number of parameters to be estimated in function
21 # define nsim 1000        // number of simulations
22
23 gsl_rng * r;  /* global generator */
24
25 //This is my Weibull function
26 double weibull_pdf(double dat,double theta[nparam]) {
27   double weibull;
28   weibull=theta[0]/(pow(theta[1],theta[0]))*pow(dat,(theta[0]-1))*exp(-pow(dat/theta
         [1],theta[0]));
29   return(weibull);
30 }
31
```

```
32
33 // ##### DEFINE FUNCTION TO BE MINIMIZED
34 // params = data
35 // v = vector of parameters to be identified
36 double weibull_ll (const gsl_vector *v, void *params)
37 {
38   double *p = (double *)params; // convert data into readable format
39   double weibll_val = 0;        // initialize value of neg loglikelihood
40   double gamma, beta;
41   gamma = gsl_vector_get(v, 0);    // assign values of param vector to specific
         variables
42   beta = gsl_vector_get(v, 1);
43   if(gamma<=0){
44     return(1e10);
45   }else if(beta<=0){
46     return(1e10);
47   }else{
48     int i;
49     for(i=0;i<npop;i++) {
50       weibll_val =
51         weibll_val -
52             log(gamma/(pow(beta,gamma))) - (gamma-1)*log(p[i]) + pow((p[i]/beta),
                 gamma);
53     }
54         return(weibll_val);
55   }
56 }
57
58
59 //write a program to sort vectors
60 void sorting(double x[npop]){
61   int i;
62   int j;
63   double temp;
64   for (i=0;i<npop;i++){
65     for (j=i+1;j<npop;j++){
66       if (x[j] < x[i]){
67         temp=x[i];
68         x[i]=x[j];
```

```
69            x[j]=temp;
70          }
71        }
72      }
73      return;
74  }
75
76  //Secant Method root finding.
77  double bisect(double lo, double hi, double e, int max, double mu[2])
78  {
79      double mid;
80      int i;
81      for(i=0;i<50;i++){
82      mid = (hi+lo)/2;
83      if((gsl_sf_gamma(1+2/hi)/pow(gsl_sf_gamma(1+1/hi),2) - mu[1]/pow(mu[0],2))
84            *(gsl_sf_gamma(1+2/mid)/pow(gsl_sf_gamma(1+1/mid),2) - mu[1]/pow(mu[0],2)) <=
                  0){
85        lo=mid;}
86      else if((gsl_sf_gamma(1+2/lo)/pow(gsl_sf_gamma(1+1/lo),2) - mu[1]/pow(mu[0],2))
87            *(gsl_sf_gamma(1+2/mid)/pow(gsl_sf_gamma(1+1/mid),2) - mu[1]/pow(mu[0],2)) <
                  0){
88        hi=mid;}
89      }
90      return mid;
91  }
92
93
94
95  //Function to find minimum between two numbers.
96  double min(double val1, double val2){
97  double minimum;
98  if(val1<val2){
99            minimum=val1;
100 }else{
101           minimum=val2;
102 }
103 return(minimum);
104 }
105
```

```
106 // ##### GENERATE DATA
107 void gendata (double theta[nparam], double par[npop], FILE *sim_data,int j)
108 {
109   /* ###### set up random number generator ############################ */
110   const gsl_rng_type * TT;
111   gsl_rng_env_setup();
112   TT = gsl_rng_default;
113   r = gsl_rng_alloc (TT);
114   gsl_rng_set(r,j);  // set seed based on simulation number
115   gsl_rng_free (r);  // function frees all the memory associated with the generator
        r
116   /* ###################################################################### */
117   int i;
118   double runif;
119   double alpha;
120
121   for(i=0;i<npop;i++) {
122     runif= gsl_rng_uniform(r);
123     par[i]=theta[1]*pow(-log(runif),1/theta[0]);
124     fprintf(sim_data,"%f,",par[i]);
125   }
126   fprintf(sim_data,"\n");
127   return;
128 }
129
130
131
132 /************************Least Squares Regression Function
        ****************************/
133 // This is a program in C to conduct least squares regression with
134 // the following data points.
135
136 void regress(double y[npop],double x[npop],double beta[2]){
137   int points=npop;
138   int i;
139   double xbar=0;
140   double ybar=0;
141
142   double div=points;
```

```
143     //compute xbar & ybar
144     for(i=0;i<points;i++){
145       xbar=xbar+x[i]/div;
146       ybar=ybar+y[i]/div;
147     }
148     double fract[2];
149     fract[0]=0;
150     fract[1]=0;
151     //now multiply to get our beta extimates
152     for(i=0;i<points;i++){
153       fract[0]=fract[0]+(x[i]-xbar)*y[i];
154       fract[1]=fract[1]+pow((x[i]-xbar),2);
155     }
156     beta[1]=fract[0]/fract[1];
157     beta[0]=ybar-beta[1]*xbar;
158
159   return;
160 }
161
162
163           /** Median Rank Regression Method **/
164 void medianrank(double data[npop], double thta[nparam]){
165           double MRR[nparam];
166      MRR[0]=MRR[1]=0;
167           int k;
168           double ynew[npop];
169           double xnew[npop];
170           for(k=0;k<npop;k++){
171             //compute y's
172             // log(log(1/(1-(i-.3)/(npop+.4))))
173             ynew[k]=log(log(1/(1-((k+1)-.3)/(npop+.4))));
174
175             //compute x's
176             // log(x[i])
177             xnew[k]=log(data[k]);
178           }
179           //fit the model.
180           regress(ynew,xnew,MRR);
181
```

```
182         //compute parameters.
183         //gamma=c1
184         thta[0]=MRR[1];
185         //beta=exp(-c0/gamma)
186         thta[1]=exp(-MRR[0]/thta[0]);
187
188     return;
189 }
190
191
192 int
193 main(void)
194 {
195
196
197   /* ###### set up multimin ########################################### */
198   const gsl_multimin_fminimizer_type *T =
199   gsl_multimin_fminimizer_nmsimplex2;
200   gsl_multimin_fminimizer *s = NULL;
201   gsl_vector *ss;
202   gsl_vector *x;
203   gsl_multimin_function minex_func;
204   size_t iter;
205   int status;
206   double size;
207   /* ################################################################## */
208
209   double par[npop];
210   double pargamma[npop];
211   double theta[nparam];
212   //theta=2,2.5
213
214   //gamma
215   theta[0] = 2;
216   //beta
217   theta[1] = 2.5;
218
219   int j;
220   //for MRR
```

```
221   double MRRest[nparam];
222   MRRest[0]=MRRest[1]=0;
223
224   FILE *est_mle;
225   est_mle= fopen("../simdata/bigB/mle_est1_big.csv", "w");
226   FILE *est_mrr;
227   est_mrr = fopen("../simdata/bigB/mrr_est1_big.csv", "w");
228   FILE *est_mom;
229   est_mom = fopen("../simdata/bigB/mom_est1_big.csv", "w");
230   FILE *sim_data;
231   sim_data = fopen("../simdata/bigB/sim_data1_bigA.csv", "w");
232
233   for(j=0;j<nsim;j++) {
234
235     iter=0;
236     // generate data
237     gendata(theta,par,sim_data,j);
238
239         double parsort[npop];
240         int k;
241         for(k=0;k<npop;k++){
242           parsort[k]=par[k];
243         }
244
245         /** Maximum Likelihood Estimation **/
246
247         // Starting values (for optimization)
248     x = gsl_vector_alloc (nparam);
249     gsl_vector_set (x, 0, 1.0); // set initial gamma value
250     gsl_vector_set (x, 1, 2.0); // set initial beta value
251
252     // Set initial step sizes to 1
253     ss = gsl_vector_alloc (nparam);
254     gsl_vector_set_all (ss, 1.0);
255
256     // Initialize method and iterate
257     minex_func.n = nparam;     // number of parameters
258     minex_func.f = weibull_ll;     // function of interest
259     minex_func.params = parsort;  // data
```

```
260
261     s = gsl_multimin_fminimizer_alloc (T, nparam);
262     gsl_multimin_fminimizer_set (s, &minex_func, x, ss);
263
264     do
265       {
266         status = gsl_multimin_fminimizer_iterate(s);
267         if (status)
268           break;
269         size = gsl_multimin_fminimizer_size (s);
270         status = gsl_multimin_test_size (size, 1e-5);
271         if (status == GSL_SUCCESS)
272           {
273             fprintf(est_mle,"%f, %f\n",gsl_vector_get (s->x, 0),gsl_vector_get (s->x
                  , 1));
274           }
275       }
276     while (status == GSL_CONTINUE && iter < 100);
277     gsl_vector_free(x);
278     gsl_vector_free(ss);
279     gsl_multimin_fminimizer_free (s);
280
281
282     /** Median Rank Regression Method **/
283
284         //sort the data.
285         sorting(parsort);
286
287         medianrank(parsort,MRRest);
288
289         //print to file.
290         fprintf(est_mrr,"%f, %f\n",MRRest[0],MRRest[1]);
291
292         /** Method of Moments Estimation **/
293
294         double MOMest[nparam];
295         MOMest[0]=MOMest[1]=1;
296         //define moments.
297         double mu[2];
```

```
298             mu[0]=mu[1]=0;
299             for(k=0;k<npop;k++){
300                double n=npop;
301                // mu1=E(x)
302                mu[0]=mu[0]+parsort[k]/n;
303                // mu2=E(x^2)
304                mu[1]=mu[1]+pow(parsort[k],2)/n;
305             }
306
307
308             //Use root finding to estimate gamma
309             double max=1000;
310             double error=1e-100;
311       MOMest[0]=bisect(.1,101.3,error,max,mu);
312
313             //Solve for beta
314             // beta=mu1/GAMMA(1+1/gamma)
315             MOMest[1]=mu[0] / gsl_sf_gamma(1+1/MOMest[0]);
316             //print to file.
317       fprintf(est_mom,"%f, %f\n",MOMest[0],MOMest[1]);
318    }
319
320    fclose(est_mom);
321    fclose(est_mrr);
322    fclose(est_mle);
323    fclose(sim_data);
324    return;
325 }
```

B.2 Code to Generate from Gamma for Misspecification Studies

```
1  /*******************
2  Mark Nielsen
3  C Project
4  ********************/
5
6
7  #include <stdio.h>
8  #include <gsl/gsl_rng.h>          // required for random number generation
9  #include <gsl/gsl_randist.h>      // required for data generation from distributions
```

```
10 #include <gsl/gsl_multimin.h>     // required for use of multimin (optimization)
11 //#include <gsl/gsl_cdf.h>          // required for CDF values
12 //#include <gsl/gsl_statistics.h>
13 //#include <gsl/gsl_sort.h>
14 #include <gsl/gsl_sf_gamma.h>
15
16 #include <math.h>
17
18 # define npop 1000     // number of data points
19 # define nparam 2      // number of parameters to be estimated in function
20 # define nsim 1       // number of simulations
21
22 gsl_rng * r;  /* global generator */
23
24 //This is my Weibull function
25 double weibull_pdf(double dat,double theta[nparam]) {
26   double weibull;
27   weibull=theta[0]/(pow(theta[1],theta[0]))*pow(dat,(theta[0]-1))*exp(-pow(dat/theta
       [1],theta[0]));
28   return(weibull);
29 }
30
31
32 // ##### DEFINE FUNCTION TO BE MINIMIZED
33 // params = data
34 // v = vector of parameters to be identified
35 double weibull_ll (const gsl_vector *v, void *params)
36 {
37   double *p = (double *)params; // convert data into readable format
38   double weibll_val = 0;        // initialize value of neg loglikelihood
39   double gamma, beta;
40   gamma = gsl_vector_get(v, 0);    // assign values of param vector to specific
       variables
41   beta = gsl_vector_get(v, 1);
42   if(gamma<=0){
43     return(1e10);
44   }else if(beta<=0){
45     return(1e10);
46   }else{
```

```
47      int i;
48      for(i=0;i<npop;i++) {
49        weibll_val =
50          weibll_val -
51              log(gamma/(pow(beta,gamma))) - (gamma-1)*log(p[i]) + pow((p[i]/beta),
                    gamma);
52      }
53          return(weibll_val);
54    }
55  }
56
57
58  //write a program to sort vectors
59  void sorting(double x[npop]){
60    int i;
61    int j;
62    double temp;
63    for (i=0;i<npop;i++){
64      for (j=i+1;j<npop;j++){
65        if (x[j] < x[i]){
66          temp=x[i];
67          x[i]=x[j];
68          x[j]=temp;
69        }
70      }
71    }
72    return;
73  }
74
75  //Secant Method root finding.
76  double bisect(double lo, double hi, double e, int max, double mu[2])
77  {
78    double mid;
79    int i;
80    for(i=0;i<50;i++){
81    mid = (hi+lo)/2;
82    if((gsl_sf_gamma(1+2/hi)/pow(gsl_sf_gamma(1+1/hi),2) - mu[1]/pow(mu[0],2))
83          *(gsl_sf_gamma(1+2/mid)/pow(gsl_sf_gamma(1+1/mid),2) - mu[1]/pow(mu[0],2)) <=
                0){
```

```
84        lo=mid;}
85      else if((gsl_sf_gamma(1+2/lo)/pow(gsl_sf_gamma(1+1/lo),2) - mu[1]/pow(mu[0],2))
86           *(gsl_sf_gamma(1+2/mid)/pow(gsl_sf_gamma(1+1/mid),2) - mu[1]/pow(mu[0],2)) <
                0){
87        hi=mid;}
88      }
89      return mid;
90  }
91
92
93
94  //Function to find minimum between two numbers.
95  double min(double val1, double val2){
96  double minimum;
97  if(val1<val2){
98          minimum=val1;
99  }else{
100         minimum=val2;
101 }
102 return(minimum);
103 }
104
105 // ##### GENERATE DATA for Gamma(n,1)
106 void gammadata (double n, double par[npop], FILE *sim_data2,int j)
107 {
108   /* ###### set up random number generator ############################ */
109   const gsl_rng_type * TT;
110   gsl_rng_env_setup();
111   TT = gsl_rng_default;
112   r = gsl_rng_alloc (TT);
113   gsl_rng_set(r,j);  // set seed based on simulation number
114   gsl_rng_free (r);  // function frees all the memory associated with the generator
          r
115   /* ###################################################################### */
116   int i,k;
117   double runif;
118   double alpha;
119
120   for(i=0;i<npop;i++) {
```

```
121       par[i]=0;
122       for(k=0;k<n;k++){
123       runif= gsl_rng_uniform(r);
124       par[i]=par[i]-log(runif);
125           }
126       fprintf(sim_data2,"%f,",par[i]);
127     }
128     fprintf(sim_data2,"\n");
129     return;
130   }
131
132
133   /************************Least Squares Regression Function
          ****************************/
134   // This is a program in C to conduct least squares regression with
135   // the following data points.
136
137   void regress(double y[npop],double x[npop],double beta[2]){
138     int points=npop;
139     int i;
140     double xbar=0;
141     double ybar=0;
142
143     double div=points;
144     //compute xbar & ybar
145     for(i=0;i<points;i++){
146       xbar=xbar+x[i]/div;
147       ybar=ybar+y[i]/div;
148     }
149     double fract[2];
150     fract[0]=0;
151     fract[1]=0;
152     //now multiply to get our beta extimates
153     for(i=0;i<points;i++){
154       fract[0]=fract[0]+(x[i]-xbar)*y[i];
155       fract[1]=fract[1]+pow((x[i]-xbar),2);
156     }
157     beta[1]=fract[0]/fract[1];
158     beta[0]=ybar-beta[1]*xbar;
```

```
159
160   return;
161 }
162
163
164           /** Median Rank Regression Method **/
165 void medianrank(double data[npop], double thta[nparam]){
166           double MRR[nparam];
167       MRR[0]=MRR[1]=0;
168           int k;
169           double ynew[npop];
170           double xnew[npop];
171           for(k=0;k<npop;k++){
172             //compute y's
173             // log(log(1/(1-(i-.3)/(npop+.4))))
174             ynew[k]=log(log(1/(1-((k+1)-.3)/(npop+.4))));
175
176             //compute x's
177             // log(x[i])
178             xnew[k]=log(data[k]);
179           }
180           //fit the model.
181           regress(ynew,xnew,MRR);
182
183           //compute parameters.
184           //gamma=c1
185           thta[0]=MRR[1];
186           //beta=exp(-c0/gamma)
187           thta[1]=exp(-MRR[0]/thta[0]);
188
189       return;
190 }
191
192
193 int
194 main(void)
195 {
196
197
```

```
198     /* ###### set up multimin ############################################## */
199     const gsl_multimin_fminimizer_type *T =
200     gsl_multimin_fminimizer_nmsimplex2;
201     gsl_multimin_fminimizer *s = NULL;
202     gsl_vector *ss;
203     gsl_vector *x;
204     gsl_multimin_function minex_func;
205     size_t iter;
206     int status;
207     double size;
208     /* ###################################################################### */
209
210     double par[npop];
211     double pargamma[npop];
212     double theta[nparam];
213
214     int j;
215     //for MRR
216     double MRRest[nparam];
217     MRRest[0]=MRRest[1]=0;
218
219     FILE *est_mle;
220     est_mle= fopen("../simdata/gamma/mle_est.csv", "w");
221     FILE *est_mrr;
222     est_mrr = fopen("../simdata/gamma/mrr_est.csv", "w");
223     FILE *est_mom;
224     est_mom = fopen("../simdata/gamma/mom_est.csv", "w");
225     FILE *sim_data2;
226     sim_data2 = fopen("../simdata/gamma/sim_data2.csv", "w");
227
228     for(j=0;j<nsim;j++) {
229
230       iter=0;
231       // generate data
232       //gendata(theta,par,sim_data,j);
233           int n=4;
234           gammadata(n,par,sim_data2,j);
235
236           double parsort[npop];
```

```
237         int k;
238         for(k=0;k<npop;k++){
239           parsort[k]=par[k];
240         }
241
242         /** Maximum Likelihood Estimation **/
243
244         // Starting values (for optimization)
245       x = gsl_vector_alloc (nparam);
246       gsl_vector_set (x, 0, 1.0); // set initial gamma value
247       gsl_vector_set (x, 1, 2.0); // set initial beta value
248
249       // Set initial step sizes to 1
250       ss = gsl_vector_alloc (nparam);
251       gsl_vector_set_all (ss, 1.0);
252
253       // Initialize method and iterate
254       minex_func.n = nparam;     // number of parameters
255       minex_func.f = weibull_ll;     // function of interest
256       minex_func.params = parsort;  // data
257
258       s = gsl_multimin_fminimizer_alloc (T, nparam);
259       gsl_multimin_fminimizer_set (s, &minex_func, x, ss);
260
261       do
262         {
263           status = gsl_multimin_fminimizer_iterate(s);
264           if (status)
265             break;
266           size = gsl_multimin_fminimizer_size (s);
267           status = gsl_multimin_test_size (size, 1e-5);
268           if (status == GSL_SUCCESS)
269             {
270               fprintf(est_mle,"%f, %f\n",gsl_vector_get (s->x, 0),gsl_vector_get (s->x
                      , 1));
271             }
272         }
273       while (status == GSL_CONTINUE && iter < 100);
274       gsl_vector_free(x);
```

```
275     gsl_vector_free(ss);
276     gsl_multimin_fminimizer_free (s);
277
278
279     /** Median Rank Regression Method **/
280
281         //sort the data.
282         sorting(parsort);
283
284         medianrank(parsort,MRRest);
285
286         //print to file.
287         fprintf(est_mrr,"%f, %f\n",MRRest[0],MRRest[1]);
288
289         /** Method of Moments Estimation **/
290
291         double MOMest[nparam];
292         MOMest[0]=MOMest[1]=1;
293         //define moments.
294         double mu[2];
295         mu[0]=mu[1]=0;
296         for(k=0;k<npop;k++){
297           double n=npop;
298           // mu1=E(x)
299           mu[0]=mu[0]+parsort[k]/n;
300           // mu2=E(x^2)
301           mu[1]=mu[1]+pow(parsort[k],2)/n;
302         }
303
304
305         //Use root finding to estimate gamma
306         double max=1000;
307         double error=1e-100;
308     MOMest[0]=bisect(.1,101.3,error,max,mu);
309
310         //Solve for beta
311         // beta=mu1/GAMMA(1+1/gamma)
312         MOMest[1]=mu[0] / gsl_sf_gamma(1+1/MOMest[0]);
313         //print to file.
```

```
314         fprintf(est_mom,"%f, %f\n",MOMest[0],MOMest[1]);
315     }
316
317     fclose(est_mom);
318     fclose(est_mrr);
319     fclose(est_mle);
320     fclose(sim_data2);
321     return;
322 }
```

APPENDIX C

R-CODE

C.1 Code to Generate from Normals for Misspecification Studies

```
1  ############
2  # Mark Nielsen
3  # Final Project
4
5  library(spuRs)
6  library(xtable)
7  library(msm)
8
9  ############################################################
10 ####repeat this last part for misspecified gamma(4,0);
11
12 miss=read.csv("../simdata/gamma/sim_data2.csv",header=F)
13 miss=as.numeric(miss[-1001])
14
15 mle.miss=read.csv("../simdata/gamma/mle_est.csv",header=F)
16 mom.miss=read.csv("../simdata/gamma/mom_est.csv",header=F)
17 mrr.miss=read.csv("../simdata/gamma/mrr_est.csv",header=F)
18
19 #KS tests
20 lg.gamma=c(
21 ks.test(miss,'pweibull',mle.miss[1,1],mle.miss[1,2])$p.value,#MLE is best!
22 ks.test(miss,'pweibull',mom.miss[1,1],mom.miss[1,2])$p.value,
23 ks.test(miss,'pweibull',mrr.miss[1,1],mrr.miss[1,2])$p.value,
24 ks.test(miss,'pgamma',4,1)$p.value)
25
26 sm.miss=read.csv("../simdata/gamma/sm_sim_data2.csv",header=F)
27 sm.miss=as.numeric(sm.miss[-31])
28
29 sm.mle.miss=read.csv("../simdata/gamma/sm_mle_est.csv",header=F)
30 sm.mom.miss=read.csv("../simdata/gamma/sm_mom_est.csv",header=F)
31 sm.mrr.miss=read.csv("../simdata/gamma/sm_mrr_est.csv",header=F)
32
```

```
#KS tests
sm.gamma=c(
ks.test(sm.miss,'pweibull',sm.mle.miss[1,1],sm.mle.miss[1,2])$p.value,
ks.test(sm.miss,'pweibull',sm.mom.miss[1,1],sm.mom.miss[1,2])$p.value,
ks.test(sm.miss,'pweibull',sm.mrr.miss[1,1],sm.mrr.miss[1,2])$p.value,#MRR is best!
ks.test(sm.miss,'pgamma',4,1)$p.value)

names(lg.gamma)=names(sm.gamma)=c('MLE','MME','MRR','Gamma')
xtable(cbind(sm.gamma,lg.gamma),digits=4)

#avg.miss=rbind(mean(mle.miss),mean(mom.miss),mean(mrr.miss))
#var.miss=rbind(diag(var(mle.miss)),diag(var(mom.miss)),diag(var(mrr.miss)))

#Graphical Summaries
pdf("../Graphs/misspec.pdf",width=14)
par(mfrow=c(1,2))

hist(sm.miss,freq=F,breaks=5, xlab='',ylim=c(0,.25),xlim=c(0,16), main='
    Misspecification - Gamma(4,1) n=30',cex.main=1.5)
curve(dweibull(x,sm.mrr.miss[1,1],sm.mrr.miss[1,2]),from=0,to=15,add=T,col='darkred
    ',lwd=2.5,lty=2)
curve(dweibull(x,sm.mle.miss[1,1],sm.mle.miss[1,2]),from=0,to=15,add=T,col='blue',
    lwd=2.5,lty=1)
curve(dweibull(x,sm.mom.miss[1,1],sm.mom.miss[1,2]),from=0,to=15,add=T,col='orange',
    lwd=2.5,lty=2)
curve(dgamma(x,4,1),from=0,to=15,add=T,col='black',lwd=2.5,lty=3)
legend(6.5,.2,c("MLE","MME","MRR","Gamma"),col=c('blue','orange','darkred','black'),
    lwd=c(2.5,2.5,2.5,2.5), lty=c(1,2,2,3),bty='n',cex=1.5)

hist(miss,freq=F,breaks=20, xlab='',ylim=c(0,.25),xlim=c(0,16), main='
    Misspecification - Gamma(4,1) n=1000',cex.main=1.5)
curve(dweibull(x,mrr.miss[1,1],mrr.miss[1,2]),from=0,to=15,add=T,col='darkred',lwd
    =2.5,lty=2)
curve(dweibull(x,mle.miss[1,1],mle.miss[1,2]),from=0,to=15,add=T,col='blue',lwd=2.5,
    lty=1)
curve(dweibull(x,mom.miss[1,1],mom.miss[1,2]),from=0,to=15,add=T,col='orange',lwd
    =2.5,lty=2)
curve(dgamma(x,4,1),from=0,to=15,add=T,col='black',lwd=2.5,lty=3)
```

```
62 legend(7,.2,c("MLE","MME","MRR","Gamma"),col=c('blue','orange','darkred','black'),
       lwd=c(2.5,2.5,2.5,2.5), lty=c(1,2,2,3),bty='n',cex=1.5)
63 #dev.off()
64 dev.off()
65 parsort=sort(miss)
66 source('sourcecode.r')
67 MRRest=medianrank(parsort)
68 thta=MRRest[[4]]
69
70 # plot how this is estimated
71 #pdf("../Graphs/MRRgamma.pdf")
72 #plot(log(parsort),MRRest[[3]], main='Median Rank Regression',
73 #  xlab='log(t)', ylab='log(1/S(t))',pch=19,cex.main=2,cex.lab=1.5)
74 #abline(thta[1],thta[2],lty=1,lwd=2,col='blue')
75 #text(3,-1,expression(y == psi[0] + psi[1]*x),col='blue',cex=2)
76 #dev.off()
77
78 # what about for a truncated normal?
79 set.seed(100)
80 miss2=rtnorm(1000,5,1,lower=0)
81 sm.miss2=rtnorm(30,5,1,lower=0)
82
83
84 #/** Median Rank Regression Method **/
85 #//sort the data.
86 parsort2=sort(miss2)
87 sm.parsort2=sort(sm.miss2)
88 MRRest2=medianrank(parsort2)
89 sm.MRRest2=medianrank(sm.parsort2)
90 thta2=MRRest2[[4]]
91 sm.thta2=sm.MRRest2[[4]]
92 MRR2=MRRest2[[1]]
93 sm.MRR2=sm.MRRest2[[1]]
94
95 #/** Method of Moments Estimation **/
96 MOMest2=rep(1,2)
97 sm.MOMest2=rep(1,2)
98 #//define moments.
99 mu2=rep(0,2)
```

```
100 sm.mu2=rep(0,2)
101 #// mu1=E(x)
102 mu2[1]=sum(miss2)/length(miss2)
103 sm.mu2[1]=sum(sm.miss2)/length(sm.miss2)
104 #// mu2=E(x^2)
105 mu2[2]=sum(miss2^2)/length(miss2)
106 sm.mu2[2]=sum(sm.miss2^2)/length(sm.miss2)
107 #//Use root finding to estimate gamma
108 MOMest2[1]=bisection(.06, 10.2,mu2, tol = 1e-6)
109 sm.MOMest2[1]=bisection(.06,10.2,sm.mu2,tol=1e-6)
110 #//Solve for beta
111 #// beta=mu1/GAMMA(1+1/gamma)
112 MOMest2[2]=mu2[1] / gamma(1+1/MOMest2[1])
113 sm.MOMest2[2]=sm.mu2[1]/gamma(1+1/sm.MOMest2[1])
114
115 #/** Maximum Likelihood Estimation **/
116 MLEest2=nlm(weibull.ll,p=c(5,5), dat=miss2)$estimate
117 sm.MLEest2=nlm(weibull.ll,p=c(5,5), dat=sm.miss2)$estimate
118
119 pdf('../Graphs/miss_norm.pdf',width=14)
120 par(mfrow=c(1,2))
121 hist(sm.miss2, ylim=c(0,.6),xlim=c(2,10),freq=F, breaks=4, xlab='', main='
        Misspecification - Normal(5,1) n=30',cex.main=1.5)
122 curve(dnorm(x,5,1)/(1-pnorm(0,5,1)), from=0, to=10, add=T, col='black',lwd=2.5,lty
        =3)
123 curve(dweibull(x,sm.MLEest2[1],sm.MLEest2[2]), from=0, to=10, add=T, col='blue',lwd
        =2.5)
124 curve(dweibull(x,sm.MOMest2[1],sm.MOMest2[2]), from=0, to=10, add=T, col='orange',
        lwd=2.5)
125 curve(dweibull(x,sm.MRR2[1],sm.MRR2[2]), from=0, to=10, add=T, col='darkred',lwd
        =2.5,lty=2)
126 legend(7,.45,c("MLE","MME","MRR","Normal"),col=c('blue','orange','darkred','black'),
        lwd=2.5, lty=c(1,1,2,3), bty='n',cex=1.5)
127
128 hist(miss2, ylim=c(0,.45),xlim=c(2,10),freq=F, breaks=20, xlab='', main='
        Misspecification - Normal(5,1) n=1000',cex.main=1.5)
129 curve(dnorm(x,5,1)/(1-pnorm(0,5,1)), from=0, to=10, add=T, col='black',lwd=2.5,lty
        =3)
130 curve(dweibull(x,MLEest2[1],MLEest2[2]), from=0, to=10, add=T, col='blue',lwd=2.5)
```

```
curve(dweibull(x,MOMest2[1],MOMest2[2]), from=0, to=10, add=T, col='orange',lwd=2.5)
curve(dweibull(x,MRR2[1],MRR2[2]), from=0, to=10, add=T, col='darkred',lwd=2.5,lty
    =2)
legend(7,.35,c("MLE","MME","MRR","Normal"),col=c('blue','orange','darkred','black'),
    lwd=2.5, lty=c(1,1,2,3), bty='n',cex=1.5)
dev.off()


#KS tests
lg.norm=c(
ks.test(miss2,'pweibull',MLEest2[1],MLEest2[2])$p.value,
ks.test(miss2,'pweibull',MOMest2[1],MOMest2[2])$p.value,#MOM is best!
ks.test(miss2,'pweibull',MRR2[1],MRR2[2])$p.value,
ks.test(miss2,'ptnorm',5,1,0)$p.value)
sm.norm=c(
ks.test(sm.miss2,'pweibull',sm.MLEest2[1],sm.MLEest2[2])$p.value,#MLE is best!
ks.test(sm.miss2,'pweibull',sm.MOMest2[1],sm.MOMest2[2])$p.value,
ks.test(sm.miss2,'pweibull',sm.MRR2[1],sm.MRR2[2])$p.value,
ks.test(sm.miss2,'ptnorm',5,1,0)$p.value)
names(lg.norm)=names(sm.norm)=c('MLE','MME','MRR','Normal')
xtable(cbind(sm.norm,lg.norm),digits=4)

#another truncated normal
set.seed(1234)
miss3=rtnorm(10000,2,3,lower=0)
sm.miss3=rtnorm(30,2,3,lower=0)

#/** Median Rank Regression Method **/
#//sort the data.
parsort3=sort(miss3)
sm.parsort3=sort(sm.miss3)
MRRest3=medianrank(parsort3)
sm.MRRest3=medianrank(sm.parsort3)
thta3=MRRest3[[4]]
sm.thta3=sm.MRRest3[[4]]
MRR3=MRRest3[[1]]
sm.MRR3=sm.MRRest3[[1]]

#/** Method of Moments Estimation **/
```

```
168 MOMest3=rep(1,2)
169 sm.MOMest3=rep(1,2)
170 #//define moments.
171 mu3=rep(0,2)
172 sm.mu3=rep(0,2)
173 #// mu1=E(x)
174 mu3[1]=sum(miss3)/length(miss3)
175 sm.mu3[1]=mean(sm.miss3)
176 #// mu2=E(x^2)
177 mu3[2]=sum(miss3^2)/length(miss3)
178 sm.mu3[2]=sum(sm.miss3^2)/length(sm.miss3)
179 #//Use root finding to estimate gamma
180 MOMest3[1]=bisection(.06, 10.2,mu3, tol = 1e-6)
181 sm.MOMest3[1]=bisection(.06,10.2,sm.mu3,tol=1e-6)
182 #//Solve for beta
183 #// beta=mu1/GAMMA(1+1/gamma)
184 MOMest3[2]=mu3[1] / gamma(1+1/MOMest3[1])
185 sm.MOMest3[2]=sm.mu3[1]/gamma(1+1/sm.MOMest3[1])
186
187 #/** Maximum Likelihood Estimation **/
188 MLEest3=nlm(weibull.ll,p=c(5,5), dat=miss3)$estimate
189 sm.MLEest3=nlm(weibull.ll,p=c(5,5),dat=sm.miss3)$estimate
190
191
192 pdf('../Graphs/miss_tnorm.pdf',width=14)
193 par(mfrow=c(1,2))
194 hist(sm.miss3, ylim=c(0,.3),xlim=c(0,12),freq=F,breaks=10, xlab='', main='
        Misspecification - Zero Truncated Normal(2,3) n=30',cex.main=1.5)
195 curve(dnorm(x,2,3)/(1-pnorm(0,2,3)), from=0, to=12, add=T, col='black',lwd=2.5,lty
        =3)
196 curve(dweibull(x,sm.MLEest3[1],sm.MLEest3[2]), from=0, to=12, add=T, col='blue',lwd
        =2.5,lty=2)
197 curve(dweibull(x,sm.MOMest3[1],sm.MOMest3[2]), from=0, to=12, add=T, col='orange',
        lwd=2.5)
198 curve(dweibull(x,sm.MRR3[1],sm.MRR3[2]), from=0, to=12, add=T, col='darkred',lwd
        =2.5,lty=1)
199 legend(5,.25,c("MLE","MME","MRR","Truncated Normal"),col=c('blue','orange','darkred
        ','black'),lwd=2.5, lty=c(2,1,1,3), bty='n',cex=1.5)
200
```

```
hist(miss3, ylim=c(0,.3),xlim=c(0,12),freq=F,breaks=20, xlab='', main='
    Misspecification - Zero Truncated Normal(2,3) n=1000',cex.main=1.5)
curve(dnorm(x,2,3)/(1-pnorm(0,2,3)), from=0, to=12, add=T, col='black',lwd=2.5,lty
    =3)
curve(dweibull(x,MLEest3[1],MLEest3[2]), from=0, to=12, add=T, col='blue',lwd=2.5,
    lty=2)
curve(dweibull(x,MOMest3[1],MOMest3[2]), from=0, to=12, add=T, col='orange',lwd=2.5)
curve(dweibull(x,MRR3[1],MRR3[2]), from=0, to=12, add=T, col='darkred',lwd=2.5,lty
    =1)
legend(4,.25,c("MLE","MME","MRR","Truncated Normal"),col=c('blue','orange','darkred
    ','black'),lwd=2.5, lty=c(2,1,1,3), bty='n',cex=1.5)
dev.off()


#Kolmogorov-Smirnov Test
lg.trnorm=c(
ks.test(miss3,'pweibull',MLEest3[1],MLEest3[2])$p.value,
ks.test(miss3,'pweibull',MOMest3[1],MOMest3[2])$p.value,#MOM is best!
ks.test(miss3,'pweibull',MRR3[1],MRR3[2])$p.value,
ks.test(miss3,'ptnorm',2,3,0)$p.value)

sm.trnorm=c(
ks.test(sm.miss3,'pweibull',sm.MLEest3[1],sm.MLEest3[2])$p.value,
ks.test(sm.miss3,'pweibull',sm.MOMest3[1],sm.MOMest3[2])$p.value,#MOM is best!
ks.test(sm.miss3,'pweibull',sm.MRR3[1],sm.MRR3[2])$p.value,
ks.test(sm.miss3,'ptnorm',2,3,0)$p.value)
names(lg.trnorm)=names(sm.trnorm)=c('MLE','MME','MRR','TruncNorm')
xtable(cbind(sm.trnorm,lg.trnorm),digits=-4)


xtable(rbind(sm.gamma,lg.gamma,sm.norm,lg.norm,sm.trnorm,lg.trnorm))
```

C.2 Code to compute MSE

```
############
# Mark Nielsen
# Masters Project
library(spuRs)
library(xtable)

```

```
7  #setwd("C:/Users/Owner/Documents/My Dropbox/Classes/STAT 624/")
8
9
10 #comparison of pdfs
11 x=seq(from=0,to=2.5,length.out=100)
12 pdf('../Graphs/Weibull_PDF.pdf')
13 plot(x,dweibull(x,.5,1),type='l',col='blue',lwd=2.5,ylab='Density',main=expression(
       paste(gamma,' - Shape parameter'),sep=''),xlab='',cex.main=2)
14 points(x,dweibull(x,1,1),type='l',col='red',lwd=2.5)
15 points(x,dweibull(x,1.5,1),type='l',col='black',lwd=2.5)
16 points(x,dweibull(x,5,1),type='l',col='forestgreen',lwd=2.5)
17 legend(1.25,2.5,c(expression(paste(beta == 1,',', gamma == 0.5)),expression(paste(
       beta == 1,',', gamma == 1)),expression(paste(beta == 1,',', gamma == 1.5)),
       expression(paste(beta == 1,',', gamma == 5))),col=c('blue','red','black','
       forestgreen'),lty=1,lwd=2.5,bty='n',cex=1.5)
18 dev.off()
19
20 #comparison of pdfs
21 x=seq(from=0,to=2.5,length.out=100)
22 pdf('../Graphs/Weibull2_PDF.pdf')
23 plot(x,dweibull(x,2,.5),type='l',col='blue',lwd=2.5,ylab='Density',main=expression(
       paste(beta,' - Scale parameter'),sep=''),xlab='',cex.main=2)
24 points(x,dweibull(x,2,1),type='l',col='red',lwd=2.5)
25 points(x,dweibull(x,2,1.5),type='l',col='black',lwd=2.5)
26 points(x,dweibull(x,2,5),type='l',col='forestgreen',lwd=2.5)
27 legend(1.25,1.5,c(expression(paste(beta == 0.5,',', gamma == 2)),expression(paste(
       beta == 1,',', gamma == 2)),expression(paste(beta == 1.5,',', gamma == 2)),
       expression(paste(beta == 5,',', gamma == 2))),col=c('blue','red','black','
       forestgreen'),lty=1,lwd=2.5,bty='n',cex=1.5)
28 dev.off()
29
30 #pdf of our parameter estimates.
31 pdf('../Graphs/params.pdf',width=14)
32 par(mfrow=c(1,2))
33 x=seq(from=0,to=10,length.out=100)
34 plot(x,dweibull(x,2,2.5),type='l',col='blue',lwd=2.5,ylab='Density',main=expression(
       paste(beta == 2,' and ', gamma == 2.5)),xlab='',cex.main=2)
35 x=seq(from=0,to=150,length.out=100)
```

```
36 plot(x,dweibull(x,5,90),type='l',col='red',lwd=2.5,ylab='Density',main=expression(
       paste(beta == 5,' and ', gamma == 90)),xlab='',cex.main=2)
37 dev.off()
38
39
40 ################
41 # MSE for each Method
42 #par(2,5)
43 mle.ty=read.csv("../simdata/tinyA/mle_est1_tiny.csv",header=F)
44 mle.sm=read.csv("../simdata/smallA/mle_est1_small.csv",header=F)
45 mle.lg=read.csv("../simdata/bigA/mle_est1_big.csv",header=F)
46
47 mom.ty=read.csv("../simdata/tinyA/mom_est1_tiny.csv",header=F)
48 mom.sm=read.csv("../simdata/smallA/mom_est1_small.csv",header=F)
49 mom.lg=read.csv("../simdata/bigA/mom_est1_big.csv",header=F)
50
51 mrr.ty=read.csv("../simdata/tinyA/mrr_est1_tiny.csv",header=F)
52 mrr.sm=read.csv("../simdata/smallA/mrr_est1_small.csv",header=F)
53 mrr.lg=read.csv("../simdata/bigA/mrr_est1_big.csv",header=F)
54
55 avg.ty=rbind(mean(mle.ty),mean(mom.ty),mean(mrr.ty))
56 avg.sm=rbind(mean(mle.sm),mean(mom.sm),mean(mrr.sm))
57 avg.lg=rbind(mean(mle.lg),mean(mom.lg),mean(mrr.lg))
58
59 var.ty=rbind(diag(var(mle.ty)),diag(var(mom.ty)),diag(var(mrr.ty)))
60 var.sm=rbind(diag(var(mle.sm)),diag(var(mom.sm)),diag(var(mrr.sm)))
61 var.lg=rbind(diag(var(mle.lg)),diag(var(mom.lg)),diag(var(mrr.lg)))
62
63
64 #par (1,3)
65 mleb.ty=read.csv("../simdata/tinyB/mle_est1_tiny.csv",header=F)
66 mleb.sm=read.csv("../simdata/smallB/mle_est1_small.csv",header=F)
67 mleb.lg=read.csv("../simdata/bigB/mle_est1_big.csv",header=F)
68
69 momb.ty=read.csv("../simdata/tinyB/mom_est1_tiny.csv",header=F)
70 momb.sm=read.csv("../simdata/smallB/mom_est1_small.csv",header=F)
71 momb.lg=read.csv("../simdata/bigB/mom_est1_big.csv",header=F)
72
73 mrrb.ty=read.csv("../simdata/tinyB/mrr_est1_tiny.csv",header=F)
```

```
74  mrrb.sm=read.csv("../simdata/smallB/mrr_est1_small.csv",header=F)
75  mrrb.lg=read.csv("../simdata/bigB/mrr_est1_big.csv",header=F)
76
77  avgb.ty=rbind(mean(mleb.ty),mean(momb.ty),mean(mrrb.ty))
78  avgb.sm=rbind(mean(mleb.sm),mean(momb.sm),mean(mrrb.sm))
79  avgb.lg=rbind(mean(mleb.lg),mean(momb.lg),mean(mrrb.lg))
80
81  varb.ty=rbind(diag(var(mleb.ty)),diag(var(momb.ty)),diag(var(mrrb.ty)))
82  varb.sm=rbind(diag(var(mleb.sm)),diag(var(momb.sm)),diag(var(mrrb.sm)))
83  varb.lg=rbind(diag(var(mleb.lg)),diag(var(momb.lg)),diag(var(mrrb.lg)))
84
85  bias=cbind((avg.ty[,1]-5),
86             (avg.sm[,1]-5),
87             (avg.lg[,1]-5),
88             (avg.ty[,2]-90),
89             (avg.sm[,2]-90),
90             (avg.lg[,2]-90),
91             (avgb.ty[,1]-2),
92             (avgb.sm[,1]-2),
93             (avgb.lg[,1]-2),
94             (avgb.ty[,2]-2.5),
95             (avgb.sm[,2]-2.5),
96             (avgb.lg[,2]-2.5))
97  colnames(bias)=c("ty.gamma5","sm.gamma5","lg.gamma5","ty.beta90","sm.beta90","lg.
        beta90","ty.gamma1","sm.gamma1","lg.gamma1","ty.beta3","sm.beta3","lg.beta3")
98  xtable(bias,digits=5)
99
100 var=cbind(var.ty[,1],
101            var.sm[,1],
102            var.lg[,1],
103            var.ty[,2],
104            var.sm[,2],
105            var.lg[,2],
106            varb.ty[,1],
107            varb.sm[,1],
108            varb.lg[,1],
109            varb.ty[,2],
110            varb.sm[,2],
111            varb.lg[,2])
```

```
112 colnames(var)=c("ty.gamma5","sm.gamma5","lg.gamma5","ty.beta90","sm.beta90","lg.
        beta90","ty.gamma1","sm.gamma1","lg.gamma1","ty.beta3","sm.beta3","lg.beta3")
113 xtable(var,digits=5)
114
115 mse=cbind((avg.ty[,1]-5)^2+var.ty[,1],
116             (avg.ty[,2]-90)^2+var.ty[,2],
117             (avg.sm[,1]-5)^2+var.sm[,1],
118             (avg.sm[,2]-90)^2+var.sm[,2],
119             (avg.lg[,1]-5)^2+var.lg[,1],
120             (avg.lg[,2]-90)^2+var.lg[,2],
121             (avgb.ty[,1]-2)^2+varb.ty[,1],
122             (avgb.ty[,2]-2.5)^2+varb.ty[,2],
123             (avgb.sm[,1]-2)^2+varb.sm[,1],
124             (avgb.sm[,2]-2.5)^2+varb.sm[,2],
125             (avgb.lg[,1]-2)^2+varb.lg[,1],
126             (avgb.lg[,2]-2.5)^2+varb.lg[,2])
127
128 mse2=cbind((avg.ty[,1]-5)^2+var.ty[,1],
129             (avg.sm[,1]-5)^2+var.sm[,1],
130             (avg.lg[,1]-5)^2+var.lg[,1],
131             (avg.ty[,2]-90)^2+var.ty[,2],
132             (avg.sm[,2]-90)^2+var.sm[,2],
133             (avg.lg[,2]-90)^2+var.lg[,2],
134             (avgb.ty[,1]-2)^2+varb.ty[,1],
135             (avgb.sm[,1]-2)^2+varb.sm[,1],
136             (avgb.lg[,1]-2)^2+varb.lg[,1],
137             (avgb.ty[,2]-2.5)^2+varb.ty[,2],
138             (avgb.sm[,2]-2.5)^2+varb.sm[,2],
139             (avgb.lg[,2]-2.5)^2+varb.lg[,2])
140 colnames(mse2)=c("ty.gamma5","sm.gamma5","lg.gamma5","ty.beta90","sm.beta90","lg.
        beta90","ty.gamma1","sm.gamma1","lg.gamma1","ty.beta3","sm.beta3","lg.beta3")
141 xtable(mse2,digits=5)
142
143 #GRAPH FOR 10, 100, 1000, 10000, 100000
144 mle10A=read.csv("../simdata/A/mle_10.csv",header=F)
145 mle1000A=read.csv("../simdata/A/mle_1000.csv",header=F)
146 mle100000A=read.csv("../simdata/A/mle_100000.csv",header=F)
147
148 mom10A=read.csv("../simdata/A/mom_10.csv",header=F)
```

```
149 mom1000A=read.csv("../simdata/A/mom_1000.csv",header=F)
150 mom100000A=read.csv("../simdata/A/mom_100000.csv",header=F)
151
152 mrr10A=read.csv("../simdata/A/mrr_10.csv",header=F)
153 mrr1000A=read.csv("../simdata/A/mrr_1000.csv",header=F)
154 mrr100000A=read.csv("../simdata/A/mrr_100000.csv",header=F)
155
156 msefunc=function(dat,param){
157   (mean(dat)-param)^2+var(dat)
158 }
159
160 mse.mle=rbind(c(msefunc(mle10A[,1], 5),mse[1,1],mse[1,3],msefunc(mle1000A[,1], 5),
      mse[1,5]),
161               c(msefunc(mle10A[,2],90),mse[1,2],mse[1,4],msefunc(mle1000A[,2],90),
                    mse[1,6]))
162
163 mse.mom=rbind(c(msefunc(mom10A[,1], 5),mse[2,1],mse[2,3],msefunc(mom1000A[,1], 5),
      mse[2,5]),
164               c(msefunc(mom10A[,2],90),mse[2,2],mse[2,4],msefunc(mom1000A[,2],90),
                    mse[2,6]))
165
166 mse.mrr=rbind(c(msefunc(mrr10A[,1], 5),mse[3,1],mse[3,3],msefunc(mrr1000A[,1], 5),
      mse[3,5]),
167               c(msefunc(mrr10A[,2],90),mse[3,2],mse[3,4],msefunc(mrr1000A[,2],90),
                    mse[3,6]))
168
169 pdf('../Graphs/mseA.pdf',width=14)
170 par(mfrow=c(1,2))
171 plot(c(10,20,100,1000,10000),mse.mom[1,],xaxt='n',type='b',log="x",xlab='Sample Size
      ',ylab='Mean Square Error',main=expression(gamma==5),col='orange',lwd=3,pch=20,
      lty=1,cex.main=2)
172 points(c(10,20,100,1000,10000),mse.mrr[1,],type='b',col='darkred',lwd=3,pch=20,lty
      =2)
173 points(c(10,20,100,1000,10000),mse.mle[1,],type='b',col='blue',lwd=3,pch=20,lty=3)
174 legend('topright',c('MLE','MME','MRR'), lty=c(3,1,2), pch=20,lwd=2,col=c('blue','
      orange','darkred'),cex=1.5)
175 axis(1,c(10,20,100,1000,10000))
176
```

```
177 plot(c(10,20,100,1000,10000),mse.mrr[2,],xaxt='n',type='b',log="x",xlab='Sample Size
        ',ylab='Mean Square Error',main=expression(beta==90),col='darkred',lwd=3,pch=20,
        lty=2,cex.main=2)
178 points(c(10,20,100,1000,10000),mse.mom[2,],type='b',col='orange',lwd=3,pch=20,lty=1)
179 points(c(10,20,100,1000,10000),mse.mle[2,],type='b',col='blue',lwd=3,pch=20,lty=3)
180 legend('topright',c('MLE','MME','MRR'), lty=c(3,1,2), pch=20,lwd=2,col=c('blue','
        orange','darkred'),cex=1.5)
181 axis(1,c(10,20,100,1000,10000))
182 dev.off()
183
184 #GRBPH FOR 10, 100, 1000, 10000, 100000
185 mle10B=read.csv("../simdata/B/mle_10.csv",header=F)
186 mle1000B=read.csv("../simdata/B/mle_1000.csv",header=F)
187 mle100000B=read.csv("../simdata/B/mle_100000.csv",header=F)
188
189 mom10B=read.csv("../simdata/B/mom_10.csv",header=F)
190 mom1000B=read.csv("../simdata/B/mom_1000.csv",header=F)
191 mom100000B=read.csv("../simdata/B/mom_100000.csv",header=F)
192
193 mrr10B=read.csv("../simdata/B/mrr_10.csv",header=F)
194 mrr1000B=read.csv("../simdata/B/mrr_1000.csv",header=F)
195 mrr100000B=read.csv("../simdata/B/mrr_100000.csv",header=F)
196
197 msefunc=function(dat,param){
198   (mean(dat)-param)^2+var(dat)
199 }
200
201 mse.mleb=rbind(c(msefunc(mle10B[,1],   2),mse[1,7],mse[1,  9],msefunc(mle1000B[,1],
        2),mse[1,11]),
202                c(msefunc(mle10B[,2],2.5),mse[1,8],mse[1,10],msefunc(mle1000B
                    [,2],2.5),mse[1,12]))
203
204 mse.momb=rbind(c(msefunc(mom10B[,1],   2),mse[2,7],mse[2,  9],msefunc(mom1000B[,1],
        2),mse[2,11]),
205                c(msefunc(mom10B[,2],2.5),mse[2,8],mse[2,10],msefunc(mom1000B
                    [,2],2.5),mse[2,12]))
206
207 mse.mrrb=rbind(c(msefunc(mrr10B[,1],   2),mse[3,7],mse[3,  9],msefunc(mrr1000B[,1],
        2),mse[3,11]),
```

```
208                     c(msefunc(mrr10B[,2],2.5),mse[3,8],mse[3,10],msefunc(mrr1000B
                            [,2],2.5),mse[3,12]))
209
210 pdf('../Graphs/mseB.pdf',width=14)
211 par(mfrow=c(1,2))
212 plot(c(10,20,100,1000,10000),mse.momb[1,],ylim=c(0,.65),xaxt='n',type='b',log="x",
        xlab='Sample Size',ylab='Mean Square Error',main=expression(gamma==2),col='
        orange',lwd=3,pch=20,lty=1,cex.main=2)
213 points(c(10,20,100,1000,10000),mse.mrrb[1,],type='b',col='darkred',lwd=3,pch=20,lty
        =2)
214 points(c(10,20,100,1000,10000),mse.mleb[1,],type='b',col='blue',lwd=3,pch=20,lty=3)
215 legend('topright',c('MLE','MME','MRR'), lty=c(3,1,2), pch=20,lwd=2,col=c('blue','
        orange','darkred'),cex=1.5)
216 axis(1,c(10,20,100,1000,10000))
217
218 plot(c(10,20,100,1000,10000),mse.mrrb[2,],xaxt='n',type='b',log="x",xlab='Sample
        Size',ylab='Mean Square Error',main=expression(beta==2.5),col='darkred',lwd=3,
        pch=20,lty=2,cex.main=2)
219 points(c(10,20,100,1000,10000),mse.momb[2,],type='b',col='orange',lwd=3,pch=20,lty
        =1)
220 points(c(10,20,100,1000,10000),mse.mleb[2,],type='b',col='blue',lwd=3,pch=20,lty=3)
221 legend('topright',c('MLE','MME','MRR'), lty=c(3,1,2), pch=20,lwd=2,col=c('blue','
        orange','darkred'),cex=1.5)
222 axis(1,c(10,20,100,1000,10000))
223 dev.off()
224
225
226
227
228 #MLE
229 pdf("../Graphs/norm_mle_big.pdf",width=14)
230 par(mfrow=c(1,2))
231 hist(mle.lg[,2],freq=F,main='',xlab=expression(beta),breaks=20)
232 curve(dnorm(x,avg.lg[1,2],sqrt(var.lg[1,2])),from=88,to=91,add=T,col='red',lwd=2)
233 hist(mle.lg[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
234 curve(dnorm(x,avg.lg[1,1],sqrt(var.lg[1,1])),from=4.85,to=5.15,add=T,col='red',lwd
        =2)
235 dev.off()
236
```

```
237 pdf("../Graphs/norm_mleb_big.pdf",width=14)
238 par(mfrow=c(1,2))
239 hist(mleb.lg[,2],freq=F,main='',xlab=expression(beta),breaks=20)
240 curve(dnorm(x,avgb.lg[1,2],sqrt(varb.lg[1,2])),from=2.45,to=2.55,add=T,col='red',lwd
        =2)
241 hist(mleb.lg[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
242 curve(dnorm(x,avgb.lg[1,1],sqrt(varb.lg[1,1])),from=1.95,to=2.05,add=T,col='red',lwd
        =2)
243 dev.off()
244
245 pdf("../Graphs/norm_mleb_small.pdf",width=14)
246 par(mfrow=c(1,2))
247 hist(mleb.sm[,2],freq=F,main='',xlab=expression(beta),breaks=20)
248 curve(dnorm(x,avgb.sm[1,2],sqrt(varb.sm[1,2])),from=2,to=4.1,add=T,col='red',lwd=2)
249 hist(mleb.sm[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
250 curve(dnorm(x,avgb.sm[1,1],sqrt(varb.sm[1,1])),from=1,to=3,add=T,col='red',lwd=2)
251 dev.off()
252
253 pdf("../Graphs/norm_mle_small.pdf",width=14)
254 par(mfrow=c(1,2))
255 hist(mle.sm[,2],freq=F,main='',xlab=expression(beta),breaks=20)
256 curve(dnorm(x,avg.sm[1,2],sqrt(var.sm[1,2])),from=84,to=96,add=T,col='red',lwd=2)
257 hist(mle.sm[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
258 curve(dnorm(x,avg.sm[1,1],sqrt(var.sm[1,1])),from=3.5,to=7,add=T,col='red',lwd=2)
259 dev.off()
260
261 pdf("../Graphs/norm_mleb_tiny.pdf",width=14)
262 par(mfrow=c(1,2))
263 hist(mleb.ty[,2],freq=F,main='',xlab=expression(beta),breaks=20)
264 curve(dnorm(x,avgb.ty[1,2],sqrt(varb.ty[1,2])),from=1,to=6,add=T,col='red',lwd=2)
265 hist(mleb.ty[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
266 curve(dnorm(x,avgb.ty[1,1],sqrt(varb.ty[1,1])),from=.5,to=4.5,add=T,col='red',lwd=2)
267 dev.off()
268
269 pdf("../Graphs/norm_mle_tiny.pdf",width=14)
270 par(mfrow=c(1,2))
271 hist(mle.ty[,2],freq=F,main='',ylim=c(0,.1),xlab=expression(beta),breaks=20)
272 curve(dnorm(x,avg.ty[1,2],sqrt(var.ty[1,2])),from=74,to=105,add=T,col='red',lwd=2)
273 hist(mle.ty[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
```

```
curve(dnorm(x,avg.ty[1,1],sqrt(var.ty[1,1])),from=2,to=12,add=T,col='red',lwd=2)
dev.off()

#MOM
pdf("../Graphs/norm_mom_big.pdf",width=14)
par(mfrow=c(1,2))
hist(mom.lg[,2],freq=F,main='',xlab=expression(beta),breaks=20)
curve(dnorm(x,avg.lg[2,2],sqrt(var.lg[2,2])),from=88,to=91,add=T,col='red',lwd=2)
hist(mom.lg[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
curve(dnorm(x,avg.lg[2,1],sqrt(var.lg[2,1])),from=4.85,to=5.15,add=T,col='red',lwd
    =2)
dev.off()

pdf("../Graphs/norm_momb_big.pdf",width=14)
par(mfrow=c(1,2))
hist(momb.lg[,2],freq=F,main='',xlab=expression(beta),breaks=20)
curve(dnorm(x,avgb.lg[2,2],sqrt(varb.lg[2,2])),from=2.45,to=2.55,add=T,col='red',lwd
    =2)
hist(momb.lg[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
curve(dnorm(x,avgb.lg[2,1],sqrt(varb.lg[2,1])),from=1.95,to=2.05,add=T,col='red',lwd
    =2)
dev.off()

pdf("../Graphs/norm_momb_small.pdf",width=14)
par(mfrow=c(1,2))
hist(momb.sm[,2],freq=F,main='',ylim=c(0,3.25),xlab=expression(beta),breaks=20)
curve(dnorm(x,avgb.sm[2,2],sqrt(varb.sm[2,2])),from=2,to=3,add=T,col='red',lwd=2)
hist(momb.sm[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
curve(dnorm(x,avgb.sm[2,1],sqrt(varb.sm[2,1])),from=1.5,to=2.6,add=T,col='red',lwd
    =2)
dev.off()

pdf("../Graphs/norm_mom_small.pdf",width=14)
par(mfrow=c(1,2))
hist(mom.sm[,2],freq=F,main='',xlab=expression(beta),breaks=20)
curve(dnorm(x,avg.sm[2,2],sqrt(var.sm[2,2])),from=84,to=96,add=T,col='red',lwd=2)
hist(mom.sm[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
curve(dnorm(x,avg.sm[2,1],sqrt(var.sm[2,1])),from=3.9,to=6.5,add=T,col='red',lwd=2)
dev.off()
```

```
309
310 pdf("../Graphs/norm_momb_tiny.pdf",width=14)
311 par(mfrow=c(1,2))
312 hist(momb.ty[,2],freq=F,main='',xlab=expression(beta),breaks=20)
313 curve(dnorm(x,avgb.ty[2,2],sqrt(varb.ty[2,2])),from=0,to=6,add=T,col='red',lwd=2)
314 hist(momb.ty[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
315 curve(dnorm(x,avgb.ty[2,1],sqrt(varb.ty[2,1])),from=1,to=4.5,add=T,col='red',lwd=2)
316 dev.off()
317
318 pdf("../Graphs/norm_mom_tiny.pdf",width=14)
319 par(mfrow=c(1,2))
320 hist(mom.ty[,2],freq=F,main='',xlab=expression(beta),breaks=20)
321 curve(dnorm(x,avg.ty[2,2],sqrt(var.ty[2,2])),from=75,to=105,add=T,col='red',lwd=2)
322 hist(mom.ty[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
323 curve(dnorm(x,avg.ty[2,1],sqrt(var.ty[2,1])),from=2,to=12,add=T,col='red',lwd=2)
324 dev.off()
325
326
327 #MRR
328 pdf("../Graphs/norm_mrr_big.pdf",width=14)
329 par(mfrow=c(1,2))
330 hist(mrr.lg[,2],freq=F,main='',xlab=expression(beta),breaks=20)
331 curve(dnorm(x,avg.lg[3,2],sqrt(var.lg[3,2])),from=89,to=91,add=T,col='red',lwd=2)
332 hist(mrr.lg[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
333 curve(dnorm(x,avg.lg[3,1],sqrt(var.lg[3,1])),from=4.8,to=5.2,add=T,col='red',lwd=2)
334 dev.off()
335
336 pdf("../Graphs/norm_mrrb_big.pdf",width=14)
337 par(mfrow=c(1,2))
338 hist(mrrb.lg[,2],freq=F,main='',xlab=expression(beta),breaks=20)
339 curve(dnorm(x,avgb.lg[3,2],sqrt(varb.lg[3,2])),from=2.45,to=2.55,add=T,col='red',lwd
      =2)
340 hist(mrrb.lg[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
341 curve(dnorm(x,avgb.lg[3,1],sqrt(varb.lg[3,1])),from=1.9,to=2.1,add=T,col='red',lwd
      =2)
342 dev.off()
343
344 pdf("../Graphs/norm_mrrb_small.pdf",width=14)
345 par(mfrow=c(1,2))
```

```
hist(mrrb.sm[,2],freq=F,main='',xlab=expression(beta),breaks=20)
curve(dnorm(x,avgb.sm[3,2],sqrt(varb.sm[3,2])),from=2,to=3.5,add=T,col='red',lwd=2)
hist(mrrb.sm[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
curve(dnorm(x,avgb.sm[3,1],sqrt(varb.sm[3,1])),from=1.3,to=2.7,add=T,col='red',lwd
    =2)
dev.off()

pdf("../Graphs/norm_mrr_small.pdf",width=14)
par(mfrow=c(1,2))
hist(mrr.sm[,2],freq=F,main='',xlab=expression(beta),breaks=20)
curve(dnorm(x,avg.sm[3,2],sqrt(var.sm[3,2])),from=84,to=97,add=T,col='red',lwd=2)
hist(mrr.sm[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
curve(dnorm(x,avg.sm[3,1],sqrt(var.sm[3,1])),from=3,to=6.5,add=T,col='red',lwd=2)
dev.off()

pdf("../Graphs/norm_mrrb_tiny.pdf",width=14)
par(mfrow=c(1,2))
hist(mrrb.ty[,2],freq=F,main='',xlab=expression(beta),breaks=20)
curve(dnorm(x,avgb.ty[3,2],sqrt(varb.ty[3,2])),from=1,to=4.5,add=T,col='red',lwd=2)
hist(mrrb.ty[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
curve(dnorm(x,avgb.ty[3,1],sqrt(varb.ty[3,1])),from=.25,to=4,add=T,col='red',lwd=2)
dev.off()

pdf("../Graphs/norm_mrr_tiny.pdf",width=14)
par(mfrow=c(1,2))
hist(mrr.ty[,2],freq=F,main='',xlab=expression(beta),breaks=20)
curve(dnorm(x,avg.ty[3,2],sqrt(var.ty[3,2])),from=73,to=105,add=T,col='red',lwd=2)
hist(mrr.ty[,1],freq=F,main='',xlab=expression(gamma),breaks=20)
curve(dnorm(x,avg.ty[3,1],sqrt(var.ty[3,1])),from=1.5,to=10,add=T,col='red',lwd=2)
dev.off()
```

C.3 Code to read in wind data

```
##############
# Mark Nielsen
# Masters project
# creating data sets with only variables of interest.


#setwd('/home/owner/Desktop/Dropbox/Classes/MASTERS PROJECT')
```

```
8
9  ab09=read.csv('../winddata/ab_2009.csv',header=T)
10 ab10=read.csv('../winddata/ab_2010.csv',header=T)
11 ca09=read.csv('../winddata/ca_2009.csv',header=T)
12 ca10=read.csv('../winddata/ca_2010.csv',header=T)
13 gc09=read.csv('../winddata/gc_2009.csv',header=T)
14 gc10=read.csv('../winddata/gc_2010.csv',header=T)
15 sb09=read.csv('../winddata/sb_2009.csv',header=T)
16 sb10=read.csv('../winddata/sb_2010.csv',header=T)
17 rtmet09=read.csv('../winddata/rtmet_2009.csv',header=T)
18 rtmet10=read.csv('../winddata/rtmet_2010.csv',header=T)
19
20 ab=rbind(ab09[,c(1,80)],ab10[,c(1,80)])
21 ab$WindS_ms_Ave[ab[,2]==-8999]=NA
22 hist(ab[,2],freq=F,breaks=40)
23 ab.nm=ab[!is.na(ab[,2]),]
24 plot(density(ab.nm[,2]))
25
26 ca=rbind(ca09[,c(1,80)],ca10[,c(1,80)])
27 ca$WindS_ms_Ave[ca[,2]==-8999]=NA
28 hist(ca[,2],freq=F,breaks=40)
29 ca.nm=ca[!is.na(ca[,2]),]
30 plot(density(ca.nm[,2]))
31
32 gc=rbind(gc09[,c(1,80)],gc10[,c(1,80)])
33 gc$WindS_ms_Ave[gc[,2]==-8999]=NA
34 hist(gc[,2],freq=F,breaks=40)
35 gc.nm=gc[!is.na(gc[,2]),]
36 plot(density(gc.nm[,2]))
37
38 sb=rbind(sb09[,c(1,80)],sb10[,c(1,80)])
39 sb$WindS_ms_Ave[sb[,2]==-8999]=NA
40 hist(sb[,2],freq=F,breaks=40)
41 sb.nm=sb[!is.na(sb[,2]),]
42 plot(density(sb.nm[,2]))
43 curve(dweibull(x,1.4,2.2),from=0, to=14, add=T, col=2)
44
45 rtmet=rbind(rtmet09[,c(1,13)],rtmet10[,c(1,13)])
46 rtmet$WindS_ms_Ave[rtmet[,2]==-9999]=NA
```

```
47 hist(rtmet[,2],freq=F,breaks=40)
48 rtmet.nm=rtmet[!is.na(rtmet[,2]),]
49 plot(density(rtmet.nm[,2]))
50
51 dim(ab)[1]
52 #28285
53 dim(ca)[1]
54 #28284
55 dim(gc)[1]
56 #28283
57 dim(sb)[1]
58 #28283
59 dim(rtmet)[1]
60 #9158
61
62 write.csv(ab,'../winddata/ab_wind.csv',row.names=F)
63 write.csv(ca,'../winddata/ca_wind.csv',row.names=F)
64 write.csv(gc,'../winddata/gc_wind.csv',row.names=F)
65 write.csv(sb,'../winddata/sb_wind.csv',row.names=F)
66 write.csv(rtmet,'../winddata/rtmet_wind.csv',row.names=F)
```

C.4 Code for parameter estimation

```
1  library(xtable)
2
3  gc=read.csv('../winddata/gc_wind.csv',header=T)
4  par=gc[(!is.na(gc[,2])),2]
5  par[par==0]=.000001
6  par2=gc[(!is.na(gc[,2]) & gc[,2]!=0),2]
7  par3=gc[(!is.na(gc[,2])),2]
8  par3[par3==0]=.0001
9  par4=gc[(!is.na(gc[,2])),2]
10 par4[par4==0]=.01
11
12 #change the data to bin format (i.e. frequencies of wspeed from 0-.25, .25-.5, etc.)
13 frq=ceiling(4*gc[!is.na(gc[,2]),2])/4
14 frq[frq==0]=.25
15
16 n1=length(par)
17 n2=length(par2)
```

```
18 ratio=n2/n1
19
20 source('sourcecode.r')
21
22 #/** Maximum Likelihood Estimation **/
23 MLEest=nlm(weibull.ll,p=c(1,3), dat=par)$estimate
24 MLEest2=nlm(weibull.ll,p=c(1,3), dat=par2)$estimate
25 MLEest3=nlm(weibull.ll,p=c(1,3), dat=par3)$estimate
26 MLEest4=nlm(weibull.ll,p=c(1,3), dat=par4)$estimate
27 # here are the frequecy formatted data...
28 MLEest5=nlm(weibull.ll,p=c(1,3), dat=frq)$estimate
29
30 #/** Maximum Likelihood Estimation with mixture distribution **/
31 par1=gc[(!is.na(gc[,2])),2]
32 MLEmix=nlm(mix.ll,p=c(3,3,.9),dat=par1)$estimate
33
34 #plot the curves and see which one fits best...
35 pdf('../Graphs/gc_MLE.pdf')
36 hist(gc[,2],freq=F,breaks=40, main='Maximum Likelihood', xlab='Wind Speed',cex.lab
       =1.5,cex.main=2)
37 curve(dweibull(x,MLEest[1],MLEest[2]),from=0,to=10,add=T,col='blue',lwd=2.5)
38 curve(dweibull(x,MLEest3[1],MLEest3[2]),from=0,to=10,add=T,col='orange',lwd=2.5)
39 curve(dweibull(x,MLEest4[1],MLEest4[2]),from=0,to=10,add=T,col='green',lwd=2.5)
40 curve(dweibull(x,MLEest5[1],MLEest5[2]),from=0,to=10,add=T,col='black',lwd=2.5)
41 curve(ifelse(x==0,1-MLEmix[3],0)+MLEmix[3]*dweibull(x,MLEmix[1],MLEmix[2]),from=0,to
       =10,add=T,col='red',lwd=2.5)
42 legend(5.5,.2,c("Mixture Model","0.000001","0.0001","0.01","Bins"),col=c('red','blue
       ','orange','green','black'),lty=c(1,1,1,1,1),lwd=2.5,cex=1.5, bty='n')
43 dev.off()
44
45 tblMLE=rbind(MLEmix[1:2],MLEest2,MLEest5,MLEest,MLEest3,MLEest4)
46 colnames(tblMLE)=c("gamma","beta")
47 rownames(tblMLE)=c("MLEmix","MLEdel","MLEbin","MLE000001","MLE0001","MLE01")
48 xtable(tblMLE,digits=5)
49
50
51 #/** Median Rank Regression Method **/
52
53 #//sort the data.
```

```
54 parsort=sort(par)
55 parsort2=sort(par2)
56 parsort3=sort(par3)
57 parsort4=sort(par4)
58 parsort5=sort(frq)
59
60 MRRest=medianrank(parsort)
61 MRR=MRRest[[1]]
62 MRR2=medianrank(parsort2)
63 grph=MRR2[[3]]
64 MRR2=MRR2[[1]]
65 thta=medianrank(parsort2)[[4]]
66 MRR3=medianrank(parsort3)[[1]]
67 MRR4=medianrank(parsort4)[[1]]
68 MRR5=medianrank(parsort5)[[1]]
69
70 #plot the curves and see which one fits best...
71 pdf('../Graphs/gc_MRR.pdf')
72 hist(gc[,2],freq=F,breaks=40, main='Median Rank Regression', xlab='Wind Speed',cex.
       lab=1.5,cex.main=2)
73 curve(dweibull(x,MRR[1],MRR[2]),from=0,to=10,add=T,col='blue',lwd=2.5)
74 curve(dweibull(x,MRR3[1],MRR3[2]),from=0,to=10,add=T,col='orange',lwd=2.5)
75 curve(dweibull(x,MRR4[1],MRR4[2]),from=0,to=10,add=T,col='green',lwd=2.5)
76 curve(dweibull(x,MRR5[1],MRR5[2]),from=0,to=10,add=T,col='black',lwd=2.5)
77 curve(ifelse(x==0,1-ratio,0)+ratio*dweibull(x,MRR2[1],MRR2[2]),from=0,to=10,add=T,
       col='red',lwd=2.5)
78 legend(6,.2,c("Delete","0.000001","0.0001","0.01","Bins"),col=c('red','blue','orange
       ','green','black'),lty=c(1,1,1),lwd=2.5,cex=1.5, bty='n')
79 dev.off()
80
81 # plot how this is estimated
82 pdf("../Graphs/MRR_gcline.pdf")
83 plot(log(parsort2),grph, main='Median Rank Regression',
84   xlab='log(t)', ylab='log(1/S(t))',pch=19,cex.main=2,cex.lab=1.5)
85 abline(thta[1],thta[2],lty=1,lwd=2,col='blue')
86 text(1,-4,expression(y == psi[0] + psi[1]*x),col='blue',cex=2)
87 dev.off()
88
89 tblMRR=rbind(MRR2,MRR5,MRR,MRR3,MRR4)
```

```
90  colnames(tblMRR)=c("gamma","beta")
91  rownames(tblMRR)=c("MRRdel","MRRbin","MRR000001","MRR0001","MRR01")
92  xtable(tblMRR, digits=5)
93
94
95  #/** Method of Moments Estimation **/
96  MOMest=rep(1,2);
97  MOMest2=rep(1,2);
98  MOMest3=rep(1,2);
99  MOMest4=rep(1,2)
100 MOMest5=rep(1,2)
101
102 #//define moments.
103 mu=rep(0,2)
104 mu2=rep(0,2)
105 mu3=rep(0,2)
106 mu4=rep(0,2)
107 mu5=rep(0,2)
108
109 #// mu1=E(x)
110 mu[1]=sum(par)/length(par)
111 mu2[1]=sum(par2)/length(par2)
112 mu3[1]=sum(par3)/length(par3)
113 mu4[1]=sum(par4)/length(par4)
114 mu5[1]=sum(frq)/length(frq)
115
116 #// mu2=E(x^2)
117 mu[2]=sum(par^2)/length(par)
118 mu2[2]=sum(par2^2)/length(par2)
119 mu3[2]=sum(par3^2)/length(par3)
120 mu4[2]=sum(par4^2)/length(par4)
121 mu5[2]=sum(frq^2)/length(frq)
122
123 #//Use root finding to estimate gamma
124 MOMest[1]=bisection(.06, 10.2,mu, tol = 1e-6)
125 MOMest2[1]=bisection(.06, 10.2,mu2, tol = 1e-6)
126 MOMest3[1]=bisection(.06, 10.2,mu3, tol = 1e-6)
127 MOMest4[1]=bisection(.06, 10.2,mu4, tol = 1e-6)
128 MOMest5[1]=bisection(.06, 10.2,mu5, tol = 1e-6)
```

```
129
130 #//Solve for beta
131 #// beta=mu1/GAMMA(1+1/gamma)
132 MOMest[2]=mu[1] / gamma(1+1/MOMest[1])
133 MOMest2[2]=mu2[1] / gamma(1+1/MOMest2[1])
134 MOMest3[2]=mu3[1] / gamma(1+1/MOMest3[1])
135 MOMest4[2]=mu4[1] / gamma(1+1/MOMest4[1])
136 MOMest5[2]=mu5[1] / gamma(1+1/MOMest5[1])
137
138 tblMOM=rbind(MOMest2,MOMest5,MOMest,MOMest3,MOMest4)
139 colnames(tblMOM)=c("gamma","beta")
140 rownames(tblMOM)=c("MOMdel","MOMbin","MOM000001","MOM0001","MOM01")
141 xtable(tblMOM, digits=5)
142
143 #plot the curves and see which one fits best...
144 pdf('../Graphs/gc_MOM.pdf')
145 hist(gc[,2],freq=F,breaks=40, main='Method of Moments', xlab='Wind Speed',cex.lab
      =1.5,cex.main=2)
146 curve(dweibull(x,MOMest[1],MOMest[2]),from=0,to=10,add=T,col='blue',lwd=2.5)
147 curve(dweibull(x,MOMest3[1],MOMest3[2]),from=0,to=10,add=T,col='orange',lwd=2.5,lty
      =3)
148 curve(dweibull(x,MOMest4[1],MOMest4[2]),from=0,to=10, add=T,col='green',lwd=2.5, lty
      =4)
149 curve(dweibull(x,MOMest5[1],MOMest5[2]),from=0,to=10,add=T,col='black',lwd=2.5, lty
      =5)
150 curve(ifelse(x==0,1-ratio,0)+ratio*dweibull(x,MOMest2[1],MOMest2[2]),from=0,to=10,
      add=T,col='red',lwd=2.5,lty=2)
151 legend(6,.2,c("Delete","0.000001","0.0001","0.01","Bins"),col=c('red','blue','orange
      ','green','black'),lty=c(1,2,3,4,5),lwd=2.5,cex=1.5, bty='n')
152 dev.off()
153
154 tbl=rbind(MLEmix,c(MOMest2,ratio),c(MRR2,ratio))
155 colnames(tbl)=c("gamma","beta","pi")
156 rownames(tbl)=c("MLEmix","MOMdel","MRRdel")
157 xtable(tbl,digits=5)
```

C.5 Source code

```
1
2 ##### DEFINE FUNCTION TO BE MINIMIZED
```

```
3  # dat = data
4  # theta = vector of parameters to be identified
5  weibull.ll=function(theta,dat)
6  {
7    if(theta[1]<=0){
8      return(1e10)
9    }else if(theta[2]<=0){
10     return(1e10)
11   }else{
12     f = -length(dat)*log(theta[1]/theta[2]^theta[1]) - (theta[1]-1)*sum(log(dat)) +
           sum((dat/theta[2])^theta[1])
13     return(f)
14   }
15 }
16
17
18 ##### Define function to be minimized with mixture distn.
19 # dat = data
20 # theta = vector of parameters to be 'ID'ed
21 mix.ll=function(theta,dat)
22 {
23   n=length(dat)
24   m=length(dat[dat!=0])
25   if(theta[1]<=0){
26     return(1e10)
27   }else if(theta[2]<=0){
28     return(1e10)
29   }else if(theta[3]<0){
30     return(1e10)
31   }else if(theta[3]>1){
32     return(1e10)
33   }else{
34     f = (n-m)*log(1-theta[3])+m*log(theta[3])+m*log(theta[1]/theta[2]^theta[1]) + (
           theta[1]-1)*sum(log(dat[dat!=0])) - sum((dat[dat!=0]/theta[2])^theta[1])
35     return(-f)
36   }
37 }
38
39
```

```
#       /** Median Rank Regression Method **/
medianrank=function(dat){
  ynew=rep(0,length(dat))
  for(k in 1:length(dat)){
  #//compute y's
  #// log(log(1/(1-(i-.3)/(npop+.4))))
  ynew[k]=log(log(1/(1-(k-.3)/(length(dat)+.4))))
}
  #//compute x's
  #// log(x[i])
  xnew=log(dat)

  #//fit the model.
  #//compute xbar & ybar
  beta=coefficients(lm(ynew~xnew))
  #//compute parameters.
  #//gamma=c1
  thta=rep(0,2)
  thta[1]=beta[2]
  #//beta=exp(-c0/gamma)
  thta[2]=exp(-beta[1]/thta[1])
  return(list(thta,xnew,ynew,beta))
}



#//Use root finding to estimate gamma
bisection=function (x.l, x.r, mu, tol = 1e-09)
{
    if (x.l >= x.r) {
        cat("error: x.l >= x.r \n")
        return(NULL)
    }
    f.l <- gamma(1+2/x.l)/gamma(1+1/x.l)^2-mu[2]/mu[1]^2
    f.r <- gamma(1+2/x.r)/gamma(1+1/x.r)^2-mu[2]/mu[1]^2
    if (f.l * f.r > 0) {
        cat("error: ftn(x.l) * ftn(x.r) > 0 \n")
        return(NULL)
    }
```

```
79      n <- 0
80      while ((x.r - x.l) > tol) {
81          x.m <- (x.l + x.r)/2
82          f.m <- gamma(1.+2/x.m)/(gamma(1.+1/x.m))^2-mu[2]/mu[1]^2
83          if (f.l * f.m < 0) {
84              x.r <- x.m
85              f.r <- f.m
86          }
87          else {
88              x.l <- x.m
89              f.l <- f.m
90          }
91          n <- n + 1
92          cat("at iteration", n, "the root lies between", x.l,
93              "and", x.r, "\n")
94      }
95      return((x.l + x.r)/2)
96  }
```