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Abstract: In the context of global warming, the problem of stabilization of the concentration of carbon in the atmosphere is widely discussed. Serious difficulties in the design of reliable stabilization strategies arise due to the uncertainty of the underlying physical model. In this paper, we suggest a pattern to construct model-robust feedback carbon emission strategies that stabilize the atmospheric carbon concentration at a prescribe target value irrespective of a particular admissible carbon cycle model governing the "real" dynamics. Specific qualitative features of the carbon cycle dynamics, including automatic stabilization under vanishing inputs are employed.

Keywords: Global warming, stabilization of carbon in the atmosphere, stabilization of uncertain systems.

1 INTRODUCTION

The tolerable window approach aimed at preventing the occurrence of harmful impacts of global warming (see [WBGU, 1995], [Bruckner, et. al, 1999]) views a carbon emission scenario as acceptable if it keeps the average annual temperature within a certain "window". Implicitly, this imposes constraints on the amount of carbon in the atmosphere, which is – as often assumed – positively related to the annual surface/air temperature. However, the exact identification of those constraints (to be met con-

stantly during a long period of time) can be a difficult task, since there is yet no clear understanding of the mechanism establishing quantitative relations between the growth in the carbon concentration and the raise of the annual temperature. The identification of some "central" or "target" point within those (fuzzy) constraints can be a much easier task. In this manner, one arrives at the problem of stabilization of the carbon concentration around a chosen target value within an infinite time horizon.

In [Svirezhev, et. al., 1999] a stabilization scenario is sought using a simplified ODE model of the global carbon cycle. The model's state variables include the amounts of carbon in the atmosphere, $x(t)$, and in the ocean, $y(t)$; here t is the time variable. The state variables are scaled so that their

zero values correspond, respectively, to the absolute value of carbon in the atmosphere and the absolute value of carbon in the ocean in the pre-industrial period. Annual antropogenic emissions of CO_2, $\varphi(t)$, act as controls. The carbon cycle model has the form

$$
\begin{aligned}
\dot{x}(t) &= \varphi(t) - \alpha_1 x(t) + \alpha_2 y(t), \\
\dot{y}(t) &= \alpha_1 x(t) - \alpha_2 y(t)
\end{aligned}
\tag{1}
$$

where α_1 and α_2, are positive parameters. The initial state of the model represents the amounts of carbon in the atmosphere and in the ocean at time 0 corresponding to the year 2000. A sought stabilization emission scenario $\varphi(t)$ ensures

$$
\lim_{t \to \infty} x(t) = \hat{x}
\tag{2}
$$

where $\hat{x}$ is a prescribed target value for the amount of carbon in the atmsophere.

Our study relates to "post-planning" decisionmaking. Assuming that a stabilization scenario $\varphi(t)$ is identified, we address the question of a practical realization of (2). We emphasise the fact that the model (1) that serves as a basis for forming the stabilization scenario $\varphi(t)$ is inaccurate and most likely does not describe the real dynamics. It is clear that (2) is violated if we implement $\varphi(t)$ for even a sligtly perturbed model. In practice, the uncertainties in the model (reflecting highly complex processes in the environment) should be viewed as large enough. An adequate assumtion is that the "real" model is not known to us; instead, we are given a (relatively broad) class of "admissible" models, which includes the "real" one. This assumption implies that a desired emission policy should guarantee (2) for every admissible model chosen beforehand. The admissible models describing a variaty of admissible dynamics for $x(t)$ and $y(t)$ can certainly include nonlinear models much more complex than (1). In this study, we assume that the admissible models have the form

$$
\begin{aligned}
\dot{x}(t) &= \varphi(t) + u(t) + g(x(t), y(t)), \\
\dot{y}(t) &= -g(x(t), y(t))
\end{aligned}
\tag{3}
$$

where $g(x, y)$ is an (uncertain) function decreasing in x and increasing in y. The parameter $u(t)$ acts as a "scenario correction" input intended to compensate the uncertainty of the model. Using currently available data on the trajectory of the "real"

model, the planner forms $u(t)$ and modifies the original emission scenario $\varphi(t)$ with the intension to ensure (2). The initial state

$$
x(0) = x^0, \quad y(0) = y^0
\tag{4}
$$

can also be given inaccurately. It is clear that in order to guarantee that every admissible model of the form (3) (4) is stabilized (in the sence that (2) is ensured), one should impose further constraints on the functions g and initial states (4). Such constraints will be specified later.

2   DECISIONMAKING PATTERN

First, we note that for a carbon cycle model of the form (3), (4) it holds that

$$
x(t) + y(t) = x^0 + y^0 + cw(t) + \Phi(t)
\tag{5}
$$

where

$$
w(t) = \int_0^t u(\tau) d\tau, \quad \Phi(t) = \int_0^t \varphi(\tau) d\tau.
$$

We will treat functions $w(t)$ representing the accumulated scenario correction increments as controls, and write $\dot{w}(t)$ instead of $u(t)$. We suppose that in the planned emission scenario the accumulated emission, $\Phi(t)$, has a finite positive limit at infinity and the emission input, $\varphi(t)$, vanishes at infinity:

$$
\lim_{t \to \infty} \Phi(t) = \bar{\Phi}, \quad \lim_{t \to \infty} \varphi(t) = 0.
\tag{6}
$$

Accordingly, we assume that every admissible control, $w(t)$, has a finite limit at infinity and its derivative, $\dot{w}(t)$, vanishes at infinity:

$$
\lim_{t \to \infty} w(t) = \bar{w}, \quad \lim_{t \to \infty} \dot{w}(t) = 0.
\tag{7}
$$

Expressing $y(t)$ from (5) and substituting into the first equation in (3), we can represent the model (3), (4) in the form

$$
\dot{x}(t) = f(t, x(t), w(t), \dot{w}(t)), \quad x(0) = x^0
$$

where $f(t, x, w, \dot{w}) = \varphi(t) + c\dot{w} + g(x, -x + x^0 + y^0 + cw + \Phi(t))$. We note that the limit relations

(6) and (7) imply that every admissible control $w(t)$ determines the "limit" model

$$\dot{x}(t) = \bar{f}(x(t), \bar{w}) \qquad (8)$$

where $\bar{f}(x, \bar{w}) = g(x, -x + x^0 + y^0 + c\bar{w} + \bar{\Phi})$.

We assume that there is a "real" model of the form (3), (4), and the planner needs to design an admissible control $w(t)$ that ensures (2) for the trajectory $x(t)$ of the "real" model. The "real" model is not known to the planner; instead the planner is given a class of "admissible" models of the form (3), (4) which contains the "real" one. When forming an admissible control $w(t)$ the planner observes the actual values of the carbon concentration, $x(t)$. The control process is started from the (known) initial state x^0 at time 0.

Our solution pattern suggests to update current admissible controls by periodic switching to new extensions. In what follows, an extension of an admissible control $w(t)$ beyond τ is understood as an admissible control $v(t)$ that coincides with $w(t)$ on the interval $[0, \tau]$. We start with the observation that if the original dynamics is linear (i.e., given by (1)), the trajectory $x(t)$ converges, as time goes to infinity, to the rest point $\bar{x}$ of the limit model (8), which is uniquely defined by the equation $\bar{f}(\bar{x}, \bar{w}) = 0$. Assuming that this stabilization propety holds for every admissible model, we treat the planner's task as forming an admissible control $w(t)$ such that the corresponding rest point $\bar{x}$ of the "real" limit model (8) coincides with the prescribed target value: $\bar{x} = \hat{x}$. If at some point in time the planner finds that the latter equality is incompatible with the current admissible control, he/she makes a decision to switch to another extension. Following this control pattern, the planner periodically updates extensions of the current admissible controls.

A planner's control strategy is implemented as follows. At the initial time 0 the planner chooses an initial admissible control $w_0(t)$ and estimates a set $\bar{W}_0$ of the limit values $\bar{w}$ of "inconsistent" admissible controls $w(t)$ that are unable to solve the stabilization problem. The motion of the "real" model starts under $w_0(t)$ and goes along a trajectory $x_0(t)$. At each time $t \geq 0$ the planner observes $x_0(t)$ and decides if w_0 must be switched to another extension, w_1. If the planner decides to switch at a time t_0^*, he/she fixes a delay $\delta(t_0^*) \geq 0$ for the switch and switches to $w_1(t)$ at time $t_1 = \delta(t_0^*)$. The planner decides to switch as soon as he/she understands that the admissible control $w_0(t)$ is inconsis-

tent with the target equality $\bar{x}_0 = \hat{x}$; in this situation we shall say that the planner receives the inconsistency signal. Upon the receipt of the inconsistency signal at time t_0^* the planner adds $\bar{w}_0$ to the initial set $\bar{W}_0$ of inconsistent limit values and forms a new set of inconsistent limit values, $\bar{W}_1$, which can however (due to some further considrations) contain also new elements differing from $\bar{w}_0$. If the decision on a switch is made and a time t_1 for the switch is fixed, the planner chooses a new extension, $w_1(t)$, for $w_0(t)$ beyond t_1 using inormation on the hystory of the process, including the inconsistency set $\bar{W}_1$. This completes the first step in the control process. Note that $w_0(t)$ is never changed if the planner never receives the inconsistency signal. The performance of m steps of the control process results in the formation of admissible controls $w_0(t), w_1(t), \ldots, w_m(t)$ switched on sequentially at times $0, t_1, \ldots, t_m$ and a set estimate $\bar{W}_m$ for inconsistent limit values of admissible controls. On each time interval in the past, $[t_i, t_{i+1})$, the "real" model goes along a trajectory $x_i(t)$ corresponding to $w_i(t)$. Starting from t_m the planner observes the current value $x_m(t)$ and decides if $w_m(t)$ must be switched to another extension, $w_{m+1}(t)$. If the planner decides to switch at a time t_m^*, he/she fixes a delay $\delta(t_m^*)$ for the switch and switches to $w_{m+1}(t)$ at time $t_{m+1} = t_m^* + \delta(t_m^*)$. The fact that the planner decides to switch implies that he/she receives an inconsistency signal, i.e., understands that $w_m(t)$ is no longer consistent with the target equality $\bar{x}_m = \hat{x}$. Upon the receipt of the inconsistency signal the planner adds $\bar{w}_m$ (and possibly some other values) to the set of inconsistent limit values and extends $\bar{W}_m$ to $\bar{W}_{m+1}$. The planner chooses a new extension, $w_{m+1}(t)$, for $w_m(t)$ beyond t_{m+1} using inormation on the hystory of the process, including the set $\bar{W}_{m+1}$. This completes step $m+1$ of the control process (which is never termnated if the planner never receives the new inconsistency signal).

The described control srategy produces a sequence $(t_m, w_m(t))$ of switching times and admissible controls, which is generally infinite (it is finite if the planner does not receive the inconsistency signal at some step; this situaion is cleraly not typical); we will call $(t_m, w_m(t))$ the *control flow*. In parallel with the control flow, the sequence $(t_m, x_m(t))$ is produced; here $x_m(t)$ is the trajectory of the "real" model, which is driven by the admissible control $w_m(t)$ between the sitching times t_m and t_{m+1}; we will call $(t_m, x_m(t))$ the *trajectory flow*. The trajectories $x_m(t)$ defined on the intervals $[t_m, \infty)$ switch sequentially and form the entire *trajectory* $x(t)$: $x(t) = x_m(t)$ for $t \in [t_m, t_{m+1})$ (for $t \in [t_m, \infty)$

if m is the last index in the finite trajectory flow). Let us stress that the control flow, trajectory flow and entire trajectory depend on the unknown "real" model.

3 ROBUST STABILIZATION STRATEGY

Now we implement the suggested decisionmaking pattern under some additional assumptions. Let us note that the described control procedure recommens to add correction quantities $\dot{w}_m(t)$, to the planned emission $\varphi(t)$ during the time intervals $[t_m, t_{m+1})$. Clearly, it is advisable to make $\dot{w}_m(t)$ considerably smaller than $\varphi(t)$, which, in turn, vanishes at infinity. Therefore, we impose the constraint $|\dot{w}_m(t)| \leq \gamma(t_m)$ where $\gamma(s)$ is a prescribed upper bound for the size of every new correction quanity switched on at time $s = t_m$, and set $\lim_{s \to \infty} \gamma(s) = 0$. Clearly, the constraint is met if

$$\dot{w}_m(t) = \begin{cases} +\gamma(t_i) \text{ or } -\gamma_i(t) & \text{if} \quad t_m \leq t \leq \tau_m, \\ 0 & \text{if} \quad t \geq \tau_m \end{cases}$$

with some $\tau_m \geq t_m$. We fix this structure, which assumes that the correction input $\dot{w}_m(t)$ is extremal in absolute value up to the stopping time τ_m and it vanishes afterwards. We also require that the new extension $w_{m+1}(t)$ is switched on not earlier than at τ_m: $t_{m+1} = t_m^* + \delta(t_m^*) \geq \tau_m$; in other words, every time the planner runs the current emission correction program $w_m(t)$ it up to the planned stopping time τ_m. Moreover, we assume that the delay function grows infinitely: $\lim_{s \to \infty} \delta(s) = \infty$. We also require that the limit values for the extensions $w_m(t)$ are uniformly bounded:

$$w^- \leq \bar{w}_m \leq w^+ \tag{9}$$

with some fixed w^- and w^+. Finally, we impose the following constraints on the class of admissible models: for every admissible model (3), (4) the function $g(x, y)$ is continuously differentiable and satisfies $g(0, 0) = 0$ and

$$-a_2 \leq \frac{\partial g(x, y)}{\partial x} \leq -a_1, \ b_1 \leq \frac{\partial g(x, y)}{\partial y} \leq b_2 \tag{10}$$

with some fixed positive a_1, a_2, b_1 and b_2, and the initial state satisfies

$$x^- \leq x^0 \leq x^+, \quad y^- \leq y^0 \leq y^+ \tag{11}$$

with some fixed x^-, x^+, y^- and y^+. Note that apart of all linear dynamics (1) with $a_1 \leq \alpha_1 \leq a_2$ and $b_1 \leq \alpha_2 \leq b_2$, the class of admissible models admits a variaty of nonlinear dynamics (3). In what follows, it is assumed that all the above constraints are satisfied. For the initial set of inconsistent limit values of admissible controls, $\bar{W}_0$, we take the complement to the interval $[w^-, w^+]$. The next two statement are key for the design of a stabilization strategy.

Proposition 1 *Let $(t_m, w_m(t))$ and $(t_m, x_m(t))$ be the conrol flow and trajectory flow corresponding to an arbitrary admissible model (3), (4). Then for each m the trajectory $x_m(t)$ converges to the unique rest point $\bar{x}_m$ of the limit model (8) and $\bar{x}_m$ is a monotonicaly increasing function of the limit value $\bar{w}_m$ for the admissible control $w_m(t)$.*

Proposition 2 *There exists a positive continuous function of time, $\nu(s)$, such that $\lim_{s \to \infty} \nu(s) = 0$, and the trajectory flow $(t_m, x_m(t))$ corresponding to an arbitrary admissible model satisfies $|x_m(t) - \bar{x}_m| < \nu(t - t_m)$ for all m and all $t \geq t_m$.*

Now let us come back to decisionmaking in step $m + 1$. If the current admissible control, $w_m(t)$, is (by chance) such that the "real" model driven by $w_m(t)$ goes to the target value, i.e., $\bar{x}_m = \hat{x}$, then by Proposition 2 $|x_m(t) - \hat{x}| < \nu(t - t_m)$ for all $t \geq t_m$ and the "real" model is stabilized. Otherwise, by Proposition 1 the planner observes $|x_m(t) - \hat{x}| = \nu(t - t_m)$ at some time $t \geq t_m$. Hence, by Proposition 2, the limit point $\bar{x}_m$ differs from the target point $\hat{x}$; therefore, the limit value of the current admissible control, $\bar{w}_m$, is inconsistent with the target equality $\bar{x}_m = \hat{x}$. This immediately produces an inconsistency signal, and at time $t = t_m^*$ the planner decides to switch to a new extension $w_{m+1}(t)$. In order to find $w_{m+1}(t)$, let us come back to the equality $|x_m(t_m^*) - \hat{x}| = \nu(t_m^* - t_m)$ specified as one of two cases:

case 1: $x_m(t_m^*) = \hat{x} - \nu(t_m^* - t_m),$
case 2: $x_m(t_m^*) = \hat{x} + \nu(t_m^* - t_m).$

Suppose case 1 takes place. Then by Proposition 2 $\bar{x}_m < \hat{x}$. By Proposition 1 $\bar{x}_m$ increases if we increase $\bar{w}_m$. Therefore, any admissible control $w(t)$ whose limit value $\bar{w}$ does not exceed $\bar{w}_m$ brings the "real" model to an $\bar{x} < \hat{x}$. Hence, the entire interval $[w^-, \bar{w}_m]$ is inconsistent and can be added to the

current set estimate $\bar{W}_m$ of inconsistent limit values. Therefore, we set

$$\bar{W}_{m+1} = \bar{W}_m \cup [w^-, \bar{w}_m] \quad \text{in case 1.}$$

Similarly, we set

$$\bar{W}_{m+1} = \bar{W}_m \cup [\bar{w}_m, w^+] \quad \text{in case 2.}$$

The suggested method to form the "inconsistency set" $\bar{W}_{m+1}$ implies that its complement, the "consistency window", is an interval:

$$[w^-, w^+] \setminus \bar{W}_{m+1} = [v^-_{m+1}, v^+_{m+1}]$$

(note that the "consistency window" may not contain its boundary points). In step $m + 1$ let us place the new limit value $\bar{w}_{m+1}$ in the middle of the "consistency window":

$$\bar{w}_{m+1} = (v^-_{m+1} + v^+_{m+1})/2.$$

Then the "consistency window" $[v^-_{m+2}, v^+_{m+2}]$ formed in step $m + 2$ is two times shorter than $[v^-_{m+1}, v^+_{m+1}]$ (unless step $m + 2$ terminates the control process) As a result, the "consistency window" $[v^-_m, v^+_m]$ shrinks gradually to the unique point $\bar{w} = \hat{w}$, for which the rest point of the "real" limit model (8) coincides with the target value $\hat{x}$. In parallel, the limit values $\bar{w}_m$ converge to $\hat{w}$. Thus, the described control strategy gradually identifies the unique target point $\hat{w}$ in the space of the limit values of admissible controls. We will call it the *target identification strategy*. The argument used above is to a considerable extent informal. A detailed analysis based on a theoretical background elaborated in [Kryazhimskiy and Maksimov, 2003] leads to the following final statement.

Proposition 3 *Let the interval $[w^-, w^+]$ containing the limit values $\bar{w}_m$ for $w_m(t)$ (see (9)) be wide enough, namely, for every admissible model (3), (4) it hold that $g(\hat{x}, -\hat{x}+x^0+y^0+\bar{\Phi}) + b_1 w^- \leq 0$ and $g(\hat{x}, -\hat{x} + x^0 + y^0 + \bar{\Phi}) + b_1 w^+ \geq 0$. Then for every admissible model, its trajectory $x(t)$ generated by the target identification strategy converges to the prescribed target value:* $\lim_{t \to \infty} x(t) = \hat{x}$.

4 ILLUSTRATION

In this section we give a numerical illustration of the work of the target ientification strategy. For a basis,

we take the linear model (1), (4) and the reference values for the model's parameters α_1 and α_2 and initial quantities x^0 and y^0, given in [Svirezhev, et. al., 1999]:

$$
\begin{array}{llll}
\alpha_1 & = & 1.5 \cdot 10^{-2} & (\text{yr}^{-1}), \\
\alpha_2 & = & 0.25 \cdot 10^{-2} & (\text{yr}^{-1}), \\
x^0 & = & 145 & (\text{Gt}), \\
y^0 & = & 76 & (\text{Gt}).
\end{array}
\tag{12}
$$

The set of admissible models we define by

$$
\begin{array}{llllll}
a_1 & = & 10^{-2}, & x^- & = & 0, \\
a_2 & = & 2 \cdot 10^{-1}, & x^+ & = & 200, \\
b_1 & = & 10^{-4}, & y^- & = & 0, \\
b_2 & = & 4 \cdot 10^{-2}, & y^+ & = & 5000
\end{array}
$$

(see (10) and (11)); the set includes all linear models (1), (4) with α_1 and α_2 ranging in $[a_1, a_2]$ and $[b_1, b_2]$, respectively, and initial states x^0, y^0 ranging in $[x^-, x^+]$ and $[y^-, y^+]$, resecively; in particular, it includes the reference linear model with the parameter values (12) and leaves much space for parametric uncertianties. In [Svirezhev, et. al., 1999] 900 Gt is viewed as an approxmate estimate for the accumulated emission over a reasonable time horizon. In our simulations we take 500 Gt for the total accumulated emission, $\bar{\Phi}$, and assume the exponential emission scenario: $\varphi(t) = \bar{\Phi}e^{-t}$. For the prescribed limit value of the amount of carbon in the atmosphere we take $\hat{x} = 710$ Gt. The upper bound for the correction inputs and the delay function are defined as $\gamma(s) = 1/(t + 1)$ and $\delta(s) = s$; the estimate function $\nu(s)$ (see Proposition 2) is given explicitly; for brievity we omit the formula. The initial admissible control, $w_0(t)$, is zero; thus we let the planned emission scenario remain unchanged up to the first switching time, t_1. Figure 1 shows the trajectories of three admissible linear models under the planned emission scenario $\varphi(t)$. The parameters of model 1 are close to the reference ones (12) and the parameters of models 2 and 3 are extremal for the chosen set of admissible models (see Table 1).

	model 1	model 2	model 3
α_1	$1.5 \cdot 10^{-2}$	10^{-2}	$2 \cdot 10^{-1}$
α_2	$0.25 \cdot 10^{-2}$	10^{-4}	$4 \cdot 10^{-2}$
x^0	100	100	100
y^0	70	5000	0

Table 1.

In Figure 1 the curves marked $x1$, $x2$ and $x3$, show the trajectories of models 1, 2 and 3, respectively.

The straight lines show the limit values for the trajectories. We see that if the planned emission scenario is never updated, the range of the expected limit values is wide enough.
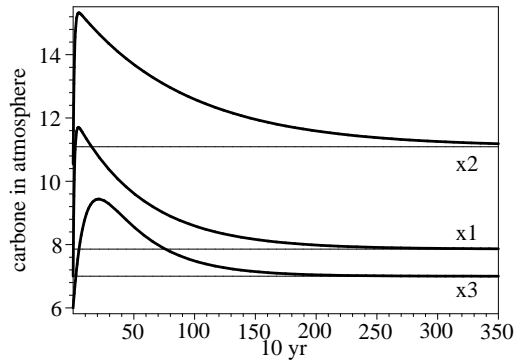

Figure 1: The trajectories of models 1 ($x1$), 2 ($x2$) and 3 ($x3$) under the planned emission scenario, and their limit values (the straight lines), Gt $\cdot 10^{-2}$.

Figure 2 shows the trajectories of models 1 (marked $x1$), 2 (marked $x2$), and 3 (marked $x3$), which are generated by the target identification strategy. All the trajectories converge to the prescribed limit value $\hat{x} = 7.1$ (Gt $\cdot 10^{-2}$), illustrating the fact that the strategy is model-robust.
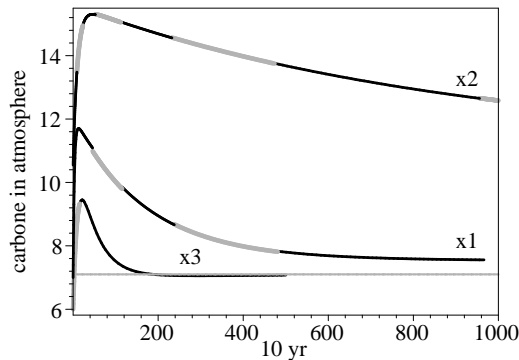

Figure 2: The trajectories of models 1 ($x1$), 2 ($x2$) and 3 ($x3$), generated by the target identification strategy, and the prescribed limit value (the straight line), Gt $\cdot 10^{-2}$. In black and grey periods, in which the planned scenario is corrected with different admissible controls, are shown.

Figure 3 shows first switching times t_m and the graphs of the inputs $\dot{w}_m(t)$ correcting the planned emission scenario $\varphi(t)$ in accordance with the target identification strategy; the illustration is given for model 1. We see that the switching time t_1, at which the planned emission scenario is updated for the first time, is 137 years.
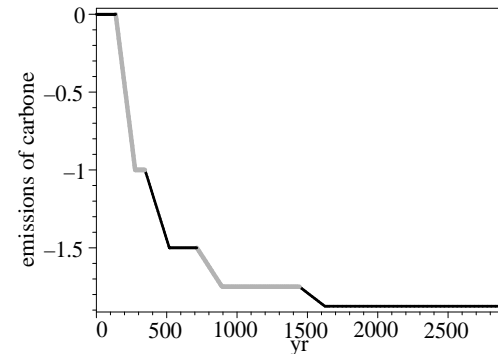

Figure 3: Figure 3. The switching times and the scenario correction inputs, for model 1, Gt $\cdot 10^{-2}$.

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