Rank-Equivalence Method for Sensitivity Analysis of an Integrated Model of a River Catchment

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Abstract: Integrated Assessment Modelling (IAM) incorporates knowledge from different disciplines to provide an overarching assessment of the impact of different management decisions. Integrated models generally require numerous parameters from varying sources, many not known with certainty. Rapid increases in model size and complexity, particularly in the case of integrated models for decision-making, pose new challenges for effective sensitivity analysis. Some of the identified shortcomings of existing sensitivity analysis methods in the context of IAM include: computational inefficiency, failure to assess parameter interactions, excessive data requirements (e.g. requiring parameter probability distributions), assumptions of model linearity and monotonicity and, in particular, difficulty of use in decision-making. To overcome these shortcomings, a new, rank-equivalence method of sensitivity analysis is proposed. The method operates on the assumption that model outputs will be used for ranking of management options. Where models are used for decision-making it is important to ensure that the solution is robust and that rankings will not alter with small changes in model parameters or inputs. The Rank-Equivalence method incorporates parameter bounding as well as numerical optimisation methods in order to find the minimum combined change in parameters or inputs that will result in the ranking of two management options becoming equal. This allows a translation of the set of acceptable model outcomes into a corresponding range of model inputs, thus allowing decision-makers to directly assess whether the current uncertainties of model parameters and inputs are adequate for differentiating between management options. The Rank-Equivalence method is tested using a case study of an integrated catchment model of the Namoi River. The SA results from the case study indicate that while there are several solutions of similar fitness, the solutions may be comprised of different changes in several parameters.

Keywords: *Sensitivity analysis, integrated assessment modelling, decision-making*

1 INTRODUCTION

Integrated Assessment Modelling (IAM) incorporates knowledge from several disciplines into one model, in order to provide an assessment of policy impact prior to implementation. The large and varied amount of data required for IAM means that frequently data are incomplete and model inputs are not known with certainty. This is particularly the case when considering environmental models. For this reason, and because models do not always behave intuitively, sensitivity analysis (SA) is an important stage of model development. Sensitivity analyses can be carried out on the parameters of the model, as well as decision variables and model inputs. For simplicity, and as this research deals with all of these factors, they will all be referred to as model parameters.

SA methods have not kept pace with rapid increases in computational power and the resulting increases in model complexity. Analyses which would have been simple to comprehend for models of, for example, five or fewer parameters become increasingly complex as the number of parameters grows, causing interpretation of the SA results to become a task which is almost as large as the SA itself. Current SA techniques tend to either rank variables in order of those that a particular output is most sensitive to, or ascribe values of the sensitivity to each parameter [Cukier, et al. 1978; Sobol' 1993]. While these can be useful in assessing which parameters are most sensitive and hence which parameters should be selected with particular care, it does not give decision makers an indication of the parameter ranges over which the model results will not alter significantly. Furthermore, the difficulty of assessing the sensitivity of models for decision-making is amplified when different management options require different parameters, thus causing the model to have different sensitivities for different management options. This is of particular importance as often models used to assess new management options have been calibrated to data different from those which are relevant to the option in question.

Ravalico et al. [2005] developed criteria for SA of integrated models based on the specific requirements of these models. The criteria were outlined as being: ease of use in decision-making, taking into account parameter interactions, realistic data requirements, taking into account of model non-linearity and non-monotonicity and computational efficiency. The same study showed that, based on these criteria, current SA methods are deficient and hence there is a need for new methods of SA. In particular, there are no current SA methods that are applicable specifically to decision-making.

The research described in this paper addresses this deficiency through the proposal of a new method of sensitivity analysis for integrated models. The Rank-Equivalence method allows the user to quantify sensitivity by assessment of whether a preferred policy decision, made based on management options rankings, would still be correct given changes to the model parameters. The ability of the Rank-Equivalence method to relate directly to the decision is an important step in facilitating use of sensitivity analysis by decision makers, as well as increasing the understanding, and hence use of, model outputs.

The method is intended for general use in sensitivity analysis of complex models where several parameters to the model may be changing at one time. The SA provided by this method is specifically suited to decision-making bodies that are attempting to select between different management options for natural resource management, for example water catchment management boards. This research is the first stage in the development of an SA method specifically for use by the Murray Darling Basin Commission, Australia. The method will be used for the flow and salinity model of the River Murray, BIGMOD, which itself is used to assess available management options for the river. It will enable assessment of the sensitivity of ranking of management options to the parameters that control the BIGMOD model.

In order to asses the efficacy of the Rank-Equivalence method, an integrated model of the Namoi River catchment (NSW) proposed by Letcher [2000] is used. This model combines a non-linear flow model, policy model, economic model and extraction model in order to represent the operation of the entire system, including human activity. The non-linear component of the model and the interactions between different parts of the model make it an ideal case study for assessing SA methods used for IAM.

2 RANK-EQUIVALENCE METHOD

Sensitivity analysis most often concentrates on the change to the model output that is caused by alteration to model parameters. When considering sensitivity analysis for decision making, there is often a particular level of sensitivity that must be adhered to; in the case where a decision maker is trying to select a management option to put into practice, the required level of sensitivity is such that any reasonable changes to the parameter values should not affect the ranking of the management options. Thus, in order to ensure that there is going to be no change in the ranking of the management options, the minimum combined change in parameter values that will alter the ranking, or result in rank-equivalence of the different management alternatives, must be determined.

The normalized Euclidean distance can be used to assess the size of the combined change in parameter values between different parameter vectors. Hence searching for the minimum normalized Euclidean distance between the original parameter vector and any parameter vector that will result in rank equivalence of two management options will identify the minimum change in parameters to cause a decision made based on management option rankings to be incorrect.

The method takes the following mathematical form:

Given a model

$$y = f(\mathbf{x}, \mathbf{z}) \tag{1}$$

where $\mathbf{x}$ is a vector

$$\mathbf{x} = [x_1 x_2 \dots x_k]^T \tag{2}$$

of k parameters and $\mathbf{z}$ is the vector of management options available, we can represent a realization of the parameters as $\mathbf{x}_A$ with corresponding model output $y_A(z)$. Two management options z_1 and z_2, yield model outputs $y_{A,1}$ and $y_{A,2}$.

The ranking of the management options is changed as we cross over the parameter set

$$\mathcal{B} = \{\mathbf{x} \in \mathcal{P} : f(\mathbf{x}, z_1) = f(\mathbf{x}, z_2)\} \tag{3}$$

where $\mathcal{P}$ denotes the feasible parameter set. The set $\mathcal{B}$ is a (k-1)-dimensional manifold, the boundary of the k-dimensional set

$$\mathcal{B}^\star = \{\mathbf{x} \in \mathcal{P} : f(\mathbf{x}, z_1) \geq f(\mathbf{x}, z_2)\} \tag{4}$$

To find the minimum combined change in parameters that will alter the ranking of the management options z_1 and z_2, we search $\mathcal{B}$ for the point(s) $\mathbf{x}_B$ closest to the original point $\mathbf{x}_A$. This point can be found by minimizing the normalized Euclidean distance:

$$d_2 = \sqrt{\sum_{i=1}^{k} \left(\frac{x_{Ai} - x_{Bi}}{x_{i\max} - x_{i\min}} \right)^2} \qquad (5)$$

between $\mathbf{x}_B$ and $\mathbf{x}_A$, where x_{imin} is the minimum and x_{imax} the maximum value that x_i can take.

As this method considers changes in all parameters at once, it implicitly takes into account interactions between parameters of the model. Investigation of the changes in the individual parameters should also give an idea of the model sensitivity to changes in individual parameters. Further, in searching the parameter space for the smallest possible change, any non-linearities or non-monotonicity in the model structure are accounted for.

A graphical illustration of the parameter space for a two-dimensional example is shown in Figure 1. The shaded area on the diagram represents parameter set $\mathcal{B}^*$, the region of parameter space where the management option ranks are unchanged. Also visible in the diagram is the boundary of the shaded region representative of the set $\mathcal{B}$ and the point $\mathbf{x}_B$, the realisation of parameters within $\mathcal{B}$ that is the minimum distance from the original model parameters $\mathbf{x}_A$.

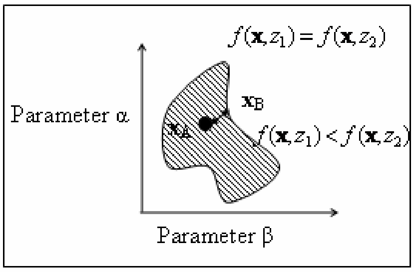

Figure 1: Rank-Equivalence boundary in 2-dimensional parameter space

Given the likely roughness and irregularity of the boundary surface representing equality in management option rankings, traditional search methods are likely to converge to local minima and have difficulty finding a global optimum. Evolutionary algorithms (EAs) are search algorithms that mimic natural biological processes, such as evolution, in order to optimize an objective function. EAs have been found to outperform traditional mathematical optimisation techniques [Elbeltagi, et al. 2005], and for this reason a genetic algorithm [Goldberg 1989] has been selected to perform the search in this research.

The benefits of this method of SA arise because the analysis is conducted in the context of ranking potential management scenarios. Rather than simply analyzing the effects of parameter variation on a specific, or several of the, model outputs, the rank-equivalence method allows decision-makers to assess the sensitivity of the model parameters in relation to the decision the model is being used to assist with making. Where the output from traditional methods of SA is often complex and difficult to relate to model requirements, the rank-equivalence method gives a range of variation for each parameter, over which the results of the model relating to the decision selection will not change.

3 IMPLEMENTATION OF THE RANK-EQUIVALENCE METHOD

To investigate the efficacy of the Rank-Equivalence method, it has been applied to the case study of the Namoi River catchment.

The model used is a simplified version of the integrated water-use policy model presented by Letcher [2002]. The integrated model incorporates numerous interactions, including streamflow, rainfall, land use, crop profits and water extraction policy. The original model incorporates considerable complexity, but in order to simplify the initial trial of the Rank-Equivalence method, the model has been simplified, while maintaining its integrated nature. This enables evaluation of the rank-equivalence method for the particular case of complex integrated models.

3.1 Namoi River Model Outline

The model used consists of IHACRES, a flow model with a non-linear component [Croke and Jakeman 2004], a policy model that determines allowable extractions based on flow, an economic model which incorporates land use, and an extraction model which calculates the actual extraction based on a combination of land use, allowed extractions and river flow. The model is run to simulate one year, with flow calculated daily.

In the context of its use for decision-making and determining appropriate management options, two versions of the policy model, representing different management options, have been employed. The first option bases the allowed irrigation extractions on the level of flow in the river, giving three different allowed extractions for each of three minimum flow levels. The extractions occur on a daily basis. The areas planted with irrigated and dry crops are then determined based on the amount of water available for extraction in that year. The second policy option does not limit the extraction, but requires that a minimum percentage of the area be planted with the dry crop so as to limit the level of irrigation. The crop areas are based on the amount of water available in the river, assuming there is no limit on extraction, beyond being able to remove what is currently there. Consideration of two different policy models allows assessment of changing sensitivities as the management options are altered. The importance of this rests in the necessity to use models to assess management options whilst ensuring that each assessment has the same level of accuracy.

The full Namoi flow network consists of several sub-catchments, each identified as a particular node. A single node of the model will be used in this assessment.

3.2 Optimisation

For this implementation of the Rank-Equivalence method a genetic algorithm is used to find the minimum combined change in parameters. The genetic algorithm is an evolutionary algorithm, based on Darwinian principles of survival of the fittest. Each set of parameters is considered a chromosome, with each individual parameter considered to be a gene on the chromosome. The fitness of the chromosome is the sum of the normalised Euclidean distance between the parameter set contained within the chromosome, and the calibrated parameter set for the model, and a penalty function to ensure that the search is occurring on the set boundary where there is rank-equivalence.

An initial population of chromosomes is generated, randomly assigning uniformly distributed parameter values within the established parameter ranges. The fitness of each chromosome is determined and the population undergoes tournament selection. During tournament selection the population is replicated and each chromosome from the copy of the population is paired with a randomly selected member of the original population. Each pair then competes for selection, with the fitter chromosome being selected for crossover. During crossover the winning population is replicated to create sets of parent chromosomes. Part of the genetic information from each of the parent chromosomes is selected, and the information from both parents combined, to produce two child chromosomes, each containing complementary fractions of the parent chromosomes' genetic information. The fitness of each child chromosome is evaluated, and the fitter half selected as the population for the next generation.

The genetic algorithm used in this instance is real coded, each gene on the chromosome contains a parameter value, rather than the chromosome being composed of binary genes which represent the real parameter values. In order to prevent repetition of results through parameter inheritance, the child parameter value which would be inherited from the parent is randomly selected from a normal distribution with a mean corresponding to the parent value, and a standard deviation of one sixth the distance between the two parent values [Gibbs, et al. 2005]. The value of standard deviation is selected such that there will be only minor overlap (less than 0.5%) between the distributions generated from each parent chromosome.

Elitism is incorporated within the GA, such that the fittest chromosome from one generation is preserved and included in the tournament of the next generation, replacing the least fit of the tournament winners. A mutation operator is also included to increase diversity of the solutions. Once a chromosome is selected for mutation, one of the parameters of the chromosome is randomly selected to be replaced by a parameter randomly generated from the parameter distribution.

The genetic algorithm used in this instance has been coded using the object oriented C++ programming language, as has the Namoi model.

4 ANALYSES CONDUCTED

Two versions of the model using differing management options to maximize the environmental flows in the river, while also maintaining profit levels among farmers, were investigated. The two models utilize the same parameter values for the flow and economic models; however, the policy and extraction models have different parameter values. This analysis allows testing as to whether the alteration in parameter values increases the model's sensitivity to the common parameters.

Management option 1 uses flow levels to determine maximum allowable irrigation extraction from the river, with three specific levels set (L1, L1 + L2 and L1 + L2 + L3) and corresponding allowed extractions (M1, M1 + M2, M1 + M2 + M3). The sum of the daily allowed extractions, further limited by a maximum annual extraction (maxE), is then used to determine the maximum area of irrigated (and more profitable) crop that can be grown with the available water. Management option 2 sets a minimum requirement for the percentage of the area which must be planted with the dry crop. In this case, given the flow in the river, as much water as possible may be removed. The area of each crop is determined in a similar way to management option 1, but with a minimum area requirement of dry crop to be planted.

The total annual flow after extractions is the only model output used to rank the two management options considered in this analysis. This output is regarded as giving an indication of the ability of the management option to both minimise water-use and maximise environmental flow in the catchment. These are both key outcomes that would be potentially desirable to alter through manipulation of the system. In this situation the management option which results in a higher annual post-extraction flow is considered to be the preferred option and thus ranked first.

Ranges for the non-linear loss module parameters (f, e, d, τ_q) were selected based on model calibration studies, while the ranges of the decision variables of the model (L1, L2, L3, M1, M2, M3, DCR, WR, MaxE) were based on values used by Hicks [2003].

The initial run of the Namoi model using the calibrated parameter values found management option 1 to outrank management option 2. The search of the parameter space then attempted to locate solutions where the outputs from both options were equal within a tolerance 0.1% of the original output values being used to assess the rank.

Using the same random number seed, the genetic algorithm was tested with population sizes of 25, 50, 75, 100 and 200, with varying rates of mutation. Population sizes of 200 with a mutation rate of 0.5 per chromosome (equating to 0.04 per parameter) were found to give better solutions. Lower population sizes and lower mutation rates were found to converge rapidly to sub-optimal solutions.

The genetic algorithm was run with a population size of 200 and run for 200 generations, although convergence generally occurred after around 60 generations. The GA run was repeated 10 times with different random number seeds to account for the stochasticity of the method.

5 RESULTS & DISCUSSION

5.1 Sensitivity Results of the Namoi model

From ten runs of the genetic algorithm, ten different parameter combinations were found, which were of similar fitness. The changes in parameter for each run are shown in Figure 2. From the figure it is apparent that while some of the parameter changes were quite consistent, some varied considerably over the different combinations, in particular changes in the crop water requirement parameter. There was also some fluctuation in the changes to the f parameter, as well as the τ_q parameter. Small changes in the parameters e and d identified them as being influential on the output of the model, however, there were also very small changes in the L2, L3, M2 and M3 parameters. These parameters are known to have little effect on the model for the given rainfall data, due to the river being unable to reach the levels prescribed by L2 and L3 for the given rainfall, thus not activating the extraction levels prescribed by M2 and M3. Due to their lack of impact on the model these parameters are able to be maintained at their initial value, without preventing the management options from reaching rank-equivalence.

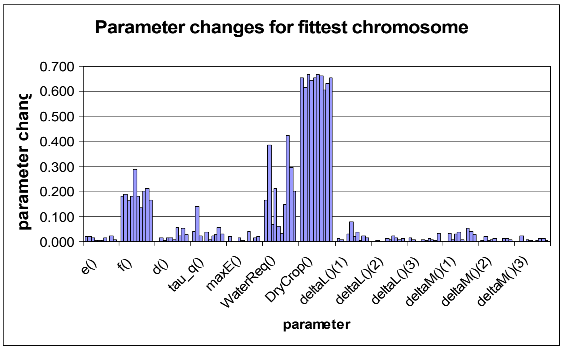

Figure 2: Individual parameter changes for fittest chromosome from each GA run

This property of the output poses potential problems in the interpretation of the results, in particular when parameters with no effect are present in the model. This can be overcome by searching for the maximum Euclidean distance that will still achieve rank equality. In this instance, parameters having no effect would be maximised, with the corresponding variation in changes between the two different stages indicating that the parameters may have no effect on the model.

Despite using different measures, the parameter sensitivities were similar to those determined by Ravalico et al. [2005], using established methods of sensitivity analysis. With the exception of the M2, M3, L2 and L3 parameters for the reasons noted above, the same parameters were highlighted as being of high or low sensitivity.

It should be noted that the minimal parameter changes found for each run of the genetic algorithm, represent a change in parameters that causes rank-equivalence. However, the individual parameter changes are specific to movement in one direction through parameter space. In the situation presented here, the results show that there are several solutions in different directions in parameter space that have a similar fitness. This indicates that there are parameter combinations located on the rank-equivalence boundary at similar Euclidean distances from the original model parameters, despite having different individual parameter changes.

5.2 Use of the Rank-Equivalence method

One of the main advantages of the Rank-Equivalence method lies in the ability of users to interpret the output of the SA without considerable difficulty. In its most simple form the method outputs a single parameter combination that corresponds to the minimum combined parameter change which will result in an incorrect selection of management option. These results provide the user with a range over which the parameters should not change. In this sense, interpretation of results is simple, however, there are currently still some complicating factors in the interpretation of the outputs from the SA. For example, the method is currently unable to determine whether a parameter is highly sensitive, or simply has no effect on the model output at all.

Another advantage is the simplicity of use, parameter combinations are chosen at random, and the search for the minimum change in parameter values is directed by the genetic algorithm, removing the task of parameter selection from the user.

The Rank-Equivalence method has been developed for use by the Murray Darling Basin Commission to assess the sensitivity of the BIGMOD modelling suite, an integrated model of the Murray Darling river system in Australia. It is planned to be used by modellers within the commission to assist with both model improvement and decision-making for natural resource management.

6 CONCLUSIONS

The Rank-Equivalence method addresses the deficiencies of current SA methods used for IAM. In particular the Rank-Equivalence method is appropriate for use in decision-making. Advantageously the method does not require parameter probability distributions, however does require knowledge of the parameter ranges in order to compute the normalised change in each parameter, which may be problematic if unknown. While the Rank-Equivalence method is reasonably computationally efficient, its efficiency may be improved by an alternative search method.

Based on the results obtained in this study, it is evident that there may be problems identifying whether the model is particularly sensitive to a parameter or if in fact that parameter has no effect on the model. In order to counter this problem, a second stage of research is proposed. This next stage explores the parameter space to find the parameter combination which lies on the rank-equivalence boundary, and is the maximum Euclidean distance from the original parameter set. In this exploration, those parameters that have no effect on the model should be set at their maximum value. The large variation between the two situations then gives an indication that the parameters are not having a considerable effect on the model.

Overall, the Rank-Equivalence method shows promise as a new method of SA for IAM, and goes some way towards bridging the gaps between model design and use.

7 ACKNOWLEDGMENTS

This research was possible due to funding from the Australian Research Council, the Murray Darling Basin Commission and the Department of Water, Land, Biodiversity and Conservation of the Government of South Australia. The author would also like to acknowledge Robert May and Matthew Gibbs for their assistance with coding the GA.

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