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T. Ka'eo Duarte

M. Robba

Charles F. Harvey

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Hydrologic-Economic Optimization of a Brackish Coastal Pumping Well

T. Ka'eo Duarte[a], **Michela Robba**[b,d,*], **Charles F. Harvey**[c]
[a]*University of Hawai'i at Manoa, Honolulu, USA*
[b]*CIMA-Interuniversity Center of Research in Environmental Monitoring*
[d]*DIST- Department of Communication, Computer and System Sciences*
[c]*MIT-Massachusetts Institute of Technology, Boston, USA*
[*]*Corresponding author: michela.robba@unige.it*

Abstract: Saltwater intrusion and upconing phenomena affect coastal aquifers worldwide. Over-exploitation leads to high salinity levels which threaten sustainable use of these freshwater resources. There is an increasing need to accurately, and practically, model the physics of these coastal pumping systems while also incorporating the relevant economics. We formalize a decision model able to define the optimal pumping trajectories for a coastal, brackish water well that minimizes overall costs, including desalinization and pumping costs, while keeping track of aquifer head and salinity levels near the well. A binary decision variable is defined to determine when the well is turned off or on, and a state equation defined to relate pumping level and pumping duration to the salinity of pumped water. Unlike most published models on coastal well optimization, pumping is related to salinity empirically. Real pumping and rebound curves are derived from on-site pump tests at Kuki'o, Hawaii. Further hypothetical cases are developed based on these field-derived salinity curves.

Keywords: Coastal aquifer management, optimisation, saltwater intrusion, decision support system, optimal pumping pattern

1. INTRODUCTION

Worldwide, development activities and population growth are stressing our coastal water resources [Postel, 1999, Bear, 2000]. Proper management of these resources, especially coastal aquifers, is important to satisfy demand, to maintain supplies, and to sustain associated terrestrial and marine ecosystems. In coastal zones, where the intensive extraction of groundwater may upset the balance between freshwater and saltwater potentials [Bear, 2000], water resource managers need to satisfy demand while limiting seawater intrusion and saltwater upconing.

Seawater intrusion and the related phenomena of saltwater upconing occur when a well is over-pumped and saline water is increasingly drawn into the pumping well's capture zone; thus degrading water quality and value. Together with recharge, these water quality problems are often the limiting factors to pumping yield in coastal aquifers. The options if a well becomes saline are to (1) decrease or stop the rate of pumping to allow water quality recovery, which can be on the order of decades in low permeability aquifers, (2) desalt via a treatment process such as reverse osmosis (RO), or (3) abandon the well.

Of course, there are economic and socio-political costs that need to be included in any real "sustainable" model of water use. As such, energy, treatment, maintenance, and capital costs are often included in such analyses. The challenge is incorporating both the hydrology and the economics into a realistic, practical and solvable optimization framework. This entails simplifying complicated partial differential equations and embedding the relevant economics, while maintaining the model's reliability and physical-economic validity.

Specifically, it is necessary to integrate an optimisation model able to minimize the supply costs (or maximize benefits) with a simulation model able to dynamically calculate the changes in local hydrology and well salinity. Many approaches exist in the literature.

Shamir and Bear [1984] determine optimal annual operation of a coastal aquifer by using a multiple objective linear programming model based on a multi-cell model of the aquifer and a network representation of the hydraulic distribution system. The decision variables are pumping and/or recharge quantities in each cell.

Willis and Finney [1988] define a planning model for the control of seawater intrusion in regional groundwater systems, structuring the management model as a problem in optimal control. The hydraulic response equations relate the movement of the interface to the magnitude and location of groundwater pumping and recharge. Finite difference methods are used to approximate the solution of the aquifer's response to management strategies.

Naji et al. [1999] focus attention on the fact that the saltwater-freshwater interface location in coastal aquifer is subject to the uncertainty of input parameters such as hydraulic conductivity, freshwater outflow, and pumping rate. The boundary element method, combined with an optimisation technique, is used to search for the interface location.

Das and Datta (1999) use the nonlinear finite-difference form of the steady-state density-dependent miscible flow and salt transport model for seawater intrusion, embedding it within the constraints of the management model. The management objectives represent plausible scenarios for planned withdrawal and/or salinity control in coastal aquifers.

In this work, we formalize a decision model able to define the optimal pumping trajectories for a coastal, brackish water well that minimizes overall costs, including desalinization and pumping costs, while dynamically keeping track of salinity levels in the well. A binary decision variable is defined to determine when the well is turned off or on, and a state equation defined to relate pumping level and pumping duration to the salinity of pumped water. Unlike most published models on coastal well optimization, pumping is related to salinity empirically. Actual pumping and rebound curves are derived from on-site pump tests at Kuki'o, Hawai'i. There is no attempt to model the area's very complicated and poorly understood flow dynamics. Additional hypothetical cases are developed based on these field-derived salinity curves.

2. THE OPTIMIZATION MODEL

The model developed determines the optimal pumping trajectory for a single coastal pumping well on an hourly time scale. Moreover, the model is also able to determine the optimal pumping rate for the well. A cost function to be minimized has been formulated on the basis of binary decision variables that define when to turn the pump on or off and the well pumping rate. The main constraints, described below, regard water-use demand and local hydrology/salinity characteristics of the aquifer. The state equation that relates the control variables (when pumping) and the state variable (salinity) is directly embedded as a constraint in the model.

2.1 The objective function

The objective is to minimize the energy costs to both pump the water and the additional costs to desalinate. Energy costs for pumping depend on the specific energy schedule and on the level of piezometric head in the well. Desalinization costs depend primarily on the energy costs to drive the reverse osmosis process. The objective function to be minimized, where C is total costs, is therefore:

$$C = \sum_{time} C_S + C_P \qquad (1)$$

where:

- C_S is the cost due to salinity/quality
- C_P is the cost related to pumping

2.2 The decision variables

The primary decision variables are (1) the hourly pumping schedule and (2) the fixed pumping rate. More specifically:

- $\boldsymbol{\delta_P^t}$ is the binary decision variable that indicates that at the beginning of the t-th time interval the pump is on
- $\boldsymbol{\Lambda_z}$ is the decision variable that indicates the use of the z-th available pumping rate

2.3 Economics constraints -- Costs

Energy is needed in the RO process to provide additional pressure to force water through the RO membrane. This additional desalinisation pressure may be easily equated to a depth of water (Clark et al.,2002)

$$P_{RO} = 7.8 S_t \tag{2}$$

where P_{RO} is in meters and salinity $\mathbf{S}_t$ in ppt.
Then, the overall cost due to salinity is given by:

$$C_S = 7.8 \cdot \sum_{t=0}^{T-1} \sum_{z=1}^{Z} \mathbf{S_z^t} \cdot Q_z \cdot C_u K_e \Lambda_z \tag{3}$$

where :

- Q_z is water that can be pumped by the well [m^3/s]
- K_e is a conversion factor equal to 14 KWH/(hr*m^4/s)
- C_u represents energy unit costs for the energy schedule in \$/KWH

The pumping costs depend on the necessary energy to lift the water and are given by:

$$C_P = \sum_{t=0}^{T-1} \sum_{\mathbf{z=1}}^{\mathbf{Z}} \boldsymbol{\delta}_P^t Q_z h C_u K_e \Lambda_z \tag{4}$$

where h is the head [m] for the pump.

2.4 The state equation

$\mathbf{S}_z^t$ is the state variable that represents the salinity at time t, caused by pumping rate z. Importantly, during the t-th time interval, the salinity changes accordingly to empirical equations that track salinity behaviour in time. The state equation relating the salinity change in time and the control variables is defined as,

$$\mathbf{S}^t = \mathbf{S}^{t-1} + \sum_{z=1}^{Z} \Delta S_{\mathbf{P,z}}^{\mathbf{t\text{-}1}} \boldsymbol{\delta}_P^{t-1} \Lambda_z + \sum_{z=1}^{Z} \Delta S_{R,z}^{t-1} (1 - \boldsymbol{\delta}_P^{t-1}) \Lambda_z$$

$$t=1,\ldots,T \tag{5}$$

where:

· $\Delta S_{\mathbf{P,z}}^{\mathbf{t\text{-}1}}$ is the increment in the time interval when, at the beginning of the interval (t-1), salinity was S^{t-1} and the pump was turned on

· $\Delta S_{R,z}^{t-1}$ is the decrement in the time interval when, at the beginning of the interval (t-1), salinity was S^{t-1} and the pump was turned off

Note that $\Delta S_{\mathbf{P,z}}^{\mathbf{t\text{-}1}}$ and $\Delta S_{R,z}^{t-1}$ can be computed when an expression for the pumping curve, which corresponds to the change in salinity with time when pumping, and an expression for the rebound curve, which corresponds to the decline in salinity with time when pump is off, are given. Specifically,

$$\Delta S_{\mathbf{P,z}}^{\mathbf{t\text{-}1}} = S_{P,z}(\tau_{P,z}^{t-1}) - S_{P,z}(\tau_{P,z}^{t-1} - 1) \tag{6}$$

$$\tau_{P,z}^{t-1} - 1 = S_{P,z}^{-1}(S^{t-1}) \tag{7}$$

Similar expressions can be computed for the rebound curve substituting the lower case P with R. As explained in Section 3, these empirical equations are determined via pump tests at coastal wells in Hawaii.

2.5 Other constraints

Several other constraints are necessary for model feasibility. Section 2.5.1 describes a constraint on user demand, section 2.5.2 is a constraint related to specific energy schedule costs, section 2.5.3 is a constraint that fixes the final optimal well pumping rate, and bounds on the coastal aquifer salinity characteristics are formalized in section 2.5.4.

2.5.1 Satisfaction of a daily water demand

The water extracted from the coastal aquifer satisfies a daily demand.

$$\sum_{z=1}^{Z} \sum_{t=i\cdot 24}^{23+i\cdot 24} \boldsymbol{\delta}_P^t Q_z \Lambda_z \geq Q_D \qquad \text{i=0,\ldots,D-1} \tag{8}$$

where Q_D [m^3/day] is the water demand for the specific well and D is the number of days in the time horizon.

2.5.2 Constraints related to the energy schedule

For the study area, energy price is determined by the number of hours one intends to use energy and the timing of that use during the day. We explore the benefits of different energy schedules through the following constraint.

$$\sum_{t=i\cdot 24}^{i\cdot 24+23} \boldsymbol{\delta}_P^t \leq E \qquad i=0,\ldots,D-1 \tag{9}$$

where D is the number of days of the time horizon and E is the number of admitted daily pumping hours for the energy schedule.

2.5.3 Admittance of only one pumping rate

Though different pumping rate options are considered when running the optimization model, only one fixed pumping rate can be admitted in the solution. It is unrealistic to develop a pumping schedule that requires pumping rates at a single well to be changing hourly. This is constrained via (10).

$$\sum_{z=1}^{Z} \Lambda_z = 1 \qquad (10)$$

2.5.4 Salinity requirements

The must be a maximum and minimum salinity level for model feasibility. The maximum is average seawater salinity of 35 ppt, and the lower bound is initial conditions in the aquifer.

$$\overline{S} \le S^t \le 35 \qquad t=0,\ldots,23D \qquad (11)$$

3. STUDY SITE AND FIELD METHODS

In the Kona coast region of Hawai'i groundwater is the primary source to satisfy the daily water demand. Due to the area's unique hydrogeology, there is a thin basal lens and many coastal wells are brackish even though located kilometres inland. Pumping tests were conducted on brackish water wells at Kuki'o, Hawai'i to determine empirical equations relating salinity to pumping.

The two wells tested behaved similarly and were pumped continuously for 7 days and then the pump was turned off and the wells' salinities allowed to rebound for 7 days. It was found that, given the region's very high hydraulic conductivity, successful rebound could be achieved in a week's time.

These pumping and rebound curves were then fit using the Matlab 6.0 Curve Fitting Toolbox; RMS > 98%. The general form for the pumping curve is

$$S_P(t) = y_0 + a(1-e^{-bt}) + c(1-e^{-dt}) \qquad (12)$$

and the inverse curve is given by,

$$t(S_P) = \frac{1}{b+d}\ln\left[\frac{a+c}{y_0+a+c-S_P}\right] \qquad (13)$$

The equations for the rebound curve are given by:

$$S_R(t) = y_{0r} + a_r e^{-b_r t} + c_r e^{-d_r t} \qquad (14)$$

$$t(S_R) = \frac{1}{b_r+d_r}\ln\left[\frac{a_r+c_r}{S_R-y_0}\right] \qquad (15)$$

Table 1 reports the parameter values for the different pumping rates. Note that only one rebound curve is necessary. The wells rebounded along a single trajectory no matter what salinity the well was at when the pump was turned off.

Q [m^3/s]	**Y_0**	**a**	**B**	**c**	**d**
Field 0.0340	**3.4846**	**1.5656**	**0.0037**	**0.3176**	**0.0003**
Low 0.0095	**3.4846**	**0.5**	**0.0037**	**0.1**	**0.0003**
High 0.0442	**3.4846**	**5**	**0.0037**	**8**	**0.0003**
Rebound	**3.4791**	**0.4689**	**0.0021**	**0.4111**	**0.0003**

Table1. Parameter values for different pumping rates

4. RESULTS

The model was applied to two levels of daily user demand based on current consumption at the site:

- Case 1, 1600 m^3/d (high demand)
- Case 2, 340 m^3/d (low demand)

There are three energy schedules available in the region; 0.13, 0.19, and 0.21 $/KWH for 10, 20, and 24 hours of energy use respectively. The optimal pumping trajectory is sensitive to these rates based on the hours of pumping necessary to meet demand.

Case 1: The lowest pumping rate is not an option in this case because, even pumping 24 hours, daily demand cannot be met. The cost per day for a 72 hr period using the high pumping rate is 1057 $. The optimal pumping pattern is shown in Figure 1. The pump should be on for ten hours per day at this rate. Therefore, one may use the lowest energy rate, 0.13 $/KWH.

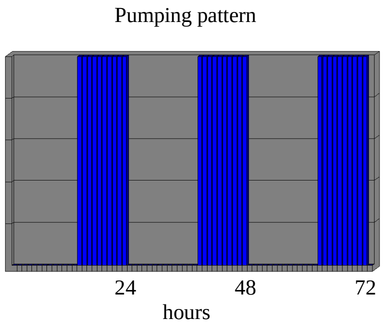

Figure 1: Optimal pumping trajectory; Case 1

Figure 2 plots the salinity trajectory corresponding to the optimal pumping pattern shown in Figure 1. Salinity rises gradually until the end of the time horizon. This is because the salinity cannot rise over the asymptotic values imposed by equation (12) and (14); which act effectively as constraints in the optimisation problem.

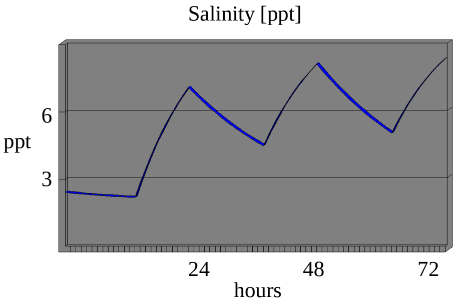

Figure 2: Salinity slope for the optimal solution

The results for the field pumping rate have a similar pumping pattern but it is necessary to pump 13 hours a day to meet demand. Salinity is slightly higher (1 ppt more) at the end of the same 72h time period, and the costs are higher (1130$) because we must use the 20 hour energy schedule and its associated higher hourly rate.

However, the result plotted in Figure 2, which indicates a fairly rapid salinization of the well, highlights an important time-scale issue when using models such as this. The time horizon of optimization must be long enough to effectively eliminate the temporal boundary effect and produce a well-defined, reasonable pumping schedule. If the time horizon is too short the model may produce a solution that appears to degrade the resource. When the time horizon is increased to one month, again using the high pumping rate, a steadier salinity pattern emerges; Figure 3 (the corresponding pumping schedule is not shown).

The optimal solution indicates that salinity at the well will rise from an initial condition of 3 ppt to an average working rate of 6 ppt. This is sustainable over the long-term given the problem constraints and water demand.

The more the time horizon is augmented the more the complexity of the model solution increases with the run time. Alternatively, the model can be run iteratively, changing the initial conditions of the new optimisation period according to the new real-time value of salinity from the previous run.

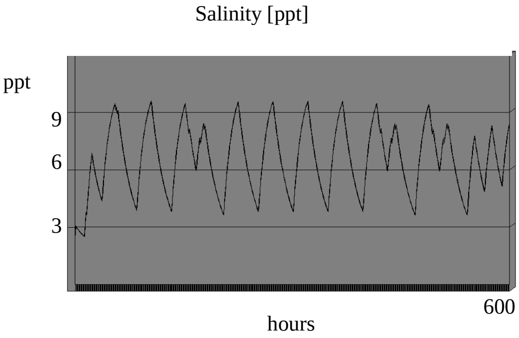

Figure 3: Salinity trajectory for Case 1 for a time horizon equal to one month

Case 2: For a daily demand of 340 m^3/d we compare results for the low and high pumping rates. The salinity state equation for the low pumping rate has strict existence conditions because pumping and rebound are tightly restricted by (12) and (14).

For this case the optimisation model was run iteratively, adjusting the salinity initial conditions for every 24 hr period to see if the solution set changes for the various days. This approach is useful to test the model, and will also allow one the ability to take real-time information from the field site to update pumping strategies.

The minimized cost, in this case, is about a 60$ per day. Figure 4 plots the salinity profile for day three of pumping. The salinity profile oscillates between 2 and 2.4 ppt every day (entire time horizon not shown). This solution is acceptable because it minimizes costs while salinity oscillates around acceptable values.

For the same water demand, the optimal solution for a high pumping rate reports a daily cost of about 123$ for the first day and 160$ for subsequent days. The results for the field pumping rate report a daily cost of about 95$ for the first day and 115$ for subsequent days. Salinity in the well for these last two cases is greater than with the low pumping rate. As a consequence, for Case 2, it is obvious that a low pumping rate, though one must pump for a longer period of time, is best from an economical and environmental point of view.

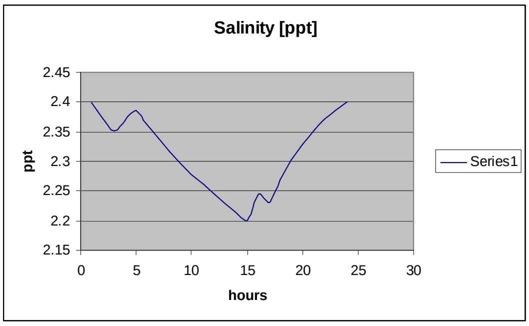

Figure 4. Optimal salinity slope for the 3rd day of pumping; Case 2, low pumping rate.

5. CONCLUSIONS AND FUTURE DEVELOPMENT

An optimisation model to define the optimal pumping pattern for a single brackish pumping well in a coastal aquifer has been presented. The model is able to dynamically track salinity with time, consider alternate pumping rates, consider alternate energy schedules/pricing, and incorporate the energy costs of desalinization via reverse osmosis.

The collection of pumping and rebound curves at Kuki'o, Hawaii allowed us to build the model on the basis of real data. This eliminates the effort and uncertainty associated with the use of analytical or numerical models of saltwater upconing and/or intrusion.

Results indicate that for high demand areas, assuming one well, it is advisable to pump at high rates for a short period of time. This allows the use of a lower energy rate and more time for aquifer recovery between pumping periods. For low demand areas, a low pumping rate for a longer period of time is advisable. Though the energy rate may be higher, RO costs are kept very minimal since salinity barely rises in the well.

Future work will include a comparison of the use of field-derived curves and well-known analytical solutions of upconing. Moreover, new data can be collected to test the model and different resolution techniques for the optimisation problem can be used in order to compare with the obtained results.

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