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Optimal Groundwater Exploitation and Pollution Control

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Abstract: Aquifer management is a complex problem in which various aspects should be taken into account. Specifically, there are conflicting objectives that should be achieved. On one side, there is the necessity to satisfy the water demand, on the other the resource water should be protected by infiltration of pollutants or substances that could reduce its availability in terms of short term and long term management. The aim of this paper is to develop a management model that is able to define the optimal pumping pattern for p (p=1,…,P) wells that withdraw water from an aquifer (characterized by pollutant contamination) and hydraulically interact, with the objectives of satisfying an expressed water demand and control pollution. In order to formalize and solve the management problem, it is necessary to consider the equations governing flow and mass transport of the biodegradable pollutants characterizing the aquifer. Such equations may be solved by using a finite-difference numerical scheme. In this work, the numerical scheme is embedded in the management model. The decision (control) variables that are considered in the optimisation problems are the water flows pumped at each well p, at time interval t. Such flows influence the state variables of the system, that is, the hydraulic head and the pollutant concentrations in the aquifer. The objective function to be minimized in the optimisation problem includes three terms: water demand dissatisfaction, pollutant concentrations in the extracted water, and pollutant concentrations in all cells of the discretized aquifer. Finally, the optimisation problem has been solved for a specific case study (Savona District, Italy), relevant to a confined aquifer affected by nitrate pollution deriving from agriculture activities.

Keywords: Groundwater management, optimisation, pollution, decision support system, optimal pumping pattern.

1. INTRODUCTION

Water is essential for human life and its protection and sustainable exploitation are crucial tasks. Specifically, it is necessary to identify the possible water bodies that could be exploited (surface water, groundwater, reservoirs, etc.) and, according to water demand needs, it is vital to define strategies that preserve the water resource from depletion and pollution and that are environmentally sustainable. The application of optimization techniques in groundwater quantity and quality management has been deeply investigated by Das and Datta (2001). In that work, they present a complete state of the art of the different optimisation approaches that have been applied to groundwater management. Specifically, the combined use of simulation and optimisation techniques is shown to be a powerful and useful method to determine planning and management strategies for optimal design and operation of groundwater systems. The simulation model can be combined with the management model either by using the system state equations as binding constraints in the optimisation model or by using a response matrix or an external simulation model. In literature, different techniques may be found to help in finding solution to the various management problems. Katsifarakis et al. (1999) combine the boundary element method (BEM) and genetic algorithms (GAs) to find optimal solution in three classes of commonly encountered groundwater flow and mass transport problems: determination of transmissivities in zoned aquifers, minimization of pumping cost from any number of wells under various constraints, hydrodynamic control of a contaminant plume by means of pumping and injection wells. Psilovikos (1999) analyses the possibility of solving two management problems formulated as linear programming and mixed integer linear programming through the integration of simulation and optimization packages.

The aim of this paper is to develop a management model that is able to define the optimal pumping pattern for *p* (*p=1,…,P*) wells that withdraw water from an aquifer, characterized by a point source pollutant contamination, with the objective of satisfying the requested water demand and control pollution. Specifically, three different objectives (minimization of water demand dissatisfaction, minimization of pollution in the aquifer and minimization of pollution in the extracted water) have been considered. The state equations that describe the physical behaviour of the system are embedded as constraints in the optimisation model.

2. THE PHYSICAL-CHEMICAL MODEL

The overall model of the considered system may be decomposed into a hydraulic component and a chemical one. As regards the hydraulic component, The adopted model is drawn by Theim (1906) and particularly focuses on the behaviour of the piezometric head at local scale, and specifically on the interaction among the various wells. The pollutant mass transport equation is solved using a finite difference scheme. The hypotheses under which our model is applied are:

1. confined, homogeneous and isotropic aquifer;
2. source terms represented by pumping wells with $Q_p(t)$ discharge pattern for $p=1,\dots P$;
3. wells completely penetrating and located in (x_p, y_p), $p=1,\dots P$.

The third hypothesis means that the fluid flow in the aquifer is only bi-dimensional, since the vertical component of the velocity field is close to zero when the wells pump from all the aquifer thickness. The flow equation with the relative initial and boundary conditions are:

$$\begin{cases} K\dfrac{\partial h}{\partial x}+K\dfrac{\partial h}{\partial y}=S_s\dfrac{\partial h}{\partial t}\displaystyle\sum_{p=1}^{P}Q_p(t)\delta(x-x_p, y-y_p) \\ h(x,y,t=0)=H \\ h(x,y,t)=H \quad if\ \left\{(x,y)\,\middle|\,x^2+y^2=R\right\} \end{cases} \tag{1}$$

where H is the undisturbed piezometric level, R is the influence radius, K is the hydraulic conductivity, h is the piezometric head in the aquifer, S_s is the specific storativity, and δ is the Kronecker Delta .

The characteristic time scale of equation (1) is:

$$T_t = \frac{S_S\ R}{K}$$

It represents the time scale of the transition behaviour of the piezometric head within a regulation interval T_R of the pumping flow. When the transition time scale T_t is negligible with respect to the regulation time step T_R it is possible to consider the flow equation under steady state in each regulation time interval. In this work we consider steady state conditions for successive regulation time steps.

Integrating eq. (1), it is possible to evaluate the piezometric head, in stationary condition:

$$h(x,y,t) = H + \sum_{p=1}^{P}\frac{Q_P(t)}{2\pi T}\ln\frac{\sqrt{(x-x_P)^2+(y-y_P)^2}}{R} \tag{2}$$

where $T=KB$ is the trasmissivity of the homogeneous aquifer and B is its thickness. Deriving equation (2) and using the Darcy law, it is possible to write an analytical expression for the velocity field due to P pumping wells spread in the domain and having a different pumping rate Q_w. Let n the soil porosity, and u and v the pore scale velocities of the fluid flowing in the aquifer along x and y directions, respectively. The pore scale velocities may be expressed as follows:

$$u(x,y,t)=\frac{-1}{2\,n\,\pi\,B}\sum_{p=1}^{P}\frac{Q_p(t)\quad(x-x_p)}{(x-x_p)^2+(y-y_p)^2} \tag{3}$$

$$v(x,y,t)=\frac{-1}{2\,n\,\pi\,B}\sum_{p=1}^{P}\frac{Q_p(t)\quad(y-y_p)}{(x-x_p)^2+(y-y_p)^2} \tag{4}$$

The knowledge of the velocity field is needed in order to solve the mass transport equation. In this work, a contaminant transport simulation model is used, which is able to predict the concentration behaviour in the aquifer for a biodegradable pollutant. Since in many application concerning the monitoring of groundwater quality, the only concentration measures that are often available are the mean value over the thickness of the sampling well, the averaged mass transport equation is taken into account in this work. These equations can be obtained by vertically averaging the classical advection-dispersion equation over the thickness of the aquifer system (Willis et al.,1998; Bear, 1972). These authors have found out the results under the following conditions:

- horizontal flow
- porosity and dispersion coefficients are constant in all the aquifer
- the source and sink terms are represented by pumping wells
- recharging phenomena are negligible because the aquifer is confined
- the bio-degradation coefficient is constant in all the aquifer.

The partial differential equation for the averaged concentration $\overline{C}$ is

$$\frac{\partial \overline{C}}{\partial t} = -\frac{\partial (u\overline{C})}{\partial x} - \frac{\partial (v\overline{C})}{\partial y} + D\frac{\partial^2 \overline{C}}{\partial x^2} + D\frac{\partial^2 \overline{C}}{\partial y^2} \pm \sum_{p=1}^{P} \frac{\overline{C}\,Q_p(t)}{B}\delta(x-x_p, y-y_p) - k\,\overline{C} \qquad (5)$$

where k is the bio-degradation coefficient for the pollutant concentration, considering a first order kinetics.

The boundary and initial conditions needed to solve equation (5) are:

$$\overline{C}(x,y,t) = \begin{cases} C_o & if \quad (x,y) = (x_o, y_o) \\ 0 & otherwise \end{cases} \qquad (6a)$$

$$\begin{cases} \left.\dfrac{\partial \overline{C}}{\partial x}\right|_{x=0,L} = 0 & (6b) \\ \left.\dfrac{\partial \overline{C}}{\partial y}\right|_{y=0,L} = 0 & (6c) \end{cases}$$

where x_0, y_0 is the point corresponding to the pollutant source.

The mass transport equation (5) can be solved by using the classical central finite difference scheme in space, and an implicit method in time (Fletcher, 1991). The stability of the methods is controlled by the dispersion and advection Currant number, defined as

$$C_{adv} = \frac{v\,\Delta t}{\Delta L} \quad \text{and} \quad C_{disp} = \frac{D\,\Delta t}{\Delta L^2}.$$

The finite difference representation of equation (5) is for any point i,j, (a generic point (x,y) on the grid) at any time t) is :

$$\frac{\overline{C}_{i,j}^{t+1} - \overline{C}_{i,j}^{t}}{\Delta t} + u_{i,j}^{t+1}\frac{(\overline{C}_{i+1,j}^{t+1} - \overline{C}_{i-1,j}^{t+1})}{2\Delta x} + v_{i,j}^{t+1}\frac{(\overline{C}_{i,j+1}^{t+1} - \overline{C}_{i,j-1}^{t+1})}{2\Delta x}$$
$$= D\frac{(\overline{C}_{i+1,j}^{t+1} - 2\,\overline{C}_{i,j}^{t+1} + \overline{C}_{i-1,j}^{t+1})}{\Delta x^2} + D\frac{(\overline{C}_{i,j+1}^{t+1} - 2\,\overline{C}_{i,j}^{t+1} + \overline{C}_{i,j-1}^{t+1})}{\Delta y^2}$$
$$- \sum_{p=1}^{P} \frac{\overline{C}_{i,j}^{t+1}\, Q_p^{t+1}}{B\Delta x \Delta y}\delta^*(i - i_{p,} j - j_{p,}) - k\,\overline{C}_{i,j}^{t+1} \qquad (7)$$

where (i_p, j_p) is the location of the wells on the grid.

3. THE MANAGEMENT MODEL

The main purpose of this paper is to present a decision model able to manage groundwater resources, satisfying the water demand and controlling the aquifer pollution. Specific control and state variables have been defined in order to formalize suitable objective functions and constraints. The control variables that characterize the system are the quantity of water that is extracted in each well p in time interval t. These quantities influence both the hydraulic head and the concentration distributions in the aquifer. The state variables of the system correspond to the pollutant concentration to the hydraulic head in the aquifer. Let $Q_{p,t}$ be the control variable that represents the quantity of water that is extracted in each well p at time t. These quantities influence both the hydraulic head and the concentration distributions in the aquifer. The evolution in time and space of pollutant and hydraulic head in the aquifer, that are the system state variables, has to be modelled as proved by (2) and (7). Moreover let $\overline{C}_{i,j,t}$ represent the pollutant concentration in the aquifer at time t in point (i,j). In this work, the pollutant concentration in the extracted water from wells corresponds to the pollutant concentration $\overline{C}_{i,j,t}$ in the nodes of the grid where the wells are located. Specifically, $\overline{C}_{i,p}$ represents the pollutant concentration in well p ($p=1,\ldots,P$) at time t ($t=1,\ldots,T$), where $p=(i_p,j_p)$.

The objective function considered in this paper is composed by three terms: minimization of water demand dissatisfaction, minimization of pollutant concentration in extracted water, minimization of pollutant concentration in all nodes of the discretized aquifer. Every objective is weighed with specific coefficients in an overall objective function. The optimisation problem turns out to be non linear.

3.1 Minimization of water demand dissatisfaction

The water demand dissatisfaction corresponds to the difference between the requested water and the extracted water from the wells, when such a difference is positive or zero. Thus this objective function (to be minimized) among this difference and zero, can be expressed as

$$\max\left[\left(Q_{REQ}-\sum_{p=1}^{N}\sum_{t=1}^{T}Q(p,t)\right),0\right] \tag{8}$$

where:

- Q_{REQ} represents the overall requested water flow, expressed in l/s, over the whole decision horizon;
- N is the number of available wells;
- T is the planning horizon.

3.2 Minimization of pollutant presence in extracted water

Another objective of the optimization problem is to minimize the impact of the pollutant in the water extracted from wells. Let $\overline{C}(p,t)$ be the pollutant concentration, expressed in mg/l, of the water extracted from well p in the *t-th* time interval. This objective function can be formalized as follows:

$$\sum_{p=1}^{N}\sum_{t=1}^{T}Q(p,t)F\left[\overline{C}(p,t)\right] \tag{9}$$

where $F\left[\overline{C}(p,t)\right]$ is a function of pollutant concentration and has been considered to be

$$F\left[\overline{C}(p,t)\right]=\overline{C}(p,t)^2 \tag{10}$$

3.3 Minimization of pollutant concentration in the aquifer

The aquifer pollution should be limited for two important reasons: the preservation of the water resource and the possibility to satisfy water demand for a longer time in the future. Indicating with $\overline{C}(i,j,T)$ the pollutant concentration [mg/l] at node *(i,j)* at the end of the optimisation period, the objective function to be minimized is

$$\sum_{i=0}^{I}\sum_{j=0}^{J}\overline{C}(i,j,T) \tag{11}$$

where i and j are the coordinates of the nodes of the grid representing the aquifer.

3.4 The overall objective function

The overall objective function to be minimized is given by the weighted sum of functions (8), (9), and (11), each one multiplied by a specific weighting factor. Then, the overall objective function is the minimization of by

$$\min imize\left\{\ \alpha\cdot\max\left[\left(Q_{REQ}-\sum_{p=1}^{N}\sum_{t=1}^{T}Q(p,t)\right),0\right]+\beta\cdot\sum_{p=1}^{N}\sum_{t=1}^{T}Q(p,t)F\left[\overline{C}(p,t)\right]+\gamma\cdot\sum_{i=0}^{I}\sum_{j=0}^{J}\overline{C}(i,j,T)\ \right\} \tag{12}$$

where α, β, and γ are suitable weighting coefficients.

3.5 The constraints

There are different kinds of constraints that should be considered in the model. The first class of constraints represents the state equations that represent the dynamics of the pollutant concentrations and of the hydraulic head, as driven by the control variables.

The other constraints are: the hydraulic head limitations due to hydraulic conditions that must be respected, the wells capacity, and the constraints that avoid to extract water from wells when the pollutant concentration exceeds the one imposed by regulations.

Besides, one can make that the equation on which the physical model is based hold only under specific hypothesis. One of them is that the aquifer is “in pressure”, that is to say:

$$h(i,j,t) > B \qquad (13)$$

where B is the aquifer thickness.

Besides the water flow extracted from a well must be less or equal to its capacity, namely

$$Q_{p,t} \le W_p \qquad p = 1,\dots,P \qquad t = 1,\dots,T \quad (14)$$

Finally, the water extracted must have a concentration of pollutant not exceeding a specific bound defined by regulations. In other words, this means that:

$$\overline{\mathbf{C}}_{p,t} > C^{*} \Rightarrow \mathbf{Q}_{p,t} = 0 \quad p=1,\dots,P \;\; t=1,\dots,T \; (15)$$

where C* is the maximum pollutant concentration allowed by regulation.

4. THE CASE STUDY

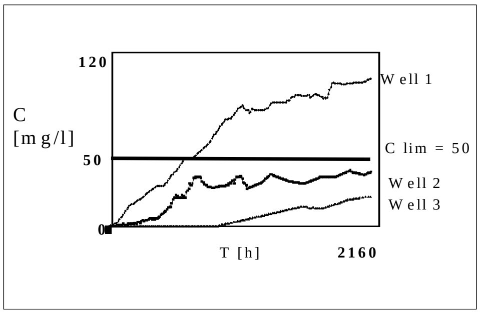
The model has been applied to a study area of 50mx50m in which three wells pump water from a confined aquifer that is affected by nitrate pollution. The spatial location of the pumping wells respect to the source of pollution sees well 1 as the nearest to the pollutant source, while well 3 is the most far. The case study is located within the Ceriale Municipality (Savona, Italy), and the confined aquifer is affected by nitrate pollution due to agricultural practices. The well field is used to extract water for drinking use, but it is periodically closed because of the pollution due to nitrates infiltration. The application of the optimisation model allows finding the optimal pumping pattern in order to satisfy the water demand needs and to control the advancing of the pollutants in the aquifer.

The optimisation problem has been solved over a three months period. The aquifer has been discretized in space (1 m), and in time (10 hours). The total water demand is 60 l/min, while the pollutant concentration is 150 mg/l. The initial value of hydraulic head is 20 m, while the aquifer thickness is equal to 15 m. The problem has been solved for two different cases: Case1 (each well is able to pump the total amount of the water demand 10 l/s), and Case2 (the three wells can pump at maximum the same quantity of water (3.33 l/s)).

The management problem formalized in the previous section has been solved, in both cases, over a time horizon of three months.

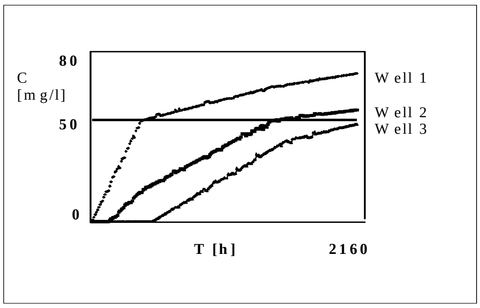
In Case 1, only well 1 (the nearest to the pollution source) overcomes the law limit (50 mg/l) reaching a concentration value of about 106 mg/l. Well 2 and well 3 (the farthest from the pollution source) reach a maximum concentration of 40 and 21 mg/l, respectively, far below the threshold for the whole length of time horizon. Figure 1 shows the pattern of the concentration over time, for the three wells.

Figure 1. Pollutant concentration in the extracted water (first case)



In Case 2, see Figure 2, no management policy can be applied, as, in order to satisfy the water demand, every well has to pump the maximum flow for the whole management horizon. Note that water pumped bywell 1 overcomes the limit after 17 days, whereas, well 2 reaches such a limit after 60 days. Only well 3 can work over 3 months.

Figure 2. The pollutant concentration in the extracted water (Case 2)



Finally, a sensitivity analysis has been performed on the weight coefficients of the objective function. Specifically, the value of the weighting factors is changed in order to make one objective more significant respect to the others.

Figure 3 reports the results, assuming the weighting factor α, relevant to water demand satisfaction objective (equation (8)), as prevailing. This assumption forces the system to maximize the extracted water quantity, setting each well pumping rate to the maximum and stopping extraction when pollutant concentration is over the limit. This strategy causes a very quick overcoming of the fixed potability threshold in the extracted water: well 1 is over the limit in 7 days, well 2 in about 46 days, well 3 in 59.

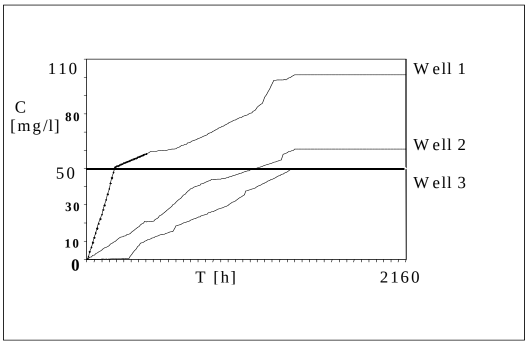


Figure 3. The pollutant concentration in the extracted water (maximum extraction case)

In order to see how solution varies when the minimization of the pollutant concentration in the extracted water is taken as the primary objective, the weighting factor β relevant to equation (9) is increased in order to be predominant respect to the other coefficients. Figure 4 reports the results for the optimisation problem. Specifically, this strategy can satisfy the quality of the pumped water (see Figure 4) but it turns out that in the most part of the time interval the overall pumped water is far below the request.

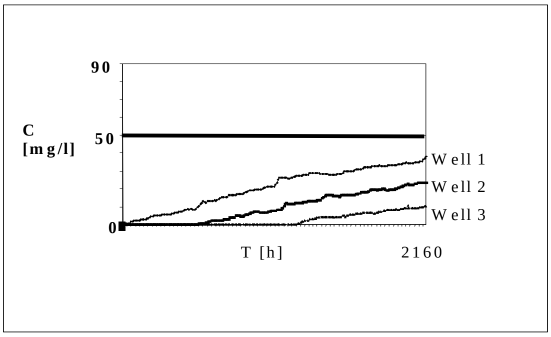


Figure 4. Pollutant concentrations in pumped water

5. CONCLUSIONS

The management of an aquifer is a very complex task since it is necessary to link together optimisation and simulation models in order to find strategies that are able to take into account several aspects (physical, economic, environmental, etc.). In this work, a mathematical formulation of the management problem has been presented, with reference to the extraction of water form wells, in order to satisfy three conflicting objectives (namely, the minimization of water demand dissatisfaction, the minimization of pollutant concentration in extracted water, and the minimization of pollutant concentration in all nodes of the discretized aquifer). Every objective is weighted by a factor whose value is set by the decision maker. Optimal solutions, which may support the decision makers in the evaluation of an extraction strategy, can be obtained solving the related mathematical programming problem that embeds a simplified simulation model of the aquifer dynamics. A preliminary sensitivity analysis on the parameters representing the weighting factors is reported.

Future developments may regard the definition of other decision variables that can give the possibility of considering the possibility of the installation of a treatment plant or of the introduction of wells for the injection of water in the aquifer (in order to control the direction of the contaminant plume). Moreover, the physical model complexity may be increased: the most restrictive assumption is the homogeneity of the hydraulic conductivity of the aquifer (the management model can be adapted to this case using an appropriate solution for the velocity field). Finally, a different approach for groundwater management might be the identification of empirical models (both from simulation runs and real data collection) able to describe the response of the aquifer in every grid point (in terms of hydraulic head and pollutant concentration) to the pumping from the different wells.

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