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Vadose-zone Leaching and Saturated-zone Mixing Model in Heterogeneous Layers

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Abstract: A screening level model was developed for simulation of pollutant migration through the vadose-zone and subsequent mixing within the saturated-zone. This one-dimensional finite difference model simulates the transport processes of liquid-phase advection, liquid- and vapor-phase dispersion, sorption, and decay of the contaminant. Using a simple mass-balance technique, the saturated-zone module estimates mixing of the vadose-zone leachate with groundwater. The model can be a useful tool in making preliminary assessments of the potential impacts of contaminants in the subsurface. The model can handle vertical heterogeneity of the soil columns and non-uniform initial contaminant concentration. The model was verified by comparing to an analytical solution and laboratory soil column experiments. Three different soil sample sizes of Ottawa quartz sand and 480 ppm saline water as groundwater contamination were used to be validated successfully in a lab steady state soil column study. The graphical user interface based on the Microsoft Window function was added in the model so that input data preparation and output data visualization processes are automated.

Keywords: Vadose-zone; Saturated-zone; Leaching model; Groundwater; Modelling; GUI

1. INTRODUCTION

In the model, one-dimensional finite difference scheme was employed for solving the vadose-zone transport equation, and a mass-balance principle was used for the mixing calculation in the saturated aquifer underneath the soil columns. Although several computer codes (VLEACH, VLEACHSM 2.0, EPACML, etc.) are available which can incorporate the heterogeneity of the soil properties (e.g., volumetric water content and corresponding infiltration rate), often many sites do not have the same degree of sophistication in the field-measured data. Even when there is a site with a reasonable amount of field data, a screening level of estimation is often necessary before conducting a comprehensive simulation. Moreover, often times when the pollutant migrates vertically to the ground water, a quick estimation of this mixing is necessary. The Vadose-zone Leaching and Saturated-zone Mixing Model was designed to meet these needs by combining a one dimensional heterogeneous vadose zone transport model and saturated zone mixing model.

In 1993, Ravi and Johnson [1993] developed one dimensional transport program called VLEACH, which handles only vertical migration of pollutant in a homogeneous soil column. Later, Lee [1996], Lee [1999], and Lee & In [2005] developed several versions of VLEACHSM by adding the liquid-phase dispersion, decay terms in the vadose zone, and the saturation zone mixing into VLEACH. This Vadose-zone Leaching and Saturated-zone Mixing Model is improved further by implementing the heterogeneous soil property and the Graphic User Interface (GUI). GUI relieves a user from tedious and error-prone processes of input data preparation and output visualization. In addition, the Vadose-zone Leaching and Saturated-zone Mixing Model allows the specification of two different types (Dirichlet’s and Cauchy’s) of boundary conditions at the top of soil column. Using a simple mass-balance technique, the saturated zone module estimates the concentration of contaminants by mixing of leachate from the vadose zone with groundwater. This module uses the effluent concentration at the bottom of the soil column, which is estimated from the vadose zone module. A complete mixing of the leachate with the groundwater is assumed.
2. Governing Equation and Boundary and Initial Conditions

2.1 Vadose Zone Transport

Considering the three equilibrium phases of pollutants in an unsaturated soil column, its one-dimensional governing transport equation is expressed as follows:

\[
\theta_w \frac{\partial C_w}{\partial t} + \theta_a \frac{\partial C_a}{\partial t} + \rho_b \frac{\partial C_s}{\partial t} =
\left( \frac{\partial}{\partial z} \left( \theta_w D_w \frac{\partial C_w}{\partial z} \right) + \frac{\partial}{\partial z} \left( \theta_a D_a \frac{\partial C_a}{\partial z} \right) \right) - \frac{\partial}{\partial z} \left( q_w C_w \right) - \mu_a \theta_a C_a - \mu_a \theta_a C_a - \mu_a \rho_b C_s
\]

where, \( C_w \) = concentration of a contaminant in liquid (water) phase [mg/L], \( C_a \) = concentration of a contaminant in vapour (air) phase [mg/L], \( C_s \) = concentration of a contaminant in solid phase [mg/Kg], \( \theta_w \) = volumetric water content (volume of water / total volume) [m^3/m^3], \( \theta_a \) = air-filled porosity (volume of air / total volume) [m^3/m^3]. Note that the total porosity (n) equals the sum of the water filled porosity and the air filled porosity. \( q_w \) = water flow velocity (recharge rate) [m/yr].

The air flow velocity (\( q_a \)) is assumed to be zero in this study. \( D_w \) = dispersion coefficient for the liquid phase contaminant in the pore water [m^2/yr]. \( D_a \) = gaseous phase diffusion coefficient in the pore air [m^2/yr]. \( \mu_w \) = first order decay rate of a contaminant in water phase [l/yr], \( \mu_a \) = first order decay rate of a contaminant in gaseous phase [l/yr], \( \mu_s \) = first order decay rate of a contaminant in solid phase [l/yr]. For simplicity, it is assumed that \( \mu_w = \mu_a = \mu_s = \mu \). \( \rho_b \) = bulk density of the soil [gr/cm^3], \( z \) = vertical coordinate with positive being downward, \( t \) = time [yr].

Instantaneous equilibrium (partitioning) of the contaminant among the phases is assumed according to the following linear relationships:

Liquid-solid phase equilibrium is

\[
C_s = K_d C_w
\]

Liquid-gas phase equilibrium is

\[
C_a = HC_w
\]

where \( K_d \) [ml/g] is the distribution coefficient between the solid phase and liquid phase, and \( H \) [dimensionless] is the partition coefficient between the air phase and water phase. Using the empirical relationship, \( K_d \) can be expressed as

\[
K_d = K_{oc} \cdot f_{oc}
\]

where \( K_{oc} \) [ml/g] is the organic carbon-water partition coefficient and \( f_{oc} \) [g/g] is the fraction organic carbon of the soil.

The dimensionless form of the Henry’s partition coefficient, \( H \), can be determined from the more common form having the units of atmospheres-cubic meters per mole (atm-m^3/mol) using the following equation

\[
H = \frac{K_H}{(RT)}
\]

where \( K_H \) [atm-m^3/mol] is the dimensional form of Henry’s Law constant, \( R \) is the universal gas constant \( (R = 8.2 \times 10^{-5} \text{ atm-m}^3/\text{mol-K}) \), and \( T \) is the absolute temperature in Kelvin (°K = 273.16 + °C).

The dispersion coefficient in the unsaturated zone is regarded as a linear function of the pore water velocity as:

\[
D_w = \alpha_L \left( \frac{q_w}{\theta_w} \right)
\]

where \( \alpha_L \) is the longitudinal dispersivity [feet] of the vadose zone.

The gas phase diffusion coefficient (\( D_a \)) in the porous medium is calculated by modifying the free air diffusion coefficient using the Millington model [1959]:

\[
D_a = D_{air} \left( \frac{n - \theta_a}{n^2} \right)^{7/3}
\]

where \( D_{air} \) is the diffusion coefficient of the contaminant in the free air, and \( n \) is the total porosity of the soil.

By substituting equation (2) and (3) into (1), and neglecting the air flow velocity (\( q_a = 0 \)), the governing transport equation can be simplified as:

\[
\theta \frac{\partial C_w}{\partial t} = D \frac{\partial^2 C_w}{\partial z^2} + \left( \frac{\partial D}{\partial z} - q_w \right) \frac{\partial C_w}{\partial z} - \left( \frac{\partial q_w}{\partial z} + \mu \theta \right) C_w
\]
where,
\[ \theta = \theta_s + \theta_H + \rho_0 K_d \]
and
\[ D = \theta_s D_w + \theta_H D_a H \]
To solve the above equation (7), an initial condition is given as follows.
\[ C_w(z,0) = \frac{C_s(z,0)}{K_d} \quad (K_d > 0) \quad (8a) \]
\[ C_w(z,0) = C_0 \quad (K_d = 0) \quad (8b) \]
where \( C_s(z,0) \) is the initial solid-phase concentration specified by the user. When the distribution coefficient \( K_d = 0 \), liquid-phase concentration must be entered as an initial concentration to avoid the program run-time error (division by zero).
The most common type of boundary condition to be applied at the top of the soil column is either the first type (Dirichlet’s) or the third type (Cauchy’s) of boundary condition as shown below.
\[ C_w(z,t) = C_0 \exp(-\gamma t) \quad (t \leq t_0) \quad (9a) \]
\[ C_w(z,t) = 0 \quad (t > t_0) \quad (9b) \]
or
\[ -D \frac{\partial C_w}{\partial z} + q_w C_w \bigg|_{t=0} = 0 \]
\[ q_w C_0 \exp(-\gamma t) \quad (t \leq t_0) \quad (10a) \]
\[ -D \frac{\partial C_w}{\partial z} + q_w C_w \bigg|_{t=0} = 0 \quad (t > t_0) \quad (10b) \]
where \( C_0 \) is the liquid phase solute concentration in the infiltration water, \( \gamma \) is the decay rate \([1/yr]\) of the solute source due to either degradation or flushing by the infiltration, and \( t_0 \) is the duration of solute release \([yr]\) which can be selected to simulate either “slug” or continuous input.
At the bottom of the soil column, the second type boundary condition (Neuman’s) is applied.
\[ \frac{\partial C_w}{\partial z} = 0 \quad (z = \infty) \quad (11) \]
In applying this boundary condition, equation (11) is actually implemented at a finite column length \((i.e., \ z \neq \infty)\). To reduce the finite length effect, dummy cells are added at the bottom of the soil column automatically in the numerical calculation in the model. After evaluation of \( C_w(z,t) \), the total contaminant mass \( (M) \) per unit volume of the soil is calculated as:
\[ M(z,t) = M_q + M_w + M_s = [\theta_s H + \theta_H + \rho_0 K_d] C_w = \theta C_w \quad (12) \]

### 2.2 Saturated Zone Mixing

After estimating the liquid phase solute concentration \( (C_w) \) at the bottom of the soil column, the mixed concentration in the aquifer can be calculated using a mass-balance technique as below (USPEA, [1989] and Summers et al., [1980]):
\[ C_{mix} = \left( C_{aq} q_{aq} A_{aq} + C_w q_w A_{soil} \right) / \left( q_{aq} A_{aq} + q_w A_{soil} \right) \quad (13) \]
where \( C_{aq} \) is the concentration of horizontal groundwater influx, \( q_{aq} \) is the Darcy velocity in the aquifer, \( A_{aq} \) is the cross-sectional aquifer area perpendicular to the groundwater flow direction, and \( A_{soil} \) is the cross-sectional area perpendicular to the vertical infiltration in the soil column. The aquifer area \( (A_{aq}) \) is determined by multiplying the horizontal width of the soil column with the vertical solute penetration depth.

Procedure for the mixing calculation is different depending on the type of soil column arrangement. In the case of the transverse (right angle) arrangement, the mixing calculation is straightforward: simply apply equation (13) at the each mixing element underneath the soil columns. For the parallel arrangement case, however, the mixed concentration at the upgradient cell is considered as an influx concentration to the next cell. The mixing concentration at the next cell is estimated by reapplying the equation (13) using the two inflow concentrations: one from the effluent from the upgradient cell and the other from the leaching from the soil column.

The solute penetration depth is the mixing thickness of the contaminant in the aquifer beneath the soil column. An estimation of the plume thickness in the aquifer can be made using the relationship below [USEPA, 1990]:

\[ \text{Penetration Depth} = \frac{C_{aq} q_{aq} A_{aq} + C_w q_w A_{soil}}{q_{aq} A_{aq} + q_w A_{soil}} \]
\[ H_d = \sqrt{2\alpha L} + B \left[ 1 - \exp\left( -\frac{Lq_w}{q_{av}B} \right) \right] \] (14)

where, \( H_d \) is the penetration depth [m], \( \alpha \) is the transverse (vertical) dispersivity [m] of the aquifer, \( L \) is the horizontal length dimension of the waste [m], and \( B \) is the aquifer thickness [m]. In equation (14) the first term represents the thickness of the plume due to vertical dispersion and the second term represents that due to displacement from infiltration water. When implementing this relationship, it is necessary to specify that in the event the computed value of \( H_d \) is greater than \( B \), the penetration thickness, \( H_d \) is set equal to \( B \).

3. Numerical Implementation

3.1 Vadose Zone Leaching

The governing solute transport equation (7) is solved using the finite difference method. Differential equations dealing with liquid contaminant concentration \( C_w \) as a function of time and depth are converted into the finite difference equations dealing with the corresponding variable \( C_i^k \) centered on time between two time steps:

\[
C_w \rightarrow \left( \frac{C_i^{k+1} + C_i^k}{2} \right),
\]

\[
\frac{\partial C_w}{\partial t} \rightarrow \left( \frac{C_i^{k+1} - C_i^k}{\Delta t} \right) \Delta t
\]

(15)

where \( \Delta t \) is the time increment, the subscript \( i \) refers to the discretized soil column cell and the superscript \( k \) refers to the time level. The subscript \( w \) is dropped for simplicity. Converting the other terms into finite difference form, the governing equation can be written as:

\[
\left( -M_i + M_i' - N_i^{' \prime} \right) C_{i-1}^{k+1} + \left( 1 + 2M_i + N_i^{' \prime} + L_i^{' \prime} \right) C_i^{k+1} + \left( M_i - M_i' + N_i \right) C_{i+1}^{k+1} + \left( -M_i - M_i' + N_i \right) C_{i-1}^{k+1} + \left( 1 - 2M_i - N_i^{' \prime} - L_i^{' \prime} \right) C_i^{k+1} + \left( M_i + M_i' - N_i \right) C_{i+1}^{k+1} \]

(16)

where the dimensionless constants \( M_i, M_i^{' \prime}, N_i^{' \prime}, N_i, L_i \) and \( L_i^{' \prime} \) are: depth.

\[
M_i = \frac{\Delta t}{2(\Delta z)^2} \frac{1}{\theta_i} D_i,
\]

\[
M_i^{' \prime} = \frac{\Delta t}{2(\Delta z)^2} \frac{1}{\theta_i} \frac{D_{i+1} - D_{i-1}}{4}
\]

\[
N_i = \frac{\Delta t}{4\Delta z} \frac{1}{\theta_i} q_i,
\]

\[
N_i^{' \prime} = \frac{\Delta t}{4\Delta z} \frac{1}{\theta_i} (q_{i+1} - q_{i-1})
\]

\[
L_i \equiv \frac{\Delta t}{2} \mu
\]

(17)

Similarly, the finite difference form of the initial condition for the liquid phase solute concentration is

\[
C_i^1 = \frac{C_s}{K_d} (K_d > 0, 2 \leq i \leq n-1)
\]

(18a)

or

\[
C_i^1 = \left( C_i^0 \right)_i (K_d = 0, 2 \leq i \leq n-1)
\]

(18b)

The finite difference forms of the top boundary conditions for the soil column are:

First Type Top Boundary Condition,

\[
C_i^k = \frac{C_s}{K_d} \exp\left\{ -\gamma (k - 1)\Delta t \right\} \quad (k = 1, 2, \ldots)
\]

(19)

\[
C_s^k \neq 0, \quad \text{when } t \leq t_0
\]

\[
C_s^k = 0, \quad \text{when } t > t_0
\]

Third Type Top Boundary Condition,

\[
C_i^{k+1} - \Psi C_{2i}^{k+1} = \frac{-\Omega}{\Phi} C_i^k + \frac{\Psi}{\Phi} C_{2i}^k + \frac{q_u C_0}{\Phi} \exp(-\gamma t)
\]

(20)

where
\[
\Phi' = \frac{D(2M + L + 1)}{4(\Delta z)(M + N)} + \frac{q_w}{2},
\]
\[
\Psi' = \frac{DM}{4(\Delta z)(M + N)},
\]
\[
\Omega' = \frac{D(2M + L - 1)}{4(\Delta z)(M + N)} + \frac{q_w}{2}.
\]

\(M, N, \) and \(L\) were defined in equation (17).

The second type bottom boundary condition is used in this model as follows:
\[
\frac{C_{n+1} - C_{n+1}}{\Delta z} = 0
\]

The above finite difference form of simultaneous equations are computer coded in C++ to solve for the value of \(C_{i+1}^{k+1}\) by the Thomas algorithm [Press et al., 1992].

### 3.2 Numerical Stability

Often, the efficiency of a numerical technique is limited due to the instability, oscillation, and mass-balance problems. Several methods have been proposed to determine the stability criteria of finite difference calculation (e.g., Fourier expansion method, matrix method, and other, [Hirsch, 1989]). The Fourier expansion method, developed by von Neumann, relies on a Fourier decomposition of the numerical solution in space neglecting boundary conditions. It provides necessary conditions for stability of constant coefficient problems regardless of the type of boundary condition [Mitchel and Griffiths, 1980]. The matrix method, however, takes eigenvectors of the space-discretization operator, including the boundary conditions, as a basis for the representation of the spatial behaviour of the solution [Hirsch, 1989 & Ames, 1997]. Based on the von Neumann method, Crank-Nicolson scheme of finite difference equation can be derived as:

\[
\Delta z < \frac{2D}{q_w}
\]

\[
\Delta t < \frac{20(\Delta z)^2}{\sqrt{(2D)^2 - (q_w\Delta z)^2}}
\]

According to the stability criteria expressed in equations (22) and (23), it is clear that the combined dispersion coefficient \((D)\) must be greater than zero. In natural soil conditions, it is rare to have a value of \(D\) as zero or close to zero. Specifically, if there is downward infiltration in the vadose zone, hydrodynamic dispersion of contaminant is inevitable. In addition, the air diffusion coefficients of selected organic compounds are must greater than zero.

Note that the numerical criteria presented in equations (22) and (23) are general guidelines and may not work for all situations. To ensure a stable result, the output should be checked thoroughly. If the results show any oscillation or negative concentration, a smaller time step and/or cell size than the values from the above equations should be tried. After having “stable” results, additional runs are recommended to check convergence of the simulation results. In this case, even a smaller time step and/or cell size can be used to check whether the solution converges or not.

### 3.3 Saturated Zone Mixing

Based on the mass balance principle of equation (13) and the mixed concentration are estimated as:

\[
C_{mx1} = \frac{q_{aq}L_{1}C_{aq} + q_{aq}\bar{H}_{d1}C_{aq}}{q_{mx1}(\bar{H}_{d1})}
\]

\[
C_{mx2} = \frac{q_{aq}L_{2}C_{aq} + q_{aq}\bar{H}_{d1}C_{aq} + q_{mx1}(\bar{H}_{d1} + \bar{H}_{d2})}{q_{mx1}(\bar{H}_{d1} + \bar{H}_{d2})}
\]

where \(C_{mx(i)}\) is the “Mixed Concentration in Groundwater”.

### 4. PROGRAM EXECUTION

A hypothetical example with two soil columns is simulated to demonstrate the impact of soil heterogeneity [Press et al., 1992].

#### 4.1 Problem Descriptions

The example depicts two soil columns arranged perpendicular to the ground-water flow direction (see Figure 1). The vadose zone soil is divided into four soil layers whose total porosity decreases (from 0.44 to 0.38) and water-filled porosity increases (from 0.26 to 0.32) along with the depth shown in Figure 1. The bulk density is adjusted according to the total porosity change (contribution from the water content change is disregarded). The soil column 1 has 1st type top boundary condition. The soil column 2 has 3rd
type top boundary condition. This assumed set of parameters are derived based on a filed geologic

top boundary condition. This assumed set of parameters are derived based on a filed geologic

situation where the total porosity of soil decreases along with the depth due to gravitational pressure while the soil becomes wetter (water-filled porosity increases) along with the depth because water sinks down to the lower layers. Recharge rate $q$ is kept constant (0.3048 m$^3$/yr/m$^2$) in order to keep water-filled porosity of each layer constant. Organic content $f_{oc}$ is also kept constant (0.005 g/g).

4.2 Results

To demonstrate the soil heterogeneity effect in the column the homogeneous soil columns were simulated (Figures 2 and 3). Note that, for homogeneous soil, uniform values soil property were used (bulk density 1.6, total porosity $n = 0.4$, and water filled porosity = 0.3). In both Soil Column 1 and Soil Column 2, the effects of putting four different layers instead of one vertically homogenous layer are obvious as shown in Figures 4 and 5. The liquid-phase contaminant
Vertically Heterogeneous Soil Column I
1st type of Boundary Condition

Liquid-Phase Contaminant Concentration (mg/L)

Distance from Surface (feet)

at 10 years
at 20 years
at 30 years
at 40 years

1st Layer
2nd Layer
3rd Layer
4th Layer

Figure 4. Simulation results of heterogeneous soil column I, 1st type of boundary condition at 10, 20, 30, and 40 years of liquid-phase contaminant concentration (mg/L) vs. depth from ground surface (ft).

Vertically Heterogeneous Soil Column II
3rd type of Boundary Condition

Liquid-Phase Contaminant Concentration (mg/L)

Distance from Surface (feet)

at 10 years
at 20 years
at 30 years
at 40 years

1st Layer
2nd Layer
3rd Layer
4th Layer

Figure 5. Simulation results of heterogeneous soil column II, 3rd type of boundary condition at 10, 20, 30, and 40 years of liquid-phase contaminant concentration (mg/L) vs. depth from ground surface (ft).

A homogeneous soil column was simulated using Vadose-zone Leaching and Saturated-zone Mixing Model and the results were compared with an analytical transport solution [Cleary and Ungs, 1994] (see Figure 7). The close matched of the results indicates Vadose-zone Leaching and Saturated-zone Mixing Model program works correctly in homogeneous case. For the heterogeneous case, Vadose-zone Leaching and Saturated-zone Mixing Model results showed reasonable match with column experiment data [Cleary and Ungs, 1994], which is available.
through Internet (http://www.vadose.net) [Lee, 1999].

![Figure 7. Verification of Vadose-zone Leaching and Saturated-zone Mixing Model with homogeneous soil column by comparing with an analytical solution (PRINCE) modified from Cleary and Ungs [1994].](image)

5. SUMMARY

Vadose-zone Leaching and Saturated-zone Mixing Model was developed. This program can handle vertical heterogeneity of soil. The Graphic User Interface (GUI) used in Vadose-zone Leaching and Saturated-zone Mixing Model has made it easy to create the input data file and view the simulation results. Because Vadose-zone Leaching and Saturated-zone Mixing Model can deal with vertically heterogeneous soil columns which require more parameters to describe the soil properties of each layer, such an easy-to-use interface is even more important.

Verification of this program showed good match between Vadose-zone Leaching and Saturated-zone Mixing Model results and an analytical solution (for homogeneous column) and the soil column experiment (for heterogeneous soil). Development of Vadose-zone Leaching and Saturated-zone Mixing Model was funded partially by the United State Environmental Protection Agency.

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