Jul 1st, 12:00 AM

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Construction of a Degree-Day Snow Model in the Light of the "Ten Iterative Steps in Model Development"

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Abstract: Jakeman et al. [2006] discuss minimum standards for model development and reporting and offer an outline of ten iterative steps to be used in model development. They present the main steps and give examples of what each step might include (especially what choices are to be made), without attempting the formidable task of compiling a comprehensive check list of the model-development process. This study reports construction of a simple degree-day snowmelt model in the light of the ten iterative steps. Such a modelling approach has been widely used in operational hydrology, where the motivation is to produce as reliable as possible snowmelt discharge predictions for streamflow forecasting. There were meteorological and snow cover data available from a research site in southern Finland. These data were used in the development, parameterisation and diagnostic checking of the model in the manner presented in the ten steps. The ten step procedure was found to provide an incentive to a more systematic model analysis – including dianostic checks and uncertainty analyses – that often receives less attention in environmental modelling studies.

Keywords: Model development; snow processes

1. INTRODUCTION

In constructing and using mathematical models it is essential to be aware of the purpose of the model, as well as the limitations, uncertainties, omissions and subjective choices that warrant attention. The best way to improve the quality of modeling studies is to generate wider awareness of what the whole modelling process entails, what choices are made, what constitutes good practice for testing and using models, how the results of using models should be viewed, and what questions model users should be asking of model developers. This amounts to specifying good modelling practice, in terms of development, reporting and critical reviewing of models.

Jakeman et al. [2006] discuss minimum standards for model development and reporting and offer an outline of ten iterative steps to be used in model development. They name the main steps and give examples of what each step might include (especially what choices are to be made), without attempting the impossible task of compiling a comprehensive check list of the model-development process. This study reports construction of a degree-day snow model in the light of the ten iterative steps of Jakeman et al. [2006].

2. DEFINITION OF THE PURPOSES FOR MODELLING (STEP 1)

A snow model is an essential part in quantifying the hydrological cycle in cold regions and therefore can have many uses. Its utility will be enhanced if its development is tailored to the intended purpose, available data and prior knowledge. A significant proportion of the annual runoff in cold regions may occur during a period of just a few weeks in spring, arising from snowmelt. A typical purpose for snow modelling is to provide an estimate of snowmelt input to be used in streamflow forecasting. Streamflow forecasts are necessary for issuing flood warnings and making water regulation decisions. The main objective is to produce daily snowmelt discharge series, but the model must also produce an estimate of the water stored in the snow pack. The latter information can be compared against field measurements, which allow updating of the estimated water storage.

3. SPECIFICATION OF THE MODELLING CONTEXT: OBJECTIVES, SCOPE AND RESOURCES (STEP 2)

The model needs to describe snow accumulation in the affected area of the watershed. When there is snow accumulated on the ground and enough energy available, the model should describe snowmelt. The model is developed with the specific focus of producing as reliable as possible daily snowmelt discharge predictions for streamflow forecasting. It does not need to be capable of describing the physics of snowmelt, e.g. heat content, liquid water percolation within the snowpack or metamorphism of snow. The model structure is aimed to be transferable to other regions with seasonal snow cover, but parameter values should preferably be calibrated for each new site.

The model needs to operate at a daily time-scale, for decisions are required on such a basis, and it produces estimates of

- total snowmelt discharge [mm d^{-1}]
- average areal snow water equivalent (SWE) [mm]

The prediction lead time will depend on what decisions are to be based on the predictions, and could be as short as one day ahead. The maximum practicable lead time is determined by the quality of the available weather forecast, and is unlikely to be much longer than 5 days. The areal snowmelt discharge is required for streamflow forecasting in snow-affected areas, and areal SWE is necessary for calibrating and validating the model satisfactorily.

Kuusisto [1984] studied data from eight snow courses in Finland, and concluded that the relative standard error in measuring the snow water equivalent along the course was typically in the range from 1 to 4 mm during the accumulation period, and from 4 to 8 mm during the melt period. The model performance will be assessed against these accuracies. When snowmelt prediction more than one day ahead is undertaken, the greatest source of uncertainty is likely to be the inaccuracy inherent in the weather forecast.

As driving data the model requires

- daily precipitation [mm d^{-1}] over the area
- daily index of energy [depends on the input variable used, usually air temperature [°C]

The spatial scope/boundary is the catchment located upstream of that point for which a streamflow forecast is required. We assume that inputs which cross the boundaries (e.g. as wind-blown snow drifts) are negligible.

4. CONCEPTUALISATION OF THE SYSTEM, SPECIFICATION OF DATA AND OTHER PRIOR KNOWLEDGE (STEP 3)

Figure 1 shows a schematic conceptualising the two main snow processes to be described, namely accumulation of snow and snowmelt.

It seems reasonable to start by assuming that the above processes are homogeneous over the catchment, warranting a catchment description lumped to the extent that each parameter can be treated as a single value applying to the entire catchment. This assumption requires the following prerequisites be fulfilled. Firstly, the meteorological variables (precipitation, air temperature) should not exhibit large spatial variability; this assumption is easily jeopardized e.g. in mountainous terrain. Secondly, there should not be significant spatial differences in the snow processes within a catchment. This assumption is violated if the catchment comprises substantial areas that differ in land use (e.g. mature forest vs. open field). However, even then it may be justifiable to use areally-averaged single parameter values. For streamflow forecasting, snowmelt predictions are only an input to an eventual forecasting model that can to some extent, via its updating facility, correct for errors arising from the lumped catchment description.

Snowfall always accumulates to the snowpack. Snowpack can also retain liquid water, and hence rainfall also accumulates in the snowpack until the liquid water-retention capacity of the snowpack is exceeded.

Snow melts when there is sufficient energy available for the phase change from the solid to the liquid phase. When snow melts, liquid water is retained in the snow pack until its retention capacity is exceeded, then the excess is discharged. The retention capacity is related to the amount of ice in the snowpack.

Clearly, there is a need to determine the form of precipitation (snowfall or rain). This is rarely observed directly, so the form is estimated usually based on the air temperature. Gauging precipitation is quite challenging, because typically not all precipitation is captured in the gauge. In the case of snowfall, this gauging error can be up to 80% [Førland et al., 1996]. Typically rainfall is also corrected for undercatch, but the correction is smaller than for snowfall.

Snowmelt can be thought to occur when air temperature is above 0 °C. Air temperature can be used as a surrogate for energy available to melt snow, and snowmelt can be thought to occur always when air temperature is above 0 °C. It is noteworthy that radiation absorbed in the snowpack can cause snowmelt even when the air temperature above the snow surface is negative. The liquid water retained in the snowpack can freeze when the temperature is below 0 °C.

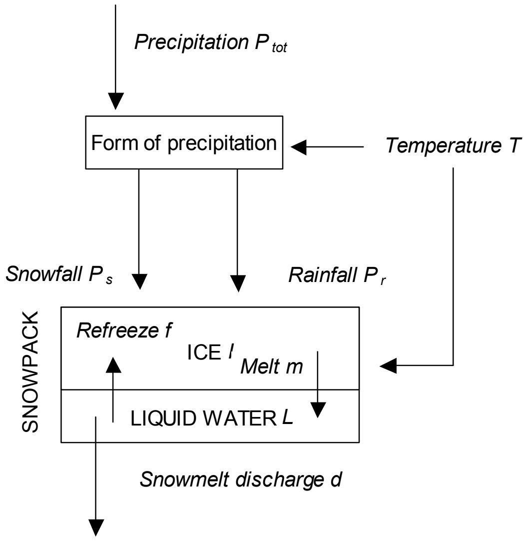

Figure 1. Schematic conceptualisation of snow processes.

For this exercise, there are data available from an open site in Siuntio, southern Finland. There are daily measurements of precipitation and air temperature for the period extending from Dec 1, 1996 to Apr 30, 2000. For the same period there are SWE measurements at intervals varying from 2 to 20 days (average 6 days). The daily temperature measurement is time-averaged from more frequent measurements. When considering single melt events, the rate of snowmelt is not necessarily a linear function of temperature, and hence time-averaging does affect results. Time-averaging of temperature is perhaps even more critical with regard to determining the form of precipitation. The time-average is not necessarily a good estimate of the temperature at the time of a precipitation event. However, it can be presumed that when averaging over several events the errors caused by using time-averaged daily temperatures become smaller.

Precipitation was measured with a weighing gauge, and temperature was measured at two metres above the ground with a Vaisala Humicap sensor. Snow water equivalent was estimated from a matrix of 12 snow sticks, where snow depth was measured at each stick and snow density was measured at three locations. The annual maximum snow water equivalent has an important role in determining the flood potential in springtime. The model uncertainty will later be assessed in terms of the annual maximum snow water equivalent.

5. SELECTION OF MODEL FEATURES (STEP 4)

Following the principle of starting simply, the model is a lumped conceptual model, based on the prior knowledge and assumptions listed in Section 4. It is composed of storage compartments with contents updated daily. Output predictions are made at the same time step. It is not necessary to impose detailed physics in the model [e.g. Morris, 1983; Jordan, 1991] that might lead to a complex numerical scheme for its solution. The modest data used in this exercise do not warrant application of complex models. Also, the end purpose of forecasting, where corrective updating can improve model performance, makes it unnecessary. We also do not see a role for empirical artificial intelligence type model families such as neural nets. While these have been applied to similar problems such as prediction of runoff from rainfall [e.g. Anctil et al., 2004], they tend to perform best in data-rich situations and where there is little theoretical knowledge of how to represent the processes.

6. CHOICE OF HOW MODEL STRUCTURE AND PARAMETER VALUES ARE TO BE FOUND (STEP 5)

Model structure has been determined according to the conceptualisation of the snow accumulation and snowmelt process described in Section 4. The model structure is tested in Section 9 by studying the identifiability and sensitivity of model parameters. If necessary, the model structure can be modified. Alternative model structures might coalesce some of the processes or expand their details. Those parameters whose values cannot reliably be fixed a priori are calibrated against SWE measurements and checked against the values suggested in the literature. When SWE measurements are not available, we rely on the literature values.

7. CHOICE OF ESTIMATION PERFORMANCE CRITERIA AND TECHNIQUE (STEP 6)

The model is calibrated using the sum of squared errors (SSE) as an optimisation criterion:

$$SSE = \sum_i \left(y_i^{obs} - y_i^{sim}\right)^2 \qquad (1)$$

where y_i^{obs} and y_i^{sim} are the observed and simulated SWE values, respectively, at time step i. The calibration parameters are adjusted to minimise the SSE. Parameter estimation is conducted simply by sampling the feasible parameter domain and selecting those parameter values that yield the best model performance. This is computationally demanding but is viable with this simple model. The advantage is that exhaustive sampling will deliver valuable information about parameter correlations, and it will reliably locate the global optimum instead of a local optimum, assuming that the selected parameter sampling domain includes the global optimum and that the sampling discretisation is fine enough.

8. IDENTIFICATION OF MODEL STRUCTURE AND PARAMETERS (STEP 7)

Model structure is first determined according to the conceptualisation of the snow accumulation-snowmelt process, but it can be altered based on the results of model diagnostics in Section 9.

8.1 Form of precipitation

Below a certain threshold temperature T_p [ºC], all precipitation is assumed to fall as snow, and above the same temperature as rain. In mathematical terms,

$$f_r = \begin{cases} 0 & T \le T_p \\ 1 & T > T_p \end{cases} \qquad (2)$$
$$f_s = 1 - f_r$$

where f_r [-] is the fraction of rainfall, f_s [-] is the fraction of snowfall, and T [ºC] is the air temperature.

8.2 Correcting the precipitation for gauging error

For rainfall P_r [mm d^{-1}] and snowfall P_s [mm d^{-1}]

$$\begin{aligned} P_r &= c_r f_r P_{tot} \\ P_s &= c_s f_s P_{tot} \end{aligned} \qquad (3)$$

where c_r [-] and c_s [-] are the correction coefficients for rainfall and snowfall, respectively, to correct for the proportions of rainfall and snowfall not registered by the gauge. P_{tot} [mm d^{-1}] is the gauged precipitation. According to Førland et al. [1996], the correction coefficients for measuring precipitation in an open site, depending on wind conditions at the gauge, range from 1.02 to 1.14 for rainfall, and from 1.05 to 1.80 for snowfall. In forested sites the effect of interception can be accounted for in the correction coefficients, which may lead to values below unity.

8.3 Snowmelt

It is proposed that the rate of snowmelt is linearly related to the air temperature above the melting temperature. This can be written as

$$\begin{cases} m = k_d\left(T - T_{melt}\right) & T > T_{melt} \\ m = 0 & T \le T_{melt} \end{cases} \qquad (4)$$

where m [mm d^{-1}] is the melt rate, k_d [mm °C^{-1} d^{-1}] is the degree-day factor for melt, and T_{melt} [°C] is the temperature where melting of snow is initiated. The value of T_{melt} is close to 0 °C, but can be allowed to differ slightly from it. Such a deviation can, for instance, account for a systematic difference between the air temperature at the measurement station and the temperature at the site where snow water equivalent is modelled. Air temperature has a strong relationship with altitude, and hence a difference in the elevation of the weather station and the modelling site easily results in a systematic bias of temperature measurements. Bergström [1990] reports that the degree-day factor for melt ranges from 1.5 to 4 mm °C^{-1} d^{-1} in operational streamflow forecasting applications in Sweden.

8.4 Freezing

Analogously to snowmelt, the rate of freezing f [mm d^{-1}] is written as

$$\begin{cases} f = k_f\left(T_{melt} - T\right) & T < T_{melt} \\ f = 0 & T \ge T_{melt} \end{cases} \qquad (5)$$

where k_f [mm °C^{-1} d^{-1}] is the degree-day factor for freezing.

8.5 Liquid water retention capacity of a snowpack

The liquid water retention capacity of a snowpack is related to the water equivalent of ice in the snow pack, i.e.

$$L_{\max} = rI \qquad (6)$$

where L_{max} [mm] is the maximum amount of liquid water in the snow pack, r [-] is the retention parameter, and I [mm] is the water equivalent of ice in the snowpack.

8.6 Mass balance for the snowpack

The following equation gives the mass balance for the water equivalent of ice in the snowpack (see Figure 1).

$$\frac{dI}{dt} = P_s + f - m \qquad (7)$$

The following equation gives the mass balance for the liquid water L [mm] retained in the snow pack (see Figure 1).

$$\frac{dL}{dt} = P_r + m - f \qquad L \le L_{\max} \qquad (8)$$

When liquid water input ($P_r + m - f$) cannot fit into the liquid water store, i.e. the value of L exceeds L_{max}, the excess liquid water above L_{max} becomes snowmelt discharge d [mm d^{-1}].

8.7 Rain/melt

Rain/melt is snowmelt discharge when there is snow on the ground, and rainfall in snow-free periods.

8.8 Parameter values

T_{melt}, c_s, k_d, k_f, and r are calibrated against measured SWE values. T_f is set equal to T_{melt}. c_r is fixed to 1.05, according to the value suggested in Førland et al. [1996].

9. MODEL TESTING INCLUDING DIAGNOSTIC CHECKING (STEP 8)

This section explores how well model parameter values can be identified from the available data, and how sensitive the model output is to parameter values.

9.1 Identifiability of model parameters

To test the model structure, the identifiability of model parameters is explored by uniformly sampling the calibration parameter space at the intervals shown in Table 1. The minimum and maximum values were selected in such a manner that values found in literature reside within the prescribed ranges [e.g. Vehviläinen, 1992; and Kuusisto, 1984]. The step values were set in such a manner that the total number of trials did not grow overly large to be run on a personal computer.

In this section the time period from Dec 11, 1996, to Apr 28, 1999 is used in assessing the capability of the model in reproducing the measured SWE values. The model performance is evaluated in terms of the Nash and Sutcliffe [1970] efficiency

$$E_{NS} = 1 - \frac{\sum_{i=1}^{N}\left(y_i^{obs} - y_i^{sim}\right)^2}{\sum_{i=1}^{N}\left(y_i^{obs} - \bar{y}^{obs}\right)^2} \qquad (8)$$

where y_i^{obs} is the observed SWE value at time step (day) i, y_i^{sim} is the simulated value at time step i and $\bar{y}^{obs}$ is the mean observed value, and N is the number of observations.

Table 1. Sampling space of snow model parameters (4 574 934 trials).

Parameter	Min	Max	Step	Unit
c_s	0.7	2.5	0.1	-
T_{melt}	-2	2	0.2	°C
k_d	0	10	0.4	mm °C^{-1} d^{-1}
k_f	0	1	0.05	mm °C^{-1} d^{-1}
r	0	0.8	0.04	-

The maximum E_{NS} obtained for the calibration period was 0.97. For all parameter combinations that yielded an E_{NS} greater than 0.96, correlations between parameter values were computed (Table 2), and marginal frequency distributions of the parameter values were graphed (Figure 2). Threshold of 0.96 was set in order to select only well behaving models but still to guarantee a sufficiently large sample (over 1000 parameter combinations) for the analysis shown below. Correlations were computed from

$$\rho_{x,y} = \frac{\mathrm{cov}(x,y)}{\sigma_x \sigma_y} \qquad (9)$$

where $\rho_{x,y}$ is the correlation between variables x and y, and σ_x and σ_y are standard deviations of x and y, respectively.

Table 2. Parameter correlations.

	c_s	T_{melt}	k_d	k_f	r
c_s	1				
T_{melt}	-0.54	1			
k_d	-0.09	0.81	1		
k_f	-0.03	0.19	0.17	1	
r	0.16	-0.60	-0.37	-0.44	1

From Table 2 it is evident that several pairs of model parameters are strongly correlated. This is due to compensating mechanisms in the model structure that affect snow dynamics. For example, the high correlation (0.81) between T_{melt} and k_d is explained by the fact that an increase in the value of T_{melt} increases the temperature threshold for the initiation of snowmelt, which is compensated by a higher rate of snowmelt (i.e. an increase in the degree-day factor k_d).

The spread in the parameter values yielding a nearly equal model performance (Figure 2) is partly explained by the compensating mechanisms discussed above in the context of parameter correlations, partly by existence of unmodelled dependencies of the model output, and partly by the insensitivity of the model result to a parameter value. With respect to the latter explanation, Figure 2 suggests that a good model fit is obtained even when r is zero, in which case the value of k_f has no control over the model result and any value of k_f gives an equal fit. It seems that a model without the liquid water storage (r = 0) can equally well reproduce measured SWE values, and hence the model structure could be simplified. Disregarding the liquid water storage would decrease the number of calibration parameters from five to three, as r and k_f would not be needed any longer.

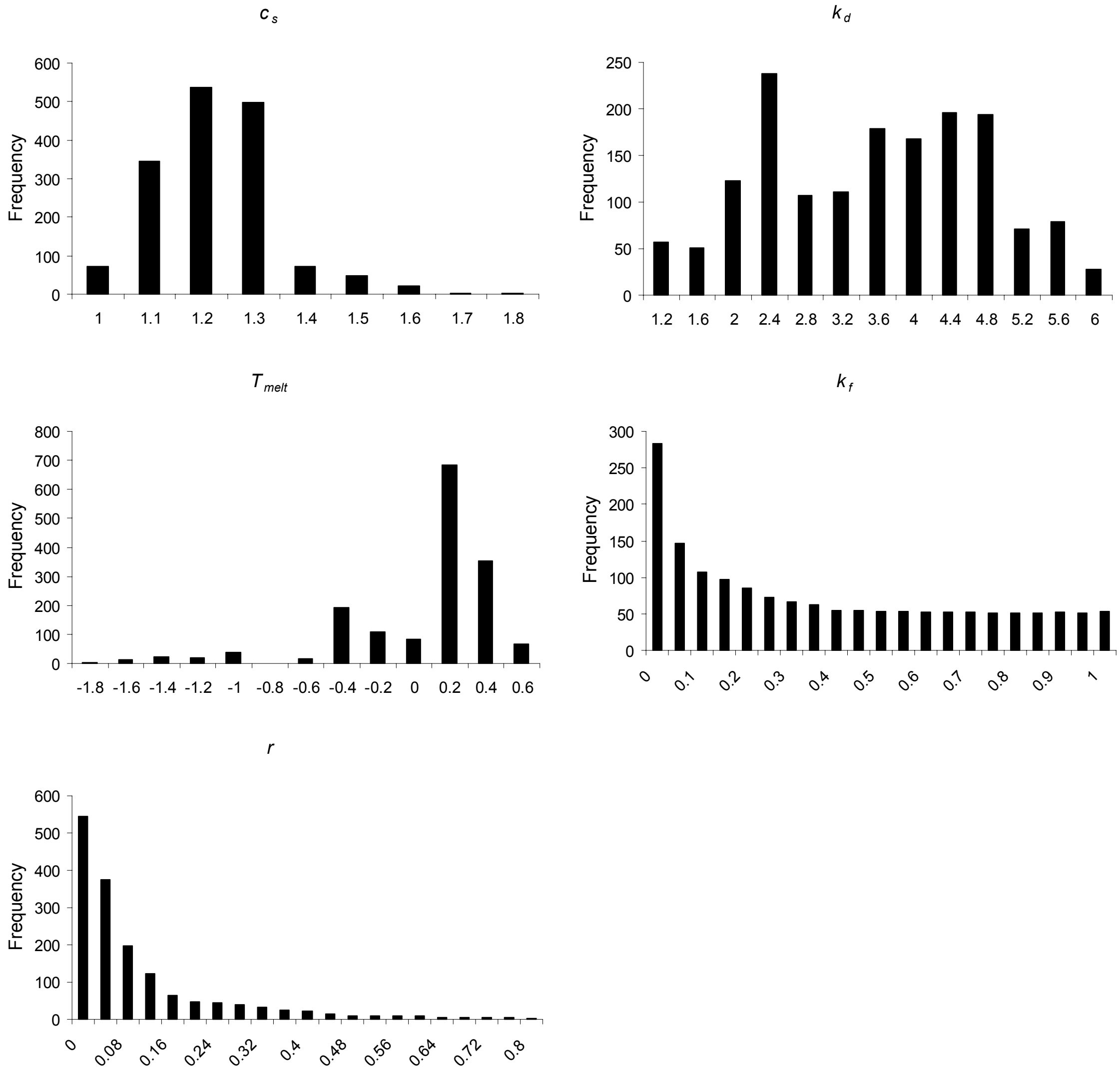

Figure 2. Marginal frequency distributions of the parameter values for all parameter combinations that yielded an E_{NS} greater than 0.96.

9.2 Sensitivity analysis

Sensitivity is assessed in terms of the average annual maximum SWE and five-day sums of snowmelt discharge. The maximum SWE reflects potential for a spring flood, and the snowmelt discharge is the model output necessary for streamflow predictions. The same period of data as in the previous section (from Dec 11, 1996, to Apr 28, 1999) is used here.

The following set of parameter values is used for a reference model run: c_s = 1.2, T_{melt} = 0.2 ºC, k_d = 4.4 mm ºC^{-1} d^{-1}, r = 0.4, and k_f = 0.05 mm ºC^{-1} d^{-1}. These values were selected as none of the values is at its limit, leaving some space for perturbation. This mainly concerns the retention parameter r, whose value cannot be perturbed downwards if it is close or equal to zero. The set of parameter values yielded an E_{NS} of 0.95.

When simulating snowmelt discharge, the computed SWE is updated periodically to match an observed value. The updating is conducted in such a manner that in the day following a measurement, the SWE value at the previous day is fixed to the measured value. In essence updating means that the model always simulates the period between two consequtive snow measurements with the initial state fixed to the first of these two measurements.

In such a scheme we assume that SWE observations are completely accurate. If we knew the relative uncertainty in the observed and modelled values of SWE, we could correct to a value which compromises between the two. Updating of estimates normally does this. Here, however, for the sake of simplicity, the updating scheme assumes SWE observations to be perfectly accurate.

In the following sensitivity analysis, simulated SWE values at five daily intervals from the reference run serve as SWE observations. Five day sums – from the first day after the SWE observation until the next observation – of snowmelt discharge are computed. With regard to $\mathrm{SWE_{max}}$, the model is run without the five-daily updating of SWE, which represents the situation where SWE measurements do not become available early enough for continuous updating.

Table 3 shows results on how much parameters have to be perturbed from their reference values in order to increase or decrease the average annual maximum SWE ($\mathrm{SWE_{max}}$), average annual mean five-daily sums of snowmelt discharge (D_{mean}), and average annual maximum five-daily sums of snowmelt discharge (D_{max}) by 20%. These variables are defined as follows

$$\mathrm{SWE_{max}} = \frac{\sum_{a=1}^{A} \max(\mathrm{SWE})_a}{A} \tag{10}$$

$$D_{mean} = \frac{\sum_{a=1}^{A} \overline{D}_a}{A} \tag{11}$$

$$D_{\max} = \frac{\sum_{a=1}^{A} \max(D)_a}{A} \tag{12}$$

where A is the number of years, $\overline{D}_a$ is the mean five-daily snowmelt discharge in year a, and $\max(D)_a$ is the maximum five-daily snowmelt discharge in year a.

Clearly, the model result is extremely sensitive to the value of T_{melt}, only a change of 0.2 - 0.3°C is required to perturb $\mathrm{SWE_{max}}$ and D_{mean} by 20%. It is noteworthy that as the melting rate is proportional to the difference between air temperature and T_{melt}, the model result is equally sensitive to a systematic error in the air temperature measurement. Also, a systematic error in the areal precipitation estimate has an effect similar to that resulting from the perturbation of c_s. The sensitivity of the model result to perturbations in c_s and k_d is fairly simple, as a 20% change in $\mathrm{SWE_{max}}$ and D_{mean} is induced by parameter perturbations of the same order of magnitude (12-43%). The D_{max} variable seems to be somewhat less sensitive to perturbations in k_d. Note that sensitivity of snowmelt discharge to perturbations in c_s is not assessed here. When SWE observations are used to update the modelled SWE, c_s would only affect snowmelt discharge values if snow melted completely between two subsequent observations.

Regarding the liquid water storage parameters, r and k_f, the model result seems to be less sensitive to their values than to other parameters. Often it is impossible to obtain the required 20% change before the parameter value goes beyond its limits. For example, even a value of 30 mm d^{-1} °C^{-1} for k_f induces only a -16.6% change in D_{max}, although it clearly is an unrealistically high value. From the results presented earlier it is evident that having the liquid water storage in the model structure is not warranted solely on the basis of reproducing SWE measurements, and hence the values of r and k_f cannot be determined by calibration against SWE data. However, ignoring the liquid water storage entirely (i.e. setting r equal to zero) will affect the snowmelt discharge series. This is visible from the –13.6% decrease in D_{max} when r changes from 0.4 to 0. The greater value of D_{max} for the model with a non-zero liquid water storage is due to the water discharge out of the liquid water storage when SWE decreases. Decrease of SWE causes the liquid water retention capacity to decrease correspondingly (see equation 6). On the other hand, in case of a rain-on-snow event, and in the absence of a liquid water storage, all rain immediately becomes snowmelt discharge. This is unrealistic, as it is known that at the beginning of snowmelt a snow pack has liquid water retention storage, and this storage can lead to a significant delay in snowmelt-induced runoff [Vehviläinen, 1992; Kuzmin, 1961]. Immediate transformation of rainfall into snowmelt discharge causes D_{mean} to increase (as opposed to D_{max}) by 6.9% when the liquid water storage is removed by setting r equal to zero.

Table 3. Parameter values yielding a ±20% change from the reference value of the average annual maximum SWE (SWE_{max}), the average annual mean five-daily sums of snowmelt discharge (D_{mean}), and the average annual maximum five-daily sums of snowmelt discharge (D_{max}). Parameter values used in the reference run are shown in parentheses after the parameter symbol. If the required 20% change would have caused a parameter to obtain a value beyond its limit (e.g. $r < 0$ or $r > 1$), or if the model output became insensitive to further perturbation of the parameter value, the change at the limit is presented in parentheses after the parameter value.

	c_s (1.2) -	K_d (4.4) mm°C^{-1}d^{-1}	T_{melt} (0.2) °C	r (0.4) -	K_f (0.05) mm°C^{-1}d^{-1}
SWE_{max} +20%	1.34	3.27	0.40	1 (18.3%)	1.05
SWE_{max} -20%	1.04	6.28	-0.10	0.04	0 (-7.8%)
D_{mean} +20%	-	5.99	-0.13	0 (6.9%)	0 (3.6%)
D_{mean} -20%	-	3.11	0.52	1 (-4.6%)	11.95
D_{max} +20%	-	6.80	-0.88	0.99	0 (0.8%)
D_{max} -20%	-	2.57	1.01	0(-13.6%)	30(-16.6%)

The liquid water retention capacity has been suggested [Vehviläinen, 1992 ref. Kuzmin, 1961] to have a value between 0.2 and 0.3 of the SWE value at the beginning of snowmelt. In the present model the retention capacity is related to the water equivalent of ice. Now, if we assumed that at the beginning of snowmelt the snowpack was completely dry (i.e. all water would appear in the form of ice), the values suggested above would translate directly into r values. Here the r value is fixed to the middle of the given range, i.e. 0.25. The value of the refreezing parameter k_f seems to have little impact on the snowmelt discharge variables D_{mean} and D_{max}, which is in line with the result of Kuusisto [1984] who stated that having the refreezing parameter in the model gave only a small improvement in model performance. Consequently, accounting for refreezing is removed from the model structure.

9.3 Identification of parameters

After having fixed the value of r equal to 0.25 and removed k_f from the model, values for the remaining parameters (c_s, T_{melt}, and k_d) are identified. From now on, the model is always updated using observed SWE data. This calibration procedure is insensitive to errors that have occurred earlier and would otherwise affect the calibration throughout the calibration period.

In Section 9.1 it was demonstrated that c_s, T_{melt}, and k_d are correlated. Hence, it will be difficult to identify unique parameter values, and many combinations will give a similar fit against SWE data. To improve identifiability of the parameter values, calibration data are screened to isolate a subset where accumulation of snow is the dominant snow process. Such a subset is used to assign a value for c_s. Those intervals between two subsequent SWE observations where there had been more than twice as much accumulation as melt, and where more than 75% of precipitation fell at an air temperature below -1°C, were identified. The latter condition was introduced to increase the probability that precipitation had fallen in form of snow. As snowmelt has not been measured, it had to be estimated with the model. In estimating the amount of snowmelt, parameter values were set to those yielding the best fit in the sampling exercise presented in Section 9.1, on the condition that r equals 0.24 and k_f equals 0. The screening of data yielded 21 SWE measurements, and the value of c_s was estimated to be 1.04.

After having identified c_s, all calibration data are used to identify values for the remaining parameters k_d and T_{melt}. Figure 3 shows a contour plot of E_{NS} as a function of k_d and T_{melt}. A higher value of T_{melt} is clearly compensated by a higher k_d value, which is seen as a ridge running diagonally across the parameter space. The global maximum of E_{NS} is located at a point where degree-day factor k_d equals 2.7 °C^{-1} d^{-1} and T_{melt} equals 0.1 °C. Kuusisto [1984] reports that average degree-day factors in the open range from 2.8 to 4.9 mm °C^{-1}d^{-1}. Finally, recalibration of c_s with k_d equal to 2.7 mm °C^{-1} d^{-1} and T_{melt} equal to 0.1 °C gives again the value 1.04.

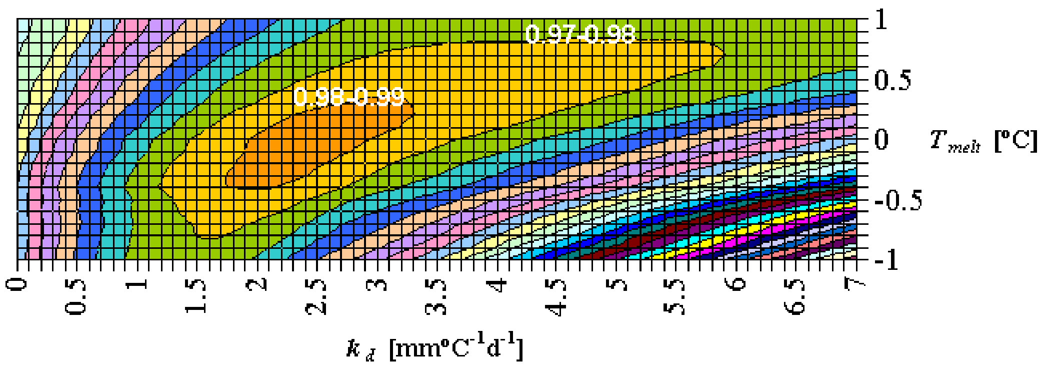

Figure 3. A contour plot of E_{NS} as a function of k_d and T_{melt}.

Limited information about the shape of the objective function would be available with little extra computation when parameter estimation techniques providing the Hessian matrix are applied. The Hessian matrix yields the curvature of the objective function at the optimum.

9. UNCERTAINTY ANALYSIS AND MODEL EVALUATION (STEPS 9 AND 10)

In addition to the Nash-Sutcliffe efficiency E_{NS}, bias B, mean absolute error E_{ma}, and maximum absolute error E_{maxa} are used for further evaluation of the model performance. Definitions for the above criteria are

$$B = \frac{1}{N}\sum_{i=1}^{N}\left(y_i^{sim} - y_i^{obs}\right) \tag{13}$$

$$E_{ma} = \frac{1}{N}\sum_{i=1}^{N}\left|y_i^{sim} - y_i^{obs}\right| \tag{14}$$

$$E_{\max a} = \max\left|y_i^{sim} - y_i^{obs}\right| \quad ,i = 1 \ldots N \tag{15}$$

Bias gives an indication whether the model has a tendency to systematically under- or overestimate SWE, and mean absolute error characterises the model performance in a similar fashion to E_{NS}, but does not give as much weight to large errors.

We have daily precipitation and temperature data from four winters covering the period from Dec 1, 1996 to April 30, 2000. For the same period there are SWE data that have been measured at an approximately weekly interval (Figure 5).

The model is calibrated over three winters, while the data from the remaining winter are reserved for model validation. All four possible combinations of three calibration winters and one validation winter are tested, and the results are shown in Table 4.

Table 4. Evaluation results for the snow model. Results for the validation winter are in bold face.

	Calibration winters	Validation winter
Case1	1997-1999	2000
Case2	1997-1998, 2000	1999
Case3	1997, 1999-2000	1998
Case4	1998-2000	1997

	Case 1	Case2	Case3	Case4
c_s [-]	1.04	1.08	1.16	0.97
T_{melt} [°C]	0.1	-0.3	-0.3	-0.1
k_d [mm°C^{-1}d^{-1}]	2.7	2.1	2.2	2.3
1997				
E_{NS} [-]	0.96	0.96	0.96	**0.96**
B [mm]	-0.22	0.19	0.05	**0.07**
E_{ma} [mm]	3.76	3.24	3.19	**3.74**
E_{maxa} [mm]	12.36	10.73	10.52	**11.03**
1998				
E_{NS} [-]	0.91	0.92	**0.90**	0.93
B [mm]	0.51	0.55	**0.42**	0.84
E_{ma} [mm]	3.06	2.88	**3.04**	2.88
E_{maxa} [mm]	5.57	6.44	**8.17**	4.51
1999				
E_{NS} [-]	0.99	**0.98**	0.98	0.99
B [mm]	0.13	**0.86**	0.71	1.05
E_{ma} [mm]	4.53	**5.15**	5.12	4.84
E_{maxa} [mm]	18.58	**17.71**	17.18	18.37
2000				
E_{NS} [-]	**0.87**	0.93	0.93	0.91
B [mm]	**-1.22**	0.25	-0.07	-0.03
E_{ma} [mm]	**5.11**	4.20	4.24	4.24
E_{maxa} [mm]	**11.91**	10.48	8.30	14.44

The results shown in Table 4 indicate that the pair of T_{melt} and k_d is well identified. Parameter values for different calibration periods are not far from each other, and a higher value of T_{melt} is accompanied by a higher value of k_d, as expected due to their positive correlation (see Figure 3). The snowfall correction factor c_s attains a fairly large range of values, depending on which of the four years is left out from the calibration data. The highest value (1.16) is 20% higher than the lowest value (0.97). According to the sensitivity analysis, such a change in c_s would induce a significant change in maximum SWE values when the model is run throughout the winter without periodical updating using SWE observations. When updating of SWE according to observations is applied, the effect of c_s on the computed SWE series is naturally much smaller. As a result of updating, the computed snowmelt discharge series is only affected by the value of c_s when the snow pack melts completely between two SWE observations, and there is precipitation in the form of snow.

The fact that c_s is more difficult to identify than the parameters controlling snow melt indicates that precipitation is more difficult to measure consistently than air temperature and SWE. The difficulty of gauging snowfall is acknowledged in the literature [e.g. Kuusisto, 1984]. Also, having just daily precipitation data available it is not possible to know whether precipitation falling in the day of an SWE observation has fallen before or after the time of the observation. This affects estimation of c_s, in particular as the number of observations used for estimation of the value of c_s is relatively small (around 20).

With respect to E_{NS}, the model performance in the validation year (in bold in Table 4) is always inferior (or equal) when compared to the case where that same year has been included in the calibration period. This is explained by the fact that both the optimisation criterion (sum of squared errors) and E_{NS} describe model performance based on squared differences between measured and simulated values. For other criteria, the model performance in the validation year can be even better than the performance for the case where the same year has been included in the calibration period.

The bias B is small, mostly well below 1 mm, which suggests that the model does not have a systematic tendency to under or over predict the change in SWE between two subsequent observations. The mean absolute error E_{ma} ranges from 2.9 to 5.1 mm, which is below the measurement accuracy given in Section 3 (up to 8 mm). The maximum absolute error is between 4.5 and 18.6 mm, where the upper limit is clearly above the accuracy of an SWE measurement. The largest error (18.6 mm) occurs for case 1 for the time period from Apr 2 to Apr 6, 1999, when the measured SWE decrease is 29 mm while the model predicts a 10 mm decrease. This error can be partly explained by snowmelt occurring on the day of the SWE observation (Apr 2) when the air temperature was well above zero (2.9 ºC). If this melt occurs after the time of SWE measurement, it results in an error in the model. This is the same problem of daily data as discussed already in the context of precipitation measurements.

Figure 4 plots daily snowmelt discharge series using the parameter set identified for case 2 against the series from case 1. Scatter plots for case 3 vs. case 1, and case 4 vs. case 1 are similar to the plot shown in Figure 4. Clearly the largest discrepancies occur when according to case 1 snowmelt discharge is zero, but results from case 2 show discharge values up to 17 mm/d. The maximum difference occurs on Dec 4, 1999, when the observed precipitation value was 15.8 mm and the air temperature was –0.1 ºC. Based on the parameter values from case 1, precipitation on that day fell in the form of snow, whereas due to the lower T_{melt} value in case 2 precipitation fell as rain. Discharge series would not show this large deviations, if the form of precipitation was not determined based on a single threshold temperature, but using a temperature range inside which precipitation is split between snow and rain. A temperature range for determining the form of precipitation is commonly used in degree-day snow models, and should be considered here, too.

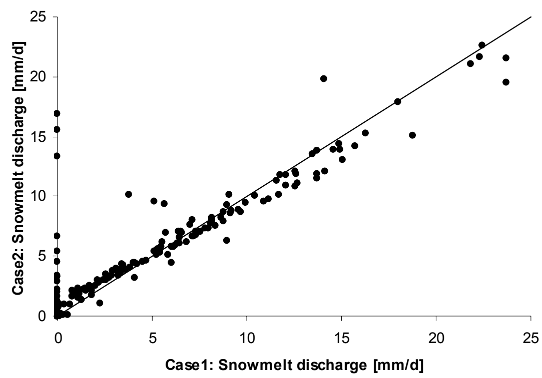

Figure 4. Daily snowmelt discharge series using the parameter values identified for case 2 against the series from case 1.

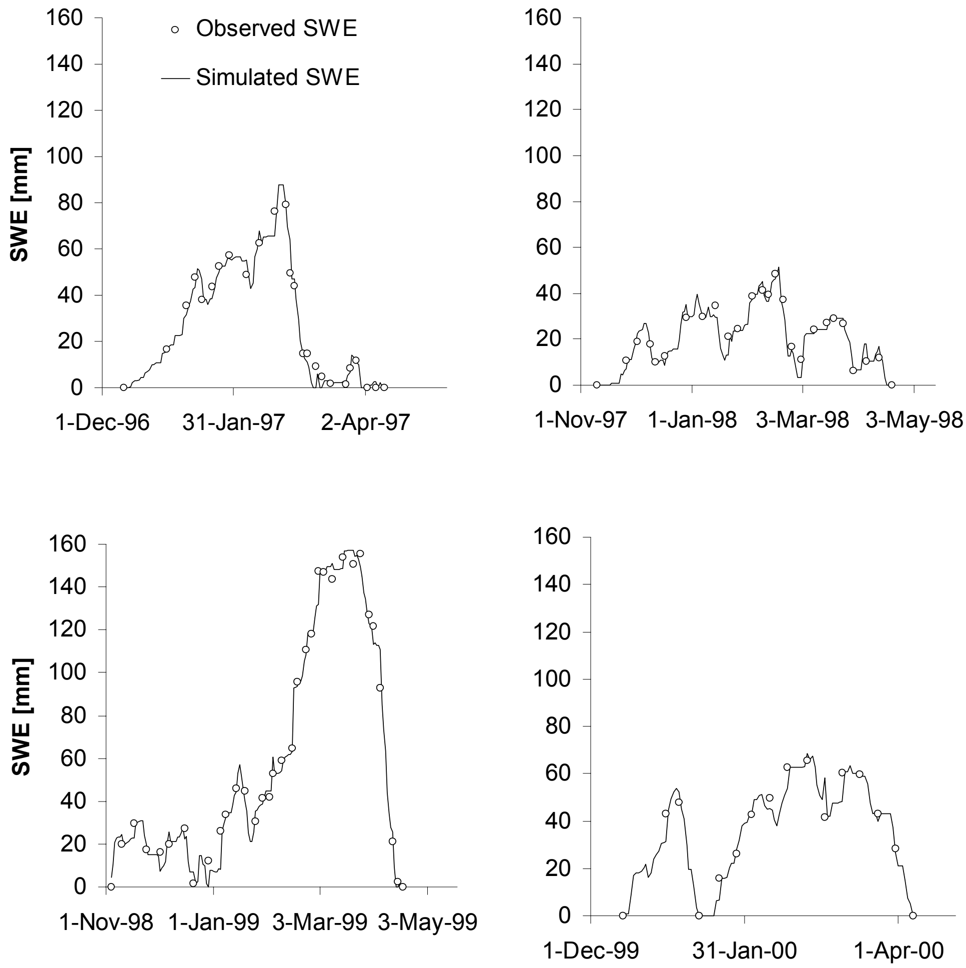

Figure 5. Observed and simulated SWE series from Dec 1996 to Apr 2000. SWE observations are used to update model result whenever an observation becomes available.

Finally, to utilise all available information in the data the model is calibrated to all four winters from Dec 1996 to Apr 2000. The following parameter values are obtained: c_s = 1.05, k_d = 2.1 mm ºC^{-1}d^{-1}, and T_{melt} = -0.3 ºC. Figure 5 shows model fit using the above parameter values.

9. SUMMARY

The purpose of this modelling exercise was to develop a simple snow model that would predict daily snowmelt discharge to be used for streamflow forecasting. The client is anyone with an interest in producing flow forecasts.

The selected model type is a simple, lumped conceptual model. As the only objective was to deliver an estimate of snowmelt discharge, it was not necessary to impose detailed physics in the model. A more physics-based model describing the energy balance of a snowpack in more detail would lead to a more complex numerical scheme for its solution, and increase requirements for input data, especially if parameters were allowed to vary in space.

Model testing revealed that while the liquid water storage had an effect on the predicted snowmelt discharge series, its parameters could not be identified from SWE observations. It was decided to fix the retention parameter to a literature value (0.25), and to remove the refreezing process by setting the refreezing parameter to zero.

When the model is used for predicting snowmelt discharge, the SWE value is updated every time a new observation becomes available. When running the model for the present data using periodical updating of SWE, the mean absolute error for SWE was found to range between 2.9 and 5.1 mm, and the maximum absolute error was up to 18.6 mm. While the mean error is below the measurement accuracy of SWE reported in the literature (see Section 3), the maximum error was considerably larger. Daily resolution of the meteorological data was found to cause errors as it was not possible to determine whether a precipitation or melt event had occurred before or after the SWE observation in the same day. Snowmelt discharge can be very sensitive to the value of T_{melt}, as it determines the form of precipitation as an abrupt threshold. Instead of a single threshold temperature, a range with two temperature limits should be considered for determining the form of precipitation.

The model reproduces the measured change in SWE between two subsequent observations with satisfactory accuracy. It is noteworthy that in this exercise the snow predictions were based on measured, instead of forecast, meteorological variables. When snow predictions are based on a weather forecast, the largest uncertainties will stem from inaccuracies inherent in the forecast. Also, it is recommended that the quality of the snow model should be further assessed against streamflow data from the same region. In such an analysis the computed snowmelt discharge can be compared against measured streamflow, which can reveal problems in the snow model that are not visible when the model performance is compared only against SWE observations.

The ten step procedure was found to provide an incentive to a more systematic model analysis – including diagnostic checks and uncertainty analyses – that often receives less attention in environmental modelling studies. The iterative nature of the modelling steps contributes to good modelling practice but poses challenges to reporting as several model versions may appear in the same report when model structure is revised in iterating through the steps.

ACKNOWLEDGEMENTS

We would like to acknowledge funding from the Academy of Finland, the Integrated Catchment Assessment and Management Centre at the Australian National University, the Land and Water Technology Foundation (Maa- ja vesitekniikan tuki ry.), and the Maj and Tor Nessling Foundation. Support from professors Pertti Vakkilainen and Tuomo Karvonen at the Helsinki University of Technology is greatly acknowledged.

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