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Abstract: The inclusion of additional information in a model should improve the model's performance and reduce associated uncertainties. The additional information may take the form of higher temporal and/or spatial resolution of data already used, or additional types of data (including soft data). However, additional data is not necessarily equivalent with additional information as a model may only be able to make use of a fraction of the information in the data, or the approach chosen to extract data might be inefficient. Consideration of the information-to-noise ratio of the data is needed in order to evaluate whether the information will indeed improve the model's performance or reduce uncertainty. Evaluating the information-to-noise ratio and the information content of data in general are often difficult tasks. One simple but effective approach to this problem is to compare the performance and uncertainty of a model with and without additional information. This workshop will focus on the theory and use of additional information in improving the performance of and reducing the uncertainty in hydrological models. This includes consideration of the uncertainty in datasets as well as techniques to evaluate the effectiveness of the additional information in improving models.

Keywords: Rainfall-runoff models; uncertainty; performance metrics; information.

1. INTRODUCTION

When mathematical models are applied to estimate hydrological variables, e.g. streamflow from rainfall and other hydroclimatalogical data, it is necessary to consider the impact of uncertainties in the input data, in model structures, and in the values of a model's parameters. The Prediction in Ungauged Basins (PUB) Decade (2003-2012) of the International Association of Hydrological Sciences (IAHS) aims to reduce predictive uncertainty in model outputs, particularly when dealing with ungauged catchments [Sivapalan et al., 2003; Wagener et al., 2004]. Several PUB Working Groups (WGs), each with different but sometimes overlapping objectives, are working towards making progress with the aims of PUB. The Top-Down modelling WG (TDWG, WG1) and the Uncertainty Estimation for Hydrological Modelling WG (WG7) have convened this workshop to review progress to date and discuss possible future PUB research. In order to set manageable boundaries for the workshop the paper introduces only some aspects of uncertainty and its reduction. A thorough review of the topic has not been attempted here. Nevertheless, by outlining some recent and ongoing work related to uncertainty in hydrological modelling, with appropriate citations, the paper sets the scene for the workshop.

A focus of the workshop is to address the information content of data, particularly additional data, in terms of whether progress towards reducing estimation uncertainty can be, or is likely to be, made. The purpose of including additional information in model inputs is to improve the performance of the model. The additional data may take a number of forms, including higher temporal and spatial resolution of existing data being used (e.g. spatial distribution and intensity of rainfall), or adding data that were not previously included [e.g. Seibert and McDonnell, 2002; Freer *et al.* 2004; Son and Sivapalan, 2007]. Note that additional information may also be in the form of constraints. For example, bounds on the steady-state gain of an autoregressive-moving average exogenous (ARMAX) model yield linear inequality constraints on the ARMAX parameters which can be used to reduce uncertainty in their estimates [Norton, 1979]. The inclusion of additional information will improve the model's performance only as long as the information content in the data (or more accurately, the information that the model is able to use) is greater than the noise that is introduced. Sorooshian *et al.* [1983] suggested that it is the quality of information contained with a dataset, rather than the quantity (i.e. length of record), which may be the more important characteristic for a data set as beyond a certain point (this will depend on the nature of the system, and the data as well as the uncertainty in the data), the additional data does not significantly improve the parameter estimates. Unfortunately, datasets often have significant uncertainty, resulting in occasions when more noise than information is added by the inclusion of the data. Therefore, the information-to-noise ratio (or more broadly the net information gain) of any datasets needs to be considered before inclusion of the data within a model.

There have been a number of studies investigating ways of addressing the question of the impact model errors have on model performance and calibration. Yapo *et al.*, [1996] conclude that "the factor currently limiting model performance is the unavailability of strategies that explicitly account for model error during calibration". The BATEA calibration method [Kavetski *et al.* 2002] is an example of allowing for model error, though there can be confusion between model error and error in the model inputs [Thyer *et al.* 2007]. Marshall *et al.* [2006] proposed the use of the hierarchical mixtures of experts (HME) framework for reducing the predictive uncertainty due to errors in the model structure by combining the predictions of two or more models. In calibration of an in-river phosphorus model, McIntyre and Wheater [2004] found that error in the pollutant load data, model structure as well as the inherent model parameter equifinality compromised the value of calibration data, concluding that expectations of model performance should be reviewed in light of the resource limitations, and that there is a need for more robust estimation of model uncertainty. In a study of the application of two rainfall-runoff models (GR4J and TOPMO) on twelve catchments in the USA (basin sizes ranged from 1021 to 4421 km^2, with the ratio of mean annual rainfall rainfall to potential evaporation ranging from 0.5 to 1.7), Perrin *et al.* [2007] found that if non-continuous calibration periods were used, in general approximately 350 calibration days selected to cover a range of conditions were sufficient to obtain robust parameter values, though for drier catchments, it was more difficult to reach stable parameter values.

The structure of this workshop position paper is based on the following sections:

- Review of recent uncertainty research

- Estimation and propagation of uncertainty

- How to estimate the reduction in uncertainty

- How to maximize information extraction from available data

- Example studies

- Discussion and conclusions

2. REVIEW OF RECENT UNCERTAINTY RESEARCH

There has been a considerable volume of work studying uncertainty and the impact on model outputs. Oudin *et al.* [2006] studied the impact of corrupted (both random and systematic) inputs (precipitation - P and potential evapotranspiration - PET) on model performance, concluding that random errors in P have potentially the largest effect on

model performance. Systematic errors in P may be accounted for in the model calibration if the model structure permits this. However, from the viewpoint of predicting flows at ungauged locations, systematic errors in rainfall are more problematic unless the error does not change significantly between the gauged and ungauged sites. The influence of temporal resolution of rainfall data on generation of infiltration excess runoff has been explored by Bronstert and Bárdossy [2003], with model outputs being most sensitive to resolution for events with rainfall intensity near the infiltration capacity.

It is important to remember that estimation of uncertainty in observed and modelled quantities may not be an end in itself. When uncertainties are used to draw conclusions, the confidence in the uncertainty estimates needs to be assessed in order to be able to define the confidence in the conclusions. In the view of Montanari [2007], this is possible when sufficient information is available. Generally, this requires consideration (e.g order of magnitude estimation) of all potential sources of uncertainty, though only significant contributors to the uncertainty need to be estimated with greater care. In hydrology, some of the sources of uncertainty that need to be considered are:

- observation errors;

- rating curves, and when these are being extrapolated;

- spatial interpolation and aggregation, including the spatial resolution of the available data, and the spatial scales of variations in the fields being interpolated.

- influence of temporal resolution (data time-step)

Only by careful consideration of the sources of uncertainty can valid conclusions be drawn regarding the utility of the data, and the impact the data have on modelled outputs and subsequent conclusions.

2.1   Uncertainty due to modelling data time-step

Recent work [Littlewood, 2007; Littlewood and Croke, submitted; Littlewood, in prep.a,b] has focussed on a discrete-time modelling artefact whereby IHACRES (and probably many other similar models') parameter values are dependent upon the data time-step used for calibrating the model. As more information was added to an IHACRES model for one of the Plynlimon research basins in the UK, by reducing the data time-step, selected model-fit statistics and precision (and accuracy) on the model parameters improved. This is not only because a smaller time step implies more information per unit time; use of a smaller time step also reduces systematic distortion of the estimated unit hydrograph by (i) averaging over a time step and (ii) aliasing, if instantaneous flow is sampled. It might be expected, therefore, that a model calibrated using a small data time-step (e.g. hourly), when applied in simulation-mode followed by aggregation of the results to a larger data time-step (e.g. daily), will give better estimates of streamflow at that larger time-step. However, care is needed since the distortions due to the sampling depend on how the sampling is done (e.g. mean values over a sampling interval, or instantaneous value for each sampling interval).

As modelling data time-step decreased from daily to hourly for the Plynlimon catchment in question, an IHACRES-derived slow flow index ($0\% < SFI < 100\%$) increased by about 20 percentage points. The shape of the curve described by the plotted changes invited extrapolation to a data time-step of effectively zero, giving an SFI* independent of data time-step. Corresponding values of an analogous baseflow index, which has been widely applied in regionalisation studies, increased by about 40 percentage points. Furthermore, the results exhibited no way of deriving a data time-step-independent value of this baseflow index. Slow flow and baseflow indices estimated from discrete-time data are unlikely to be consistently characteristic between catchments. Therefore, regionalisation studies that have assumed that SFI, the baseflow index, and rainfall–streamflow model parameters generally are robust when derived using discrete-time data for all gauged basins employed, may not have generated the best possible regionalisation equations. Better regionalisation equations, e.g. using SFI* and other data time-step-independent model parameters would lead to reduced uncertainty in hydrographs estimated at ungauged (flow) sites.

Further work is required to investigate this important aspect of regionalisation and catchment-scale modelling using parsimoniously parameterised rainfall–streamflow models. The TDWG TRUMPER (Towards Reducing Uncertainty in rainfall-streamflow Model Parameter Regionalisation) initiative has been established for that purpose. Amongst other things, TRUMPER will provide an opportunity to compare the results from Plynlimon outlined above with corresponding results from application of a data-based mechanistic (DBM) approach [Young and Romanowicz, 2004; Young and Garnier, 2006] whereby continuous-time parameter values, which should be independent of data time-step, are estimated using discrete-time data.

3. ESTIMATION AND PROPAGATION OF UNCERTAINTY

Data contain both information that will improve a model's performance and error that will degrade the model's performance. Consequently, evaluation of the impact a dataset has on the modelled output requires propagating the uncertainty through the model, as well as the net effect of information able to be used by the model. Andréassian *et al.* [2007] found that for a catchment in France, there was no difference in model performance between a lumped approach using areal rainfall estimate for the entire catchment, or a semi-distibuted approach where separate areal rainfall estimates for individual subcatchments were used.

While Monte Carlo techniques can easily be used to estimate the uncertainty in model outputs resulting from uncertainty in model inputs (both data and parameters) so long as the input uncertainties can be well described, the resulting computational load can be prohibitive. It is possible to use analytical techniques to propagate uncertainties through a model, though this is straightforward only if the model is linear and the errors additive and Gaussian. For analytical assessment of uncertainty in models that are sufficiently linear, only a measure of the uncertainty is needed (e.g. standard deviation, or 95% confidence limits). Note that the Bayesian Monte Carlo (BMC) approach also needs to have the distribution of the uncertainties defined; a weakness of the BMC method compared to the Markov Chain Monte Carlo (MCMC) method. BMC is generally inefficient compared with MCMC through poor sampling distribution [Qian *et al.*, 2003]. For likelihood-based methods [e.g. GLUE -Beven and Binley, 1992], a non-informative prior density is adopted (usually uniform), with the assumption that the information extracted from the posterior distribution is not significantly influenced by the choice of prior. However, the information introduced by a flat prior depends on how the model is parameterized; for example, a flat prior for parameter *a* in an exponential *exp(-at)* is not the same as a flat prior for an equivalent parameter, the time constant $\tau = 1/a$ in *exp(-t/τ)*. Shrestha *et al.* [2007] used fuzzy set methods to study the impact of uncertainty in the stage-discharge relationship on a one-dimensional hydrodynamic model, giving a method of assessing uncertainty in a non-probabilistic but subjective framework.

In some cases that are important within a hydrological context, computationally intensive Monte-carlo-based approaches to uncertainty analysis are not necessary since analytical solutions exist. The most important example in this regard the case of linear or near linear system models, as used in flow routing, where the Kalman filter [Kalman, 1960] provides for uncertainty propagation associated with the state and output variables, while recursive parameter estimation for transfer function models [e.g. Young, 1984, 2002, 2005] allows for uncertainty propagation in relation to the model parameters. Also, an important nonlinear example, which reduces the need for Monte Carlo-based analysis, is the 'Hammerstein' model, where the model consists of an input nonlinearity in series with a linear system. Such models are appropriate for characterizing rainfall-flow processes (e.g. IHACRES, HYMOD, DBM models), where the nonlinearity is associated with the generation of 'effective' rainfall, and these have been exploited [e.g. Young 2002, Romanowicz *et al,* 2006] within a flood forecasting context where uncertainty analysis is, of course, an important prerequisite. In this situation, state and parameter estimation for the linear component, as well as the associated uncertainty propagation, can be accomplished using slightly modified Kalman filter algorithms. The only requirement for numerical methods is for uncertainty propagation in relation to parameters of the serial nonlinearity.

In the case when the model output is a simple function of one input:

$$y = f(x) \tag{1}$$

Croke [2007] showed that, through Taylor series expansion of *f*, the standard deviation of *y* can be obtained analytically providing sufficient information is known about the distribution of the uncertainty in *x*. If the function *f* is sufficiently close to linear over the range of uncertainty in *x,* only the first term in the Taylor series expansion is needed:

$$\sigma_y^2 \approx \left(\frac{df}{dx}\bigg|_{x=\bar{x}} \right)^2 \sigma_x^2 \tag{2}$$

However, if the function *f* is sufficiently non-linear (e.g. the function $y=x^2$ near $x=0$) , then the second derivative at least will be needed:

$$\sigma_y^2 \approx \left(\frac{df}{dx}\bigg|_{\bar{x}} \right)^2 \sigma_x^2 + \frac{df}{dx}\bigg|_{\bar{x}} \frac{d^2 f}{dx^2}\bigg|_{\bar{x}} {}^3M_x - \frac{1}{4}\left(\frac{d^2 f}{dx^2}\bigg|_{\bar{x}} \right)^2 \left\{ \sigma_x^4 - {}^4M_x \right\} \tag{3}$$

$${}^nM_x = \sum_{i=1}^{N} \left\{ \frac{(x_i - \bar{x})^n}{N-1} \right\}$$

This result can be generalised to consider the distribution of uncertainty. Taking

$$x = \bar{x} + \varepsilon_x, \qquad\qquad y = \bar{y} + \varepsilon_y \tag{4}$$

the deviation ε_y of *y* from its mean is approximated by:

$$\varepsilon_y = y - \bar{y} = f(\bar{x}) + \sum_{i=1}^{N} \frac{\varepsilon_x^{\,i}}{i!} \frac{d^i f}{dx^i}\bigg|_{x=\bar{x}} - f(\bar{x}) - \sum_{i=2}^{N} \frac{\text{mean}(\varepsilon_x^i)}{i!} \frac{d^i f}{dx^i}\bigg|_{x=\bar{x}} \tag{5}$$

$$= \sum_{i=1}^{N} \frac{\varepsilon_x^{\,i}}{i!} \frac{d^i f}{dx^i}\bigg|_{x=\bar{x}} - \sum_{i=2}^{N} \frac{\text{mean}(\varepsilon_x^i)}{i!} \frac{d^i f}{dx^i}\bigg|_{x=\bar{x}}$$

for some finite *N*. For highly non-linear models, additional information on the distribution of the uncertainty is needed (e.g uniform, or normally distributed). Croke [2007] showed that if the second-order term in the Taylor series expansion of the model is needed, then the 3rd and 4th moment about the mean of the distribution is needed in order to calculate the standard deviation of the model output, therefore requiring more information regarding the distribution of the uncertainty in the input.

4.   HOW TO ESTIMATE THE REDUCTION IN UNCERTAINTY?

The relative uncertainty before and after new data have been assimilated into the model provides a quantification of the information content in the new data that the model is able to utilise. This uncertainty reduction can be measured using multiple approaches, partially depending on the approach to uncertainty estimation chosen, namely statistical versus set-theoretic approach. Another question is: Which uncertainty is reduced? This is mainly the uncertainty in the model predictions, but it could also be the uncertainty in the model parameters or in some other item which the model is required to reproduce well, e.g. flow-duration curve for a runoff model.

A simple approach that works if the distributions of parameters or individual output variables are not too complex is to compare the prior (before new data are assimilated) and posterior (after new data are assimilated) standard deviations or variances of individual distributions. The ratio of before to after standard deviations provides a direct quantification of learning [Vrugt *et al.*, 2005].

An alternative approach can be used to assess reduction in parameter uncertainty if a set-theoretic approach to parameter estimation has been utilized [Freer *et al.* 2004]. The result of a set-theoretic approach is a population of behavioural parameter values, i.e. values

causing the model to meet one or more requirements.. Adding new data should lead to a reduction in the size of the behavioural parameter population. Wagener *et al*. [2002; 2003] quantified this reduction by comparing the 90% percent confidence limits of before and after behavioural distributions of individual parameters.

If the reduction in the uncertainty in ensemble predictions of some output variable time series is of interest, then the improvement in sharpness can be calculated as suggested by Yadav et al. [2007]. The unconditioned parameter ranges for a model can be used to produce prior ensemble predictions. For each time step these predictions represent 0% sharpness. Assimilating data into the model, which reduces the parameter ranges and therefore usually the subsequent ensemble prediction uncertainty should lead to an increase in sharpness. This increase of sharpness is related to a reduction of the ensemble uncertainty range and 100% sharpness would mean the ensemble prediction has been reduced to a single trace.

The approaches mentioned in this section enable a quantification of the uncertainty reduction in a relative sense, before versus after assimilation of new data. How valuable new data are for this purpose largely depends on how successful a chosen approach is in extracting this information. The next section will discuss the role of objective functions or performance metrics or likelihood functions in this context.

5.   HOW TO MAXIMIZE INFORMATION EXTRACTION FROM DATA?

5.1   Performance metrics

A number of metrics are used to evaluate the performance of models, with many based on the sum of squared errors (SSE). In hydrology, a commonly used objective function is the Nash-Sutcliffe efficiency [NSE, Nash and Sutcliffe, 1970]. Garrick *et al.* [1978] suggested two modifications to the NSE:

- replacing the mean flow as the "no model" forecast with the mean value at the current date (and time) for each sampling interval calculated from the period being considered (averaging could be at a different resolution than that of the modelled and observed values; for example, using a seasonal average flow). Note that this requires a sufficiently long period to permit calculation of the mean, which will potentially be a problem for selection of a calibration period). Ideally, the current timestep should not be used in calculating the mean.

- including an updating procedure for real-time forecasting applications

Such objective functions do not routinely account for the uncertainty in the observed and modelled values being compared. Rather, they assume particular behaviour of the errors (most assume homoscedasticity) and as a result, their validity as performance indicators depends on the validity of this assumption. In the case of hydrological models, the assumption of homoscedasticity is usually not valid. A simple approach for correcting for the heteroscedasticity in the model residuals is through the use of the Box-Cox transformation [Box and Cox, 1964]:

$$\tau(Y;\lambda,\alpha) = \begin{cases} \left((Y+\alpha)^{\lambda}-1\right) & \text{if } \lambda \neq 0 \\ \ln(Y+\alpha) & \text{if } \lambda = 0 \end{cases} \qquad (6)$$

where α is a shift parameter (often set to zero). While this approach can significantly reduce the heteroscedasticity, the uncertainty is assumed to depend only on the current value of Y. If there is significant correlation between the uncertainties and other quatities (e.g. when there is hysteresis) the Box-Cox transformation may not be adequate. The result obtained from applying the Box-Cox transformation is also model dependent as the influence of model structure uncertainty is also removed. Consequently, different models will employ different values of λ, and comparing the performance of models will be difficult.

Another approach involves weighting the model residuals by their uncertainty [Croke, 2007]. This ensures that the variance in the weighted residuals is constant, as long as the estimates of the variability in the uncertainty are sufficiently accurate. This has similarity to optimal (inverse-covariance) weighting in minimum-covariance, linear, unbiased estimation (generalized least-squares regression) [e.g. Aitken, 1935 and Plackett, 1949].

5.1.1   Modified objective functions

Croke [2007] showed that objective functions can be modified using an optimal weighted average approach to explicitly include the influence of uncertainties in the modelled and measured quantities. Here, the following definitions will be used:

- observed value at timestep i: $x_{o,i}$

- modelled value at timestep i: $x_{m,i}$

- model residual at timestep i: $e_i = x_{o,i} - x_{m,i}$

- observed deviation from mean: $d_{o,i} = x_{o,i} - \bar{x}_o$

For example, the Nash-Sutcliffe model efficiency R^2 [Nash and Sutcliffe, 1970] is:

$$R^2 = 1 - \frac{\sum_i^n e_i^2}{\sum_i^n d_{o,i}^2} \qquad (7)$$

The modified Nash-Sutcliffe efficiency is given by:

$$R_m^2 = 1 - \left[\frac{\sum_i^n w_{e,i} e_i^2 \Big/ \sum_{i=1}^n w_{e,i}}{\sum_i^n w_{d,i} d_{o,i}^2 \Big/ \sum_{i=1}^n w_{d,i}} \right] \qquad (8)$$

where the weights $w_{d,I}$ and $w_{e,I}$ are:

$$w_{d,i} = \frac{1}{\lambda_{o,i} \gamma_{f_{o,i}^2}}, \qquad w_{e,i} = \frac{1}{\alpha_i \gamma_{e_i^2}} \qquad (9)$$

and α_i and $\lambda_{o,I}$ are:

$$\alpha_i = \left(\Delta e_i\right)^2 = \left(\Delta x_{o,i}\right)^2 + \left(\Delta x_{m,i}\right)^2 \qquad (10)$$

$$\lambda_{o,i} = \left(\Delta d_{o,i}\right)^2 = \left(\Delta x_{o,i}\right)^2 + \left(\Delta \bar{x}_o\right)^2 \qquad (11)$$

where Δ represents the uncertainty in each quantity, and γ is the correction for non-linearity. For example, for a second order correction, γ for estimation of the standard deviation (based on equation 3) is:

$$\gamma_{x_i^2} = 1 + \frac{\left.\frac{df}{dx}\right|_{\bar{x}} \left.\frac{d^2 f}{dx^2}\right|_{\bar{x}} {}^3 M_x + \frac{1}{4}\left(\left.\frac{d^2 f}{dx^2}\right|_{\bar{x}}\right)^2 \left\{\sigma_x^4 - {}^4 M_x\right\}}{2x\Delta x} \qquad (12)$$

5.1.2   Example

The modified NSE described above was tested using a Monte Carlo approach [Croke, 2007], employing synthetic data generated by the IHACRES rainfall-runoff modelling software [Croke *et al.* 2006]. Random errors in the "observed" flow were added by introducing random errors in the parameters of the rating curve, thereby introducing highly heteroscedastic errors with significant serial correlation. Only error in observed flow was considered; propagation of uncertainty in the rainfall through the model (e.g. uncertainty framework of Huard and Mailhot, [2006]) is still under development (Mungkasi *et al.* in prep). For this test, the parameters of the linear module (unit hydrograph) were fixed, with only 2 parameters in the non-linear module (τ_w and c) optimised.

Figures 1 and 2 show the resulting optimised parameter values for 100 trials, and clearly demonstrate that, given a good estimate of the uncertainty in each timestep, the modified NSE gives a much smaller uncertainty in the calibrated parameter value. This shows the importance of a good estimate in the uncertainty in the model inputs.
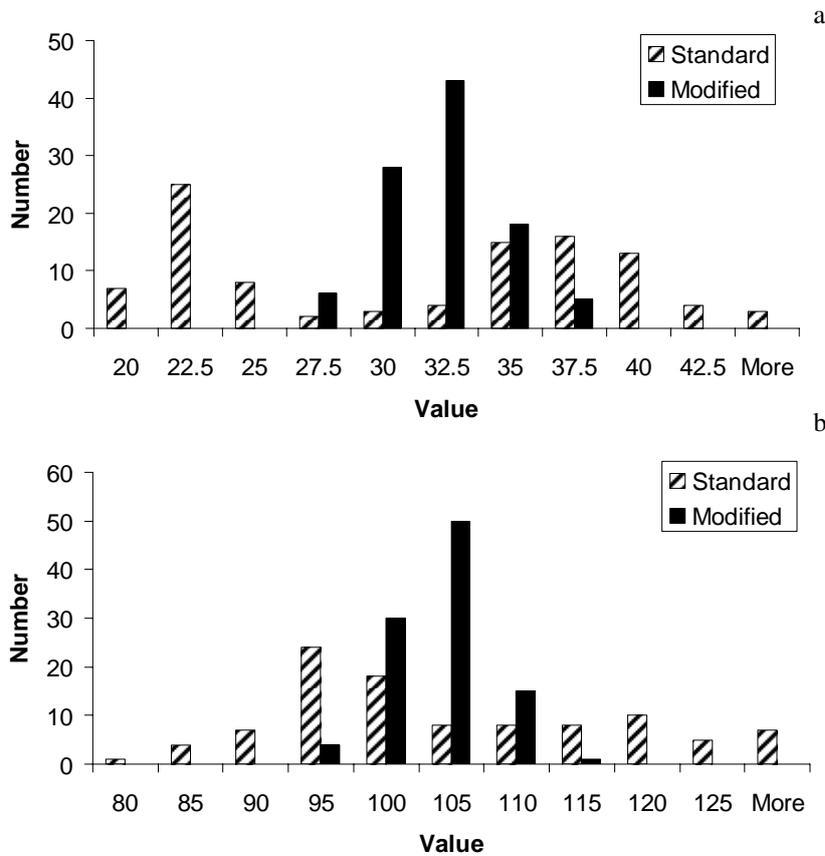

Figure 1. Calibrated parameter values for (a) the catchment drying rate parameter (τ_w, nominal value 30) and (b) the catchment storage parameter (*c*, nominal value 100).

Weighted ordinary least squares (OLS) estimation techniques have a long history [e.g Aitken, 1935, Plackett, 1949, 1950]. Williams and Yeh showed that generalised least squares (GLS) is a better technique for estimating parameter values in rainfall-runoff models than ordinary least squares (OLS), with GLS permitting weighting of inputs to the model (though they note that the effects of noise can "render the procedure inaccurate or inoperative"). There is also a long history of dealing with noise in recursive GLS or the equivalent; for example by adjoining a noise structure model [Plackett, 1950; Kalman, 1960].

There are a number of approaches that have been developed to address the influence of input uncertainty on model calibration [e.g. Beven and Binley, 1992; Kavetski *et al.* 2002; Huard and Mailhot, 2006]. However, to date the assessment of model performance has ignored uncertainties in the observed and modelled values.

5.1.3   Inclusion of Significance of Model output in Performance Metric

An extension to the modified objective functions can be made by considering the significance of different parts of the model output, given the question the model is attempting to answer. This can be done either with a binary switch (1 for time steps to be considered, 0 for time steps to be ignored) or a weight set for each time step. This is a minor correction to the modified NSE given above:

$$R_m^2 = 1 - \left[\frac{\sum_i^n s_i w_{e,i} e_i^2}{\sum_i^n s_i w_{d,i} d_{o,i}^2} \right] \left[\frac{\sum_{i=1}^n s_i w_{d,i}}{\sum_{i=1}^n s_i w_{e,i}} \right] \tag{13}$$

where s_i is the significance of each time step. The significance of each time step can be selected based on any desired criteria (e.g. discharge rate, slope of hydrograph etc). This gives the modeller the ability to focus the calibration and testing of the model to specific parts of the hydrograph. The result is that the original Nash-Sutcliffe efficiency assumes that the significance is inversely proportional to the weighting applied. As the weights are themselves inversely proportional to the square of the uncertainty, the conclusion is that the original Nash-Sutcliffe efficiency adopts a significance for each point that is proportional to the square of the uncertainty in the corresponding values for that point. Further, the weights for the numerator and denominator are the same, implying negligible uncertainty in the modelled flow (and mean of the observed flow).

This approach allows for considerable flexibility in the objective function at the cost of difficulty with comparing results for different studies. As a guideline, it is recommended that the objective function be evaluated using a uniform significance. While this will permit direct comparison of model performance, care should be exercised in this comparison as model parameters will not necessarily have been optimised for the case of uniform significance.

5.2   Model structure and complexity

Dividing the residuals by their estimated uncertainty will enable assessment of the suitability of the model structure and parameterisation providing that the uncertainties are sufficiently well known. There are two possible tests that can be performed: a check for heteroscedasticity in the scaled residuals (testing the model structure) and assessment of the magnitude of the scatter in the scaled residuals (testing model complexity).

If the scaled residuals (each residual divided by its estimated uncertainty) show significant heteroscedasticity, this indicates either that the model is introducing heteroscedasticity (e.g. by not adequately representing the impact of rainfall intensity, or antecedent conditions), or that there is a systematic error in the estimated relative uncertainties. As with the modification to objective functions discussed in section 4.1, this test only requires sufficiently accurate estimation of the relative uncertainty (i.e. the variations in the uncertainty) in the residual at each time step. Statistical approaches such as the Box-Cox transformation or the BATEA methodology cannot be used to assess the model structure as these approaches combine the influence of such errors with the uncertainty in the modelled and measured flows.

The second test of a model demands more accuracy in the estimation of the uncertainty in the residuals, as the absolute uncertainty rather than the relative uncertainty is needed. By dividing each residual by its estimated uncertainty, the resulting scaled residual has an uncertainty given by the ratio of the actual and estimated uncertainty in the residual. Providing that the magnitude of the estimated uncertainty is sufficiently close to that of the actual uncertainty, the result is a set of scaled residuals, each with an uncertainty of approximately 1. The distribution of the ensemble of scaled residuals can therefore be investigated to test whether the complexity of the model is appropriate for the system being modelled. Assuming, for example, that the uncertainty estimates correspond to the standard deviation, the standard deviation of the scaled residuals should be approximately 1. If the standard deviation is significantly greater than 1, then providing that the estimated

uncertainties are sufficiently close to the actual uncertainties, the model is not adequately representing the response of the system, and is subsequently too simple. Conversely, if the standard deviation of the scaled residuals is significantly less than 1, then the model is over-fitting the data, and the model is too complex. Similar conclusions will result from other definitions of the uncertainty (e.g. 95% confidence bounds). Unfortunately, the uncertainty in the residuals cannot be accurately known, so the utility of this model test is reduced. In order to use this technique to assess model parameterisation, the modeller needs to be able to place confidence limits on the uncertainty estimates. If the uncertainty values are expected to be accurate to within 10%, then an acceptable model would have a value for the standard deviation of the scaled residuals between roughly 0.9 and 1.1.

5.3 Increasing information extraction through breaking –up and transforming streamflow time-series

Two other approaches can be used to increase the amount of information that can be extracted from time series during the calibration process. Breaking up the time series into components [e.g. Boyle *et al*., 2000; Wagener *et al.*, 2001; 2003; Vrugt *et al.*, 2006] and transforming time series to put the emphasis on calibrating the model to different aspects of the system response [e.g. Gupta *et al*., 1998; van Werkhoven *et al.*, 2008]. Both approaches can utilize standard performance metrics as discussed above. In the context of this paper it is assumed that the streamflow is the response variable of interest for simplicity.

Wagener *et al*., [2003] showed that calculating a performance metric like the Root Mean Squared Error (RMSE) on the residuals of the whole time series leads to a loss of information that could otherwise discriminate between different models or parameter sets. This problem can be reduced if the hydrograph is broken up into different periods and the performance metric is calculated for each period separately [e.g. Boyle *et al*., 2000; Wagener *et al*., 2001; 2003; Vrugt *et al*., 2006]. The break-up can be based on: [1] experience with a specific model so that different periods are likely controlled by different parameters, [2] based on hydrological understanding so that different periods represent different system response modes, or [3] based on an analysis of the information content so that periods with high information content for different parameters are separated out.

An alternative approach is the use of data transformations before model residuals are calculated. One method which has already been mentioned is the use of Box-Cox transformation [Gupta *et al.*, 1998]. Calculating the RMSE on the untransformed data leads to a focus on high flows (where the uncertainty is greatest), while doing the same on the transformed data leads to a focus on lower flows. A more process-based approach is the transformation of the observed and simulated streamflow data into hydrologically relevant signatures, and to calculate the performance metric on these signatures. Van Werkhoven *et al.* [2008] provides an example where performance metrics were calculated on water balance signatures (runoff ratio) and on signatures of the flow duration curve (central slope). The study showed that significantly more information became available for model identification than only using a traditional RMSE measure.

6. EXAMPLES

6.1 Assimilation of soil wetness data

The European project AIMWATER (Analysis, Investigation, and Monitoring of WATER Resources, http://dataserv.cetp.ipsl.fr/AIMWATER/) investigated the assimilation of additional soil wetness information into parsimoniously parameterized rainfall–streamflow models. For a catchment in France Aubert et al. [2003] showed how assimilation of soil wetness information from at-a-point Time Domain Reflectivity (TDR) measurements led to improvements in streamflow forecasting.

However, when a 'standard' IHACRES loss module was replaced by a more deterministic loss module, which in turn was calibrated using 'soil wetness' derived from satellite observations, unpublished work indicated that there was no clear improvement in model-fit to streamflow for a catchment in the UK. The law of diminishing returns appeared to apply. Given that the 'standard' IHACRES model with inputs of only rainfall and air temperature

already gave a reasonably good model-fit to streamflow, it would always be difficult to improve upon that. The fact that 'soil wetness' derived from satellite data, applied in a replacement loss module, did not lead to improved estimation of streamflow could be because the remotely-sensed data did not contain sufficiently good information, or because of the more deterministic loss module, or both. However, from a PUB perspective, remotely-sensed data may assist in setting parameter values of the water balance component of a rainfall runoff model when streamflow data is not available.

6.2   Impact of rainfall data quality on model performance and parameter values

The impact of rainfall and streamflow data quality on model performance has been extensively studied [e.g. Faurés *et al.*, 1995; Hansen *et al.*, 1996; Chaubey *et al.*, 1999; and Andréassian et al., 2001]. Faurés *et al.*, [1995] found that the uncertainty in modelled flow was related to the number of raingauges, while Hansen *et al.* [1996] found that the primary factors affecting model performance were rain gauge density, stream gauge rating quality, catchment response dynamics (particularly the baseflow fraction). In a study based in the Little Washita Basin, Chaubey *et al.* [1999] found that the uncertainty in the AGNPS parameter values were strongly dependent on the spatial variability of the rainfall. Andréassian *et al.* [2001] studied the impact of imperfect rainfall data for three catchments on three hydrological models (GR4J, and derivatives of IHACRES and TOPMODEL), finding that the models used were able to correct for imperfect rainfall input estimates. For predicting flows at ungauged locations however, the variations in the imperfections in the rainfall data between catchments need to be taken into consideration.

Vaze et al. [2008] report the results of a study in which two models, SIMHYD and Sacramento were calibrated to ten catchments in the Murray-Darling Basin (see Figure 1) using rainfall data derived using four different methods:

1.   Single raingauge;
2.   Arithmetic mean of multiple raingauges;
3.   Weighted mean of multiple raingauges (Theissen polygon);
4.   5km gridded rainfall.

Vaze et al. [2008] showed that the quality of the rainfall-runoff models (as measured with Nash-Sutcliffe efficiency, E) improved as the quality of the input rainfall improved (as determined by greater spatial representation of rainfall). They also showed that this improvement was greater for larger catchments than for smaller catchments and hypothesised that this was because individual raingauges are more likely to miss rain events entirely in larger catchments.

The performance of the models was evaluated, both over the calibration and an independent simulation period and the suite of model parameters produced using each method was derived.

The impact of this possible range of model parameters on the ability to regionalise hydrologic response was evaluated by building models with all possible model parameter sets and comparing the modelled streamflow from each of these models with observations over the simulation period.

7.   DISCUSSION AND CONCLUSIONS

The uncertainty in the output from a model is a result of uncertainty in model inputs, parameter values, the observed outputs of the system, the suitability of the model structure, and the limitations in the information provided to the model by the input data. The impact additional data has on model uncertainty depends on the information content and uncertainty in the data used. In particular, higher spatial and/or temporal resolution leads to an increase in the information content, but at the cost of a potential increase in the uncertainty. For example, estimates of areal rainfall at a higher spatial resolution without increasing the number of raingauges will result in an increase in the uncertainty of the resulting rainfall estimates due to the reduction in averaging between gauges. Similarly, estimates at higher temporal resolution will have higher uncertainty due to the more variable nature of rainfall at shorter timescales. Introducing additional data (e.g. more

raingauges, or radar estimates of rainfall) has the potential to reduce the uncertainty in the rainfall estimates. The influence of additional data on a model's performance depends on:

1. the model's ability to extract information from the data

2. the sensitivity of the model performance to the uncertainty in the data (it may not be possible to separate this from the previous item).

3. how the model uses the additional data. If the data is used to constrain model parameters, then model performance will be improved providing the constraints become active at some point, or the model is used to bound the output and performance is measured by the width of those bounds. If the model is modified to incorporate the new data, then the model performance may or may not be improved depending on points 1 and 2 above.

The aim of model calibration is not to fit the data; rather it is to use the data to quantify (in terms of the model structure) the catchment response characteristics. This can only be achieved if the assessment of a model's performance takes into account the uncertainty in the model's output and the observations with which they are being compared. All performance criteria make some assumption regarding the distribution of the uncertainty (most assume constant additive uncertainty). If the distribution of the uncertainties is significantly different than what is assumed, then the performance criterion is biased. In the example given here, due to the highly heteroscedastic nature of the uncertainty in flow data, the NSE is biased towards data with highest uncertainty, with the bias proportional to the uncertainty squared. The result is that the NSE is biased towards high flows, as these have the greatest additive uncertainty.

The Box-Cox transformation attempts to correct for this bias, by transforming the quantities being compared (in this case, observed and modelled flow) to significantly reduce the heteroscedasticity in the model residuals. While this is a simple and effective technique to reduce the bias in the performance criteria, the adopted transformation is based on the aggregated impact of all contributors to the heteroscedasticity in the model residuals. Consequently, the significance of each contributor cannot be determined without additional information or assumptions.

An alternative approach is to consider the uncertainties in the observed output(s) of the system, as well as all model inputs and how these propagate through the model. One such approach is BATEA, where the distribution of uncertainties is fitted at the same time as the model parameters are estimated. A different approach to consideration of the uncertainties is to explicitly represent these in the performance criteria. While this is a considerable task, the advantage is that there is considerably more flexibility regarding the nature of the uncertainties, and a measure of the suitability of the model complexity to the application can be derived (it should be noted however, that the conclusions depend on the confidence in the uncertainty estimates). A Monte Carlo trial with synthetic data with error in the rating curve added showed that this approach yields significantly less uncertainty in the parameter estimates, and hence in the model predictions.

There are a number of techniques that have been developed for estimating the reduction in uncertainty resulting from the assimilation of new data, including measuring the reduction in the variances of distributions of model parameters or outputs, reduction in the volume of a behavioural set to estimation of the sharpness of the model predictions. These techniques allow the relative reduction in uncertainty to be quantified.

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