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Optimal Combined Long-Term Facility Design and Short-Term Operational Strategy for CHP Capacity Investments

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Abstract

This work presents a detailed case study for the optimization of the expansion of a district energy system evaluating the investment decision timing, type of capacity expansion, and fine-scale operational modes. The study develops an optimization framework to find the investment schedule over 30 years with options of investing in traditional heating sources (boilers) or a next-generation combined heat and power (CHP) plant that provides heat and electricity. In district energy systems, the selected capacity and type of system is dependent on demand-side requirements, energy prices, and environmental costs. This work formulates capacity planning over a time horizon as a dynamic optimal control problem considering both operational modes and capital investment decisions. The initial plant is modified by the dynamic optimization throughout the 30 years to maximize profitability. The combined optimal controller and capital investment planner solves a large scale mixed integer nonlinear programming problem to provide the timing and size of the capacity investment (30 year outlook) and also guidance on the mode of operation (1 hour time intervals). The optimizer meets optimal economic, environmental, and regulatory constraints with the suggested design and operational guidance with daily cyclical load following of heat and electricity demand.

Keywords: Combined heat and power, Dynamic optimization, Optimal design, Optimal control, Combined scheduling and control

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1. Introduction

Energy producers and end-users face many challenges as they balance energy demand, economic uncertainty, and environmental regulations. Both private and public utility companies are often pressured into making investment decisions due to the economic and energy demand variability in the presence of increasing environmental regulation [1, 2]. Sometimes these sub-optimal decisions include delayed investments which can result in losses in long-term economic benefits. Combined heat and power (CHP) plants, or cogeneration plants, provide an efficient method to balance the multiple energy requirements [3, 4, 5]. CHP plants accomplish this feat by simultaneously providing heat and generating power through a single fuel source [6]

While considering CHP implementation, stakeholders must consider expanding traditional combustion units, pollution reduction equipment [7, 8, 9], inclusion of renewable generation [10, 11], electricity-only systems, energy storage [12], and constructing new CHP units entirely [2]. When these options are coupled with daily dynamics such as demand, fuel cost, and uncertainties such as regulation and price, a dynamic optimization approach which can account for short-term operational scenarios as well as long-term capacity planning is one possible method to deliver improved solutions for the problem. This work presents a precise case study demonstrating the use of such an approach to deliver an improved solution. Much research has been done to effectively account for both short-term operation and long-term strategy [13, 14, 15, 16, 17, 18, 19, 20, 21]. The model in this work provides a long-term planning horizon for a CHP plant in a district energy system that uses constrained differential equations to describe capacity investments over the horizon. This long-term model is coupled with an hourly dynamic optimization approach to provide methodology that considers short term operational constraints as well as long-term investment strategy that is optimized simultaneously alongside demand and price forecasts. This formulation solves as a single-cycle of model-predictive control considering a 30 year horizon as well as hourly operations.

2. District Energy Systems

District energy systems provide a combination of electrical distribution, heating, and cooling to surrounding areas from a central location [22, 23, 24, 25]. These systems often include energy storage [26, 27] or shifts in production to accommodate for changing energy demands [28, 29]. The energy demand, system constraints, and system dynamics of the given energy network determines the overall effectiveness of the plant. A CHP system is identified by the type of equipment, or prime mover, that drives the overall process. Combustion gas turbines, steam turbines, reciprocating engines, and fuel cells all represent various CHP prime movers that use a variety of fuel sources [3].

CHP systems with combustion turbine generators (CTG) are popular prime movers for systems requiring more than 1 MW electric load. CTG systems employ a compressor, combustor, and turbine to supply electric loads from 1

44 MW to 100 MW [3]. Leftover energy from the CTG hot exhaust gas is used
 to create necessary hot water or steam from a heat recovery boiler, instead of
 46 using separate utilities as the heat source. Supplemental burners and boilers
 are used if extra heating is needed to meet the system heating requirements.
 48 Separate heat and power generation systems have relatively low overall efficiency
 of about 49% [6]. Whereas, the heat recovery portion of a CTG CHP system
 50 has about 80% efficiency, giving the CHP an overall efficiency of about 75% [30].
 District energy systems yield more optimization opportunities by simultaneously
 52 providing for electrical, heating, and cooling demands [28, 31]. However, the
 new demands and constraints of these systems offer more complex problems
 54 and require a more diverse range of system models. Optimization methods have
 been used to increase energy savings and reduce costs, but similar methods
 56 have yet to be applied to a district energy system for simultaneous capacity
 planning and dynamic capacity utilization. This work demonstrates the use of
 58 a formulation that simultaneously accounts for long-term capacity planning and
 hourly capacity utilization.

60 **3. Optimization of District Energy Systems**

The design capacity of a power generation system is the maximum produced
 62 energy per unit time [32]. Energy generation systems tend to be oversized over
 typical load scenarios because enough adequate energy must be generated when
 64 unusual peak demand episodes occur. Meanwhile, in most cases, thermal energy
 cannot be imported from an outside source as easily as electricity is imported
 66 from the power grid. This forces district energy systems with heating loads to
 provide dispatchable capacity to meet thermal demands. The design capacity
 68 of an energy system can be fulfilled with a single large capacity unit or multi-
 ple smaller capacity units, which can be staged on or off as demand changes.
 70 Growing energy demands and regulations on existing energy production fleets
 require that new investments in design capacity be considered. At the same
 72 time, energy demand is uncertain because of weather and economic factors. Ca-
 pacity expansion projects are mostly considered irreversible investments because
 74 of high capital cost while the plants remain available for an extended period of
 time. An aging fleet of coal and natural gas burning boilers provides motivation
 76 to plan for new prime movers in district energy systems.

Lozano et al. [33] reports that traditionally there are two main approaches
 78 to the design capacity and integration of an energy system: 1) a deterministic
 thermodynamic approach and 2) a mathematical programming approach. A de-
 80 terministic thermodynamic approach takes advantage of the physical equations
 that govern the thermodynamics of the system. This approach solves the equa-
 82 tions based on energy demands and design equations of the system components,
 such as boilers, heat exchangers, evaporators, etc. Pinch analysis methods, also
 84 known as heat integration or process integration, are used to find the most
 efficient configuration of a prescribed energy system based on thermodynamic
 86 approaches [34]. A pinch analysis approach is often applied to optimize energy
 consumption when designing chemical or energy generation processes. Instead

88 of providing heating and cooling utilities, energy consumption can be reduced
by calculating thermodynamically feasible energy targets because energy targets
90 are absolute thermodynamic values [35].

Mathematical programming has been used extensively for decades to opti-
92 mize and plan complex systems in many different areas, including energy sys-
tems. The mathematical programming approach to design and integration of
94 energy systems uses optimization algorithms to search for a feasible design ac-
cording to specified criteria [36]. If the aim is to minimize cost and plan for
96 optimum capacity of a district energy system, the program must calculate cost
and represent the capacity with mathematical equations of the energy output of
98 the system. The economic and mathematical representation of the dynamics and
physical system can be done through first principles or empirical (data-driven)
100 models or a mixture of both (hybrid, or semi-empirical) models.

Reviews in the design and planning of complex energy generation systems by
102 Chicco [37] and Mancarella [38] detail mathematical programming formulations
used in the literature to solve optimal energy capacity and planning problems.
104 The mathematical formulations included linear programming, genetic and evo-
lutionary algorithms [39], branch and bound for discrete formulations, Pareto
106 optimizations, and Lagrangian-based algorithms. The survey also indicates that
problems are typically solved either by looking at short-term or long-term time
108 frames in the optimization time horizon. This separation between long-term
and short-term time horizon in the optimization formulation illustrates that a
110 multi-time scale, also known as multi-stage, optimization approach framework
is yet to be fully developed, thus providing greater motivation for the value of
112 the optimization framework utilized in this work.

Within the mathematical programming approaches, mixed integer program-
114 ming (MIP) [40], mixed integer linear programming (MILP) [33, 41, 42, 43, 44],
or mixed integer nonlinear programming (MINLP) methods [45] are used ex-
116 tensively in the literature for deterministic optimal capacity and configuration
of energy systems. Enterprise-wide dynamic optimization seeks similar objec-
118 tives but with the use of large-scale nonlinear programming (NLP) solvers [46].
P. Arcuri et al. [41] present an approach that separates the short-term (annual
120 year) versus the long-term (lifetime) use of the energy system. The optimal so-
lution is then derived through an iterative scheme, which allows for the optimal
122 size and configuration of the system that maximizes operational margins over
the operational lifetime. Du et al. [13] present a method to bridge time scales
124 for moderate-term (scheduling) and short-term (control) decisions. Combining
scheduling and control is an active area of focus with several recent contribu-
126 tions [14, 15, 16, 17, 18, 19]. Contributions by Ricardez-Sandoval [47] and Kim
et. al [48] have developed frameworks to simultaneously design and control
128 large-scale systems under process parameter uncertainty through a sequential
method of simulations of the dynamic control problem and the subsequent non-
130 linear optimization of the steady state first principles model. An optimal design
of the system is found after iterations of perturbation of the control simulation
132 and the subsequent optimization of the system model to update the dynamic
control parameters. Such an approach provides a framework for optimal system

134 design that can satisfy both steady state and transient operations. Long term
 135 uncertainty in costs or demand forecast for the system are omitted, thus leav-
 136 ing a system that can be economically attractive within the short term dynamic
 horizon, but economically infeasible in the lifetime of the system when economic
 138 and demand conditions may change drastically.

It is evident from the examples above that researchers have attempted to
 140 resolve the long-term and short-term challenges of optimal equipment configura-
 tion and energy system capacity. This work demonstrates the use of a method
 142 to simultaneously optimize a system with short term (1 hour operational in-
 tervals) dynamic objectives and long term (30 year planning period) economic
 144 and demand forecasts through an optimal control framework. This method is
 demonstrated in a precise case study of capacity expansion for a CHP facility
 146 over a 30-year horizon.

4. Long-term Capacity Investment Planning

148 Decision making involves consideration of multiple objectives and uncertain
 outcomes; and in many situations, decisions must be made at different times
 150 and at different levels. These types of problems are generally referred to as
 multi-objective decision processes under uncertainty [49]. Planning for capacity
 152 and configuration upgrades is important because stakeholders must balance the
 opposite objectives of economic viability, security of supply, and environmental
 154 regulations over time horizons spanning decades. In the utility power generation
 sector, the objective of generation planning is to determine the number of units,
 156 the timing of the investment, and the capacity of the overall system, all while
 minimizing fixed and production costs [50]. The same challenges are faced by
 158 district energy systems when expansion or retrofits to more efficient options,
 such as CHP, are contemplated.

160 The long-term energy system expansion planning problem is a highly nonlin-
 ear discrete dynamic problem [50]. In past decades the majority of the methods
 162 to solve problems dealing with power generation planning have been related
 to stochastic mathematical programming [51]. These stochastic methods in-
 164 clude dynamic programming, multi-stage stochastic programming (MSSP), and
 stochastic optimal control [1, 52, 53, 54]. The stochastic programming meth-
 166 ods have the advantage of explicitly incorporating the probabilistic nature of
 the problem into the optimization algorithm. The disadvantages include lin-
 168 earization of the problem, which is then solved through linear programming,
 and simplifications to the probabilistic trees that account for the uncertainty
 170 [50]. Without simplification, stochastic programming methods suffer from the
 curse of dimensionality, which makes the problem intractable in finding a so-
 172 lution [49, 50, 55]. The absence of the nonlinear fidelity and dynamics of the
 problem may increase the difficulty of finding an optimal solution that will have
 174 the resolution to fulfill the short term (hourly energy consumption) and long
 term (capacity and profitability) requirements of the capacity planning problem.
 176 Fuss et al. [55] used a real options valuation approach to find a solution in other

178 similar problems of investment and planning for power generation systems un-
 179 der uncertainty. They reported that a stochastic programming approach would
 180 have resulted in the same outcomes as those obtained in a real options approach.
 181 Their work also reports that the main reason for not using stochastic methods
 182 was the increased computational intensity due to the dimensionality increase
 when there are many periods and scenarios as well as a modest state space.

Other less common, but reported, methods in the literature that account
 184 for uncertainty in power generation systems expansion and investment planning
 include fuzzy logic [56] and Monte Carlo simulations [57]. Others have used
 186 a combination of methods to account for uncertainty [12]. For example, joint
 probabilistic programming and fuzzy possibility programming are used by Liu
 188 et al. [51] in an optimization approach for power generation planning under un-
 certainty in a mixed integer linear programming (MILP) framework. Another
 190 combined method to account for uncertainty is to use an MSSP and fuzzy linear
 programming in an MILP framework [58]. The benefits of such formulations is
 192 in describing uncertainties in terms of interval values, fuzzy sets, and probabili-
 ty distributions. In energy system planning under uncertainty, the combined
 194 method approach can reflect dynamic decisions for facility-capacity expansions
 and energy supply over a multistage context [58].

196 Multistage approaches refer to mathematical models that integrate spatial
 and temporal dimensions, or two or more temporal dimensions with signifi-
 198 cantly different scales [59]. In planning and scheduling problems [60], the han-
 dling of multiple time scales is mostly handled through simplification of the
 200 asynchronous time scales to a single representative scale or the separation of
 the problem into discrete periods. Such simplifications mask the short term
 202 response of the system and shift the optimization towards systems that overes-
 timate capacity and favor long term inputs.

204 5. Dynamic Optimization for Energy Systems

Dynamic optimization constitutes a methodology to optimize systems rep-
 206 resented by dynamic models in the form of differential and algebraic equations
 (DAE). The optimization algorithms for dynamic optimization may have non-
 208 linear objective functions and constraints with continuous or integer variables.
 Dynamic optimization is an integral part of some advanced control algorithms
 210 such as Model Predictive Control (MPC). MPC is an accepted advanced control
 technique that utilizes deterministic process models to predict future responses
 212 of a system [61]. The process models used in MPC are sometimes dynamic
 and nonlinear and capture the dynamic and static interactions between inputs,
 214 outputs, and disturbances affecting the system [62]. In control applications of
 complex chemical and energy processes, MPC technology is well utilized because
 216 the algorithms not only attempt to optimize the present optimal control moves,
 but also optimize future system behavior by computing a sequence of future
 218 decision variables adjustments [61]. The ability of MPC to predict future var-
 iable moves through optimization has similarities to the objectives of capacity
 220 planning over a future horizon where economic, environmental, and operational

222 targets must be achieved while the capacity of the system must be optimally
planned out under the constraints and uncertainty of the system.

224 Ricardez-Sandoval et al. [63] reviewed different approaches to integrate op-
timal process design and control of large systems under process parameter un-
certainty. Large systems such as chemical plants are usually designed based
226 on steady state economic calculations, while the control aspects are studied in-
dependently. The sequential fashion of the approach from design and control
228 gives rise to unforeseen constraints and limitations that can hinder the eco-
nomic operation of the system once online. The simultaneous optimization of
230 dynamic control variables and design variables can thus greatly reduce the effect
of under-sizing or over-sizing the capacity of the system and improve profitable
232 operation under different market conditions [63]. From an uncertainty point of
view, the dynamic behavior of system parameters and variables is a factor that
234 must be addressed in power systems optimization [64]. This work does not de-
tail the instantaneous process dynamics of the energy system because a longer
236 term multi-decade time scale dominates the process dynamic effects. However,
one-hour energy responses are simulated through simple first order differential
238 equations which are explained further in this work. The motivation to include
one-hour system changes to optimize capacity planning of energy systems comes
240 from practical experience reported in previously planned and constructed plants
in which owners find that the planned capacity is not fully utilized, is insuffi-
242 cient, or is no longer cost effective given changing economic conditions and
the load following system requirements [65]. This work also demonstrates the
244 use of novel methods for using soft constraints to integrate various control sce-
narios into the capacity planning optimization problem to further reduce the
246 possibilities of the aforementioned problems of insufficient capacity, loss of cost
effectiveness, and sub-optimal utilization.

248 6. Nonlinear Dynamic Modeling

A general model form for a nonlinear dynamic problem can be formulated
250 as follows

$$\min J(x, y, p, d, u) \quad (1a)$$

$$0 = f\left(\frac{dx}{dt}, x, y, p, d, u\right) \quad (1b)$$

$$0 = g(x, y, p, d, u) \quad (1c)$$

$$0 \leq h(x, y, p, d, u) \quad (1d)$$

252 These equations represent a model that may include differential $(x, \frac{dx}{dt})$
or algebraic (y) variables. Of these, some may be binary (0, 1), integer (e.g.
-1, 0, 1, 2), or discrete (e.g. 0, 1.71, 5.1, 7.2) variables. The solution to

the equations can be given by the initial state x_0 , parameters p , trajectory of disturbances $d = (d_0, d_1, \dots, d_{n-1})$, and control moves $u = (u_0, u_1, \dots, u_{n-1})$ [66]. The solutions for variables x and y are solved for each step n of the time horizon from the differential (Eq. 1b), algebraic equality (Eq. 1c), or algebraic inequality (Eq. 1d) equations in the model. The formulation outlined in Eq. 1 is especially suitable to handle the complexities of an energy system capacity planning problem. The DAEs can describe differential and nonlinear physical features, while the explicit handling of disturbances as trajectories mimics the hourly energy demands over the time horizon. The solution method for a DAE model is summarized in the following section.

7. Numerical Solution of DAE Systems

Usually the time horizon of an energy capacity planning problem is on the order of decades because of the magnitude of the investment associated with capacity increases. The problem can become very large as the time horizon increases.

To solve dynamic optimization problems researchers and practitioners have used simultaneous [67] and sequential solution methods [68]. More detail about each approach can be found in the literature [69], but the important aspect of the simultaneous method is the computational advantage for problems with many decision variables and a moderate number of state variables [66] as is the case with capacity planning problems over a long horizon. State variables can be seen as a set of variables used to describe the mathematical state of a dynamic system to determine the future behavior of the system [70]. Simultaneous methods solve the DAE model by converting it to only algebraic equations in a method known as direct transcription [71]. This changes the problem to a nonlinear programming problem that can then be solved by large-scale optimization solvers [72].

8. Dynamic Optimization Framework

DAE models are used in nonlinear predictive control and estimation applications such as industrial process fouling [73], unmanned aerial systems [74], drilling automation [75], systems biology [76], batch distillation [77], pipeline flow assurance [78] and many other applications. In a control environment, multiple objectives may be desired within a single control application. These objectives can have different priorities or rates at which they must be achieved. Similar multi-objective tradeoffs can exist in capacity planning because although cost minimization is the ultimate goal, certain environmental and operational objectives are also desired over the lifetime of the plant. These additional objective considerations have a significant effect on the capacity of the system [20, 21]. Optimizing capacity investment timing and size alongside these important operational objectives is demonstrated in this work. One approach to handle the multi-objective challenge is the use of the ℓ_1 -norm formulation for nonlinear

dynamic optimization. The ℓ_1 -norm formulation simultaneously optimizes the multiple objectives by selectively manipulating the degrees of freedom that have the highest sensitivity on the most important objective, and then meeting the lower ranking objectives with the remaining degrees of freedom [79]. Priorities are assigned by giving higher weighting to the most important objectives. The dynamic optimization ℓ_1 -norm objective formulation is shown in Eq. 2.

$$\min_{x, y_m, u} \Phi = w_{hi}^T (e_{hi}) + w_{lo}^T (e_{lo}) + (y_m)^T c_y + (u)^T c_u + (\Delta u)^T c_{\Delta u} \quad (2a)$$

$$\text{s.t. Eqs. 1b} - 1d \quad (2b)$$

$$\tau_c \frac{dy_{t,hi}}{dt} + y_{t,hi} = sp_{hi} \quad (2c)$$

$$\tau_c \frac{dy_{t,lo}}{dt} + y_{t,lo} = sp_{lo} \quad (2d)$$

$$e_{hi} \geq (y_m - y_{t,hi}) \quad (2e)$$

$$e_{lo} \geq (y_{t,lo} - y_m) \quad (2f)$$

[Table 1 about here.]

The objective function for the ℓ_1 -norm formulation, Eq. 2, is a specific case of Equation 1. The most important distinction beneficial to a capacity planning problem is the ability to include other objectives such as trajectory targets or dead-bands that can be met at different response rates and levels of importance. Based on the ℓ_1 -norm formulation, the capacity planning problem can be framed as one cycle of an MPC problem. Manipulated variables such as CHP or boiler set points are moved by the optimizer to meet operating targets such as minimizing the error between energy production and demand over the entire planning horizon. At the same time, the capital and operating costs are minimized as part of the highest ranking objective function.

9. The CHP Investment Model

A district energy system may provide cooling and electricity with waste heat used to provide for the heating demand. In such arrangement, the buildings and cooling system represent the heating and cooling loads. The heat is provided by coal-fired or gas-fired boilers while the electric power supply comes from the CHP or the regional electrical grid. The heat produced from the boilers is used to provide all the energy for heating during the winter months as well as any auxiliary uses such as kitchens, showers, laboratories, etc. During the summer months, the boilers continue to operate to provide heat for absorption chillers

and auxiliary uses. A conversion of a district energy system to one that includes a CHP arrangement is illustrated in Figure 1. The arrangement includes important components such as 1.) gas turbine with generator for electricity, 2.) heat recovery boiler, 3.) back-up boiler capacity.

[Figure 1 about here.]

[Figure 2 about here.]

In this work, the objective function represents the sum of operating and capital cost over a 30 year horizon period as a function of key variables and parameters in the system. The 30 year energy demand horizon as shown in Figure 2 is built from average daily demand profiles and demand increase forecasts. Demand data for each year is reduced to demand profiles for an average winter day and an average summer day. To account for the total energy demand, the model multiplies each average hours value by 365/2 to give the horizon for a simplified season (winter or summer) in a given year. To further reduce problem size, every two years' average daily profiles are averaged to obtain 15 summer day profiles and 15 winter day profiles. This assumes that the energy demand will be relatively constant every two years, and after a two-year period, demand increases. The handling of the demand data for model reduction is shown in Equation 15 which is discussed later in this section. Although the discretization of the horizon into average daily profiles gives 15 separate periods, the figures in this work are displayed as continuous for visualization purposes. For a complete description on the method to generate the electricity and heat demand horizon see [80]. This work utilizes data from a turbine vendor [81] to construct empirical relationships through first-order and second-order polynomials for the important physical relationships. The data from the vendor is for small-size to medium-size industrial combustion turbines in the range of the capacity requirements for the district energy system considered in this study (1-30 MW). This work does not account for mechanical equipment degradation over time. It is assumed that to some degree, degradation effects are reversed by maintenance to turbine and boiler systems [82]. In future work, a relationship to degradation over time could be explored. The objective and the equations that make up the deterministic energy system expansion model are first summarized in the following qualitative model formulation.

- Minimize: present value of capital and operating costs
- Subject to:
 - Combined electricity production and electricity import supply must be greater than or equal to electricity demand.
 - Utilized heat production capacity must be greater than or equal to heat demand.
 - Utilized turbine and supplemental boiler capacity cannot exceed the maximum capacity of their respective systems.

- Capacity investments are irreversible.
- Other physical and logical constraints that make the model feasible.
- CO₂ calculations for environmental and cost constraints.

It is important to note that for this proposed optimization framework, there is no need to index or discretize the system by time and allocation of capacity. In the following model, the use of indices is only used to abbreviate equations and simplify notation. The subscript s is used to denote the season (summer or winter) in which the system operates, but the entire model is solved simultaneously. In order to simplify the system, energy demand data for the summer and winter months are analyzed to derive two different representative summer day and representative winter day demand profiles. For a complete description of the method to generate the electricity and heat demand horizon see [80]. The symbols are described in Table 2.

[Table 2 about here.]

The semi-empirical mathematical description of the dynamic model is stated as follows.

Minimize :

The total operating and capital costs is the objective function given by:

$$\sum_s OC_s + \sum_p CC_p \quad (3)$$

The operating cost (OC_s) for each season “ s ” is summed for all steps in the time horizon as well as the capital costs (CC_p) of capacity increases for system type “ p ” at any step in the time horizon. The operating and capital cost functions are explained later in this section.

Subject to Equations :

Empirical first order differentials:

$$\tau_{chp} \cdot \frac{dx_s}{dt} + x_s = u_s \quad (4)$$

$$\tau_{boiler} \cdot \frac{db_s}{dt} + b_s = v_s \quad (5)$$

This work uses a first order differential equation model to fit turbine (Eq. 4) and boiler (Eq. 5) dynamic responses to power and heat generation respectively. A constrained differential term to describe both the timing and size of capacity investments is also demonstrated in this work. This work uses first order differential equations to model the hourly energy production as a response to a set point change. In a higher fidelity, shorter time-scale model, the fitting of τ would reveal the process time constant. The fitting process for the time constant can be accessed in common process control textbooks [83]. In essence, τ or the time constant describes numerically how fast a system responds to a set point change. This can have large effects on the results of optimization as the time

constant determines the dynamic response of a system. In this work, the τ for both the boiler and the CHP turbine is set to 0.024 hours for model simplicity. This means that the change in response to a set point is very fast. However, though the modeled change in response to a set point is too fast to model detailed process dynamics within each hour block, the hourly changes (or hourly dynamics) are incorporated into the long-term capacity planning problem. In a higher fidelity, shorter time-scale model, the time constants can and should be fit according to the actual dynamics of the system. However, the use of differentials and the ℓ_1 -norm formulation in this work adds simplicity in modeling and allows operational scenarios to be considered in the capacity investment problem rather than accounting for responses from the short-time-scale systems such as turbine and boiler dynamics.

Though the detailed dynamics on the minute and second scales are not accurately represented in this work, the formulation with first order differential equations means this formulation could be used with high-fidelity models given a large amount of computational power. The formulation also enables continuous time that is used for the capacity timing and sizing differential equations. The formulation as a differential equation also allows the use of the ℓ_1 -norm formulation to incorporate various operational scenarios and multiple objectives into the optimization problem. The APMonitor modeling and optimization package [66] allows the implicit use of time differentials to account for any time changes, both short and large scale, without further discretizing the system into large blocks. Future work should consider the more detailed dynamic effects of turbine and boiler system responses combined with large scale capacity allocation.

$$y_s = -0.0817 \cdot x_s^2 + 5.6547 \cdot x_s \quad (6)$$

$$0 \text{ MW} < x_s < 30 \text{ MW}$$

Eq. 6 is a nonlinear interaction that describes the recoverable heat y in a heat recovery boiler as a function of power production x from the gas turbine. First principles models of turbines are available to detail the heat versus power response [28]. A polynomial fit is sufficient in this work because heat recovery performance data for different turbine capacities operating at typical operating conditions is available from turbine manufacturers (see [80] for data and polynomial fit). The polynomial fit for this data, if extrapolated beyond the available data range, gives unrealistic properties because of the negative value coefficient; nevertheless, the optimization program is constrained to search only within the acceptable range. Exponential correlations of the data have better agreement than the polynomial fits, especially at smaller turbine capacities; however, exponential fits are more difficult to solve with optimization solvers because of strong nonlinear behavior. When exponential or logarithmic equations are coupled with other nonlinear equations in relatively large optimization problems, optimization solvers may not be able to find a solution. The total available heat supply to the district system is given by Eq. 7 which sums the boiler and CHP

436 heat generation.

$$hs_s = y_s + b_s \quad (7)$$

Eq. 8a indicates the excess heat generation which is the difference between
438 total heat production and heat demand. Because heat production can only come
from the on-site system, the energy system is constrained to have a difference
440 equal or greater than zero as indicated by Eq. 8b.

$$he_s = hs_s - hd_s \quad (8a)$$

$$he_s \geq 0 \quad (8b)$$

Usable heat production from a CHP system is limited by turbine capacity
442 utilization, which means that at maximum load the maximum amount of heat
is generated. Excessive heat production beyond the heat demand is discarded
444 to the atmosphere through the flue combustion gases at higher temperatures.
Although usually there are design and regulatory limitations on the flue gas
446 temperatures, generally if too much excess heat is being discarded to the at-
mosphere, heat integration and efficiency goals of a CHP system are not being
448 achieved. In the results and discussion section, optimization formulations to
minimize excess heat he_s are explored. The net power relationship is given by

$$np_s = x_s - ed_s \quad (9)$$

Many CHP systems have the convenience of interconnection with a larger
450 power grid. Interconnection with a larger grid provides backup power, or if
452 the CHP capacity is designed to partially fulfill the electric load, a larger grid
can meet the remaining load. In certain markets net metering measures the
454 difference between the electricity from the utility used by the customer and the
electricity generated and provided back to the utility [84]. Depending on the
456 market where the CHP is located, net metering rates and incentives may make
the CHP investment more financially attractive even after fuel and efficiency
458 savings. Eq. 9 defines the net power usage of the system to simulate the net
metering options of certain markets. When the CHP power production x_s is
460 greater than the electric demand ed_s then the excess electricity is sold back
to the grid. On the other hand, if electric demand is greater than the current
462 CHP power production, the resulting negative np_s denotes that power is being
imported from the external electrical grid. This flexibility makes for even greater
464 optimization trade-offs that can affect capacity planning of CHP systems. In
this work, both net metering and electricity import only options are explored
466 in different optimization cases. For simplicity when net metering is allowed,
the same electric rates are considered for either import or export of electricity.
468 Although this is a simplification of more complicated net metering rules, it
provides a good basis to explore the realistic effects of such trade-offs in energy
470 system capacity planning.

Turbine load fraction and efficiency are given by:

$$lf_s = \frac{x_s}{mc_{chp}} \quad (10a)$$

$$\begin{aligned} \eta c_s &= 0.9044 \cdot lf_s + 0.0956 \\ 0 &< lf_s < 1 \end{aligned} \quad (10b)$$

472 The natural gas consumption for CHP and boiler are given by:

$$\begin{aligned} fc_s &= \frac{-0.1361 \cdot x_s^2 + 13.347 \cdot x_s}{\eta c_s} \\ 0 \text{ MW} &< x_s < 30 \text{ MW} \end{aligned} \quad (11a)$$

$$fb_s = \frac{b_s}{\eta b} \quad (11b)$$

474 When less than full power is required from a gas turbine, the output is
 475 reduced by lowering the turbine inlet temperature. This reduction also reduces
 476 efficiency [6]. The load fraction lf_s is the ratio of instantaneous CHP output
 477 and CHP capacity. When the turbine is operating at its maximum capacity load
 478 fraction is 1. Turbines in the range of capacity considered in this work generally
 479 reach a maximum efficiency around 30%. The turbine load versus efficiency
 480 relationship is normalized to 1 (100%) for maximum efficiency and a linear
 481 relationship is approximated by Eq. 10b. This is done because fuel consumption
 482 from manufacturer data for different turbine capacities is rated at the maximum
 483 load, at the most efficient state. In other words, the maximum capacity of
 484 the turbine as defined in this work is the functional maximum capacity, not
 485 the theoretical maximum capacity. The CHP fuel consumption (Eq. 11a) is
 486 also represented by a polynomial fit from manufacturer data for gas turbines
 487 running at optimal full capacity. Operation below the turbine's rated capacity
 488 consumes more fuel, thus the polynomial for fuel consumption is divided by the
 489 normalized CHP efficiency ηc_s to estimate the effects of part load operation on
 490 fuel consumption. When the normalized CHP efficiency is less than one, fuel
 491 consumption increases thus quantifying the effects of operating the gas turbine
 492 at less than full capacity. Fuel consumption by the gas boiler (Eq. 11b) is
 493 estimated by directly dividing the boiler energy output by the expected boiler
 494 efficiency of 80%.

Carbon dioxide calculations are given by:

$$ce_{chp,s} = cn/2000 \quad (12a)$$

$$ce_{boiler,s} = cn/2000 \quad (12b)$$

The CO₂ emissions rate from fuel consumption estimated by Eqs. 12a and 12b are based on a stoichiometric relationship of natural gas combustion. When there is a cost associated with CO₂ emissions, the cost is calculated by:

$$ct_s = (ce_{chp,s} + ce_{boiler,s}) \cdot cp \cdot (907.185/1000) \quad (13)$$

The capital and operating cost functions are given by:

$$\begin{aligned} CC_{chp} = & (-0.0069 \cdot (\Delta mc_{chp})^2 + 1.3351 \cdot (\Delta mc_{chp})) \cdot (10^6) \\ & \cdot (1 + ir)^{yr} \cdot (1 + dr)^{-yr} \quad (14a) \\ & 0 \text{ MW} < \Delta mc_{chp} < 30 \text{ MW} \end{aligned}$$

$$\begin{aligned} CC_{boiler} = & 2 \cdot (-39.504 \cdot (\Delta mc_{boiler})^2 + 29900 \cdot (\Delta mc_{boiler})) \\ & \cdot (1 + ir)^{yr} \cdot (1 + dr)^{-yr} \quad (14b) \\ & 0 \text{ MMBTU/hr} < \Delta mc_{boiler} < 200 \text{ MMBTU/hr} \end{aligned}$$

Correlating the capital costs is important for an accurate objective function. Polynomial approximation of manufacturer data on capital costs for a range of CHP capacities is used for the CHP capital cost function. The data comes from a survey of estimated total capital costs for typical gas turbine-based CHP systems [81, 80]. One of the innovations of this work is in using differential equations that explicitly consider system changes with respect to time for capacity planning problems. Over the entire time horizon the energy generation capacities may increase when the energy demands requires expansion, but those changes are also limited by the economic objective of minimizing costs. In Eqs. 14a and 14b, Δmc_p denotes the change in maximum capacity of system p . To describe this in the optimization program, the multiplication operation $\Delta t \cdot \frac{dmc_p}{dt}$ yields an approximation to Δmc_p , the magnitude of the maximum capacity change at a particular time period. The differential time change dt is approximately equal to the time discretization Δt interval used in the model. The optimization software used in this work explicitly handles differential equations, which allows for the calculation of capital cost at any time period of the horizon at which the optimizer finds optimal to have a change in capacity in the energy system. The capital cost equations are multiplied by $(1 + ir)^{yr}$ to account for inflation and and by $(1 + dr)^{-yr}$ to account for the time value of money. The capital costs are summed together and included as minimization terms in the objective function Eq. 3. The operating cost is evaluated by:

$$OC_s = (ng \cdot fc_s + ng \cdot fb_s - ee \cdot np_s + ct_s) \cdot (0.5 \cdot 365 \cdot n) \cdot (1 + dr)^{-yr} \quad (15)$$

The operating expenses CHP fuel, boiler fuel, net electric power, and CO₂ emissions are summed and multiplied by $(0.5 \cdot 365 \cdot n)$ to approximate the number of days (half a year) in each season S . The letter n is the number of years

each demand cycle represents, and is used to reduce the number of individual cycles the optimization program must solve. In this study each demand cycle represents two years ($n = 2$). The operating cost equation also contains the term $ee \cdot np_s$ which quantifies the cost of electricity. This term has a negative sign because net power (np_s) is positive when excess electricity is produced. Excess electricity production reduces operating cost because it is sold to the grid. When electricity is imported np_s is negative which causes the $-ee \cdot np_s$ term to be positive and the operating cost to increase. Operating costs are also discounted to the present value by $(1 + dr)^{-yr}$. There is no inflation factor in Eq. 15 because the natural gas and electricity price data already considers inflation. It is important to note that CO₂ emissions cost (ct_s) is only considered in economic scenarios that give CO₂ a cost associated per ton emitted. The baseline scenarios in this work do not consider CO₂ emission cost but some scenarios with CO₂ cost are explored in this work. To specify the irreversible nature of change in capacity, the differential terms are directly constrained by:

$$\frac{dmc_{chp}}{dt} \geq 0 \quad (16a)$$

$$\frac{dmc_{boiler}}{dt} \geq 0 \quad (16b)$$

Directly constraining differential terms is a feature that few optimization software packages have, but is extensively exploited in this work by utilizing the APMonitor package [66]. Limiting $\frac{dmc_p}{dt}$ to be ≥ 0 allows for the change in maximum capacity of system p to remain positive meaning there is no decrease in capacity once a decision is made. The maximum capacity always remains greater than or equal to the instantaneous generation rates of the CHP and boiler system respectively through the following constraints:

$$mc_{chp} \geq x_s \quad (17a)$$

$$mc_{boiler} \geq b_s \quad (17b)$$

In summary, the dynamic model includes all the potential energy generation systems to fulfill the heating and electric demands. The optimizer decides which arrangement and capacity is needed. The optimizer can either decide to have a CHP only system, a boiler only, or a combination of the two along with the choice of import capacity from the grid for electricity. The optimizer uses the model to find timing and potential overall capacity in a 30 year horizon by minimizing total present value costs as stated in the objective function Eq. 3.

10. Uncertainty in Natural Gas and Electricity Prices

With uncertainty in natural gas and electricity prices, there is motivation to optimize the problem to the expected value of the total capital and operating

cost over the time horizon. Natural gas and electricity prices are stochastic inputs to the model represented by the uncertain future spread of prices over the optimization horizon as illustrated in Figure 3 below. The 30 year horizon from 2010 to 2040 is chosen for this study due to availability of long-horizon projection data (current available projections only span through 2040). Commercial end-user price projections from the U.S. Energy Information Administration (EIA) are utilized (see [85, 86]). Based on the likelihood of each economic model, there is a specific probability associated with each value of natural gas and electricity price. At any particular point in the time horizon the probability of all the possible values of a random variable can be summarized by a probability mass function (PMF) if the random variable is discrete [87]. It is often desirable to summarize the probability information in a single representative number. That is accomplished by a deterministic value, also called the expectation or the mean, of a random variable, which is a weighted average of the possible values of the random variable [87].

[Figure 3 about here.]

The expectation of a random variable X is mathematically defined as

$$E[X] = \sum_x x \cdot px(x) \quad (18)$$

where $px(x)$ is the probability associated with each variable x . For linear function of random variables it can be verified that the expected value of a function of the form $a \cdot X + b \cdot Y + c$ is given by $a \cdot E[X] + b \cdot E[Y]$ where X and Y are random variables and a , b , and c are scalars and x , u , p , d , and u are other non-stochastic variables affecting the system. This property is extended to find the expected value of the total cost. If the total present cost (PC) function is of the form shown in Eq. 19.

$$PC = f_1\left(\frac{dx}{dt}, x, y, p, d, u\right) \cdot A + f_2\left(\frac{dx}{dt}, x, y, p, d, u\right) \cdot A + f_3\left(\frac{dx}{dt}, x, y, p, d, u\right) \cdot B + f_4\left(\frac{dx}{dt}, x, y, p, d, u\right) \quad (19)$$

f_1 through f_4 in Equations 19 and 20 represent nonlinear functions for CHP fuel consumption (Eq. 11a), boiler fuel consumption (Eq. 11b), net power production (Eq. 9), and capital costs (Eqs. 14a14b), respectively. The parameters A and B are the stochastic natural gas price and electricity price respectively. Although the PC is the result of a highly nonlinear system, each function f does not depend on the stochastic A and B parameters. Because this dynamic optimization simultaneously solves each time step and does not depend on feedback from previous solutions, the expected value of the PC at each time step

reduces to

$$\begin{aligned}
E[PC] = & f_1\left(\frac{dx}{dt}, x, y, p, d, u\right) \cdot E[A] + f_2\left(\frac{dx}{dt}, x, y, p, d, u\right) \cdot E[A] \\
& + f_3\left(\frac{dx}{dt}, x, y, p, d, u\right) \cdot E[B] + f_4\left(\frac{dx}{dt}, x, y, p, d, u\right)
\end{aligned} \tag{20}$$

Eq. 20 shows that the uncertainty associated with natural gas and electricity prices on the PC at each time step is captured only by the expected value of each random variable as defined in Eq. 18. This is true after the assumptions established earlier that the energy demand is considered a deterministic input and there is negligible uncertainty on model parameters and initial conditions. Otherwise all functions f are random functions whose PMFs would be very difficult to obtain because of the nonlinear nature.

To find the expected natural gas and electricity price, the probabilities of each price data point must be known. Statistical analysis using the MATLAB statistical toolbox constructs probability density functions (PDFs) and cumulative density functions (CDFs) of the price data for each year in the horizon to estimate the probability of each data point. MATLAB statistical toolbox uses a kernel distribution to estimate PDFs. A kernel distribution is a nonparametric representation of the PDF of a random variable. This method is employed when a parametric distribution cannot describe the data or to avoid making assumptions about the distribution of the data [88]. Eq. 19 is invoked to find the expected value of natural gas and electricity prices. If each economic model is assumed to have equal probability, the deterministic value of natural gas and electricity prices at each year is the arithmetic mean, a very simple computation. The calculated deterministic values for both natural gas and electricity prices at each year are used as the price inputs in Eq. 15 to calculate the operating cost. This makes the objective function Eq. 3 similar to Eq. 20, the expected value of the total net present value costs. The stochastic information is encapsulated in the deterministic natural gas and electricity prices and incorporated in the optimization directly through the objective function.

11. Results and Discussion

[Table 3 about here.]

The objective of this work is to evaluate the optimal CHP capacity investment schedule for a district energy system through dynamic optimization and the ℓ_1 -norm formulation (see Eq. 2). This approach opens the flexibility of posing the capacity investment problem as an optimal control problem. This approach aims at addressing some of the disturbances that may affect approaching a single investment scenario through a single optimization formulation. Through this approach stakeholders can evaluate not only the common global cost minimization objective but also introduce control objectives to operational variables that can affect the overall capacity and investment schedule over a large time

624 horizon. The different optimization evaluations to a single economic scenario
 are first summarized in Table 3. The table compares six different optimization
 626 cases of the district energy model described earlier in this work. Each case eval-
 uates a different aspect of the control objectives with its corresponding results
 628 in capacity, costs, and levelized cost of usable energy. It is worth noting that
 cases A-F do not take CO₂ cost into account, but section 12 includes cases which
 630 account for multiple possible scenarios with variable CO₂ cost. Parameters used
 for cases A-F are detailed in Table 4.

632 [Table 4 about here.]

The last column of Table 3 is the levelized cost of energy. This is an impor-
 634 tant parameter used to compare, through a single number, the cost of energy
 over the lifetime of the system. It is defined as

$$\frac{\sum_{t=1}^n CC_t + OC_t}{\sum_{t=1}^n E_t} \quad (21)$$

636 where CC_t is the discounted capital cost at time t , OC_t is the discounted
 operating cost at time t , E_t is the usable energy generation at time t , n is the life
 638 of the system. The sum of usable energy (E_t) is the sum of both electric and
 usable thermal energy in megawatt-hours, but does not account for thermal
 640 energy that is wasted through the flue gas. The levelized cost of energy is a
 useful tool to evaluate which generation system option produces energy in the
 642 most cost effective way.

The overall monetary and capacity results alone may provide interesting in-
 644 sights, but the evaluation of additional objectives within the same model can
 provide stake holders information in mode of operation, utilization of capac-
 646 ity, and time dependent parameters that affect the overall cost, capacity, and
 schedule of the investment. Such an approach provides granularity and insights
 648 into the variables affecting the size and schedule of the investment that oth-
 erwise would be unknown, thus providing greater certainty to large, upfront
 650 investments.

[Figure 4 about here.]

652 [Figure 5 about here.]

[Figure 6 about here.]

654 [Figure 7 about here.]

[Figure 8 about here.]

656 Table 3 summarizes the final capacity for the different optimization cases,
 but Figure 4 shows the exact timing and magnitude of the investments over the
 658 30 year horizon period. For CHP Capacity investment, every case suggests an
 investment of capacity within the first year of evaluation, in other words as soon

as investment is possible. It is notable that there are small subsequent jumps in capacity expansion for 3 of the cases that eventually converge to an installed capacity of 11 MW. The smaller increments are mainly a function of the models inability to quantify for the larger cost per MW of capacity investment at scales smaller than typical large-scale CHP projects. In a more realistic scenario, the incremental CHP capacity increases across the horizon suggested by the optimization could be combined to a larger increase at the time of the first forecast need. Future work will address the smaller increments through model enhancements to the capital cost equations for CHP investment.

The boiler capacity schedule shows a more staggered approach to capacity increases, with some models requiring significant increase 10 to 15 years after the initial CHP installation. Small, modular boiler capacity increases across the horizon are more realistic than small capacity increases in CHP turbine systems. Additionally, boiler systems can be reconfigured for small increases in capacity with the addition of burners and piping to increase heat transfer with a smaller investment cost to existing equipment. The ability of this optimization approach to holistically account for the capital cost as well as the operating cost relationship to turbines and boiler efficiency are key in evaluating more realistic investment scenarios and reducing the risk of over-investment or under-investment. Next is a discussion for the specific differences in optimization approach between cases and their influence on the capacity and schedule result.

11.1. Case A

This case is considered the base case scenario as it only includes the economic objective of reducing all cost in present value while meeting the power and heat demands of the district energy system. As shown in equation 22 the heat demand is constrained to be met by constraining error heat to be non-negative and net power is constrained to be less than or equal to zero, meaning no electricity may be sold to the grid.

$$\begin{aligned} \min E[PC] \\ he_s \geq 0 \\ np_s \leq 0 \end{aligned} \tag{22}$$

The optimal CHP capacity investment begins from the beginning of the horizon and remains stable without calling for additional potential increases in CHP capacity throughout the horizon, as seen in Figure 4. The CHP also remains operationally stable as electricity is produced at maximum capacity for most of the winter demands and for all of the summer demands as indicated in Figures 7 and 8. The results show that the turbine capacity is set by the optimizer based on the lowest base load demand of the horizon. This allows for the turbine to be utilized at its highest most efficient load over the entire horizon at all times. Figures 5 and 6 show that during the summer months the heat demand is met by the turbine heat supply while for the winter months additional heat is needed to meet demand. Thus, expanded boiler capacity is

required later in the horizon. The boilers are operational only during the winter months to meet peaking demand.

11.2. Case B

As indicated by Table 3, every case in this study evaluates the economic objective equation in the model, or Equation 3 which calculates the sum of all operating and capital costs in present dollars. Case B introduces the ℓ_1 -norm formulation to keep a control variable, excess heat production (Equation 8a), above a lower bound soft constraint. These lower bounds are referenced as soft constraints because they can be violated with a cost to the objective function proportional to the difference with the given bounds. Evaluating this optimization problem with a dead-band or just upper or lower bounds on excess heat production removes the need for the explicit excess heat constraint (Equation 8b). As part of the ℓ_1 -norm formulation, the control variables can be set to follow a band where they are free to fluctuate without penalty. Violation of the control bounds by the control variable (CV) is penalized by a weight factor that can be set at different magnitudes for the upper or lower bounds set points as shown in Equation 2 where w_{hi} and w_{lo} are penalty weights. These upper and lower bound set point weights act on the objective function when the control variable crosses the set bounds. The base model contains the excess heat constraint Equation 8b, but in Case B Equation 8b is removed to evaluate the results of utilizing only the lower bound set point on the excess heat CV. The lower band set point is set at 10 MMBTU/HR to attempt to guarantee slight excess heat. The lower band violation has a weight of 10^6 while the upper has a weight of zero. The lower band boundary penalty must be more severe as excess heat below zero would trigger under-production of heat energy. A weighting factor of zero is set for the upper band to indicate that excess production of heat energy is not penalized.

$$\begin{aligned} \min E[PC] \\ -\infty < he_s < \infty \\ np_s \leq 0 \end{aligned} \tag{23a}$$

$$\begin{aligned} w_{hi} &= 0 \\ w_{lo} &= 10^6 \\ sp_{hi} &= 0 \\ sp_{lo} &= 10 \end{aligned} \tag{23b}$$

This case demonstrates the viability and flexibility of soft constraints through dead-band guidance on CV in a capacity investment schedule optimization problem. Violations on excess heat energy below zero as shown in Figures 9 and 10 for Case B indicate that the optimization finds under-production of heat during brief periods optimal. In a real scenario, under-production of heat is generally unacceptable; however, this formulation provides deeper insight into other options. In evaluating power generation capacity investments, stake holders many

times must assess the viability of always meeting energy demands through increases in capacity which may only be used during peak periods, or manage the peak usage through other more cost effective ways such as shifting energy demands to other times of the day or shutting down equipment. Stake holders can also consider energy storage capabilities for shifting energy demands. Further work should explore investment and operational models which integrate these peak load shifting options.

[Figure 9 about here.]

[Figure 10 about here.]

11.3. Case C

This case maintains the same formulation as Case A with the exception of the removal of the inequality constraint on net power. If left unconstrained, Equation 9 in the model indicates the variable is allowed to go either negative or positive. If positive, the system is producing electric power above the systems demand. The excess electric power is allowed to be sold to the grid and thus counts as positive cash flow in the operating cost evaluation.

$$\begin{aligned} \min E[PC] \\ he_s \geq 0 \\ -\infty < np_s < \infty \end{aligned} \tag{24}$$

As expected, the results show that the optimization finds driving the CHP capacity to the given maximum 30MW optimal, as shown in Figure 4. Without the inequality constraint, the system behaves as a power plant for export rather than meeting the systems demand at the lowest cost. Although attractive due to overall cost reduction resulting from revenue, this system has higher cost per megawatt hour than Case A, as indicated by the levelized cost of energy evaluation. In addition to the higher cost per MW-hr, the model greatly simplifies the revenue structure which does not account for the very complex environmental regulations and uncertainties in the business of power generation as well as the additional maintenance costs. The higher levelized cost of energy for this case can also be explained by the fact that although the solution suggests increasing capacity to behave as a major power plant, the limitations in size and scope for the CHP system restrain it from fully attaining the economies of scale and tri-generation benefits of utility scale power plants which usually have the lowest levelized cost of energy. This case highlights the ability of this optimization framework to easily evaluate operational scenarios through explicit constraint changes that do not require major reconfiguration on the model and its assumptions.

11.4. Case D

Case D applies an upper bound limit to restrain heat energy over-production. The upper bound limit on excess heat is set at 20 MMBTU/hr with a weight of 10^5 . If the excess heat energy crosses the 20 MMBTU/HR upper bound, the penalty adds cost to the combined objective function as illustrated in Equation 2.

$$\begin{aligned} \min E[PC] \\ he_s \geq 0 \\ np_s \leq 0 \min E[PC] \end{aligned} \quad (25a)$$

$$\begin{aligned} w_{hi} &= 10^5 \\ w_{lo} &= 0 \\ sp_{hi} &= 20 \\ sp_{lo} &= 0 \end{aligned} \quad (25b)$$

The result of the upper bound CV limit is a dramatic reduction of excess heat production. Figures 9 and 10 show that the excess heat energy curve remains at zero in both seasons over the entire horizon. This is the only case where excess heat production levels out at zero, constrained by Equation 8b, for both seasons. It is also important to analyze that the effect of the CV bounds have a noticeable effect on the overall problem, and thus differences in operational scenarios arise to meet the constraints, or violate them, both have an effect on the utilization of capacity. For example, Figure 9 shows Case D is the only instance where the electricity production for the summer season follows the heat production, rather than topping off at the CHP maximum or following the electricity demand. The levelized cost of electricity is the highest among all cases, indicating that perturbation on the problem by the CV upper bound soft constraint, even with small penalties, can have a significant effect on the utilization of capacity but do not affect the overall CHP and boiler capacity and timing of the investment.

11.5. Case E

Case E was formulated to observe the effects of a weak lower bound set point penalty. The hard constraint in the model on excess heat, Equation 8b, remains active, but a lower bound set point on the excess heat CV is set at 10 MMBTU/HR and no upper bound set point is applied.

$$\begin{aligned} \min E[PC] \\ he_s \geq 0 \\ np_s \leq 0 \end{aligned} \quad (26a)$$

$$\begin{aligned}
w_{hi} &= 0 \\
w_{lo} &= 10^6 \\
sp_{hi} &= 0 \\
sp_{lo} &= 10
\end{aligned} \tag{26b}$$

The result is that due to the weak lower set point weight, the optimization
 finds keeping excess heat above the hard constraint optimal but does not find
 keeping the excess heat above the soft constraint of 10 MMBTU/hr optimal.
 The CV violated the soft constraint to provide lower overall cost.

11.6. Case F

Case F contains both a lower and upper set point on the excess heat CV
 dead-band at 10 MMBTU/HR and 20 MMBTU/HR respectively. The weight
 factor for CV band violations for both upper and lower bounds is set at 10^5 .

$$\begin{aligned}
\min E[PC] \\
he_s &\geq 0 \\
np_s &\leq 0
\end{aligned} \tag{27a}$$

$$\begin{aligned}
w_{hi} &= 10^5 \\
w_{lo} &= 10^5 \\
sp_{hi} &= 20 \\
sp_{lo} &= 10
\end{aligned} \tag{27b}$$

As shown in Figures 9 and 10 summer and winter excess heat supply re-
 spectively track very closely with minor variations the results for Case E. The
 results never violate the upper bound of 20 MMBTU/HR but the lower bound is
 consistently violated as is in Case E. The likely explanation is that the lower set
 point band creates a tendency for the excess heat CV to rise but not enough to
 rise above the lower bound CV set point of 10 MMBTU/HR. The upper bound
 of 20 MMBTU/HR is never violated, but perhaps the perturbation this upper
 bound creates in the optimization space creates the small differences that tend
 to push the excess heat CV lower as opposed to Case E which does not have
 an upper bound set point. In other words, Case E has a tendency to have more
 excess heat than Case F because there is no upper bound limiting Case E.

11.7. Overall Discussion

Overall, the addition of excess heat energy as a control variable on top of
 the monetary cost minimization incorporates operational scenarios with the op-
 timal capacity problem. These operational what-if scenarios are a novel method
 to evaluate the robustness of the economic-only cost minimization solution. As

shown in the results of cases B and D through F, when attempting to control the excess heat CV within a band or above or below a target, the CHP capacity is lowered to 11 MW and boiler capacity is greatly increased. These evaluations show additional flexibility of the capacity investment with a system that reacts to changes in either electricity or heat generation regardless of the target value. The mix of CHP and larger boiler capacity adds overall cost, but operational flexibility, especially in uncertain regulatory and economic environments, may be more cost effective if taken into account early in the investment schedule instead of over-investing in CHP capacity and paying in the form of sub-optimal operation of a CHP turbine system. Integration of operational and capacity decisions is seldom utilized, yet it provides greater granularity to help confirm or question capacity investment when only cost reductions are evaluated. In conclusion, the cases in this study show that a CHP system with 11MW power generation maximum capacity and boiler capacity with a minimum of 50 MMBTU/HR would be a reasonable investment to cover different operational scenarios around excess heat generation for the district energy system. The exploration of the effects of additional combinations of operational objectives on capacity investment scheduling is beyond the scope of this paper. However, the demonstrated formulation can be applied to other operational control objectives in district energy systems as well as other types of systems to evaluate capacity investment decisions under variable operational scenarios.

12. Optimization Under Uncertainty with Future Economic Scenarios

For long-term energy system planning, it is important to understand the sensitivity of the result to uncertain future energy prices or policies. This section contains optimization results based on different economic and policy scenarios. The results directly reflect the effect of different economic data with the same equations and objective function. Case A is selected for the sensitivity study although any of the cases could be used for this analysis. Case A does not restrict excess heat generation, allows import of electricity from grid, and does not allow export of electricity to the larger grid.

12.1. Economic Scenarios

Optimizing capacity planning under different economic scenarios beyond the expected natural gas and electricity prices generate results that give insights on the sensitivity of the capacity and timing of investment on energy prices. Figure 11 shows a distribution of future costs for natural gas (\$/MMBTU), electric grid power (\$/MW-hr), and CO₂ emissions (\$/tonne).

[Figure 11 about here.]

The optimization results are restricted to integer increments (e.g. 10 or 11 instead of 10.2) for CHP capacity and increments of 5 for boiler capacity (e.g. 0, 5, 10, etc.). Integer formulations are computationally harder to solve, but give more realistic results because of discrete sizes that are common from vendors.

834 For example, turbine capacities are usually rated by integer number (e.g. 10
 835 MW) instead of a mixed number (e.g. 10.14 MW). Results for the mixed integer
 836 nonlinear programming (MINLP) problem are summarized in Figure 12 for the
 set of 27 economic scenarios. Optimization of a wide range of economic scenarios
 838 gives insights about the feasibility of a CHP system.

[Figure 12 about here.]

840 For most of the economic scenarios, the CHP maximum capacity recommen-
 dations are closely aggregated between 9 MW and 13 MW capacity, a difference
 842 of just 4 MW in capacity (see Figure 14. Although a 4 MW spread translates to
 differences in several million dollars in capital investment, the optimization so-
 844 lution provides less uncertain capacity options because more information about
 the system is encapsulated and solved simultaneously. It is important to mention
 846 that for all cases where CHP capacity is cost effective, the capacity investment
 starts from the beginning of the horizon. Figure 13 also shows the frequencies
 848 of the maximum boiler capacities. The boiler capacity increases are distributed
 at different points of the horizon.

850 There are five cases where the optimization finds a lower objective function
 without CHP investment. These results are less intuitive to understand, but
 852 underscore the benefits of large scale dynamic optimization formulations that
 utilize large sets of data and solve the problem simultaneously for the entire
 854 horizon. The economic evaluations are performed at every step of the horizon,
 thus the hidden trade offs from the economic data inputs at different time
 856 points are exploited to find an overall lower cost objective. Future work should
 investigate the details of these solutions.

858 [Figure 13 about here.]

[Figure 14 about here.]

860 For a district energy system, the size of the capacity investment strongly de-
 pends on the financial incentive that the optimizer finds based on the uncertain
 862 fuel cost and electricity price data. In contrast, without a simultaneous opti-
 mization with uncertain parameters, CHP designs may be arbitrarily oversized
 864 to deal with the uncertainty. Oversized CHP capacity options not only increase
 already expensive capital expenditures, but also CHP utilization will be more
 866 expensive because there is a greater opportunity for the gas turbine to operate
 at lower, less efficient loads over the lifetime of the system.

868 Difficulty in solving large and highly nonlinear dynamic optimization prob-
 lems is something that is still challenging and the subject of continuous research.
 870 This limitation is visible in this work in using second degree polynomials in-
 stead of better fitting exponential functions for capacity vs. cost relationships.
 872 The optimizer in certain cases recommends smaller increases in boiler capac-
 ity throughout the horizon, which may be an artifact of the slightly nonlinear
 874 polynomial functions. Close to the origin, the polynomial fits are almost linear;
 therefore, smaller capacities give smaller cost and potentially an artificially lower

876 objective. Limitations also exist in this work when formulating the problem to
 878 explicitly account for uncertainty by introducing larger amounts of data into the
 problem. The optimizer handles 20463 state variables, 18309 total equations,
 880 and 2872 slack variables, with over 700 degrees of freedom when considering a
 single economic case. When different economic cases are simultaneously solved,
 the optimization solver is unable to solve the problem. Future work should
 882 explore improving optimization solver start up procedures as well as considera-
 tion of alternative large scale optimization solvers to diminish both the highly
 884 nonlinear and large problem limitations.

To further understand and differentiate the effects of important economic
 886 parameters, future work should explore global optimization techniques to ver-
 ify global optimality from given economic inputs. This work does not include
 888 a comprehensive study on the global optimality of the solutions, but mainly
 focuses on a gradient based dynamic optimization strategy for energy system
 890 capacity expansion problems. This work does not attempt to show an exhaustive
 capacity investment schedule for a district energy system, but rather demon-
 892 strate that a dynamic optimization framework is a feasible approach to guide
 capacity planning of energy systems. This approach can be expanded to larger
 894 systems such as smart grid applications where a number of interactive energy
 systems with unique dynamics such as coal power plants, wind and solar re-
 896 newable power sources, energy storage, and carbon capturing systems interact
 to meet demands. The intrinsic dynamics, economic, and operational targets
 898 affect capacity and timing of investments of system components. This optimiza-
 tion framework brings a holistic approach to handle multiple targets and system
 900 data within a single optimization formulation.

13. Conclusions

902 This work presents a dynamic optimization case study for capacity planning
 of a district energy system. The modeling of the district energy system along
 904 with the formulation to find the capacity investment over a 30 year horizon is
 facilitated by the use of differential and algebraic equations, including directly
 906 constrained differential equations describing the capacity investment timing and
 size. The differential elements explicitly account for changes in capacity over
 908 time without the need to explicitly discretize the model to account for time as
 is done in traditional linear programming formulations. For this application,
 910 differential terms describe both long time-scale capacity investment decisions
 and short time-scale operational scenarios. This cannot be done in models that
 912 do not treat time as a continuous, explicit variable. This work also shows that
 within the dynamic optimization framework the objective function can include
 914 other control objectives with the economic objective. The ℓ_1 -norm formula-
 tion inserts operational control objectives in the optimization formulation to
 916 observe the short term effects on long term capacity planning. Different eco-
 nomic cases also affect capacity and timing of the investment. This optimization
 918 framework is a tool to guide timing of capacity investments that reflect oper-
 ational strategies and multiple objectives. This optimization framework may

920 guide decision-makers to more effectively avoid issues of sub-optimal capacity
utilization, insufficient capacity, loss of cost-effectiveness, as well as other issues
922 encountered in the past.

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Table 1: Nomenclature [66]

Φ	objective function
y_m	model values $(y_{m,0}, \dots, y_{m,n})^T$
$y_t, y_{t,hi}, y_{t,lo}$	desired trajectory dead-band
w_{hi}, w_{lo}	penalty outside trajectory dead-band
$c_y, c_u, c_{\Delta u}$	cost of variables y, u , and Δu , respectively
u, x, d	inputs (u), states (x), parameters or disturbances (d)
f, g, h	equation residuals (f), output function (g), and inequality constraints (h)
a, b	lower and upper limits
τ_c	time constant of desired controlled variable response
e_{lo}, e_{hi}	slack variable below or above the trajectory dead-band
sp, sp_{lo}, sp_{hi}	target, lower, and upper bounds to final set-point dead-band

Table 2: Optimization problem nomenclature

Indices		
p	plant investment types (CHP, boiler)	
s	season(summer,winter)	
Parameters		
cn	CO ₂ content per MMBTU of natural gas	$\frac{lb\ of\ CO_2}{MMBTU\ of\ natural\ gas}$
cp	CO ₂ price	$\frac{\$}{metric\ ton\ CO_2}$
dr	discount rate	
ed_s	electric demand $[MW]$	
ee	electricity price	$\frac{\$}{MW\ hr}$
hd_s	heat demand	$\frac{MMBTU}{hour}$
ir	inflation rate	
n	number of years being averaged	
ng	natural gas price	$\frac{\$}{MMBTU}$
ηb	boiler efficiency assumed constant at 80%	
ηc	CHP efficiency	
Δt	time step	
yr	year in time horizon	
Variables		
SV = state variable, CV = control variable, MV = manipulated variable		
b_s	boiler thermal energy production	$\frac{MMBTU}{hour}$ CV
CC_p	capital costs [\$]	SV
OC_s	operating costs [\$]	SV
ce_p	CO ₂ emissions from boiler or CHP	$\frac{short\ tons\ of\ CO_2}{hour}$ SV
ct_s	total CO ₂ cost in [\$]	SV
fb	fuel consumption boiler	$\frac{MMBTU\ of\ natural\ gas}{hour}$ SV
fc	fuel consumption CHP	$\frac{MMBTU\ of\ natural\ gas}{hour}$ SV
he_s	excess heat production	$\frac{MMBTU}{hour}$ CV
hs_s	total heat supply	$\frac{MMBTU}{hour}$ SV
lf_s	load fraction of CHP turbine	SV
mc_{boiler}	maximum capacity of boiler	$\frac{MMBTU}{hour}$ SV
mc_{CHP}	maximum capacity of CHP $[MW]$	SV
np_s	net power $[MW]$	SV
u_s	CHP turbine setpoint $[MW]$	MV
v_s	Boiler setpoint	$\frac{MMBTU}{hour}$ MV
x_s	CHP turbine electricity production $[MW]$	CV
y_s	Recoverable heat through Heat Recovery Boiler	$\frac{MMBTU}{hour}$ SV

Table 3: Results for six different dynamic optimization problems for the expected energy prices economic case.

Case	Economic Objective	Electricity Net Metering	Excess Heat Hard Constraint	Excess Heat Lower Band	Excess Heat Upper Band	Max CHP Capacity (MW)	Max Boiler Capacity (MMBTU/hr)	Capital Cost (MM USD)	Operating Cost (MM USD)	Levelized Cost of Usable Energy (\$/MWh)
A	✓	✗	✓	✗	✗	13	20	16.2	230.8	23.99
B	✓	✗	✗	✓	✗	11	70	15.4	288.3	26.42
C	✓	✓	✓	✗	✗	30	0	32.0	178.8	27.40
D	✓	✗	✓	✗	✓	11	80	16.2	296.9	27.83
E	✓	✗	✓	✓	✗	11	80	16.1	290.5	27.13
F	✓	✗	✓	✓	✓	11	80	16.2	290.8	27.12

Table 4: Parameters utilized for Cases A-F

Parameter Values		
Parameter	Value	Description
dr	0.06	discount rate
ed_s	See Figure 2*	electric demand $[MW]$
ee	See Figure 3*	electricity price $\left[\frac{\$}{MW \text{ hr}}\right]$
hd_s	See Figure 2*	heat demand $\left[\frac{MMBTU}{hour}\right]$
ir	0.03	inflation rate
n	2	number of years being averaged
ng	See Figure 3*	natural gas price $\left[\frac{\$}{MMBTU}\right]$
η_b	80%	boiler efficiency
η_c	See Eq. 10b*	turbine efficiency
Δt	1 hour	time step
$\tau_{boiler}, \tau_{chp}$	0.024 hr, 0.024 hr	time constants for boiler and CHP
* For parameters which change over time the same data or equation is utilized for each case. A figure plotting the data points or an equation describing the parameter is referenced in this table.		