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Quaternion Based Attitude Error for a Tailsitter in Hover Flight

Matthew E. Argyle, Jason M. Beach, Randal W. Beard, Timothy W. McLain, and Stephen Morris

Abstract—The tailsitter is a promising airframe that can take off and land on its tail and transition to level flight. While this ability provides vertical takeoff and landing capabilities with no additional moving parts, it introduces interesting control challenges. In this paper, we look at the attitude control system of a tailsitter in hover flight and show that the behaviour of the aircraft relies on the method used to compute the attitude error. We investigate three different methods of computing the attitude error, quaternion feedback, resolved tilt twist, and the resolved Euler angles, and compare them through simulated hover flight.

I. INTRODUCTION

The tailsitter is an airframe that has recently received renewed interest because of its promising capabilities. By design, a tailsitter can takeoff and land on its tail, providing the name, which provides vertical takeoff and landing capabilities without any additional moving parts. One downside is that there are several distinct flight modes, hover, level, and the transitions between them, that need to be controlled.

Hover flight, for a tailsitter, is inheritably unstable and must be effectively controlled for a tailsitter to be a practical airframe. There have been several attitude control for hover flight approaches including adaptive control [1] [2], LQR [3], and backstepping [4], but one of the more common approaches is the PID controller [5] [6] [7]. While the controllers differ, they all require a method of calculating the attitude error.

Because tailsitters can conceivably be at almost any attitude, including where all of the Euler angle sequences have singularities, many developers use a unit quaternion to represent the aircraft's attitude. The quaternion representation has many advantages over the standard Euler angle sequence including, but not limited to,

increased computational efficiency and no singularities. In addition, it is very easy to compute the single rotation, axis and angle, needed to drive the system to a desired attitude. While this method, sometimes known as quaternion feedback (QF), can and has been used [6], it does not take into account the dynamics of a hovering tailsitter which can cause, as will be shown, undesired motion. Matsumoto et al. briefly mention this problem and present a possible alternative [7]. In this paper we explore in more detail where the QF attitude error method fails and present a slightly modified form of the resolved tilt twist algorithm proposed by Matsumoto et al. to get around this issue [7]. We also propose an alternative to the RTT, which we call the resolved Euler angle method (REA), that can provide almost identical results with a significantly smaller computational cost.

The paper is organized as follows. Section II will briefly discuss the coordinate frames used in the paper. Section III describes the attitude error methods. These methods are compared in Section IV which is followed by some conclusions in Section V.

II. COORDINATE FRAMES

Several coordinate frames are used throughout this paper and are defined here. The inertial frame is centered at an arbitrary position with the x axis pointing north, the y axis pointing east and the z axis pointing down. Let $\mathbf{K}_i = [i_i \ j_i \ k_i]$ be the unit vectors for the inertial x, y, z axes respectively. The body frame is centered at the center of mass of the aircraft with the x axis pointing out the nose of the aircraft, the y axis pointing out the right wing and the z axis pointing out the belly of the aircraft. Let $\mathbf{K}_b = [i_b \ j_b \ k_b]$ be the unit vectors for the body x, y, z axes respectively.

III. ATTITUDE ERROR

Let $\boldsymbol{\eta} = \begin{bmatrix} \eta_0 \\ \vec{\eta} \end{bmatrix}$ be the quaternion representing the current attitude of the aircraft and $\boldsymbol{\eta}_d$ be the desired attitude. Furthermore, let $\mathbf{K}_b^i = [i_b^i \ j_b^i \ k_b^i]$ be the current body frame unit vectors expressed in the inertial frame and $\mathbf{K}_d^i = [i_d^i \ j_d^i \ k_d^i]$ be the desired body frame unit vectors expressed in the inertial frame. The attitude error will be denoted by $\tilde{e}$.

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Fig. 1: Top down view of a tailsitter pitching down 20° and changing its heading by 180°

A. Quaternion Feedback

Let $\tilde{\eta}$ be the quaternion error which describes the rotation between the current and the desired attitude, i.e.

$$\eta_d = \tilde{\eta} \otimes \eta. \quad (1)$$

Equation (1) can be solved for $\tilde{\eta}$ to obtain

$$\tilde{\eta} = \eta_d \otimes \eta^{-1}. \quad (2)$$

The vector component of $\tilde{\eta}$ corresponds to the attitude error about the body axes, $\tilde{e}_{\text{QF}} = \tilde{\eta}$.

While the QF attitude error method has been used successfully in satellite attitude control [8], there are issues that arise when applying it to aircraft controllers that most designers never take into account. The key problem can clearly be seen in an example. Let the current attitude be vertical, $\eta = \left[\frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0 \right]^T$, and $\eta_d = \text{Euler2Quat}([0 \ 70^\circ \ 180^\circ])$, where $\text{Euler2Quat}([\phi \ \theta \ \psi])$ converts a ZYX Euler angle sequence to a quaternion. Ideally, in this situation, the attitude error would lie only in the body x-y plane which would cause a rotation similar to the one in Figure 1a. However, the QF error is $\tilde{e}_{\text{QF}} = [-0.9848 \ 0 \ 0.1736]^T$ which lies in the body x-z plane. If the attitude controller used this error directly, for example in a PID loop, the aircraft would initially rotate towards the y axis, as shown in Figure 1b which would cause some undesired motion. This point is further illustrated in Figure 2 which shows $\tilde{e}_{\text{QF}}$ for $\eta = \left[\frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0 \right]^T$ and $\eta^d = \text{Euler2Quat}([0 \ 70^\circ \ \psi])$ where $\psi \in [-180^\circ \ 180^\circ]$. As can be seen, using $\tilde{e}_{\text{QF}}$ will cause the aircraft to have some undesired motion if the aircraft wants to pitch and change its heading at the same time. This problem occurs anytime the aircraft wants to rotate about a vector in the body y-z plane, which we will define as tilt, and change its heading.

Fig. 2: QF attitude error

Fig. 3: Tilt and twist error definitions for the resolved tilt twist method

B. Resolved Tilt Twist

The RTT method was developed by Matsumoto et al. [7] to compute the attitude error in a manner that suited tailsitter dynamics better than the QF method. A slightly modified version of the RTT method is presented here. Unlike the QF method which treats the attitude error as a single rotation, the RTT method divides the attitude error into two components: tilt, which depends on the error about the body frames y and x axes, and twist, which accounts for the heading error. The tilt error, Θ_{tilt} , is calculated first, and is the angle between the desired and current x axis as shown in Figure 3a. Matsumoto et al. compute Θ_{tilt} by looking at the error about the current body y and z axes separately.

The error about the y axis can be computed by

$$\Theta_y = \text{atan2}(d_{31}, d_{11}),$$

where $d_{11} = i_b^{iT} i_d^i$ and $d_{31} = k_b^{iT} i_d^i$ are the projections of the current x and z axis onto the desired x axis respectively. Likewise, the error about the z axis can

be computed by

$$\Theta_Z = \text{atan2}(d_{21}, d_{11}),$$

where $d_{21} = j_b^{iT} i_d^i$. Note that d_{11} , d_{21} , and d_{31} are elements of the direction cosine matrix that describes the rotation between the current and the desired attitude given by

$$\mathbf{D} = \mathbf{K}_d^{iT} \mathbf{K}_b^i = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}.$$

The total tilt error is

$$\Theta_{\text{tilt}} = \sqrt{\Theta_Y^2 + \Theta_Z^2}.$$

The twist error, as shown in Figure 3b, is defined to be the error about the x axis once Θ_{tilt} has been compensated for. To compensate for Θ_{tilt} , the current body x axis needs to be aligned with the desired x axis without rotating about the current body x axis. This rotation is computed using Rodrigues' rotation formula

$$\mathbf{R}_v = \mathbf{I}_3 + \hat{\mathbf{v}} \sin(\Theta_{\text{tilt}}) + \hat{\mathbf{v}}^2 (1 - \cos(\Theta_{\text{tilt}})), \quad (3)$$

where $\mathbf{I}_3$ is the 3x3 identity matrix, $\mathbf{v} = [v_x \ v_y \ v_z]^T$ is the unit vector describing the axis of rotation computed by the normalized cross product

$$\mathbf{v} = \frac{i_b^i \times i_d^i}{\|i_b^i \times i_d^i\|} \quad (4)$$

and

$$\hat{\mathbf{v}} = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}.$$

Equation (4) is undefined if $\|i_b^i \times i_d^i\| = 0$ which occurs when $\Theta_{\text{tilt}} = 0$. To handle this (3) is rewritten as

$$\mathbf{R}_v = \begin{cases} \mathbf{I}_3 + \hat{\mathbf{v}} \sin(\Theta_{\text{tilt}}) + \hat{\mathbf{v}}^2 (1 - \cos(\Theta_{\text{tilt}})), & \Theta_{\text{tilt}} \neq 0 \\ \mathbf{I}_3, & \text{otherwise} \end{cases} \quad (5)$$

Note that [7] claims that (4) breaks down when $\mathbf{D} = \mathbf{I}_3$. While this condition is sufficient it is not necessary. The condition in (5) is the proper condition.

The current attitude is compensated for Θ_{tilt} by

$$\mathbf{K}_p = \mathbf{R}_v \mathbf{K}_b^i = [i_p^i \ j_p^i \ k_p^i].$$

Note that $i_p^i = i_d^i$. The twist error is computed by finding the angle between the desired and the tilt compensated z axis by

$$\Theta_{\text{twist}} = \cos^{-1}(k_p^{iT} k_d^i). \quad (6)$$

Fig. 4: RTT attitude error

While (6) computes the angle, it does not provide the sign which is found by

$$\Theta_X = \begin{cases} \Theta_{\text{twist}}, & \Theta_{\text{sign}} \leq \frac{\pi}{2} \\ -\Theta_{\text{twist}}, & \Theta_{\text{sign}} > \frac{\pi}{2} \end{cases},$$

where

$$\Theta_{\text{sign}} = \cos^{-1}(j_p^{iT} k_d^i).$$

The last step, of the RTT method, is to express the tilt error in the current body axes by

$$\tilde{\mathbf{e}}_{\text{RTT}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Theta_X) & -\sin(\Theta_X) \\ 0 & \sin(\Theta_X) & \cos(\Theta_X) \end{bmatrix} \begin{bmatrix} \Theta_X \\ \Theta_Y \\ \Theta_Z \end{bmatrix}.$$

To illustrate the difference between the QF error and the RTT error, the attitude error is calculated for an initial attitude of $\boldsymbol{\eta} = [\frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}}]^T$ and a desired attitude of $\boldsymbol{\eta}^d = \text{Euler2Quat}([0 \ 70^\circ \ \psi])$ where $\psi \in [-180^\circ \ 180^\circ]$. As shown in Figure 4, if the RTT method is used, the aircraft will rotate in the desired plane. For example, when $\psi_d = 90^\circ$, the RTT error is in the x-z plane causing the aircraft to rotate in a way that reduces the undesired motion.

C. Resolved Euler Angle

One attractive feature of the RTT controller is that it attempts to decouple the heading error from the tilt error by treating the error as a sequence of two rotations. However, this advantage comes at the cost of significantly increased computational complexity. Luckily there are alternative approaches.

Instead of viewing the error as one or two rotations, as the QF and RTT methods do, the REA method treats the error as a sequence of three rotations. The heading error, $\tilde{\phi}$, is computed first by rotating, about i_b , k_b onto the desired body frame's x-z plane. The tilt error can be

Fig. 5: REA attitude error

computed by rotating about the y and z axis in either order, but we first rotate about the y axis by $\tilde{\theta}$ followed by rotating about the z axis by $\tilde{\psi}$. Note that this rotation sequence is the ZYX Euler angle sequence and can be computed by

$$\tilde{\phi} = \tan^{-1} \left(\frac{2(\tilde{\eta}_0\tilde{\eta}_1 - \tilde{\eta}_2\tilde{\eta}_3)}{2\tilde{\eta}_0^2 + 2\tilde{\eta}_3^2 - 1} \right) \quad (7)$$

$$\tilde{\theta} = \sin^{-1} (2(\tilde{\eta}_0\tilde{\eta}_2 - \tilde{\eta}_1\tilde{\eta}_3)) \quad (8)$$

$$\tilde{\psi} = \tan^{-1} \left(\frac{2(\tilde{\eta}_0\tilde{\eta}_3 - \tilde{\eta}_1\tilde{\eta}_2)}{2\tilde{\eta}_0^2 + 2\tilde{\eta}_1^2 - 1} \right). \quad (9)$$

Because the REA attitude error is represented as an Euler angle sequence, it has a singularity when $\tilde{\theta} = \pm 90^\circ$. While unlikely to occur, this situation can be handled by saturating $\tilde{\eta}$.

While it is possible to compute the tilt and twist error from (7)-(9) it is not necessary. The REA attitude error is given by

$$\tilde{\mathbf{e}}_{\text{REA}} = [\tilde{\phi} \ \tilde{\theta} \ \tilde{\psi}]^T.$$

Figure 5 shows the REA attitude error for an initial attitude of $\boldsymbol{\eta} = [\frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0]^T$ and a desired attitude of $\boldsymbol{\eta}^d = \text{Euler2Quat}(0 \ 70^\circ, \psi)$, where $\psi \in [-180^\circ, 180^\circ]$. Note that these results are almost identical to the attitude error computed by the RTT method.

IV. ATTITUDE ERROR COMPARISON

As the innermost control loop, the attitude control needs to be as computationally efficient as possible. However, even the best run time cannot excuse poor performance. To compare these methods we look at both their run time and their performance in several scenarios.

A. Run Time

To compare the run times, each method was implemented in Matlab and optimized for speed. Two random quaternions, that were within 20° of vertical, were generated and the attitude error was computed using each method. This was repeated one million times and the run times were analyzed. This test was performed on a computer with a Core i7 2600 processor running at 3.4 GHz. As expected, the QF method was the fastest followed by the REA method. On average the QF was 5.5 times faster than the RTT and 1.5 times faster than the REA.

B. Simulation Results

A simple hovering tailsitter simulator is used to compare the performance of the three methods. The rotation dynamics are given by

$$\begin{bmatrix} \dot{\eta}_0 \\ \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \begin{bmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix},$$

where p , q , and r are the angular rates about the body x, y, and z axes respectively. The angular rates evolve by

$$J\dot{\omega} = \omega \times J\omega + \mathbf{m},$$

where $\omega = [p \ q \ r]^T$,

$$J = \begin{bmatrix} J_x & 0 & -J_{xz} \\ 0 & J_y & 0 \\ -J_{xz} & 0 & J_z \end{bmatrix},$$

$$\mathbf{m} = \begin{bmatrix} \nu_1\delta_1 \\ \nu_2\delta_2 \\ \nu_3\delta_3 \end{bmatrix}, \quad (10)$$

ν_i are constants and δ_i are the control inputs. Note that (10) assumes that the aircraft's thrust does not change significantly and that the aircraft's velocity remains low. The control inputs are generated by the simple PID controller

$$\delta_i = - \left(k_p \tilde{e}_i + k_i \int \tilde{e}_i dt + k_d \dot{\tilde{e}}_i \right).$$

The position dynamics are likewise simplified. We assume that the only forces acting on the aircraft are gravity, the thrust from the propulsion system, and aerodynamic drag which is given by

$$F_{\text{drag}} = -\gamma \dot{x}^2 \text{sign}(\dot{x}) = -\gamma \dot{x} |\dot{x}|,$$

	Test 1.1	Test 1.2	Test 1.3
Desired Attitude	[45° 0 0]	[0 45° 0]	[0 0 45°]

TABLE I: First set of tests in which the vehicle wants to rotate about a single axis

	Test 2.1	Test 2.2
Desired Attitude	[90° 20° 0]	[180° 20° 0]

TABLE II: Second set of tests in which the vehicle wants to tilt and change its heading

where $\dot{x}$ is the velocity of the aircraft and γ is the drag coefficient. Under these assumptions the position dynamics are given by

$$\begin{aligned}\ddot{P}_n &= (\eta_1^2 + \eta_0^2 - \eta_2^2 - \eta_3^2) \frac{T}{m} - \gamma \dot{P}_n \left| \dot{P}_n \right| \\ \ddot{P}_e &= 2(\eta_1\eta_2 + \eta_3\eta_0) \frac{T}{m} - \gamma \dot{P}_e \left| \dot{P}_e \right| \\ \ddot{P}_d &= g + 2(\eta_1\eta_3 - \eta_2\eta_0) \frac{T}{m} - \gamma \dot{P}_d \left| \dot{P}_d \right|,\end{aligned}$$

where the thrust, T , is selected so the aircraft maintains altitude, i.e.

$$T = -\frac{mg}{2(\eta_1\eta_3 - \eta_2\eta_0)}.$$

Two sets of tests were run for each attitude error method. The first test set show how the aircraft reacts to a desired rotation about one of the body axes. The initial attitude, for each test, was vertical with the belly facing north. The desired attitudes, given in XYZ Euler angles with vertical being $[0 \ 0 \ 0]^T$, are shown in Table I. The simulation results are shown in Figure 6. Note that the RTT and the REA attitude error methods produced identical results. The QF attitude error caused a slightly different response but it could be made to match the other two methods by tuning the controller gains.

In the second test set, given in Table II, the aircraft wants to make a large heading change while tilting. In test set 2.1, the desired attitude would cause the aircraft to move east. Ideally the aircraft would rotate in a manner that would cause the aircraft to only move east. The desired attitude, for test set 2.2, would cause the aircraft to move south. Ideally the aircraft would not move east during this maneuver. As can be seen in Figures 7a and 7b, the QF error initially causes the aircraft to tilt in a direction that causes significant undesired motion as seen in Figures 7a and 7b. This initial rotation is inherent to the QF error and cannot be removed by tuning the controller gains. Note that while the RTT and REA methods also caused some undesirable motion, it is significantly less than the QF case, and the motion could be reduced by tuning the controller gains. However,

reducing the undesirable motion, for the RTT and REA methods, would decrease the response time.

V. CONCLUSION

In this paper, we have explored three different ways of computing the attitude error, from a quaternion representation of attitude, for a tailsitter in hover flight. We have shown that the QF error, one of the most common attitude error methods for this application, causes less than ideal maneuvers while the aircraft is simultaneously trying to tilt and change its heading. This undesired movement can be reduced by using the RTT or the REA methods. In the scenarios tested, the RTT and REA methods provided nearly identical results while the REA method is several times quicker.

If the control system can guarantee that the attitude error, or more specifically the heading error, remains small during hover flight, then the QF method makes an excellent attitude error method due to its computational efficiency and its ability to be used in all flight modes. However, if there is no guarantee that the heading error remains small then the REA method is a better choice because it reduces the undesired movement. Future work includes investigating if the RTT and the REA methods can be used in the other flight regimes. Flight tests will also be performed.

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Fig. 6: Simulation results for the first test set

Fig. 7: Simulation results for the second test set