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Quantum electrodynamics based on self-fields, without second quantization: A nonrelativistic calculation of $g-2$

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Using a formulation of quantum electrodynamics that is not second quantized, but rather based on self-fields, we compute the anomalous magnetic moment of the electron to first order in the fine-structure constant α. In the nonrelativistic (NR) case and in the dipole approximation, our result is $a_e \equiv (g-2)/2 = (4\Lambda/3m)(\alpha/2\pi)$, where Λ is a positive photon energy cutoff and m the electron mass. A reasonable choice of cutoff, $\Lambda/m = \frac{3}{4}$, yields the correct sign and magnitude for $g-2$ namely, $a_e = +\alpha/2\pi$. In our formulation the sign of a_3 is correctly positive, independent of cutoff, and the demand that $a_e = +\alpha/2\pi$ implies a unique value for Λ. This is in contradistinction to previous NR calculations of a_e that employ electromagnetic vacuum fluctuations instead of self-fields; in the vacuum fluctuation case the sign of a_e is cutoff dependent and the equation $a_e = \alpha/2\pi$ does not have a unique solution in Λ.

I. INTRODUCTION

Bethe[1] first calculated the Lamb shift for the hydrogen atom in 1947, using a method that was essentially nonrelativistic (NR), but nevertheless approximately correct. In 1948, Welton[2] gave an intuitive derivation of the Bethe result by considering the coupling of the electron to second-quantized electromagnetic vacuum fluctuations—leading to the generally held folklore that vacuum fluctuations are the physical cause of the Lamb shift. However, when the Welton approach was used to compute the anomalous magnetic moment of the electron,[2,3] the incorrect sign for $g-2$ was obtained.

Complementary to the vacuum fluctuation picture is the self-field picture, in which one views radiative corrections as arising from radiation reaction effects due to the interaction of a particle with its own self-field. The self-field picture is thus in line with the classical point of view where there are no infinite energy density zero-point fluctuations and the vacuum field is identically equal to zero.

The 1951 paper of Callen and Welton[4] on the fluctuation dissipation theorem showed that there is an intimate connection between vacuum fluctuations and the process of radiation reaction. The existence of one implies the existence of the other. In the 1970s several workers in the field[5] formulated standard QED in terms of the Heisenberg equations of motion and were able to show that the phenomenon of spontaneous emission could be interpreted as being caused by vacuum fluctuations or by radiation reaction *or* indeed by any linear combination of the two effects. These interpretations are dependent upon whether one uses symmetric ordering or normal ordering, or some linear combination of these two orderings, respectively, when writing down the field operators. This does not allow one to do away with the vacuum field operators, however. In standard QED the vacuum field must be maintained in the equations of motion of the atomic operators, which otherwise would decay to zero as the atom radiates. This would violate unitarity.

Dalibard *et al.*[6] have argued that only the symmetric ordering of the field operators can be used if one demands that the self-field and vacuum operators be separately Hermitian, which would seem to force upon us the vacuum fluctuation interpretation. It is interesting in this context to note that in the theory of stochastic or random electrodynamics the n-point correlation functions of the classical, stochastic background field agree with those of the QED vacuum field only if the QED field operators are symmetrically ordered.[7]

The radiation reaction picture, however, also has its many advocates. To quote Jaynes:[8] "This complete interchangeability of source-field effects and vacuum fluctuation effects does not show that vacuum fluctuations are "real." It shows that the source-field effects are the same *as if* vacuum fluctuations are present." He has shown that the energy density of the radiation field, over the spectral interval of the natural linewidth, is exactly the same as that of the vacuum field.[8] Finally, we quote Milonni:[9] "It seems. . . that the generalization of these ideas. . . may lead us to view the vacuum field more as a formal artifice or subterfuge than a "real" physical thing."

As mentioned above, nonrelativistic endeavors to derive the anomalous magnetic moment of the electron by coupling it to the vacuum fluctuations yield the wrong sign for $a_e := (g-2)/2$. (Here $a := b$ is symbolic logic notation for "a is being defined as equal to b;" the quantity on the side of the colon is the quantity being defined.) Grotch and Kazes[10] have managed to obtain the correct sign of a_e in such a NR vacuum fluctuation approach by including the effects of a mass renormalization term δm. However, the sign of a_e in this approach is itself cutoff dependent—for a cutoff Λ in the range $\Lambda \in (0, 4m)$ one gets $a_e > 0$, but for $\Lambda \in (4m, \infty)$ we have the incorrect sign, $a_e < 0$. Even when $a_e > 0$ the choice of Λ which solves $a_e = +a/2\pi$ is not unique. These difficulties do not occur with the self-field approach which is used herewith; the sign of a_e is correct, independent of cutoff, and $a_e = +a/2\pi$ leads to a unique choice of the parameter Λ.

Before mass renormalization (MR), we obtain, in the self-field approach,

$$a_e = -\frac{\alpha}{2\pi}\left[\frac{4\Lambda}{3m}\right] \tag{1}$$

to the first order in the fine-structure constant α. After mass renormalization is employed, the correct sign results and we have

$$a_e = +\frac{\alpha}{2\pi}\left[\frac{4\Lambda}{3m}\right] . \tag{2}$$

We see that a choice of cutoff $\Lambda = 3m/4$ in Eq. (2) yields the correct value for a_e. The sign of a_e is positive for all $\Lambda \in (0, \infty)$ and the function is linear in the parameter $x := \Lambda/m$, leading to a unique value of x for the solution of $a_e = \alpha/2\pi$. In contradistinction, the result of the vacuum fluctuation (VF) calculation[10] is a quadratic function of x with negative concavity; hence the sign of the VG formula for $a_e(x)$ depends on x, and the solution of $a_e(x) = +\alpha/2\pi$ is not unique, as $a_e(x)$ is not single valued.

The fact that the self-field approach yields in a straightforward and unambiguous manner the correct sign and magnitude for a_e, and that the result diverges only linearly rather than quadratically with the cutoff parameter x, might be viewed as evidence for the interpretation that it is the self-field of the particle itself (rather than the hypothetical fluctuating vacuum) which is the physical origin of the nonzero value of $g-2$ in free space.

II. METHOD

We briefly review the self-field approach to QED which was used recently to evaluate a number of radiative processes.[11–16] The basic idea of the approach is quite simple: one includes the self-field of the particle from the beginning, rather than introducing a second-quantized radiation field. Thus vacuum fluctuations, a direct consequence of the second quantization procedure, do not appear. The vacuum field is identically zero.

The classical field $A_\mu(x)$ surrounding the charged particle is conceptually separated into a self-field contribution A_μ^s and an external field A_μ^e if required. The A_μ are used to construct the field tensor $F_{\mu\nu}$ via the usual definition

$$F_{\mu\nu} := A_{\nu,\mu} - A_{\mu,\nu}$$
$$:= \partial_\mu A_\nu - \partial_\nu A_\mu \, .$$

The $F_{\mu\nu}$ obey the inhomogeneous Maxwell equations ($e > 0$, $h/2\pi = c = 1$)

$$F_s^{\mu,\nu}{}_{,\mu} = e j^\nu \, . \tag{3}$$

In the absence of external currents, $j^\mu(x)$ is just the four-current of the electron. The self-field A_μ^s is completely determined by j^μ and by boundary conditions. Equation (3) can be solved formally for the self-field alone as[17]

$$A_\mu^s(x) = e\int dy\, D_{\mu\nu}(x-y) j^\nu(y) \, , \tag{4}$$

where $D_{\mu\nu}$ is an electromagnetic Green's function. In the Coulomb gauge, and for empty space without cavities, we have

$$\begin{aligned} D_{ij}(x-y) &= \frac{1}{(2\pi)^4}\int dk \frac{e^{-ik\cdot(x-y)}}{k^2}(\delta_{ij} + \kappa_i\kappa_j) \, , \\ D_{00}(x-y) &= \frac{1}{(2\pi)^4}\int dk \frac{e^{-ik\cdot(x-y)}}{|\mathbf{k}|^2} \, , \\ D_{i0}(x-y) &= D_{0i}(x-y) = 0 \, , \end{aligned} \tag{5}$$

where we are using the four-vector notation: $k\cdot x := k_\mu x^\mu$, $dk := d^4k$, etc. and $\kappa_i := k_i/|\mathbf{k}|$.

We proceed to calculate all energy shifts from an action W, with corresponding action density w

$$W = \int dx\, w(x; \varphi; A) \, . \tag{6}$$

(Indices μ, ν, etc. are suppressed.) For scattering problems the dimensionless action W is related to the scattering amplitude per unit space-time G, and for bound states is related to the total invariant energy $\mathcal{E}$ of the system by

$$\begin{aligned} W_{fi} &= (2\pi)^4\delta^4(P_f - P_i) G \, , \\ W_{fi} &= (2\pi)\delta(E_f - E_i)\mathcal{E} \, . \end{aligned} \tag{7}$$

(The P's and E's are the initial or final momenta and energies in the free- and bound-state cases, respectively.) For the purpose of computing $g-2$ we take φ to be a two-component Pauli spinor field which we couple to the electromagnetic field A_μ via the Pauli Hamiltonian:

$$H = \frac{1}{2m}[\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A})]^2 + eA_0 \, . \tag{8}$$

Including the self-field Lagrangian contribution, $\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, and using integration by parts to symmetrize H, we have for the total action density

$$w = \varphi^* \left\{ \frac{1}{2m}[(\overleftarrow{\nabla} + ie\mathbf{A})\cdot\boldsymbol{\sigma}][\boldsymbol{\sigma}\cdot(\overrightarrow{\nabla} - ie\mathbf{A})] + eA_0 - i\partial_t \right\}\varphi + \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \, . \tag{9}$$

The *Euler-Lagrange equations of motion* yield the *Pauli equation* upon variation with respect to φ^*:

$$\frac{\delta W}{\delta\varphi^*} - \partial_\mu\frac{\delta W}{\delta\varphi^*{}_{,\mu}} = \left\{ -\frac{\nabla^2}{2m} + \frac{ie}{m}\mathbf{A}\cdot\nabla + \frac{ie}{2m}\nabla\cdot\mathbf{A} - \frac{e}{2m}\boldsymbol{\sigma}\cdot\mathbf{B} + \frac{e^2}{2m}A^2 \right\}\varphi = 0 \, ,$$

and they give the inhomogeneous Maxwell equations

$$\frac{\delta W}{\delta A^s_\nu}-\partial_\mu\frac{\delta W}{\delta A^s_{\nu,\mu}}=-ej^\nu+F_s^{\mu\nu}{}_{,\mu}=0\ ,$$

upon variation with respect to A_μ–so long as we define the four-current j^μ as

$$\frac{\delta W}{\delta A^s_\mu}=:-ej^\mu\ ,$$

which implies in turn that (S, M, and F label the spin, momentum, and field contributions, respectively)

$$j^\mu=\varphi^*\left[1,\frac{1}{2mi}\overleftrightarrow{\nabla}+\frac{1}{2m}(\overleftarrow{\nabla}\times\sigma-\sigma\times\overrightarrow{\nabla})-\frac{e}{m}\mathbf{A}\right]\varphi$$

$$=[\rho,\mathbf{j}_M+\mathbf{j}_{SM}+\mathbf{j}_F]\ . \tag{10}$$

With this current the interaction of the matter field with the electromagnetic field (EM) field is of the standard form $eA_\mu j^\mu$.

Equation (10) is the NR version of the Gordon decomposition of the Dirac current $e\Psi\gamma_\mu\Psi$. The vector portion of j^μ separates naturally into momentum (M), spin momentum (SM), and field (F) currents. In fact, the nonrelativistic approximation made by the separation of Ψ in $e\Psi\gamma_\mu\Psi$ into upper and lower (large and small) components yields Eq. (10).

In what follows we assume that $A^s(x)$ vanishes at infinity. Thus, in calculations involving the action density $w(x)$, equality will be understood as equality with respect to integration by parts and possible surface integrals vanishing at infinity along with A^s.

III. FIELD CONTRIBUTION

The electromagnetic field Lagrangian $F_{\mu\nu}F^{\mu\nu}$ in $w(x)$ contains both A^s and A^e. However, cross terms may be converted to surface integrals which vanish at infinity, leaving only $F_{\mu\nu}F^{\mu\nu}=F^e_{\mu\nu}F_e^{\mu\nu}+F^s_{\mu\nu}F_s^{\mu\nu}$. The external field tensor contraction is the invariant:

$$\tfrac{1}{4}F^e_{\mu\nu}F_e^{\mu\nu}=-\tfrac{1}{2}(E_e^2-B_e^2)\ , \tag{11}$$

which we shall drop from $w(x)$, because it is a nondynamical fixed quantity. The self-field tensor contraction may be written

$$\tfrac{1}{4}F^s_{\mu\nu}F_s^{\mu\nu}=\tfrac{1}{4}A^s_{[\mu,\nu]}F_s^{\mu\nu}$$

$$=\tfrac{1}{4}(A^s_{[\nu}F_s^{\mu\nu})_{,\mu]}-\tfrac{1}{4}A^s_{[\nu}F_s^{\mu\nu}{}_{,\mu]}$$

$$=-\frac{e}{2}A^s_\nu j^\nu\ , \tag{12}$$

where [,] implies an antisymmetrization with respect to the two indices. We recall that j^μ given by Eq. (10) contains $\mathbf{A}$ which is the sum of $\mathbf{A}^e$ and $\mathbf{A}^s$; keeping this in mind, we expand out the product $A^s_\mu j^\mu$. Integration by parts is used to sandwich all the ∇'s between φ^* and φ, and the spin algebra of the σ's is used to simplify various terms. The result is

$$\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}=-\tfrac{1}{2}eA^s_\mu j^\mu$$

$$=\varphi^*\left[-\frac{e}{2}A^s_0-\frac{ie}{2m}\mathbf{A}^s\cdot\nabla-\frac{ie}{4m}\nabla\cdot\mathbf{A}^s\right.$$

$$\left.+\frac{e}{4m}\sigma\cdot\mathbf{B}^s-\frac{e^2}{2m}(\mathbf{A}^e\cdot\mathbf{A}^s+A_s^2)\right]\varphi\ , \tag{13}$$

where $\mathbf{B}^s:=\nabla\times\mathbf{A}^S$.

IV. TOTAL ACTION

The kinetic portion of the action density, w_0, may be expanded and integrated by parts to give

$$w_0=\varphi^*\left[-\frac{1}{2m}\nabla^2+\frac{ie}{m}\mathbf{A}^s\cdot\nabla+\frac{ie}{2m}\nabla\cdot\mathbf{A}^s\right.$$

$$\left.-\frac{e}{2m}\sigma\cdot\mathbf{B}^s+\frac{e^2}{2m}A^2\right]\varphi\ , \tag{14}$$

keeping in mind that $\mathbf{A}=\mathbf{A}^s+\mathbf{A}^e$, so that $\nabla\times\mathbf{A}=\mathbf{B}=\mathbf{B}^s+\mathbf{B}^e$.

Combining the results of Eqs. (13) and (14), we may write the total action density $w(x)$ as

$$w=\varphi^*\left[-\frac{\nabla^2}{2m}+eA^e_0+\frac{e}{2}A^s_0-i\partial_t+\frac{ie}{m}\mathbf{A}^e\cdot\nabla\right.$$

$$+\frac{ie}{2m}\mathbf{A}^s\cdot\nabla-\frac{e}{2m}\sigma\cdot\mathbf{B}^e-\frac{e}{4m}\sigma\cdot\mathbf{B}^s+\frac{e^2}{2m}A_e^2$$

$$\left.+\frac{e^2}{2m}\mathbf{A}^e\cdot\mathbf{A}^s+\frac{ie}{2m}\nabla\cdot\mathbf{A}^e+\frac{ie}{4m}\nabla\cdot\mathbf{A}^s\right]\varphi$$

$$=:\sum_{i=1}^{12}w_i\ . \tag{15}$$

Note that the term A_s^2 drops out in the total action.

For a particle of charge e moving in a uniform external magnetic field $\mathbf{A}^e(\mathbf{x})=\frac{1}{2}\mathbf{B}^e\times\mathbf{x}$ we may set $A^e_0(x)\equiv 0$. In the Coulomb gauge, $\nabla\cdot\mathbf{A}^e=\nabla\cdot\mathbf{A}^s=0$. We drop, for weak fields, the term proportional to A_e^2 and that proportional to $\mathbf{A}^e\cdot\mathbf{A}^s$ will turn out to be of $O(\alpha^2)$ and so we drop it too. Terms w_1+w_5 will give rise to the standard Landau orbital solutions, while w_7 is the normal magnetic moment contribution. (Our choice of action does not give a spin-orbit term.) Barut and Van Huele[12] have analyzed the contributions from w_3 and w_6. The term w_3 corresponds to the electrostatic self-energy and can be written in such a way as to suggest a NR analogue to the QED vacuum polarization; it contributes here only to mass renormalization. The action density w_6 can be shown to give rise to the phenomenon of spontaneous emission, with the correct Einstein A coefficient—and also to the Bethe expression for the Lamb shift. In addition, w_6 contains a mass renormalization term.

Of primary interest for us in this work, however, is the term w_8,

$$w_8=\varphi^*\left[-\frac{e}{4m}\sigma\cdot\mathbf{B}^s\right]\varphi\ , \tag{16}$$

which contains the anomalous magnetic moment.

V. CALCULATION OF w_8

In order to evaluate w_8 we must compute $\sigma\cdot\mathbf{B}^s=\sigma\cdot(\nabla\times\mathbf{A}^s)$. The self-field $A_\mu^s(x)$ is specified by Eqs. (4) and (5). We need not use the entire expression (10) for the current j^μ. We shall eventually be making the dipole approximation (DA) and also be taking the limit of the electron momentum going to zero. Only the portion of j^μ which couples the spin to the field will then contribute to the final result:

$$\mathbf{j}_F=-\frac{e}{m}\varphi^*\mathbf{A}\varphi\ . \tag{17}$$

Using Eq. (4) for the self-field $A_\mu^s(x)$, and Eqs. (5) for the Green's function $D_{\mu\nu}(x-y)$, we have

$$-\mathbf{A}^s(x)=-\frac{e^2}{m}\int\int dy\,dk\frac{1}{(2\pi)^4}\frac{e^{-ik\cdot(x-y)}}{k^2}\times\varphi^*\{\mathbf{A}(y)-\boldsymbol{\kappa}[\boldsymbol{\kappa}\cdot\mathbf{A}(y)]\}\varphi\ . \tag{18}$$

Here $\mathbf{A}=\mathbf{A}^e+\mathbf{A}^s$, but in an iterative procedure, to within $O(\alpha)$, we may take $\mathbf{A}(y)=\mathbf{A}^e(y)$ under the integrand. Furthermore, anticipating use of the DA and the zero-momentum limit, we can take $\mathbf{A}^s(y)\cong\mathbf{A}^s(\mathbf{x})$. Writing the four-vectors $k^\mu=[\omega,\mathbf{k}]$, $x^\mu=[t,\mathbf{x}]$, $y^\mu=[u,\mathbf{y}]$ and taking the DA, $e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\cong 1$, we arrive at

$$\nabla\times\mathbf{A}^s=\frac{e^2}{m}\int\int dy\,dk\frac{1}{(2\pi)^4}\frac{e^{-i\omega(t-u)}}{k^2}\times\varphi^*[\tfrac{1}{2}\mathbf{B}^e+\tfrac{1}{2}\boldsymbol{\kappa}(\boldsymbol{\kappa}\cdot\mathbf{B}^e)]\varphi\ . \tag{19}$$

Expression (19) may be simplified by the following trick: let $\mathbf{a}$ be an *arbitrary* constant vector. Dot $\mathbf{a}$ into both sides of (19) and carry out the $d\Omega_k$ integration with the aid of the identity

$$\int d\Omega_k(\mathbf{a}\cdot\boldsymbol{\kappa})(\mathbf{b}\cdot\boldsymbol{\kappa})=\frac{4\pi}{3}\mathbf{a}\cdot\mathbf{b}\ .$$

Then, since $\mathbf{a}\cdot\mathbf{b}=\mathbf{a}\cdot\mathbf{c}\ \forall\mathbf{a}\Longrightarrow\mathbf{b}=\mathbf{c}$, we may extract $\mathbf{a}$ to obtain ($dy:=d^4y$)

$$\nabla\times\mathbf{A}^s=\frac{e^2}{m}\frac{1}{(2\pi)^4}\int dy\int_0^\infty d\lambda\int_{-\infty}^{\infty}d\omega\,\lambda^2\frac{e^{-i\omega(t-u)}}{k^2}\times\varphi^*\left[\frac{8\pi}{3}\mathbf{B}^e\right]\varphi\ , \tag{20}$$

where $\lambda^2:=|\mathbf{k}|^2$ so $k^2=\omega^2-\lambda^2$. This expression (20) is interesting in its own right; it relates the magnetic field produced by a circulating electron $\mathbf{B}^s$, to the external magnetic field $\mathbf{B}^e$ causing the circulation. We may now write w_8 as

$$w_8=\varphi^*\left[-\frac{e}{4m}\sigma\cdot\mathbf{B}^s\right]\varphi$$
$$=-\varphi^*(x)\left[\frac{4}{3}\frac{\alpha}{(2\pi)^2}\frac{1}{m}(\boldsymbol{\mu}_0\cdot\mathbf{B}^e)\int\int_0^\infty\int_{-\infty}^{\infty}dy\,d\lambda\,d\omega\,\lambda^2\frac{e^{-i\omega(t-u)}}{k^2}\rho(y)\right]\varphi(x)\ , \tag{21}$$

where $\boldsymbol{\mu}_0=\mu_0\sigma$ with $\mu_0=e/2m$ the Bohr magneton, and $\rho(y)=\varphi^*(y)\varphi(y)$ as usual. In our units $\alpha=e^2/4\pi$.

VI. THE CALCULATION OF $g-2$

We now proceed with the analysis of the action $W=\int dx\,w(x)$ in the manner set forth by Barut and Kraus[11] or Barut and Van Huele.[12] From Eq. (15) we may write the total action density as

$$\begin{aligned}w&=\varphi^*(H_0+H'-i\partial_t)\varphi\ ,\\ H_0&:=-\frac{\nabla^2}{2m}+\frac{ie}{m}\mathbf{A}^e\cdot\nabla\ ,\\ H'&:=-\frac{e}{2m}\sigma\cdot\mathbf{B}^e-\frac{e}{4m}\sigma\cdot\mathbf{B}^s+\frac{ie}{2m}\mathbf{A}^s\cdot\nabla\\ &:=H_1'+H_2'+H_3'\ ,\end{aligned}\tag{22}$$

where H_0 gives rise to the normal Landau solutions, and H' is a perturbation containing spin and self-field effects. We now perform a Fourier expansion of the Pauli field as (see Refs. 11 and 12)

$$\varphi(x)=\sum_n\varphi_n(\mathbf{x})e^{-iE_nt}\ , \tag{23}$$

where we anticipate that the $\varphi_n(\mathbf{x})$ will be, in the lowest order of iteration, the Landau solutions for H_0. We consider the H_2' contribution in detail ($dx:=d^4x$, $dy:=d^4y$):

$$W_2' = \int dx\, \varphi^* \left[-\frac{e}{4m} \sigma \cdot \mathbf{B}^s \right] \varphi$$
$$= -\frac{4}{3} \frac{\alpha}{(2\pi)^2} \frac{1}{m} \int \int dx\, dy\, \varphi^*(x)(\boldsymbol{\mu}_0 \cdot \mathbf{B}^e)\varphi(x) \int_0^\infty d\lambda\, \lambda^2 \int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega(t-u)}}{\omega^2 - \lambda^2} \rho(y)$$
$$= -\frac{4}{3} \frac{\alpha}{(2\pi)^2} \frac{1}{m} \sum_{n,m,r,s} \int \int dx\, dy\, \varphi_n^*(\mathbf{x})(\boldsymbol{\mu}_0 \cdot \mathbf{B}^e)\varphi_m(\mathbf{x}) \int_0^\infty d\lambda\, \lambda^2 \int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega(t-u)}}{\omega^2 - \lambda^2} \varphi_r^*(\mathbf{y})\varphi_s(\mathbf{y})$$
$$\times \exp[i(E_n - E_m)t + i(E_r - E_s)u] , \qquad (24)$$

where $k := [\omega, \mathbf{k}]$ and $\lambda := |\mathbf{k}|$. Carrying out the $dx^0 := dt$, $dy^0 := du$ integrations gives a δ function: $\delta(\omega_{nm} + \omega_{rs})$, where $\omega_{nm} := E_n - E_m$, etc. Carrying out the $dk_0 = d\omega$ integration and using Dirac notation for simplicity, Eq. (22) gives, with $n = s$ and $m = r$, the self-energy contribution

$$W_2' = \frac{4}{3} \frac{\alpha}{m} \int_0^\infty d\lambda \sum_n \langle n \,|\, \boldsymbol{\mu}_0 \cdot \mathbf{B}^e \,|\, n \rangle , \qquad (25)$$

where we have used orthonormality of the φ_n. (The terms with $n = m$ and $r = s$, which are vacuum polarization terms in the relativistic case, do not arise in NR calculations.)

We may analyze $W_1 = \int dx\, \varphi^*(-\boldsymbol{\mu}_0 \cdot \mathbf{B}^e)\varphi$ in the same fashion. We divide W by 2π to obtain an energy shift. [See Eq. (7).] Extracting the contribution to the nth Landau level, the total energy shift proportional to $\boldsymbol{\mu}_0 \cdot \mathbf{B}^e$ is given by

$$\Delta E_n = \frac{(W_1' + W_2')_n}{2\pi}$$
$$= -\langle n \,|\, \boldsymbol{\mu}_0 \cdot \mathbf{B}^e \,|\, n \rangle \left[1 - \frac{4}{3} \frac{\alpha}{2\pi} \frac{\Lambda}{m} \right] . \qquad (26)$$

(Recall, we have used $e > 0$ throughout.) We have introduced a cutoff Λ in the photon momentum integration $\int_0^\infty d\lambda \rightarrow \int_0^\Lambda d\lambda =: \Lambda$.

The free-space contribution to the magnetic moment is then

$$\frac{\delta\mu}{\mu} = -\frac{\alpha}{2\pi} \frac{4\Lambda}{3m} + O(\alpha^2) . \qquad (27)$$

Taking the cutoff of $\Lambda = 3m/4$ gives a value of $\delta\mu/\mu$ correct in magnitude, but incorrect in sign.

As mentioned in the Introduction, Grotch and Kazes[10] have pointed out the necessity of including mass renormalization in NR calculations such as this. In Sec. VII, we compute the mass renormalization, and show that the factor of $-\frac{4}{3}$ is actually $+\frac{4}{3}$.

VII. MASS RENORMALIZATION

From Eq. (22) we now consider H_3' which gives a piece of the action, W_3',

$$W_3' = \int dx\, \varphi^* \left[\frac{ie}{2m} \mathbf{A}^s \cdot \nabla \right] \varphi . \qquad (28)$$

We are concerned here with a change in the mass as a coefficient of inertia of the particle. Thus we will be interested in a change in the kinetic energy operator which is proportional to ∇^2. An inspection of Eq. (10) for the current j^μ shows that only $\mathbf{j}_M$ and $\mathbf{j}_{SM}$ contain the momentum operator explicitly. Detailed calculations show that the contribution from $\mathbf{j}_{SM}$ is zero; therefore we shall concentrate on $\mathbf{j}_M(y) = \varphi^* \nabla \varphi / mi$. The analysis proceeds similarly to that of W_1' and W_2' in the magnetic moment calculation. With $\mathbf{j}(\mathbf{y}) = \mathbf{j}_M(\mathbf{y})$ we compute $\mathbf{A}^s$.

$$-\mathbf{A}^s = \frac{e}{(2\pi)^4} \int \int dy\, dk \frac{e^{-ik\cdot(x-y)}}{k^2} [\mathbf{j} - \boldsymbol{\kappa}(\boldsymbol{\kappa} \cdot \mathbf{j})]$$
$$= \frac{8\pi}{3} \frac{e}{mi} \frac{1}{(2\pi)^4} \int dy \int_0^\infty d\lambda\, \lambda^2 \frac{e^{-i\omega(t-u)}}{\omega^2 - \lambda^2} \varphi^* \nabla \varphi , \qquad (29)$$

where we have used the dipole approximation, valid in the limit $\mathbf{B}^e \rightarrow \mathbf{0}$ and $\langle -i\nabla \rangle \rightarrow \mathbf{0}$. Inserting the expression (29) into Eq. (28) and expanding

$$\varphi(x) = \sum_n \varphi_n(\mathbf{x}) \exp(-iE_n t)$$

as before, we obtain

$$W_3' = \frac{4}{3} \frac{\alpha}{m^2} \sum_{m,n} \int_0^\Lambda d\lambda \left[\langle n \,|\, \nabla \,|\, n \rangle \cdot \langle m \,|\, \nabla \,|\, m \rangle - \frac{\lambda^2}{\omega_{nm}^2 - \lambda^2} \langle n \,|\, \nabla \,|\, m \rangle \cdot \langle m \,|\, \nabla \,|\, n \rangle \right] , \qquad (30)$$

with $\omega_{nm} := E_n - E_m$ and $\lambda = |\mathbf{k}|$, as was previously. Barut and Van Huele[12] have shown that the first term in the large parentheses vanishes if one does not make the dipole approximation, and so we drop it. The second term in the large parentheses can be modified by noticing that, due to the symmetry in indices n and m, we have the partial fraction expansion

$$\frac{\lambda^2}{\omega_{nm}^2 - \lambda^2} = \frac{1}{2} \left[\frac{\omega_{nm}}{\omega_{nm} - \lambda} - \frac{\omega_{nm}}{\omega_{nm} + \lambda} - 2 \right]$$
$$= \left[\frac{\omega_{nm}}{\omega_{nm} + \lambda} - 1 \right]_{n,m} , \qquad (31)$$

where the second equality is, with respect to the double sum, $\sum_{n,m}$. Thus

$$W'_3=\frac{4}{3}\frac{\alpha}{m^2}\sum_{m,n}\int_0^{\Lambda}d\lambda\left[1-\frac{\omega_{nm}}{\omega_{nm}+\lambda}\right]\times\langle n\,|\,\nabla\,|\,m\rangle\cdot\langle m\,|\,\nabla\,|\,n\rangle\ . \tag{32}$$

The constant term 1 in the large parentheses of (32) leads to our mass renormalization, while the other term gives rise to spontaneous emission and the Lamb shift (see Ref. 12).

Taking only the mass renormalization term from (32) and extracting only the contribution to a single energy level n, we get

$$\Delta E^n_{\rm MR}=\frac{W'^n_{\rm MR}}{2\pi}=\frac{\alpha}{2\pi}\frac{4\Lambda}{3m^2}\sum_m\langle n\,|\,\nabla\,|\,m\rangle\cdot\langle m\,|\,\nabla\,|\,n\rangle=-\frac{\alpha}{2\pi}\frac{8\Lambda}{3m}\left\langle n\left|-\frac{\nabla^2}{2m}\right|n\right\rangle\ . \tag{33}$$

Calling the bare mass m_0, and the renormalized mass m, we can define a new kinetic energy operator for a free electron as

$$-\frac{\nabla^2}{2m}:=-\frac{\nabla^2}{2m_0}\left[1-\frac{\alpha}{2\pi}\frac{8\Lambda}{3m_0}\right],$$

which then implies the mass renormalization

$$m=m_0\left[1-\frac{\alpha}{2\pi}\frac{8\Lambda}{3m_0}\right]^{-1}=m_0\left[1+\frac{\alpha}{2\pi}\frac{8\Lambda}{3m_0}\right]+O(\alpha^2)=:m_0+\delta m\ , \tag{34}$$

where use of the binomial expansion is valid if we assume that $\Lambda\sim m$. The total shift proportional to $\sigma\cdot\mathbf{B}$ will be the sum of contributions from w_7 and w_8 with m renormalized:

$$\Delta E^n_7+\Delta E^n_8=-\frac{e}{2m_0}\langle n\,|\,\sigma\cdot\mathbf{B}^e\,|\,n\rangle\left[1-\frac{\alpha}{2\pi}\frac{4\Lambda}{3m}\right]=-\frac{e}{2m}\langle n\,|\,\sigma\cdot\mathbf{B}^e\,|\,n\rangle\left[1+\frac{\delta m}{m}+\frac{\delta\mu}{\mu}\right]=-\frac{e}{2m}\langle n\,|\,\sigma\cdot\mathbf{B}^e\,|\,n\rangle\left[1+\frac{\alpha}{2\pi}\frac{4\Lambda}{3m}\right], \tag{35}$$

where, to $O(\alpha)$, either m or m_0 may be used in the large parentheses of the last line of (35).

Thus the mass renormalization combines with the original moment shift $\delta\mu$ to yield the correct sign for a_e. Our final expression for the gyromagnetic ratio $(g-2)/2$ is

$$a_e:=\frac{g-2}{2}=+\frac{\alpha}{2\pi}\frac{4\Lambda}{3m}+O(\alpha^2)\ . \tag{36}$$

This is our main result. Our first value for a_e, given in Eq. (27) was negative, but after including the effects of mass renormalization, it became positive with a value of the same magnitude as before. The unique choice of the value $\Lambda=3m/4$ for the cutoff parameter gives an agreement with the QED value of $a_e=+\alpha/2\pi+O(\alpha^2)$.

VIII. SUMMARY AND CONCLUSIONS

Thus we see that there are two separate effects contributing to the NR calculation of $g-2$. One must compute the actual magnetic moment change $\delta\mu$ and then include the effects of mass renormalization δm. Below, we compare the results of our self-field (SF) approach to that of the vacuum fluctuation approach [the VF results are those of Grotch and Kazes (GK), Ref. 10]. Here $x:=\Lambda/m$

	Self-field	Vacuum fluctuation
$\frac{\delta\mu}{\mu}$	$-\frac{\alpha}{2\pi}\left[\frac{4x}{3}\right]$	$-\frac{\alpha}{2\pi}\left[\frac{2x^2}{3}\right]$
$\frac{\delta m}{m}$	$+\frac{\alpha}{2\pi}\left[\frac{8x}{3}\right]$	$+\frac{\alpha}{2\pi}\left[\frac{8x}{3}\right]$
a_e	$+\frac{\alpha}{2\pi}\left[\frac{4x}{3}\right]$	$+\frac{\alpha}{2\pi}\left[\frac{8x}{3}\right]\left[1-\frac{x}{4}\right]$

where to $O(\alpha)$ either m or m_0 can be used interchangeably. The same cutoff Λ for the photon momentum $|\mathbf{k}|$ is used in all formulas.

It has been claimed that the vacuum fluctuation result is positive, but in fact the GK formula for $a_e^{\rm VF}$ as a function of Λ is only positive in the restricted range $\Lambda\in(0,4m)$; thus the sign is actually cutoff dependent (see Fig. 1). If we define the unitless parameter x by $x:=\Lambda/m$, then the formula for $a_e^{\rm VF}=a_e^{\rm VF}(x)$ yields $a_e^{\rm VF}=+\alpha/2\pi$ for the two values $x=2\pm\frac{1}{2}\sqrt{10}$

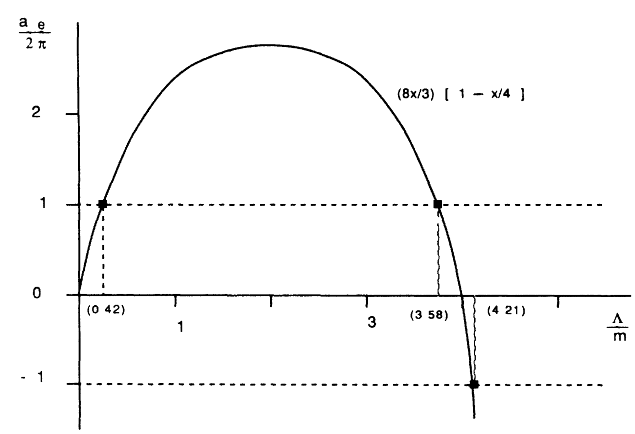

FIG. 1. Grotch and Kazes's formula for $a_e=(g-2)/2$ as a function of the unitless cutoff parameter $x:=\Lambda/m$. Notice that $a_e(x)>0$ only if x is in the interval (0,4). The equation $a_e(x)=+\alpha/2\pi$ has two solutions and $a_e(x)=-\alpha/2\pi$ has one.

$\cong \{3.58, 0.42\}$. (The value of 0.42 is quoted by GK in Ref. 10). Hence the choice of x here is not unique. In addition, one can equally arrive at the *wrong* answer of $a_e^{\mathrm{VF}} = -\alpha/2\pi$ by choosing $x = 2 + \frac{1}{4}\sqrt{78} \cong 4.21$. Since none of these choices of x appears to be physically unreasonable, it seems to the present authors that the problem of the correct sign for a_e has not been satisfactorily solved within the context of the NR vacuum fluctuation approach, mass renormalization notwithstanding. Even the choice of Λ which yields the correct positive experimental result is ambiguous there.

In contrast, our formula for $a_e^{\mathrm{SF}}(x)$, which was derived from the self-field approach, is linear in the variable $x = \Lambda/m$, and it is uniformly positive for all (physical) positive values of x. Thus in the domain $x \in (0, \infty)$ the incorrect equation $a_e^{\mathrm{SF}}(x) = -\alpha/2\pi$ satisfactorily has no solutions, since $a_e^{\mathrm{SF}}(x) > 0$. Also, $a_e^{\mathrm{SF}}(x) = +\alpha/2\pi$ has the single *unique* solution $x = \frac{3}{4}$. (See Fig. 2.)

Complications arose in the VF case due to the fact that $a_e^{\mathrm{VF}}(x)$ in the VF theory depends quadratically on x. In the SF case such complications do not occur, as $a_e^{\mathrm{SF}}(x)$ has a linear x dependence which leads to the positivity of a_e^{SF} and a unique solution for $a_e^{\mathrm{SF}}(x) = +\alpha/2\pi$.

The curious business of the alternating $\pm$ signs which appear in Eqs. (37) warrants an attempt at an interpretation. In the VF situation the electron is immersed in a stochastic background field and "feels" a drag force as its cloud of virtual positron-electron pairs encounters the *bumps* of the vacuum field. Thus the charge is a bit slow to respond inertially to external forces, leading to an increase of the effective mass by an amount δm_{VF}. Also, this same vacuum drag phenomenon tends to slow the spin angular momentum and thus decrease the effective intrinsic magnetic moment by $\delta\mu_{\mathrm{VF}}$. For a small portion of the domain of the cutoff parameter, namely, $x := \Lambda/m \in (0,4)$, the mass shift wins out over the spin momentum change to yield a positive result for $g-2$.

In the self-field scenario a similar argument holds with the drag being due to the field of the charge itself. In analog with classical radiation reaction theory, the bare mass must be renormalized by adding on the electromagnetic mass bound in the self-field, δm_{SF}. The spin angular momentum of a bare Dirac electron is slowed slightly when one includes self-field effects, as the charge is now obligated to drag its own field around with it as it spins. The loss of spin energy lowers the magnetic moment by $\delta\mu_{\mathrm{SF}}$; but the loss is always more than compensated by the mass change, regardless of the choice of cutoff, giving a net positive sign for $g-2$.

The simplicity and straightforwardness of the (NR) self-field calculation of $g-2$ over the (NR) vacuum field computation is very appealing. The self-field effect, with its classical correspondence to radiation reaction theory, gives us an intuitive physical cause for the nonzero value of $g-2$.

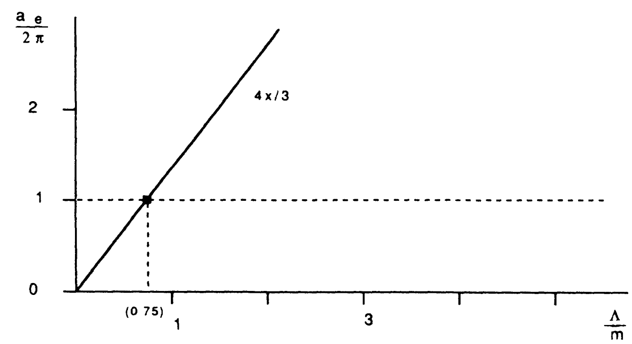

FIG. 2. Our formula for $a_e = (g-2)/2$, as a function of $x := \Lambda/m$. Notice that $a_e(x) > 0$ for all x in the interval $(0, \infty)$ and that the equation $a_e(x) = +\alpha/2\pi$ has only the unique solution $x = \frac{3}{4}$; hence the nonphysical equation $a_e(x) = -\alpha/2\pi$ has no solutions.

The self-field concept, as developed by Barut and Kraus,[11] has been used successfully to compute NR Lamb shifts and spontaneous emission,[12] spontaneous emission in cavities,[13] Lamb shift and Casimir-Polder van der Waals forces in cavities,[14] and a full relativistic account of the two-body problem, the Lamb shift, and spontaneous emission.[15,16] This approach to QED touches indeed the very foundations of quantum theory and its interpretation. If the field $\Psi(x)$ describes an objective distribution of electronic matter, as Schrödinger and de Broglie originally thought, the electric current associated with this distribution produces a field, namely, the self-field, which must be added to the external field in the Schrödinger, Pauli, or Dirac equations. The self-consistency of the coupled matter and Maxwell fields requires this.[17] Thus the successful classical calculations of the radiative processes has led to a possible revival of the Schrödinger interpretation of quantum theory.[18] Work is in progress to apply the method to $g-2$ in cavities, a fully relativistic computation of $g-2$, and the Unruh effect.

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