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A Mathematical Model for Numerical Simulation of Shallow Water Flow: Description and Practical Application of GUAD|2D

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Abstract: GUAD 2D is a finite volume based two-dimensional model for the numerical simulation and analysis of flood waves caused by different factors such as extreme rainfall, gradual or sudden dam break.. The model is able to deal with critical, subcritical or mixed flow situations over irregular topography. The computation requires initial conditions, that can correspond to dry bed or any other flow state.

Keywords: *Shallow* Water, Hydraulic simulation model, two-dimensional, finite volume.

1. INTRODUCTION

Many engineering and environmental problems involve the study of unsteady water flows. River flows, in particular, are mostly unsteady and, as they are characterized by the presence of a vertical scale much smaller than the horizontal ones, they can be described by the shallow water model (Cunge et al. 1980) which forms a set of non linear hyperbolic equations.

A great deal of work has been devoted to develop 1D and 2D numerical models for unsteady shallow flows in the last decades and various computational techniques using finite difference, finite element and finite volume methods have been reported (Cunge et al. 1980, Bellos et al. 1991, Alcrudo and García-Navarro 1993, Sleigh et al. 1998, Bermúdez et al. 1998, García-Navarro and Vázquez-Cendón 2000). Several numerical difficulties must be adequately treated to obtain an accurate solution without numerical errors. Zhao et al. (1994) provided a good historic revision and the features required for a two dimensional river flow simulation model: it should be able to handle complex topography, dry bed advancing fronts, wet-dry moving boundaries, high roughness values, steady or unsteady flow and subcritical or supercritical conditions: natural topography is the main challenge. Dominant source terms and open boundaries are two important difficulties to face when using a conservative method since they both can damage the conservative character of the solution. Bed slope and friction source terms are of special relevance in hydraulic applications based on a shallow flow model. For that reason, a considerable effort has been recently devoted to this topic in a search for the correct source term, discretization, given a particular numerical scheme with good properties for the homogeneous case (Leveque, 1998, Roe, 1981, Glaister 1992, Vázquez-Cendón 1999, Bermúdez and Vázquez-Cendón 1994 and Bermúdez et al. 1998, Murillo et al., 2006).

Another numerical problem of relevance is the modelling of wet/dry interfaces between internal cells that have traditionally represented a difficulty for modellers wanting to solve the shallow water equations over a bed of irregular geometry. Flow over dry bed involves a complicated situation that can be analyzed as a boundary condition which is dynamically

changing in time with the moving front and continuously expanding or reducing the flow domain. The alternative is to include the wet/dry interfaces in the full domain of computation in which there may be wet cells and dry cells simultaneously. In this case the numerical scheme chosen for the discretization must be able to cope with them. In general, cells being flooded or dried during the computation tend to introduce numerical instabilities in the solution, resulting for example in negative water depths or unphysical high velocities. Different approaches have been proposed to handle them (Kramer, 2001, Beffa and Connel 2001, Heniche and Secretan 2000, Kawahara and Umetsu 1986, Khan 2000, Brufau et al. 2002 and Brufau et al. 2004, Begnudelli and Sanders 2006, Bradford and Sanders 2002, Murillo et al., 2007).

The simulation model GUAD2D is based on a finite volume method developed at the University of Zaragoza (for more details see Murillo et al. 2006, Murillo et al. 2007). The resulting software package has been adapted for standard personal computers. Furthermore it has been provided with a user-friendly interface that simplifies the definition and set-up of two-dimensional simulations (GUADCreator).

The package also offers post-processing module called GUADView that is fed from the GUAD2D results and is very helpful to plot, visualize and analyze them. It includes the most necessary GIS capabilities to analyze this kind of simulations making possible the visualization of raster images and vector layers together in the same frame with the results of the simulation generated by the model, simplifying the right identification of flood zones. Also the visualization package can show time histories of depth and velocities at different locations, water sections and hydrographs. The post-processing module offers the possibility to export the results to a standard “.avi” format. Moreover, it can also start the analysis before the conclusion of the simulation process with the help of a local area network (LAN).

2. MATHEMATICAL MODEL

The water movement is governed by the basic laws of mass and momentum conservation under the shallow water hypothesis. This implies a depth average process in the equations and is associated to the assumption of hydrostatic pressure vertical distribution. The formulation takes the form of a set of non linear hyperbolic equations that, in two dimensions, involve the water depth as well as the two depth averaged components of the velocity,

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \mathbf{E} = \frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x}(\mathbf{U}) + \frac{\partial \mathbf{G}}{\partial y}(\mathbf{U}) = \mathbf{S}(x, y, \mathbf{U})$$

$$\mathbf{U} = (h, q_x, q_y)^T$$

$$\mathbf{F} = \left(q_x, \frac{q_x^2}{h} + \frac{gh^2}{2}, \frac{q_x q_y}{h} \right)^T, \quad \mathbf{G} = \left(q_y, \frac{q_x q_y}{h}, \frac{q_y^2}{h} + \frac{gh^2}{2} \right)^T \quad (1)$$

where $q_x = uh$ and $q_y = vh$ are the unit discharges in the Cartesian directions. The variable h represents the water depth, g is the acceleration of the gravity and (u, v) are the averaged components of the velocity vector \mathbf{u} along the x and y coordinates respectively. The source terms in the momentum equations are the bed slopes S_{0x} , S_{0y} and the friction losses S_{fx} , S_{fy} along the two coordinate directions,

$$\mathbf{S} = (0, gh(S_{0x} - S_{fx}), gh(S_{0y} - S_{fy}))^T \quad (2)$$

where,

$$S_{0,x} = -\frac{\partial z}{\partial x}, S_{0,y} = -\frac{\partial z}{\partial y} \quad (3)$$

and the friction losses in terms of the Manning's roughness coefficient n , with

$$S_{fx} = n^2 u \sqrt{u^2 + v^2} / h^{4/3}, S_{fy} = n^2 v \sqrt{u^2 + v^2} / h^{4/3} \quad (4)$$

3. NUMERICAL METHOD

The physical domain is discretized in grid cells for the numerical resolution. Our method is able to work over cells of different shapes (triangles, quadrilaterals, hexagons) and the choice of the grid is not a trivial step. Structured Cartesian grids are simpler and more efficient whereas unstructured triangular grids offer more geometric adaptability and better refinement possibilities.

A cell-centred finite volume method has been formulated in GUAD2D where all the dependent variables of the system are represented as piece-wise constants (first order). A discrete approximation of Equation (1) is applied in every cell at a given time so that the volume integrals represent integrals over the area of the cell and the surface integrals represent the total flux through the cell boundaries. Denoted by U_i , the average value of the conservative variables over the volume Ω_i , at a given time, from equation (1), the conservation equation can be written for every cell as follows:

$$\frac{\partial U_i}{\partial t} A_i + \oint_{\partial\Omega} (\mathbf{E} \cdot \mathbf{n}) ds = \int_{\Omega} \mathbf{S} d\Omega \quad (5)$$

where A_i is the area of the cell, \mathbf{n} is the unit normal vector pointing outward and $\partial\Omega$ is the contour line. A mesh fixed in time is assumed and the contour integral is approached by a sum over the cell edges. In all of them, the normal flux is approximated via an upwind flux difference splitting technique

$$\oint_{\partial\Omega_i} (\mathbf{E} \cdot \mathbf{n}) ds \approx \sum_{k=1}^{NE} (\delta \mathbf{E}_k \cdot \mathbf{n}_k) s_k \quad (6)$$

where, k represents the index of the cell edge, NE is the total number of edges in the cell. The vector \mathbf{n}_k is the normal outward unit to edge k , s_k is the length of the side, and $\delta \mathbf{E}_k \cdot \mathbf{n}_k$ is the numerical flux difference. Upwind schemes are based on the idea of discretizing the spatial derivatives so that the information is taken from the side it comes from. When the source terms are present, it has previously been shown that the flux derivatives and the source terms have to be discretized in a similar manner (Hubbard et al, 2000, and Brufau, et al, 2002). The evaluation of fluxes and sources at the same local state is important. The mathematical properties of the hyperbolic system of equations (1) include the existence of a Jacobian matrix, \mathbf{J}_n , of the normal flux to a given direction ($\mathbf{E} \cdot \mathbf{n}$) defined as

$$\mathbf{J}_n = \frac{\partial(\mathbf{E} \cdot \mathbf{n})}{\partial \mathbf{U}} = \frac{\partial(\mathbf{F})}{\partial \mathbf{U}} n_x + \frac{\partial(\mathbf{G})}{\partial \mathbf{U}} n_y \quad (7)$$

making possible for the generation of an approximate matrix \mathbf{J}_n^* . Using the eigenvalues $\tilde{\lambda}^m$ and eigenvectors $\tilde{\mathbf{e}}^m$ of this matrix it is possible to linearize fluxes and source terms (Brufau et al. 2002 and Hubbard and Garcia-Navarro 2000). The first order formulation of the upwind scheme is as follows (Murillo et al. 2006):

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \sum_{k=1}^{NE} \sum_m ((\tilde{\lambda}^m - \alpha^m - \beta^m) \mathbf{e}^m)_k^n \frac{l_k}{A_i} \Delta t \quad (8)$$

where α^m and β^m are coefficients of the method whose exact expression and meaning has been previously detailed (Murillo et al., 2006). The discretization of the bottom elevation source terms is successfully constructed when it ensures an exact balance between flux gradients and bed variations (Leveque 1998, Hubbard and García-Navarro, 2000). It has been demonstrated that, in first order finite volume schemes, if the upwind technique is applied to the flux and bottom terms, in the case of still water, the equilibrium is maintained for the water level surface (Bermúdez and Vázquez, 1998, Brufau et al, 2002).

The friction term dictates over any other influences in many practical situations, in particular, in wet/dry fronts, characterized by small values of water depth. A explicit upwind discretization of the friction source term that guarantees a perfect discrete balance has been used (Burguete et al., 2006).

4. GUAD 2D

4.1 General Working Issues

GUAD 2D is an application that solves the most popular problems in two-dimensional trade models. Contrary to the package presented here most models have limited simulation capabilities or are developed for flat terrain, and they can not simulate switching flow regimes over levees and embankments correctly. Also this tool is able to deal with huge terrains divided up to 2.000.000 cells. A wide variety of boundary conditions are handled by the model: input hydrographs, constant water levels, dumps, etc., allowing to the user to initiate the simulation in dry/wet condition terrain in any case. To create a simulation, the model requires at least a digital terrain model, single or global rub value for each cell terrain, one or more input/output boundary conditions. The final requirement simulation parameters are the desired time of simulation and result paths to allocate the number of images that the user should previously have defined.

4.2 A Simulation Generation

Starting up GUAD 2D based on a simulation over a modelled terrain, users should follow the steps explained in Figures 4.2.1, 4.2.2, 4.2.3 and 4.2.4. 4.3.

4.3 Simulation Outcomes

This two dimensional model will generate the results in a single extension and a proper format file (*.gds). Those GUADView readable files will contain all the generic information from the GUAD 2D simulation i.e.: modelled simulation terrain, results represented such as water layers showing depth, level, and u & v component of velocity. The user will be able to get this information at any moment in time during the simulation. Therefore, this data will be identified by component values and its effective period of time.

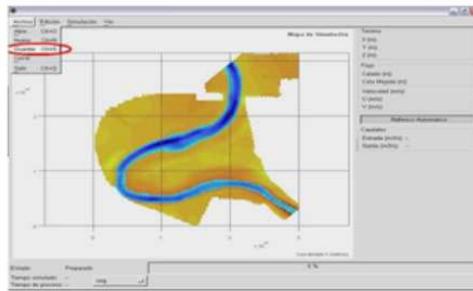


Fig. 4.2.1. Simulation Terrain in GUAD 2D

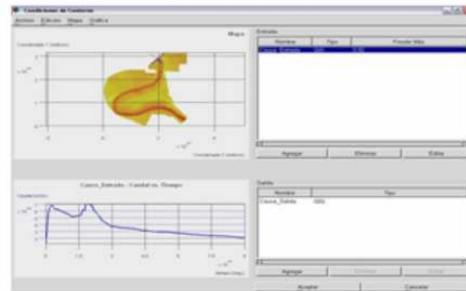


Fig. 4.2.2. Boundary Conditions List

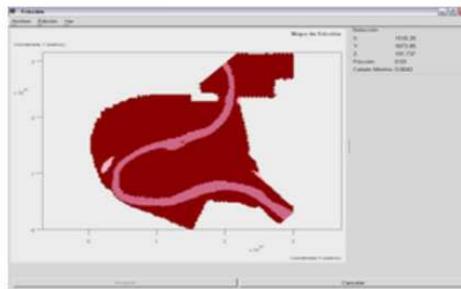


Fig. 4.2.3. Rub Grid for each cell

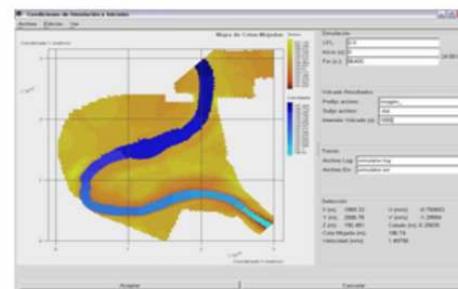


Fig. 4.2.4. Configuration of simulation parameter

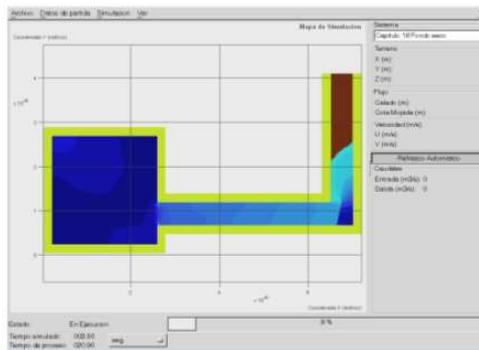


Fig. 4.2.5 Simulation Start-up

5. GUADVIEW

5.1 General Working Issues

GUADView is an analysis application to analyse GUAD 2D results. It can be considered as a GIS itself, with specific analysis capabilities added for this kind of outcomes. It is able to show in different layers data contained on each grid cell. Data is composed by depth, level and module/direction component of velocity. Thanks to the post-processing module capabilities, the analysis of hydraulic information layer is fast and simple (Fig 5.1., Fig 5.2, Fig 5.3 and Fig 5.4). GUADView can also create simulation videos, envelope layers, offers layer control tools, vector and raster layer managing and exports GUAD 2D layer to commercial GIS format.

5.2 Outcomes Analysis



Fig. 5.1. Water Layer Visualization over raster



Fig. 5.2. Terrain section hidrographs.



Fig. 5.3 Visualize section details



Fig. 5.4. Video with GUAD 2D water layer

6. PRACTICAL CASE

The case described corresponds to an urban flooding event in Miranda de Ebro (Spain). The topography was obtained with LIDAR technology. With this information a rectangular mesh, with a cell size of 2 meters, was used for the simulation. The flood is generated by the opening of a reservoir located in the north of the city. The flooding wave advances correctly through the streets, from north to south, until it reaches the riverbed. Figure 6 correspond to two calculation instants in which we can depict the evolution of the flood represented on orthophoto using different tones of blue (darker as the depth of the water is greater) for the depth.



Figure 6. Flood evolution.

7. TIME SIMULATION COMPARATION

In order to compare the performance of GUAD 2D respect to other software packages, Sobek and Mike 21, different test cases were defined in Garrote et al., 2008. For the comparison, analytical and real cases were used: a straight channel with a moving

hydraulic jump, a dam break with a 90° bend channel, a channel with an expansion and a contraction and finally a flooding event. The computational times obtained with an Intel Xeon 1.7GH Processor and 1GB RAM memory are shown in Table 1. For more details see Garrote et al., 2008.

	Sobek	Mike 21	GUAD 2D
Numerical Scheme	Finite Differences	Finite Differences	Finite Volumes
Hydraulic jump	111 s	262 s	21 s
Channel contraction	182 s	160 s	44 s
Dam break	9876 s	Inconsistent results	235 s
Flooding event (2200 ha)	119324	Unfeasible computational time	54264 s

8. CONCLUSIONS

The hydraulic tool presented in this work is able to deal with complicated phenomena. The initial and boundary conditions can be easily implemented and the results can be analysed in most commercial GIS applications. Nowadays, GUAD 2D has been used in real cases and tested in several laboratory cases, thereby obtaining results that have been contrasted with present hydraulic situations, offering to hydraulic communities a reliable, fast and functional solution.

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