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Wind plant system engineering through optimization of layout and yaw control

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ABSTRACT

Recent research has demonstrated exciting potential for wind plant control systems to improve the cost of energy of wind plants. Wind plant controls seek to improve global wind plant performance over control systems in which each turbine optimizes only its individual performance by accounting for the way wind turbines interact through their wakes. Although these technologies can be applied to existing wind plants, it is probable that the maximum benefit would be derived by designing wind plants with these capabilities in mind. In this paper, we use system engineering approaches to perform coupled wind plant controls and position layout optimizations of a model wind plant. Using several cost metrics, we compare the results of this optimization to the original plant and to plants in which the control or layout is optimized separately or sequentially. Results demonstrate that the benefit of this coupled optimization can be substantial, but it depends on the particular constraints of the optimization.

KEYWORDS
wind plant control; system engineering; wind turbine wakes; optimization

1. INTRODUCTION

Wind turbines arranged in a cluster to form a wind plant interact with each other through the wakes that they produce while extracting energy from the flow. This interaction, which has been studied thoroughly (see, for example, ¹ for a literature overview), may have a negative effect on the total electrical power production of the plant. These effects can be mitigated by layout optimization, such as placing the turbines farther away from each other and/or by using wind plant control techniques during the operation of the plant, such as coordinating the control activities of individual turbines to increase power production of the plant as a whole.

Several methods have been presented to improve wind plant power capture. In one approach, the power capture of upstream turbines is reduced to improve the performance both of power and loading of downstream turbines. In some cases, even though the upstream turbines produce less power, because of the downstream turbines’ increased performance,
this results in greater power output for the plant as a whole. This approach is evaluated in [2] and tested on a scaled model wind plant within a wind tunnel. Controllers making use of this effect are presented in [3–6]. This type of control is evaluated comparatively using models of varying fidelity, including computational fluid dynamics (CFD), in [7,8].

An alternative approach for using controls to improve wind plant performance is to redirect the wake from upstream turbines to avoid interactions with downstream turbines. In [9], a CFD model was used to investigate the potential for turbines to redirect their wake laterally using yaw misalignment. In [10,11], the Simulator fOr Wind Farm Applications (SOWFA), which is a high-fidelity simulation tool that embeds wind turbine aero-servo-elastic models in a CFD flow, was used to investigate both the ability of turbines to redirect their wakes, as well as the production and loading impacts of doing so.

The yaw wake redirection method was studied experimentally in wind tunnel tests with scaled turbines in [12] and [13]. Field tests with kilowatt-scale turbines were performed in [14] with encouraging results, although the data were scattered and yielded no clear conclusions. A correlation between yaw offset and a higher wind velocity downstream was demonstrated on a megawatt-scale turbine in [15].

Wind plant control algorithms, based on the results provided in the above literature review, have the potential to improve the power output of a wind plant by better coordinating the control activities of individual turbines. These technologies can be applied to existing wind plants, but it is probable that their benefit to global electrical production would be greatest for wind plants designed with these technologies in mind.

Traditional wind plant layout optimization has focused on maximizing energy production by placing turbines in the windiest locations while minimizing array losses caused by wake effects. However, the state of the art in plant design has been extended to include a number of objectives, such as overall cost of energy (COE), which considers plant infrastructure and operations in addition to energy production and even noneconomic objectives such as noise mitigation and minimization of environmental impacts. A recent work on the state of the art in wind plant optimization provided a comprehensive overview and examples of the various types of wind plant layout optimization studies that have been performed [16]. Generally, there is an increased recognition that coupling across the system requires more integrated approaches to wind plant design that consider a large range of decision criteria and a more holistic approach to optimization. Systems engineering and multidisciplinary analysis and optimization methods in particular hold promise for improving overall plant design for a range of system objectives [17].

The integration of wind plant controls with the design of a wind plant was not included in [16], but it has been treated at some level in past work. For example, in [18], a multilevel optimization approach was used to optimize the layout of the Middelgrunden Wind Turbine Cooperative, to maximize wind plant profit with an overall COE objective in which turbine-level controls were an integral part of the study. Still, the active combination of wind plant controls for increased energy production or even reduced turbine loads has not been directly coupled to wind plant layout optimization in past studies.

In this paper, we use an engineering model of wake effects in a wind plant to consider the possibilities of combining wind plant controls with system-engineering-based layout optimization, and we compare this coupled approach to the results from applying these methods separately. Specifically, for wind plant control we consider the technique of wake redirection through intentional yaw misalignment. For layout optimization, we use the wake model to find optimal wind plant layouts with and without active wind plant control. For this study, we model the Princess Amalia Wind Park in the Netherlands, to compare the performance of each technique.

The paper is intended as a proof-of-concept study on combining wind plant optimization techniques with wind plant controls. We use approximate means of estimating cost to allow this work to focus on the coupled optimization methods prior to increasing the complexity; however, given the potential demonstrated by this coupled approach, future research will seek to refine the method to include more detailed models.

The contribution of this paper is an investigation into the potential for both wind plant control and system-engineering approaches to improve wind energy performance. Additionally, the paper demonstrates how considering system design optimization and advanced control technologies simultaneously improves wind power plant performance more than what
is possible when applying either technique alone. This indicates that future wind plants might be designed considering 
the control systems that could be used to optimize plant-wide performance. Finally, we present analysis that indicates the 
relative importance of advanced controls compared to layout optimization given the design constraints.

The remainder of this paper is organized as follows. Section 2 presents the models used in the study. Section 3 presents 
the design cases considered, including separate control and layout optimizations and coupled optimizations. Section 4 
details the optimizations performed and defines the cost function, constrains, and optimization method used. The results of 
the optimizations are presented, analyzed, and compared in Section 5. Section 6 considers the amount of yaw misalignment 
used and impacts on loads. Finally, conclusions are given in Section 7.

2. WIND PLANT MODELS

2.1. Princess Amalia Wind Park

As described in the introduction, we use as our test case the Princess Amalia Wind Park, which is a wind plant located 
23 km off the coast of The Netherlands. Princess Amalia consists of 60 wind turbines that have a rotor diameter of 80 m and 
a rated power of 2 MW. The locations of the turbines in Princess Amalia are shown in Figure 1. The spatial configuration 
of the turbines is such that they are located farther away from each other in the prevailing wind directions.

By using wind measurements at a nearby location, an estimate is made of the annual wind rose at Princess Amalia, as 
shown in Figure 2. These wind measurements were made by the NoordzeeWind meteorological mast at a nearby location

Figure 1. Layout and location of the Princess Amalia Wind Park [6]. The spacing between some of the turbines is shown with 

$D = 80$ m.
in the North Sea during the period from July 1, 2005, to June 30, 2006 [19]. The measurements consist of 10-minute averages of the wind direction and the freestream wind speeds and are available in [20]. In this study, we focus only on the 8-m/s wind speeds and extract only this data. Note that the wind rose data is binned into $5^\circ$ increments, yielding 72 total bins.

![Figure 2. Estimated wind rose (in percentage) for the Princess Amalia Wind Park, using wind data from the nearby NoordzeeWind meteorological mast [20].](image)

From this data, we are able to develop an engineering model of the wind plant that includes wake interaction. Note, however, that the real wind plant is composed of Vestas V80 wind turbines that have a rotor diameter of 80 m and a rated power of 2 MW, whereas for this exercise we employ NREL 5-MW reference turbines that have a rotor diameter of 126 m. This substitution is because of the necessity for an open-access turbine model, which is available for the NREL 5-MW turbine. However, this substitution changes the effective spacing in terms of rotor diameters, and although we believe the comparison between optimization methods is still valid, the improvement relative to the baseline would probably be less given greater rotor diameter spacing.

2.2. FLORIS

In [11], the FLOw Redirection and Induction in Steady-state (FLORIS) model was presented, which is a control-oriented model that predicts the steady-state characteristics of wakes in a wind plant as a function of the axial inductions and yaw angles of the rotors. It is a static nonlinear model that describes the velocity profile of the wakes (based on an augmented Jensen model [21,22]), the wake deflection caused by yaw offset (based on the engineering model in [9]), and the rotational effects. The augmentation to the Jensen model mainly consists of a segmentation of the wake in different zones such that the cross-wind velocity profile of the wake can be better fitted. From the wake velocity profiles, the power of each of the wind turbines is estimated using the wake overlap weighting method described in [3].

* The measurements are undisturbed by the NoordzeeWind offshore wind plant, that was constructed later.
Although it has a small amount of parameters to describe the relevant effects, the FLORIS model can be fit to the
time-averaged results from high-fidelity simulations of situations with full or partial overlap of the turbine wakes with
downstream rotors. The parameters were fit to the SOWFA simulation results \[10\] of a row of two NREL 5-MW turbines,
in which the upstream turbine has a yaw offset or a cross-wind position offset. In \[11\], it was also shown that the predictions
of the FLORIS model can be used to optimize the yaw angles of turbines in a wind plant for maximum power production.

3. PROBLEM DEFINITION

The following sections investigate the opportunities for optimizing a wind plant using a FLORIS model of the Princess
Amalia Wind Park with NREL 5-MW turbines. We consider layout optimization, optimized control (through yaw-based
wake redirection), and finally a combined optimization of layout and control. This provides the following cases:

**Baseline:** Fixed (original) positions, turbines all yawed in mean wind direction

**Optimized yaw:** Fixed (original) positions, turbines optimally yawed for each wind direction

**Optimized location:** Position optimized, turbines all yawed in mean wind direction

**Combined optimization:** Yaw optimized for each wind direction and position optimized simultaneously

The goal of wind plant optimization is to minimize the COE. The formula for COE is

\[
\text{COE} = \frac{\text{FCR}(\text{TCC} + \text{BOS}) + \text{O&M}}{\text{AEP}}
\]

where FCR is the fixed charge rate, TCC is the turbine capital costs, BOS is the balance-of-station costs, O&M is the
operation and maintenance costs, and AEP is the annual energy production.

The trade-offs in balance-of-station costs compared to annual energy production are site specific, and this paper seeks
to explore solutions across a range of conditions. The relative cost and feasibility of changing the overall shape of a wind
plant will vary from site to site, as will the relative cabling costs. Rather than choosing numbers for a specific site, we
considered three surrogate metrics that capture trade-offs in annual energy production compared to balance-of-station and
maintenance costs in different ways. Although these metrics are not necessarily useful as absolutes, they are useful for
comparing relative changes among layouts, which is the goal of this study. The first objective is to maximize the power
density (or plant power/area). The second objective is to maximize power with a fixed total cabling length. The final
objective is to maximize power with a fixed plant boundary. In all cases, minimum separation distances among turbines
are enforced. For yaw optimization a simple power maximization is sufficient.

The power used in the objectives is an expected value of power across all wind directions according to the wind-
direction-bin percentages shown in Figure 2. This averaging was done at only one wind speed (8 m/s). The area of the
wind plant was computed as the area of the minimum-sized convex polygon using the outermost turbines as vertices. Such
a polygon is shown in Figure 4c. The polygon was computed using the SciPy scientific computing package for Python \[23\].
The cable length was computed by finding the minimum spanning tree for the given layout. The minimum spanning tree
is the set of lines that connects all the turbines in a single graph with the minimal total length of lines. The minimum
spanning tree was computed using an implementation of Prim’s algorithm \[24\]. The cable length for the baseline layout is
shown in Figure 4d.

4. OPTIMIZATION

This paper considers the maximal benefit for the model wind plant by optimizing control, layout, and a combination of
both. This section details the optimization routine employed and the specific optimizations carried out for each case.
All optimizations in this paper were performed using the software package SNOPT [25], a nonlinear optimization package based on the sequential quadratic programming method. The pyOpt wrapper [26] was used to call SNOPT from Python. Gradients were estimated using finite differencing, and all results were converged with an optimality tolerance of $5 \times 10^{-5}$ and a feasibility tolerance of $1 \times 10^{-6}$. Objectives and constraints were normalized to be of order one.

In the baseline case, all the metrics (mean power, area, cable length) were computed assuming that for each wind direction, the turbines were yawed into the wind, and the positions of the turbines were from the originally provided layout.

4.1. Yaw optimization (control only)

For each wind direction, the yaw angle of each turbine was optimized to maximize power production. Because yaw control was assumed, the optimizations for each wind direction were independent and could be optimized separately. This led to 72 optimization problems (one for each wind direction bin) of the form:

$$\begin{align*}
\text{maximize} & \quad P(x, y, \psi), \\
\text{subject to} & \quad -\pi/4 \leq \psi \leq \pi/4
\end{align*}$$

(2)

Where $x$ and $y$ are the (fixed) turbine locations and $\psi$ is the turbine yaw angle. The bound constraints on yaw angle were used to prevent exploitation of unphysical solutions that sometimes arise in this model for highly yawed solutions. This optimization problem will be referred to as $YawOpt$.

4.2. Position optimization (layout only)

For all three position optimization applications, the position of the wind turbine affects the power production for all wind directions, so evaluation across the complete wind rose needed be done at each iteration. The $x$ and $y$ locations of the wind turbines were included as design variables; however, if all $x$, $y$ variables were included, then there would be no unique solutions. Because there were no domain boundaries, the entire array of turbines could be shifted an arbitrary amount in any direction and the power output would remain unchanged. To prevent this nonuniqueness, the position of the last turbine at location $(1683.7, 0.0)$ remained fixed. All other positions were added as design variables. For convenience, the specification of the optimization problems in the below sections used $x$, $y$ to represent the vector of all turbines positions, except for the last turbine. In the optimization, all positions were normalized by 1,000 and to prevent unrealistic intermediate solutions, the positions were constrained to stay within a box of half-width 1.0 (or 1,000 in physical coordinates) from their starting point.

The interturbine spacing constraint constrained the distance between each turbine to be greater than 2 diameters. This results in $(n^2 - n)/2$ spacing constraints (1,770). This constraint is expressed below as $d_i \geq 2D$, $i = 0 \ldots 1,770$.

Next, the three separate layout position optimization cases are defined, and each one will be indicated by $PosOpt$, with a specifying suffix.

4.2.1. Case: $PosOpt_{Density}$

Case $PosOpt_{Density}$ maximizes power density (expected power/area ($E[P]/A$)), with a constraint on minimum expected power generation and a minimum turbine separation distance. A constraint on the minimum expected power production was added to prevent the optimizer from driving the solution to unrealistically small areas. The trade-off in area and power production is such that it is generally easier to decrease area than it is to increase power production. Thus, maximizing power density often results in spacings that are too dense, even with the minimum spacing constraint. Adding the minimum power constraint prevents that behavior. The new layout must produce at least as much power as the original one. Lastly, note that in the position-optimized cases, it is assumed that wind turbine yaw controllers operate normally, so $\psi$ is always chosen to be aligned with the inflow wind direction for all turbines.
maximize \( E[P](x, y, \psi)/A \)

subject to \( E[P] \geq E[P]_0 \)

\[ di \geq 2D, \ i = 0 \ldots 1,770 \]

\[ x_{i0} - 1000 \leq x_i \leq x_{i0} + 1000, \ i = 0 \ldots 59 \]

\[ y_{i0} - 1000 \leq y_i \leq y_{i0} + 1000, \ i = 0 \ldots 59 \]

4.2.2. Case: \( \text{PosOptCable} \)

Case \( \text{PosOptCable} \) maximizes the expected power with a constraint on the maximum cable length. The total cabling length between all the turbines was computed as \( l \) and was enforced to be no greater than the original total cabling length, \( l_0 \). The optimization problem is summarized as:

maximize \( E[P](x, y, \psi) \)

subject to \( l \leq l_0 \)

\[ di \geq 2D, \ i = 0 \ldots 1,770 \]

\[ x_{i0} - 1000 \leq x_i \leq x_{i0} + 1000, \ i = 0 \ldots 59 \]

\[ y_{i0} - 1000 \leq y_i \leq y_{i0} + 1000, \ i = 0 \ldots 59 \]

4.2.3. Case: \( \text{PosOptBoundary} \)

Case \( \text{PosOptBoundary} \) maximizes the expected power with a fixed original boundary. The closest normal distance from every turbine to the boundary is computed and stored in \( n_i \). If \( n_i \) is negative, it means the turbine is within the boundary. The optimization problem is given as:

maximize \( E[P](x, y, \psi) \)

subject to \( n_i \leq 0, \ i = 0 \ldots 59 \)

\[ di \geq 2D, \ i = 0 \ldots 1,770 \]

\[ x_{i0} - 1000 \leq x_i \leq x_{i0} + 1000, \ i = 0 \ldots 59 \]

\[ y_{i0} - 1000 \leq y_i \leq y_{i0} + 1000, \ i = 0 \ldots 59 \]

4.3. Yaw and position optimization

This final scenario consists of optimizing the positions of the turbines and the yaw angles of all the turbines at each wind direction simultaneously (4,438 total design variables). The combined position and yaw optimization problem is posed as a monolithic optimization problem rather than a nested problem. Even though the yaw angles can be independently chosen at each wind direction, it was deemed that finite differencing across an internal optimization problem would be less effective. Also, although the monolithic problem had a large number of design variables, the vast majority of the computational cost was in computing the finite differences and that is easily parallelized. For this problem we used 192 cores and a run time of 36 hours and made use of pyOpt’s ability to do finite differencing in a parallel manner. The optimization problem is stated below, where capital \( \Psi \) is used to denote the concatenation of all the yaw vectors (lower case \( \psi \)) across all wind directions. All three cases for the position optimization were explored as a combined position/yaw optimization. These cases will be identified as \( YP \). As above, the separate cases are identified by the subscripts: density, cable, and boundary.
maximize $E[P](x, y, \Psi)$

subject to minimum power, maximum cabling length, or fixed boundary

\[ d_i \geq 2D, \ i = 0 \ldots 1,770 \]
\[ x_{i0} - 1000 \leq x_i \leq x_{i0} + 1000, \ i = 0 \ldots 59 \]
\[ y_{i0} - 1000 \leq y_i \leq y_{i0} + 1000, \ i = 0 \ldots 59 \]
\[ -\pi/4 \leq \Psi \leq \pi/4 \]

Because of the large number of design variables, obtaining the same tight convergence tolerances as previous cases, exponentially longer computation times would have been required. Instead, the objective function was monitored until no discernable changes were observed. Figure 3 shows the objective function for each combined position/yaw optimization compared to the iteration step. It shows that with this level of computation, the solution seems to have converged.

5. RESULTS

This section reviews the results for each case. First, the analysis of the baseline case is presented.

5.1. Baseline

In the baseline case, the turbines are positioned according to their original layout, and for each wind direction each turbine is yawed into the wind. Figure 4 shows the results of the baseline analysis.

Figure 4a shows the power output of the baseline wind plant for each wind direction. As shown, in the directions in which the wind is aligned with the wind plant rows, there is significant reduction in power output. This is caused by the wake effects, which decrease power production for the turbines behind the lead row. Figure 4b shows the power is replotted after scaling by the wind rose, shown in Figure 2. Figure 4c shows the layout of the baseline wind plant, and the bounding polygon used to compute the area. The minimum spanning tree is shown in Figure 4d which indicates the length of cable required in the baseline case. Finally, Table I shows the values of the key metrics: mean power production, area, cable length, and power density.
5.2. Yaw-optimized

Figure 5 considers the control-only case (yaw-optimized). Figure 5a shows the power output of this case for each wind direction, compared to the values from the baseline case. Note that power production increased in every wind direction. Figure 5b shows the percent change in power, relative to the baseline, for each wind direction. These plots show that the yaw control improves the power performance of each wind direction, especially in the directions in which wake losses are greatest. Of course, yaw control does not impact the layout of the wind plant, so the plots for area and cabling length are unaffected and not shown. According to Table II, it follows that overall yaw control increases the mean power output (and likewise power density) by 7.7%.
5.3. Position optimization

As discussed in Section 4.3, three objectives are explored in position optimization. $PosOpt_{Density}$ seeks to maximize power density (power/area) while constraining the power production to be greater than or equal to the baseline power production. $PosOpt_{Cable}$ seeks to maximize power output while constraining the cable length to be less than or equal to the baseline length. $PosOpt_{Boundary}$ assumes that the original boundary of the wind plant is fixed and seeks to maximize power output within that boundary. The results of the three optimizations are summarized in Figure 6.

For $PosOpt_{Density}$, Figure 6 shows that the optimization succeeds in increasing the power density. Figure 6c and Table III show that this is achieved through shrinking the plant area while maintaining the mean power output, as expected. Note that in Figure 6c, the original boundary of the wind plant is shown by the black dashed line. Interestingly, the wind plant is rearranged into a more streamlined shape. In the dominant wind direction (from the southwest), the wind turbines are staggered so that wake effects are reduced on the downstream wind turbines. Further, few wind turbines are used in the front, whereas they are packed together in the back to extract as much momentum as possible because there are no downstream turbines to experience negative wake effects. In the least probable wind direction, the turbines remain in grid lines. The most probable and least probable wind directions are almost 90° apart for this wind site.

$PosOpt_{Cable}$ spaces the turbines as far apart as possible, to minimize wake effects while maintaining a constant cable length (Figure 6g). This allows for much greater power capture. The wind plant is aligned into what are essentially 5 rows facing the predominant wind direction, but spaced as far apart as possible while keeping a few wind turbines between the rows to maintain the cabling length.

Finally, the results of $PosOpt_{Boundary}$ are shown in Figure 6e. This optimization is restricted to the original boundary, so there is less flexibility. The turbines are pushed toward the outer boundary to create as much spacing as possible, and

---

**Figure 5.** Plots showing yaw-optimized power performance.

**Table II.** Key Metrics for the $YawOpt$ Case.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Baseline</th>
<th>$YawOpt$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Power (MW)</td>
<td>78.86</td>
<td>84.91</td>
</tr>
<tr>
<td>Area (km$^2$)</td>
<td>14.53</td>
<td>14.53</td>
</tr>
<tr>
<td>Cable Length (km)</td>
<td>32.74</td>
<td>32.74</td>
</tr>
<tr>
<td>Power Density (W/m$^2$)</td>
<td>5.43</td>
<td>5.84</td>
</tr>
</tbody>
</table>
Figure 6. Results of power performance, area, and cable length for position-optimized cases.

Table III. Comparison of Key Metrics for the Baseline, YawOpt and PosOpt Cases.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>YawOpt</th>
<th>PosOpt Density</th>
<th>PosOpt Cable</th>
<th>PosOpt Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Power (MW)</td>
<td>78.86</td>
<td>84.91</td>
<td>78.86</td>
<td>90.67</td>
<td>80.68</td>
</tr>
<tr>
<td>Area (km$^2$)</td>
<td>14.53</td>
<td>14.53</td>
<td>12.45</td>
<td>33.28</td>
<td>14.47</td>
</tr>
<tr>
<td>Cable Length (km)</td>
<td>32.74</td>
<td>32.74</td>
<td>27.50</td>
<td>32.74</td>
<td>28.45</td>
</tr>
<tr>
<td>Power Density (W/m$^2$)</td>
<td>5.43</td>
<td>5.84</td>
<td>6.33</td>
<td>2.72</td>
<td>5.58</td>
</tr>
</tbody>
</table>
turbines are staggered in the predominant wind direction; both of these changes reduce wake losses and increase power production compared to the baseline design.

Each optimization succeeds in finding a new layout that has better performance—in terms of the metric it seeks to optimize—compared to the baseline, as shown in Table III. Comparing only PosOpt Boundary to YawOpt shows that control optimization, rather than layout optimization, was more effective in increasing mean power for a fixed original boundary.

5.4. Combined optimization

The final set of optimizations considers those that combine yaw-control optimization and position layout optimization, (signified by YP). These results are summarized in Figure 7.

Table IV shows that the combined optimizations are superior to either the control-only or the layout-only optimizations for each case. YPDensity achieves a significantly smaller plant area compared to that of PosOptDensity while still maintaining an approximate equivalent mean power. This results in a nearly 40% increase in power density compared to that of optimizing position alone. It seems that for tightly spaced wind plants, the effect of yaw control is especially important in increasing power density. Comparing PosOptCable to YPCable shows that adding yaw optimization to the position optimization also results in an improvement, but the improvement in mean power is significantly smaller (2%). For plants that are less constrained by area and can increase spacing, it seems that the position optimization is more effective than the yaw-control optimization. Finally, for a fixed area, comparing YawOpt, PosOptBoundary, and YPBoundary, it seems that the effect of yaw control is much more important. Of the potential 8.5% in total improvement possible when optimizing yaw and position simultaneously, yaw control alone can achieve 85% of the same benefit, whereas position optimization alone can achieve only 27% of the potential benefit.
(a) Power output by wind direction for the coupled optimizations.  
(b) Power output shown in percent change from baseline.  

(c) Area of wind plant $Y_P^{Density}$.  
(d) Area of wind plant $Y_P^{Cable}$.  
(e) Area of wind plant $Y_P^{Boundary}$.  

(f) Cable length $Y_P^{Density}$.  
(g) Cable length $Y_P^{Cable}$.  
(h) Cable length $Y_P^{Boundary}$.  

**Figure 7.** Results for coupled-optimized yaw control and layout position cases.  

**Table IV.** Comparison of Key Metrics for the Baseline, $YawOpt$, $PosOpt$, and $YP$ Cases.  

|                     | Baseline | $YawOpt$ | $PosOpt^{Density}$ | $PosOpt^{Cable}$ | $PosOpt^{Boundary}$ | $YP^{Density}$ | $YP^{Cable}$ | $YP^{Boundary}$ |
|---------------------|----------|----------|--------------------|------------------|---------------------|----------------|--------------|----------------|-----------------|
| Mean Power (MW)     | 78.86    | 84.91    | 78.86              | 90.67            | 80.68               | 78.84          | 92.33        | 85.61          |
| Area (km$^2$)       | 14.53    | 14.53    | 12.45              | 33.28            | 14.47               | 8.96           | 32.53        | 14.48           |
| Cable Length (km)   | 32.74    | 32.74    | 27.50              | 32.74            | 28.45               | 23.88          | 32.74        | 27.80           |
| Power Density (W/m$^2$) | 5.43    | 5.84    | 6.33               | 2.72             | 5.58                | 8.80           | 2.84         | 5.91            |
5.5. Sequential optimization

It is expected that an integrated optimization approach will be more effective than a sequential one in which the layout is optimized first and the yaw angles are optimized afterwards for the new, fixed layout. However, it is not obvious how significant the improvement is and whether the additional complexity of the combined optimization is justified. We compared all three combined yaw-position optimization cases to a sequential approach in which the layout was first optimized and the yaw angles were optimized following the selection of the optimal layout. These sequential optimizations took significantly less computational power to perform. Typically, they can be performed in several hours by a single computer, whereas the integrated optimizations were run in 36 hours using a cluster of 192 cores.

The three sequential optimizations are denoted as $SeqOpt$, and a comparison of the results is shown in Table V.

Results of the density layout optimizations ($SeqOpt_{Density}$ and $YP_{Density}$) show that the coupled optimization was much more successful in maximizing power density than the sequential one. Conversely, results of the cable-length optimizations showed only a minor difference in performance. It seems that $YP_{Cable}$ is not fully converged, because it should be higher than $SeqOpt_{Cable}$; however the difference seems negligible, so tightening the convergence will not yield additional insight. The difference is related to the relative importance of considering control. The density optimizations attempt to contract the layout of the park, which decreases turbine spacing, and magnifies the importance of control in the optimization. The cable optimizations expand the layout, and therefore they obtain much less impact from control; thus whether or not control is included in the layout optimization is much less important. Finally, results of the boundary optimizations show that including control in the layout optimization has some benefit, but the difference between the two results is not large (0.5%).

6. YAW MISALIGNMENT

The yaw-controlled cases in this study use intentional yaw misalignment to generate the wake redirection. Although this paper is a proof-of-concept study, and fully accounting for loading impacts within the optimization is the subject of future work, this section reviews the overall magnitude of the yaw misalignments in each case and briefly considers impacts on turbine loads.

Figure 8 shows the percentage of yaw angles for each wind turbine and each wind direction (for a total of 4,320 yaw angle set points) for each of the yaw-controlled cases. The figure shows that for the majority of wind directions, the yaw misalignment on the turbines is small (less than 15°). When the turbine spacing is small along the wind direction, the optimized yaw misalignment is larger. The flowfields predicted by the FLORIS model are given in Figures 9a and 9b for the 180° wind direction, for both the baseline and $YawOpt$ case. Note that this single direction is chosen for point of illustration of the FLORIS model for a particular direction, and that the final results discussed in this paper are the wind-rose-averaged results for every direction. Note also that this direction is a worst-case, that is with many fully-waked

<table>
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<th>Table V. Comparing $Seq - Opt$ to $YP - Opt$</th>
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<tr>
<td>$SeqOpt_{Density}$</td>
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<tr>
<td>Mean Power (MW)</td>
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<td>Area (km²)</td>
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<td>Cable Length (km)</td>
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<td>Power Density (W/m²)</td>
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Figure 8. Percentage of yaw-misalignment angles occurring across each wind turbine and wind direction for the yaw-controlled cases.

Figure 8 also demonstrates that the use of larger yaw misalignments increases with increasing proximity of turbines. YP_Density has the smallest area, and have a larger proportion of larger yaw angles than does YP_Cable, which has the largest area and greatest spacing. This is intuitive in that with greater spacing, the benefit gained by downstream turbines because of wake redirection decreases—because of increased wake recovery—with respect to optimized power capture of the upstream turbine.

Given the distributions shown in Figure 8, we can consider whether the larger yaw misalignments are in fact necessary to achieve optimized results. To check this, we can re-optimize the YawOpt case, with tighter constraints on the yaw angles—specifically, limiting them to 25° instead of 45°. Figure 9c shows the resulting flow field as predicted by FLORIS for the 180° wind direction. In this case, the mean power across the annual wind rose is 84.82 MW, which represents a 0.1% reduction from 84.91 MW. This reduction indicates that allowing angles greater than 25° might not be necessary. However, it is possible the effect of this limitation would be greater in more closely spaced cases.

The impact of intentional yaw misalignment on turbine fatigue loads is a topic of ongoing study. This research is aimed at mitigating unwanted load effects by introducing possible constraints on the allowable yaw settings in the optimization problem. Current literature shows that yaw misalignment can either increase or reduce loads, depending on the particular yaw settings used [27,30]. Reduction can be a result of the fact that for some wind conditions, yaw misalignment on a wind turbine is able counteract the effect of vertical wind sheer on the periodic loads on the blades and effectively reduce them. [28,29]. Also, it was shown that standard individual pitch control techniques can be used to reduce fatigue load impacts of partial wake overlap on downstream turbines [30].

In [30], it was also proposed that if the yaw misalignment is constrained to only positive yaw angles (at least for the range of winds in which wind plant controls is expected to operate—that is, below very high winds), the impact on turbine lifetime is expected to be mostly positive. Based on this information, if the YawOpt case is again re-optimized, and the angles are limited to be between -10° and +25° (removing only the larger negative angles because limiting to only positive
angles is perhaps too constrictive), the mean power becomes 84.02 MW, which represents a 1% reduction in power. This is still a 6.6% improvement from the baseline, with mostly benign, or even helpful, impacts on loads expected. Figure 9 shows the resulting flow field as predicted by FLORIS for the 180° wind direction.

However, this is only a first consideration of loads. Future studies will strive to more thoroughly account for the optimization of loading impact (or load improvement) in the combined optimization. One way in which this could impact the analysis, for example, is that the partial-wake impact on rotor loads would be expected to increase with closer proximity. Additionally, the amount which the wind plant controller changes the percentage of time each turbine is partially-waked, compared to fully- or non-waked, should be considered in assessing the impact of wind plant control. Figure 9 shows for example that in the 180 degree case, the wakes are moved from fully- to partially- overlapping the downstream turbines. However, many directions are expected to involve moving wakes from partially overlapping a downstream turbine to less overlapped or nonoverlapped. Quantifying the overall impact will require a thorough analysis.

7. CONCLUSIONS

This paper presented the results for several optimizations of a model wind plant with control-based, position (layout)-based, and coupled optimizations (both simultaneously and sequentially). The presented results show that the best overall improvement was achieved by the coupled control and position optimizations.
The three layout scenarios—density, cable, and boundary—each provided an instructive set of results on the possibilities for optimization, and the relative importance of control compared to position layout as well as the importance of simultaneous compared to sequential optimization. The density cases, in which the optimization seeks to maintain a high mean power output while making the plant as small as possible, require full simultaneous coupling to achieve optimal results: the power density of $Y_P^{Density}$ is 28.7% greater than that of $SeqOpt^{Density}$ and 31% greater than that of $PosOpt^{Density}$. The boundary cases, which have a fixed area, demonstrated overall optimal performance for the simultaneous coupled optimization $Y_P^{Boundary}$, which has the greatest mean power. However, the cases that include control, $Y_P^{Boundary}$ and $SeqOpt^{Boundary}$, are not a substantial improvement over yaw control alone (0.8% and 0.3%, respectively), suggesting in these cases that control is primarily important and only minimal gain can be achieved with position optimization regardless of coupling to control. Finally, the cable length cases, which expand the wind farm, seem to be dominated by the position optimization in terms of both mean power and cable length. Additional improvement is achieved by applying control (1.8%); however, in this case, simultaneous optimization of control and position seems to add negligible benefit over a sequential optimization.

It is important to note that this study focused on the method for optimizing wind plant controls and layout optimization and a number of simplifications were employed to avoid overcomplicating a first study. The separately optimized metrics and constraints used for optimization (power density, cable length), are representative, but a real wind plant design could include many further constraints on layout (given realities such as topography or zoning regulations) and factors contributing to the COE. Future work will expand on these initial results to include more realistic constraints and fuller definitions of the COE. Future work will also strive to optimize the loading impact of turbines in addition to the power production.

REFERENCES

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