International Congress on Environmental Modelling and Software

5th International Congress on Environmental Modelling and Software - Ottawa, Ontario, Canada - July 2010

Jul 1st, 12:00 AM

Multilinear Diffusion Analogy Model for Stage Hydrograph Routing

Muthiah Perumal

Tommaso Moramarco

Bhabagrahi Sahoo

Silvia Barbetta

F. Melone

Multilinear Diffusion Analogy Model for Stage Hydrograph Routing

Muthiah Perumal[a], **Tommaso Moramarco**[b], **Bhabagrahi Sahoo**[c], **Silvia Barbetta**[b], **and Florisa Melone**[b]

[a] *Department of Hydrology, Indian Institute of Technology Roorkee, Roorkee-247 667, India (p_erumal@yahoo.com)*

[b] *National Research Council, IRPI, Via Madonna Alta 126, 06128 Perugia, Italy (t.moramarco@irp.icnr.it, silvia.barbetta@irpi.cnr.it, f.melone@irpi.cnr.it)*

[c] *ICAR Research Complex for NEH Region, Nagaland Centre, Jharnapani, Nagaland,, India; Formerly at Department of Hydrology, Indian Institute of Technology Roorkee, Roorkee-247 667, India (bsahoo2003@yahoo.com)*

Abstract: A multilinear stage-hydrograph routing method using the diffusion analogy model as a sub-model is proposed in this study. The diffusion analogy model can be considered as the next level of approximation to the full Saint-Venant equations. The applicability of the method is tested by studying flood wave propagation in the 15 km Pierantonio-Ponte Felcino reach of the Tiber River in Central Italy. The results demonstrate the suitability of the diffusion analogy model as a sub-model for its use in the multilinear flood routing method for field applications.

Keywords: Diffusion; Multilinear; River; Routing; Stage; Streamflow; Modelling.

1. INTRODUCTION

Information based on discharge is the basis of analysis and decision making in many hydrological applications. However, for flood forecasting, design of flood control levees, automatic operation of canals, and consideration of environmental flow issues, particularly for aquatic-habitat needs, the stream stage or flow depth is a more relevant variable than the discharge. Besides, stage can be measured easily and economically. Due to these reasons, considerable advantage is gained in routing a stage-hydrograph rather than a discharge hydrograph, except for the cases such as rainfall-runoff modelling. Once the decision is made to use the stage as the operating variable, then a decision is required on the method of routing to be employed. Though a number of generic software are available like the US National Weather Service's DAMBRK model [Fread, 1990], HEC-RAS [USACE, 2008], MIKE11 [DHI, 2008] etc., in this study it is proposed to develop and employ a simplified hydraulic routing method for performing the channel routing task. The use of simplified routing techniques in design and operational hydrology is justified on the following considerations: By analysing different river wave types, Ferrick [1985] suggested the use of appropriate wave type equations for obtaining accurate solutions without facing numerical problems, and argued that the use of more complete equations (Saint-Venant equations) may not yield more accurate river wave simulations for all wave types. This argument is now substantiated by the incorporation of the option for routing dam failure floods in steep river reaches using the variable parameter Muskingum-Cunge (VPMC) method [Ponce and Yevjevich, 1978] in the NWS DAMBRK model which earlier used only the Saint-Venant equations for obtaining routing solutions. Further, apart from the feature of easy to understand and use by the field engineers and hydrologists, the use of simplified hydraulic routing methods for many situations is also cost effective as routing can be carried out using fewer cross sections data of the considered river reach unlike that of the solution using the full Saint-Venant equations, which require many cross-section details from the perspective of numerical stability considerations.

Few simplified routing methods using stage as the operating variable were available till recently; for example, Hayami's diffusion analogy model [Hayami, 1951] was the only simplified physically based stage-hydrograph routing method well known to the hydrologists. However, recently many number of physically based stage-hydrograph routing methods have been introduced [Perumal and Ranga Raju, 1998a, 1998b; Perumal et al., 2007a, 2007b, 2009b]. The capabilities of these methods have been very well tested using the hypothetical and field data, especially using the Tiber River data of Central Italy [Perumal et al., 2007a; Perumal et al., 2009a, 2009b, 2009c, 2010]. Basically, these methods have been developed based on the variable parameter approach in which the simplicity of the conventional linear theory concept is adopted; however, the parameters of the linear model vary from one-time interval to the next time interval of routing based on the physically based relationships of these parameters developed from the Saint Venant equations, which govern the one-dimensional flow in rivers and channels. Such an approach of routing retains the mathematical simplicity of the linear models, but at the same time, incorporates the nonlinearity of the flood movement process. Such variable parameter routing methods follow either a recursive equation based approach [Perumal and Ranga Raju, 1998a, 1998b; Perumal et al., 2007a; Perumal et al., 2009a, 2009c, 2009d, 2010] or the multilinear modelling approach based on the convolution principle [Perumal et al., 2009b]. Both these approaches demonstrate superior performance over the traditional linear based approaches. However, the well known Hayami's [1951] diffusion analogy based stage-hydrograph routing method has not yet been tested using the concept of such variable parameter approaches accounting for nonlinearity in the flood wave movement to make it more useful to field applications. The usefulness of adopting the multlinear modelling approach to route discharge hydrographs using the Hayami's diffusion analogy model has been demonstrated recently by Perumal et al. [2008]. This multilinear routing method using the diffusion analogy model as a sub-model can be considered as the next level of approximation to the full Saint-Venant equations in describing the flood wave movement in rivers. In this study, the development of the multilinear diffusion analogy method for stage-hydrograph (MDS) routing in a river reach is considered and its applicability is demonstrated by studying the flood wave propagation in a 15 km reach of the Tiber River in Central Italy.

2. MULTILINEAR ROUTING MODELS

For real-time flood forecasting, the linear flood routing methods such as the Muskingum and Kalinin-Milyukov methods are widely used in literature. Since the flood waves are inherently nonlinear in nature, it is generally desirable to use a nonlinear approach to study the flood wave movement in channels, although the nonlinear models are more difficult to apply in the field than the linear models. One simple method by which the nonlinearity of the flood routing process could be accounted for is by using the multilinear modelling approach advocated by Keefer and McQuivey [1974] and followed by Becker [1976] and Becker and Kundzewicz [1987], wherein the convenience of the linear analysis is used for modelling the nonlinear hydrological processes by working within the limitation imposed by its assumption. This approach was further extended by employing the Muskingum method [Perumal, 1992] and the discrete cascade model [Perumal, 1994] as the sub-models of the multilinear routing methods. All these multilinear routing methods amply demonstrate the appropriateness of this approach by closely reproducing the solutions of the full Saint-Venant equations in off-line mode, which were considered as the benchmark solutions.

Moramarco and Singh [1999], and Perumal et al. [2008] investigated the use of the diffusion analogy model as a sub-model of multilinear methods for routing a given discharge hydrograph with the sub-model parameters varying for every inflow ordinate to be routed. While Moramarco and Singh [1999] directly employed the time varying response function (IUH) of the diffusion analogy sub-model for convoluting with the given inflow hydrograph to arrive at the outflow hydrograph, Perumal et al. [2008] employed the time varying Δt - hour unit-ordinate response function for convoluting with the given inflow hydrograph. However, when the routing time interval Δt is small, say an hour, the difference between the IUH and the corresponding Δt-hour response function may not be significant, implying that one may directly employ the IUH for convolution with the given inflow hydrograph ordinates as has been done by Moramarco and Singh [1999].

3. MULTILINEAR DIFFUSION ANALOGY BASED STAGE-HYDRGRAPH ROUTING METHOD (MDS)

The routed stage estimate at any time in response to the given input stage hydrograph, observed till that same time, may be expressed by the discrete convolution approach as:

$$y_d(j\Delta t) = y_b + \sum_{i=1}^{j} h(i\Delta t)\left(y_u\left((j-i+1)\Delta t\right) - y_b\right) \qquad (1)$$

where y_u, y_d and y_b are the upstream, downstream, and initial flow depths in the considered river reach, respectively; $j\Delta t$ denotes the time t in discrete intervals with j=1 corresponding to the first ordinate; and $h(i\Delta t)$ denotes the ordinate of the Δt-hour unit-ordinate response function of the diffusion analogy sub-model at time $i\Delta t$. The Δt-hour unit-ordinate response function of the diffusion analogy sub-model is expressed using the convolution approach as:

$$h(t) = \int_{t-\Delta t}^{t} u(\tau)\,d\tau \qquad (2)$$

where $u(t)$ is the IUH of the diffusion analogy model [Hayami, 1951] expressed as:

$$u(t) = \frac{x}{\sqrt{4\pi D t^3}} \exp\left[\frac{-(x-ct)^2}{4Dt}\right] \qquad (3)$$

where x = routing reach length; c = wave celerity in m/s, and D = hydraulic diffusivity in m^2/s expressed as:

$$D = Q_0/(2S_0B) \qquad (4)$$

where Q_0 = reference discharge computed using the Manning's friction law at time t corresponding to the reference stage, y_o; S_0 = channel bed slope; and B = water surface width corresponding to Q_0.

Although there are a number of built-in functions available in the modern day personal computers for a close estimation of the Δt-hour unit-ordinate pulse response function, the use of such a solution approach is not efficient as well as not mathematically elegant. To overcome this problem, the use of the Adam-Moulton multi-step numerical integration method [Atkinson, 2003] is used for the numerical integration of equation (2) and the expression of pulse response for $t \geq 2\Delta t$ is expressed as:

$$h(i\Delta t) = \left(8u_{t-\Delta t} + 5u_t - u_{t-2\Delta t}\right)\Delta t/12 \qquad (5)$$

where Δt is the duration of the pulse input or the routing time interval. The unit ordinate pulse response at time Δt is expressed as:

$$h(\Delta t) = 0.5\Delta t\left(u(0) + u(\Delta t)\right) = 0.5\Delta t.u(\Delta t) \qquad (6)$$

where $u(0) = 0$.

Computation of Celerity and Reference Stage in a Compound Channel Section

The flood routing in natural rivers involves both in-bank and over-bank flows. A natural river reach can be represented approximately as a prismatic compound channel reach consisting of a trapezoidal main channel section and a trapezoidal floodplain channel section [e.g., Perumal et al. 2007a]. The different geometrical elements of this symmetrical trapezoidal compound section are (see Fig. 1) defined as: b_m = bottom width of the main channel, y_m = bank-full level of the main channel section, z_1:1= side slope of the main channel section (horizontal:vertical), b_f = bottom width of the floodplain channel, and z_2:1= side slope of the floodplain channel section (horizontal:vertical). Subsequently, the wave celerity corresponding to the in-bank and over-bank flow depths are computed as follows.

For a trapezoidal channel with a bottom width b_m and side slope z_1:1 (horizontal:vertical), the reference stage, y_o and celerity, c [Perumal et al., 2007a] are estimated, respectively, as

$$y_o(t) = y_b + a_1(y_u(t) - y_b) \quad (7)$$

$$c = \frac{1}{n}\left[\frac{5}{3} - \frac{4}{3}\frac{y_o(b_m + y_o z_1)\sqrt{1+z_1^2}}{\left(b_m + 2y_o\sqrt{1+z_1^2}\right)(b_m + 2y_o z_1)}\right]\left[\frac{y_o(b_m + y_o z_1)}{b_m + 2y_o\sqrt{1+z_1^2}}\right]^{2/3}\sqrt{S_o} \quad (8)$$

where a_1 = the main channel coefficient with limits $0 \le a_1 \le 1$; n = the Manning's roughness coefficient; and S_o = channel bed slope.

Similarly, for floodplain channel flow condition, the reference stage and the corresponding celerity of the compound channel section, $c_{comp,0}$, can be expressed as [Perumal et al., 2007a]

$$y_o(t) = y_b + a_2(y_u(t) - y_b) \quad (9)$$

$$c_{comp,0} = \left[\frac{5\sqrt{S_o}}{3n}\left(\frac{A_{main}}{P_{main}}\right)^{2/3}\left(\frac{\partial A_{main}}{\partial y}\right)\right] \Big/ \left[\frac{\partial A_{comp}}{\partial y}\right]$$
$$+\left[\frac{\sqrt{S_o}}{n}\left(\frac{A_1}{P_1}\right)^{2/3}\left(\frac{5}{3}\frac{\partial A_1}{\partial y} - \frac{2}{3}\frac{A_1}{P_1}\frac{\partial P_1}{\partial y}\right)\right] \Big/ \left[\frac{\partial A_{comp}}{\partial y}\right]$$
$$+\left[\frac{\sqrt{S_o}}{n}\left(\frac{A_2}{P_2}\right)^{2/3}\left(\frac{5}{3}\frac{\partial A_2}{\partial y} - \frac{2}{3}\frac{A_2}{P_2}\frac{\partial P_2}{\partial y}\right)\right] \Big/ \left[\frac{\partial A_{comp}}{\partial y}\right] \quad (10)$$

where a_2 = floodplain coefficient with limits $0 \le a_2 \le 1$; and

$$A_{comp} = (b_m + y_m z_1)y_m + (b_f + z_2(y_0 - y_m))(y_0 - y_m) \quad (11a)$$

$$\frac{\partial A_{comp}}{\partial y} = b_f + 2(y_0 - y_m)z_2 \quad (11b)$$

$$A_{main} = (b_m + y_m z_1)y_m + (b_m + 2y_m z_1)(y_0 - y_m) \quad (11c)$$

$$P_{main} = b_m + 2y_m\sqrt{1+z_1{}^2} \quad (11d)$$

$$\partial P_{main} / \partial y = 0 \quad (11e)$$

$$A_1 = A_2 = 0.5((b_f - b_m - 2y_m z_1) + z_2(y_0 - y_m))(y_0 - y_m) \quad (11f)$$

$$P_1 = P_2 = 0.5(b_f - b_m - 2y_m z_1) + (y_0 - y_m)\sqrt{1+z_2^2} \quad (11g)$$

$$\frac{\partial A_1}{\partial y}=\frac{\partial A_2}{\partial y}=0.5\left(b_f-b_m-2y_m z_1\right)+z_2\left(y_0-y_m\right) \tag{11h}$$

$$\frac{\partial P_1}{\partial y}=\frac{\partial P_2}{\partial y}=\sqrt{1+z_2^2} \tag{11i}$$

4. PERFORMANCE EVALUATION MEASURES

The performance of the MDS routing method is evaluated by comparing its simulated routing results with the corresponding observed stage hydrograph on the basis of the following performance measures:

Model Efficiency

The closeness with which the MDS routing method reproduces the observed stage-hydrographs can be measured using the Nash–Sutcliffe criterion of model efficiency [Nash and Sutcliffe, 1970]. The measure in percentage of observed stage-hydrograph reproduction is given by

$$\eta_y=\left(1-\sum_{i=1}^{N}\left(y_{oi}-y_{ci}\right)^2\Big/\sum_{i=1}^{J}\left(y_{oi}-\bar{y}_{oi}\right)^2\right)\times 100 \tag{12}$$

where η_y = model efficiency estimate in reproducing the observed stage-hydrograph by the MDS method; y_{oi} = i[th] ordinate of the observed stage-hydrograph at the outlet; $\bar{y}_o$ = mean of the observed stage-hydrograph ordinates at the outlet; y_{ci} = i^{th} ordinate of the routed stage-hydrograph by the MDS method; and N = total number of stage- hydrograph ordinates to be simulated.

Peak Stage Reproduction

The precision in reproducing the peak stage is estimated by the percentage error in simulating the peak of the observed stage-hydrograph at the outlet of the routing reach, y_{per} (in percentage), as

$$y_{per}=\left(y_{pc}/y_{po}-1\right)\times 100 \tag{13}$$

where y_{pc} = routed peak of the stage-hydrograph by the MDS method at the reach outlet; and y_{po} = peak of the observed stage-hydrograph at the reach outlet.

Time to Peak Stage Reproduction

The precision in reproducing the time to peak stage is estimated by the error in time to peak stage, t_{pyer} (in hour), as

$$t_{pyer}=t_{ypc}-t_{ypo} \tag{14}$$

where t_{ypc} = time corresponding to the routed peak stage by the MDS method at the outlet; and t_{ypo} = time corresponding to the observed peak stage at the outlet.

5. APPLICATION

The MDS routing method described above was applied for routing the natural river stage-hydrographs. The application was made for routing in the 15 km long reach of the Pierantonio- Ponte Felcino section of the Tiber River in Central Italy. This reach is approximated by a compound trapezoidal section consisting of a main river and a flood plain section (see Figure 1) using the equivalent prismatic river reach representation procedure followed by Perumal et al. [2010] by utilizing the upstream and downstream cross-sections of the considered river reach only.

The equivalent geometrical elements of the compound trapezoidal section arrived at by Perumal et al. [2010] for the Pierantonio–Ponte Felcino reach are: y_m =5.0 m, b_m =27.31 m, z_1=1.98, b_f=57.6 m, and z_2=3.8; and this equivalent reach is characterized by the same Manning's roughness coefficient n=0.039 as used by Perumal et al [2010].

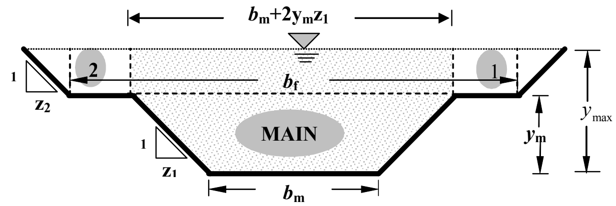

Fig. 1. Schematic diagram and compartmentalization of the compound channel section into a main channel section (shaded) and two floodplain channel sections 1 and 2 for computing the celerity, when $y > y_m$ [after Perumal et al., 2010].

6. DISCUSSION OF RESULTS

The proposed MDS routing method was used to route six flood events consisting of four in-bank events and two over-bank events, in the considered Pierantonio–Ponte Felcino reach conceptualized as the optimized equivalent compound trapezoidal section reach as described in section-5. Table 1 presents the measures of reproduction of the pertinent characteristics of the observed hydrographs at the Ponte Felcino reach based on the routing results obtained using the proposed MDS routing method. All the six simulated events using the proposed methodology are shown in Figure 2 along with the observed hydrographs at the Ponte Felcino station. It is inferred from Figure 2 that the proposed method is able to closely match the rising limb of all the observed hydrographs which suggests that the method is able to capture the convection speed of the flood wave movement in the considered reach. It is inferred from Table 1 that the simulation of the in-bank flood events could be achieved with the Nash-Sutcliffe efficiency criteria, η_y greater than 95%. The over-bank flood event of December 2000 could be simulated with an efficiency of η_y= 98.07%. However, the other over-bank flood event corresponding to November, 2005 could be reproduced only with the Nash- Sutcliffe efficiency of η_y= 92.7%. But it may be noted that this event recorded a large lateral flow and the flow measurements could not be carried out due to the higher magnitude of the event. As inferred from Table 1, only the event of November, 2005 estimates the peak of the simulated hydrograph with an underestimation of 11.7% and an early arrival of the peak by 5 hours. Barring this event, the simulation of the observed stage-hydrographs for the other five considered events suggests the suitability of the proposed MDS method for river routing applications. However, a detailed study involving the application of this method for many other rivers is required before recommending this method for field use.

7. CONCLUSIONS

The study demonstrates that the proposed multilinear diffusion analogy stage-hydrograph (MDS) routing method based on the use of diffusion analogy response function as the sub-model is capable of simulating the observed hydrographs at the Ponte Felcino station of the Pierantonio–Ponte Felcino reach of the Tiber River in Central Italy. The method is simple in form and can be easily understood by the field engineers. Further, the application of this method to natural rivers does not require the use of many river cross-sections within the routing reach as in the case of the methods based on the solution of the full Saint-Venant equations. However, a detailed study involving application of this method for many other rivers is required before recommending this method for field use.

Table 1. Summary of performance criteria showing reproduction of the pertinent characteristics of the floods of the Tiber River reach by the MDS method

Events	η_y (%)	y_{per} (%)	t_{pyer} (h)	Lateral inflow (%)	a_1	a_2
Dec-96	96.04	-5.73	-0.50	0	0.2	0.25
Apr-97	95.63	-7.96	-0.50	2	0.2	0.25
Nov-97	96.36	-3.13	-0.50	0	0.2	0.25
Feb-99	97.38	-2.32	-0.50	0	0.2	0.25
Dec-00	98.07	-1.95	-1.50	Flooding	0.2	0.25
Nov-05	92.69	-11.69	-5.00	Flooding	0.2	0.25

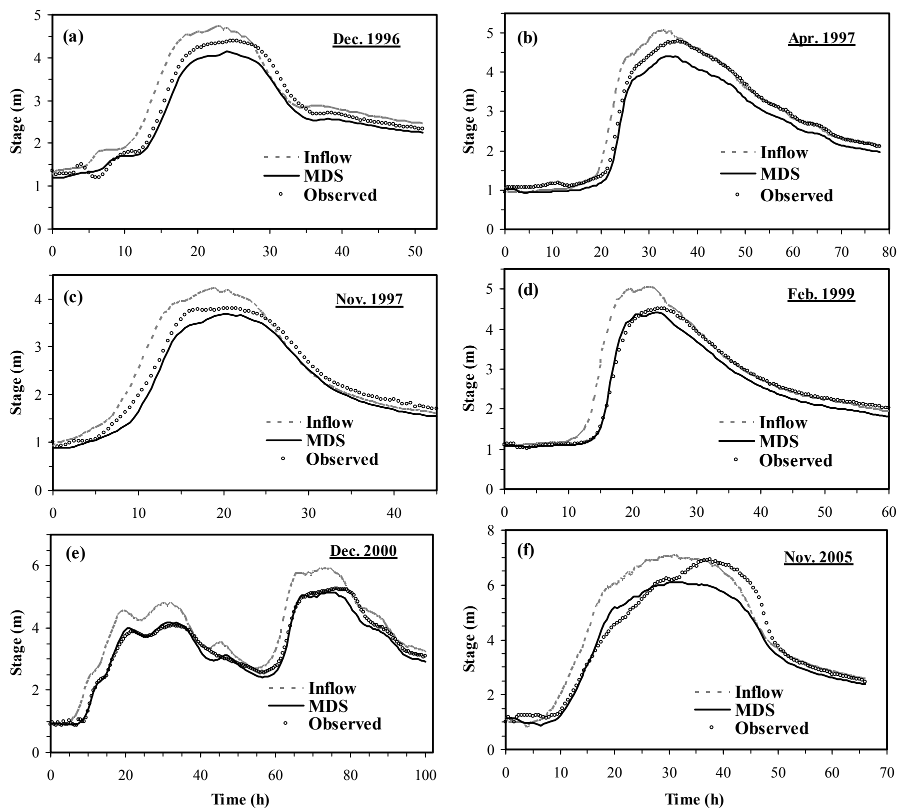

Fig. 2. Performance evaluation of the MDS method in simulating the Tiber River floods.

ACKNOWLEDGEMENTS

We thank the financial support "International short-term mobility program for scientists/researchers from Italian and foreign institutions" offered to the first author by the CNR (National Research Council) of the Italian Government to carry out this research at the CNR-IRPI office of Perugia, Italy. The authors are also grateful to the Department of Environment, Planning and Infrastructure of the Umbria Region for providing the Tiber River data.

REFERENCES

Atkinson, K., *Elementary Numerical Analysis*, John Wiley & Sons, Second edition, 2003.

Becker, A., and Z.W. Kundzewicz, Nonlinear flood routing with multilinear models, *Water Resources Research, 23,* 1043–1048, 1987.

Becker, A., Simulation of nonlinear flow systems by combining linear models, *IAHS Publication, 116*, 135–142, 1976.

DHI-Water and Environment, MIKE11–A modelling system for rivers and channels, Horsholm, Denmark: Danish Hydraulic Institute, 2008.

Ferrick, M. G., Analysis of river wave types, *Water Resources Research*, *21*(2), 209–220, 1985.

Fread, D.L., DAMBRK: The NWS dam-break flood forecasting model, Hydrologic Research Laboratory, Office of Hydrology, National Weather Service (NWS), Silver Spring, Maryland USA, 1990.

Hayami, S., On the propagation of flood waves, *Bull. Disaster Prevention Research Institute*, Kyoto Univ., Japan, *1*(1), 1–16, 1951.

Keefer, T. N., and R.S. McQuivey, Multiple linearization flow routing model, *Journal of Hydraul. Div., ASCE, 100*(HY7), 1031–1046, 1974.

Moramarco, T., and V.P. Singh, Linear routing in channel networks, In: V.P. Singh, I.W. Seo, and J.H. Sonu (Eds.), *Hydrologic Modeling*, Water Resources Publications, 153–160, 1999.

Nash, J. E., and J. V. Sutcliffe, River flow forecasting through conceptual models, Part-1: A discussion of principles, *Journal of Hydrology*, *10*(3), 282–290, 1970.

Perumal, M., and K. G. Ranga Raju, Variable-parameter stage-hydrograph routing method. I: Theory, *Journal of Hydrologic Engineering*, *ASCE, 3*(2), 109–114, 1998a.

Perumal, M., and K. G. Ranga Raju, Variable-parameter stage-hydrograph routing method. II: Evaluation, *Journal of Hydrologic Engineering*, *ASCE, 3*(2), 115–121, 1998b.

Perumal, M., B. Sahoo, T. Moramarco, and F. Melone, A multilinear discrete cascade model for stage-hydrograph routing in compound channels (Submitted to *Hydrological Sciences Journal*, IAHS for review), 2009c.

Perumal, M., B. Sahoo, T. Moramarco, and S. Barbetta, Multilinear Muskingum method for stage-hydrograph routing in compound channels, *Journal of Hydrologic Engineering, ASCE,* 14(7), 663–670, 2009b.

Perumal, M., Multilinear discrete cascade model for channel routing, *Journal of Hydrology*, *158*, 135–150, 1994.

Perumal, M., Multilinear Muskingum flood routing method, *Journal of Hydrology, 133*, 259–272, 1992.

Perumal, M., T. Moramarco, and F. Melone, A caution about the multilinear discrete lag-cascade model for flood routing, *Journal of Hydrology, 338*(3–4), 308–314, 2007b.

Perumal, M., T. Moramarco, B. Sahoo, and S. Barbetta, A methodology for discharge estimation and rating curve development at ungaged river sites, *Water Resour. Res., AGU,* 43(2), W02412, doi: 10.1029/2005WR004609, 1–22, 2007a.

Perumal, M., T. Moramarco, B. Sahoo, and S. Barbetta, Multilinear diffusion analogy model for real-time streamflow routing, In: *Proceedings of the iEMSs Fourth Biennial Meeting: International Congress on Environmental Modelling and Software (iEMSs 2008), International Environmental Modelling and Software Society, Barcelona, Catalonia, July 2008.,* M. Sànchez-Marrè, J. Béjar, J. Comas, A. E. Rizzoli, and G. Guariso (Eds.), (July 7–10, 2008), Barcelona, Catalonia, ISBN: 978-84-7653-074-0, Vol. 1, 536–543, 2008.

Perumal, M., T. Moramarco, B. Sahoo, and S. Barbetta, On the practical applicability of the VPMS routing method for rating curve development at ungauged river sites, *Water Resources Research, 46*(3), W03522, doi:10.1029/2009WR008103, 1–9.

Perumal, M., T. Moramarco, S. Barbetta, F. Melone, and B. Sahoo, Real-time flood-stage forecasting by VPMS routing method, *Hydrology Research (Formerly Nordic Hydrology)*, 2009a (in press).

Perumal, M., T. Moramarco, S. Barbetta, F. Melone, and B. Sahoo, Real-time flood forecasting using Muskingum stage-hydrograph routing method, In: *Proceedings of the International Conference on Water, Environment, Energy and Society (WEES-2009),* S.K. Jain, V.P. Singh, V. Kumar, R. Kumar, R.D. Singh, and K.D. Sharma (Eds.), (January 12–16, 2009), Allied Publishers Pvt. Ltd., New Delhi, India, ISBN: 978-81-8424-400-7, Vol. II (Statistical and System Analysis Techniques), 735–741, 2009d.

Ponce, V. M., and V. Yevjevich, Muskingum-Cunge method with variable parameters, *Journal of Hydraulic Division, ASCE*, *104*(HY12), 1663– 1667, 1978.

U.S. Army Corps of Engineers, *HEC-RAS River Analysis System Hydraulic Reference Manual* (version 3.1), *Rep. CPD-70*, Hydrol. Eng., Cent., Davis, Calif., 2002.