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2003-6

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Original Publication Citation

McLain, T. and Beard, R. Cooperative Path Planning for Timing Critical Missions, Proceedings of the American Control Conference, pp. 296-301, June 2003, Denver, Colorado.

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McLain, Timothy W. and Beard, Randal W., "Cooperative Path Planning for Timing Critical Missions" (2003). *Faculty Publications*. 1532.

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Cooperative Path Planning for Timing-Critical Missions

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Abstract

This paper presents a cooperative path planning approach for teams of vehicles operating under timing constraints. A cooperative control approach based on coordination variables and coordination functions is introduced and applied to cooperative timing problems. Three types of timing constraints are considered: simultaneous arrival, tight sequencing, and loose sequencing. Simulation results demonstrating the approach are presented.

1 Introduction

The ability to plan paths in a cooperative fashion for a system of vehicles is of great importance in a wide variety of applications. Two technical challenges must be addressed for cooperative planning methods for a distributed team of unmanned air vehicles to become viable: the inherent level of complexity in cooperative planning for multiple vehicles and the need to produce paths for a changing environment in real time.

Work on cooperative control and cooperative path planning for UAVs has only recently begun to appear. In Ref. 1, a decentralized optimization method is developed and applied to a multiple aircraft coordination problem. In Ref. 2, mixed-integer linear programming (MILP) is used to solve tightly-coupled task assignment problems with timing constraints. The advantage to this approach is that it yields the optimal solution for a given problem. The primary disadvantage is the computational burden involved. Pruning strategies for simplifying the MILP problem have been proposed to enable near-real-time solutions.

The objective of this paper is to introduce a general approach to cooperative control problems, and to specifically demonstrate its application to cooperative timing missions. The fundamental axiom of our approach is that cooperation requires the sharing of knowledge. Knowledge may be shared in a variety of ways. For example, relative position sensors may enable vehicles to construct state information of other vehicles, or knowledge may be communicated between vehicles using a wireless network, or joint knowledge might be pre-programmed into the vehicles before the mission begins. Our approach is to collect the information that must be jointly shared to facilitate cooperation into a single vector quantity called the *coordination variable*.

Although it is known by different names, the notion of

a coordination variable is found in other works on cooperative control, most commonly in the area of formation control. For example Ref. 3 introduces an “action reference” which, if known by each vehicle, facilitates formation keeping. In leader-following applications,⁴ the states of the leader constitute the coordination variable since the action of the other vehicles in the formation are completely specified once the leader states are known. In Ref. 5, the notion of a virtual structure is used to derive formation control strategies. The motion of each vehicle is causally dependent on the dynamic states of the virtual structure, therefore the states of the virtual structure are coordination variables.

The second main ingredient of the cooperative control strategy introduced in this paper is the notion of a coordination function. Coordination functions parameterize the effect of the coordination variable on the myopic objectives of each agent. The idea is to quantify how changing the coordination variable impacts the individual myopic objectives, and then to use this information to modify the coordination variable.

In the cooperative timing problems considered here, the coordination variable defines mission-critical timing information, such as estimated-time-of-arrival (ETA) at a specified destination. The coordination function describes the cost to an individual UAV of achieving different values of the coordination variable that are feasible for the UAV. Cooperative path planning is enabled by communication of coordination functions and coordination variables among UAVs participating in the mission. Preliminary investigations of this approach have been reported in Ref. 6. Although the notion of coordination variables is prevalent in many other works, the notion of a coordination function seems to be missing in most of the cooperative control literature. One of the contributions of this paper is to provide a formal mechanism for introducing the type of team feedback that coordination functions allow.

Our strategy requires information vital to the cooperative timing effort to be organized in an efficient manner. Rather than requiring the determination of UAV state trajectories for all UAVs on the team centrally, only the critical timing information represented by the coordination function must be determined. Based on the team-optimal coordination variable, UAV trajectories are determined in a decentralized fashion on the individual UAVs. This decomposition of the cooperative path planning problem results in significant simpli-

fication of team-level planning process and a substantial reduction in the volume of information communicated among UAVs.

2 Coordination Variables and Functions

Cooperative control by a team of vehicles is dependent on the environment or mission scenario in which the vehicles are acting. To characterize the significant elements of the environment, define $\mathcal{X}_i$ to be the situation state space for the i^{th} vehicle and let $\mathbf{x}_i \in \mathcal{X}_i$ be the situation state of the i^{th} vehicle. For a given scenario, each vehicle can act to influence the effectiveness of the team. Let $\mathcal{U}_i(\mathbf{x}_i)$ be the set of feasible decision, or influence values at state $\mathbf{x}_i$, and let $\mathbf{u}_i \in \mathcal{U}_i(\mathbf{x}_i)$ be the influence variable for the i^{th} vehicle.

Our basic axiom is that there is a minimum amount of information needed by the team to effect cooperation. This is termed the *coordination variable* and denoted by θ . The essential idea is that if every agent knows the coordination variable and responds appropriately, then cooperative behavior will be achieved. The coordination variable is a vector in coordination space $\mathbb{R}^c$. In this work, the coordination variable captures information about the arrival time of the team.

Representing the distillation of information from the situation state and influence variables (full information) to the coordination variable (minimal information) is central to this method. If $f_i : \mathcal{X}_i \times \mathcal{U}_i \rightarrow \mathbb{R}^c$ is a function that maps situation state and influence vector pairs to $\mathbb{R}^c$, then the set of feasible coordination variables for the i^{th} vehicle at state $\mathbf{x}_i$ is given by

$$\Theta_i(\mathbf{x}_i) = \bigcup_{\mathbf{u}_i \in \mathcal{U}_i(\mathbf{x}_i)} f_i(\mathbf{x}_i, \mathbf{u}_i). \quad (1)$$

Note that $\Theta_i(\mathbf{x}_i)$ is not necessarily a connected set.

We assume that f_i is (pseudo) invertible in the sense that there exists a function $f_i^\dagger : \mathcal{X}_i \times \Theta_i \rightarrow \mathcal{U}_i$ (called the pseudo-inverse of f_i), such that for every $\vartheta \in \Theta_i(\mathbf{x}_i)$, $f_i(\mathbf{x}_i, f_i^\dagger(\mathbf{x}_i, \vartheta)) = \vartheta$. Simply stated, if the situational state and the coordination variable are known, the decision variable is unique.

In addition to cooperative behavior, the team may have individual performance objectives. Associated with the i^{th} vehicle is a myopic performance objective $J_i : \mathcal{X}_i \times \mathcal{U}_i \rightarrow \mathbb{R}$ that is in harmony with the team objectives. The myopic cost can be parameterized as a function of the coordination variable. This can be done by using the relationship $\mathbf{u}_i = f_i^\dagger(\mathbf{x}_i, \vartheta)$, for each $\vartheta \in \Theta_i(\mathbf{x}_i)$. The function

$$\phi_i(\mathbf{x}_i, \vartheta) = J_i(\mathbf{x}_i, f_i^\dagger(\mathbf{x}_i, \vartheta)), \quad (2)$$

is a representation of the local myopic cost $J_i(\mathbf{x}_i, \mathbf{u}_i)$. Under the restriction that $f_i^\dagger$ is onto, $\bigcup_{\vartheta \in \Theta_i(\mathbf{x}_i)} \phi_i(\mathbf{x}_i, \vartheta) = \bigcup_{\mathbf{u}_i \in \mathcal{U}_i(\mathbf{x}_i)} J_i(\mathbf{x}_i, \mathbf{u}_i)$. If $f_i^\dagger$ is not restricted to be onto, it follows that $\bigcup_{\vartheta \in \Theta_i(\mathbf{x}_i)} \phi_i(\mathbf{x}_i, \vartheta)$ may only be a proper subset of $\mathcal{U}_i$, and $\phi_i(\mathbf{x}_i, \cdot)$ an

approximation of $J_i(\mathbf{x}_i, \cdot)$. As later examples will show, this approximation can be made in a way so that only good, locally optimal decisions are considered from $\mathcal{U}_i$. The function

$$\phi_i : \mathcal{X}_i \times \Theta_i(\mathbf{x}_i) \rightarrow \mathbb{R}$$

given by Equation (2) is called the *coordination function* of the i^{th} vehicle. For a given situation state $\mathbf{x}_i$, the coordination function parameterizes the myopic cost of the i^{th} vehicle verses the coordination variable.

In this paper, the cooperation problems of interest can be posed as a minimization of a team objective function, where the team objective is a function of the myopic objective functions. Let $\mathcal{J}_T : \mathbb{R}^N \rightarrow \mathbb{R}$ be the team objective function, then the cooperative control problem is to find influence variables $\mathbf{u}_1, \dots, \mathbf{u}_N$ that solve the following optimization problem:

$$(\mathbf{u}_1, \dots, \mathbf{u}_N) = \arg \min_{\mathbf{u}_1 \times \dots \times \mathbf{u}_N} \mathcal{J}_T(J_1(\mathbf{x}_1, \mathbf{u}_1), \dots, J_N(\mathbf{x}_N, \mathbf{u}_N)), \quad (3)$$

subject to

$$f_i(\mathbf{x}_i, \mathbf{u}_i) = f_j(\mathbf{x}_j, \mathbf{u}_j), \quad \forall i, j \in \{1, \dots, N\}. \quad (4)$$

This optimization problem will clearly pose computational problems as the number of vehicles increase, and for large state and influence dimensions.

Using coordination variables and coordination functions, a decomposition of the optimization problem of Equations (3) and (4) that captures the information essential for cooperation can be posed:

$$\theta = \arg \min_{\vartheta \in \Theta_i(\mathbf{x}_i)} \mathcal{J}_T(\phi_1(\vartheta), \dots, \phi_N(\vartheta)).$$

Once a team optimal value for the coordination variable is found, individual vehicle decisions can be found by solving for the influence variable from the relationship

$$\mathbf{u}_i = f_i^\dagger(\mathbf{x}_i, \theta).$$

This two-level decomposition process significantly reduces the computation and communication loads.

3 Application to Cooperative Timing

In defining the cooperative UAV timing problems that will be addressed, we will assume that the UAVs are equipped with autopilot and trajectory following capabilities that render the response to heading and velocity commands a first-order dynamic system. Therefore, assuming constant altitude, the i^{th} UAV dynamics are given by

$$\begin{aligned} \dot{z}_{ix} &= v_i \cos \psi_i \\ \dot{z}_{iy} &= v_i \sin \psi_i \\ \dot{\psi}_i &= \alpha_\psi (\psi_i^c - \psi_i) \\ \dot{v}_i &= \alpha_v (v_i^c - v_i), \end{aligned}$$

where α_ψ and α_v are known constants that depend on the implementation of the autopilot.⁷ In addition, the

underlying UAV dynamics constrain the heading rate and velocity as follows:

$$\begin{aligned} -c &\leq \dot{\psi}_i \leq c \\ v_{\min} &\leq v_i \leq v_{\max}, \end{aligned}$$

where c , $v_{\min}$ and $v_{\max}$ are positive constants that depend on the dynamic capability of the aircraft. Let $\mathcal{Z}_i(T_i) = \{\mathbf{z}_i(t) : 0 \leq t \leq T_i\}$ define a feasible trajectory where the initial position of the i^{th} UAV is given by $\mathbf{z}_{i0}$ and the desired final position is given by $\mathbf{z}_{if}$. The UAVs are required to maneuver through a threat field, where $\mathbf{h}_i$ defines the location of the i^{th} threat and the set of threats is given by $\mathcal{H} = \{\mathbf{h}_1, \dots, \mathbf{h}_M\}$.

The cooperative timing problems considered here involve finding trajectories for a team of vehicles to specified destinations. These trajectories must minimize the collective threat exposure of the team and satisfy specified timing constraints. We propose a sub-optimal, but computationally feasible approach to the problem. The architecture for cooperative path planning is composed of three complementary pieces shown in Figure 1. The Waypoint Path Planner (WPP) produces a low-cost set of waypoint paths through the threat field.⁸ The Coordination Manager (CM) receives coordination function information corresponding to the set of waypoint paths for each vehicle, and selects paths and feasible velocities (coordination variables) such that the timing constraints are satisfied. The Dynamic Trajectory Smoother (DTS) smoothes the waypoint paths such that both the dynamic constraints are satisfied and the timing constraints are maintained.⁹ The decision state space $\mathcal{X}_i$ for

Figure 1: Cooperative path planning architecture.

the cooperative timing problems considered here, consists of the Cartesian product of a UAV position vector, a target position vector, and the set of threat locations. Therefore

$$\mathbf{x}_i = \begin{pmatrix} \mathbf{z}_{i0} \\ \mathbf{z}_{if} \\ \mathcal{H}_i \end{pmatrix},$$

where $\mathbf{z}_{i0}$ is the current position of the UAV, $\mathbf{z}_{if}$ is the target position, and $\mathcal{H}_i$ is the set of threat locations known to the i^{th} vehicle.

The set of feasible influence vectors $\mathcal{U}_i(\mathbf{x}_i)$ at $\mathbf{x}_i \in \mathcal{X}_i$ is a range of feasible velocities $[v_{\min}, v_{\max}]$, and a set of waypoint paths from the UAV position to the target position. Therefore an influence vector

$$\mathbf{u}_i = \begin{pmatrix} v_i \\ \mathcal{W}_i \end{pmatrix}$$

consists of a feasible velocity v_i and a waypoint path $\mathcal{W}_i = \{\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_P\}$, where $\mathbf{w}_1 = \mathbf{z}_{i0}$ and $\mathbf{w}_P = \mathbf{z}_{if}$.

For cooperative timing problems, coordination hinges on arrival times at the target. Therefore the coordination variable θ is the estimated-time-of-arrival (ETA) if the UAV were to fly the waypoint path $\mathcal{W}_i$ at velocity v_i . For a given path $\mathcal{W} = \{\mathbf{w}_1, \dots, \mathbf{w}_P\}$, the length of the path is given by

$$L(\mathcal{W}) = \sum_{j=2}^P \|\mathbf{w}_j - \mathbf{w}_{j-1}\|.$$

The mapping from state and influence vector to the coordination variable is given by

$$f_i(\mathbf{x}_i, \mathbf{u}_i) = L(\mathcal{W}_i)/v_i. \quad (5)$$

Since v_i can vary over the feasible range $[v_{\min}, v_{\max}]$, for a given path $\mathcal{W}$, the set of possible coordination variables associated with that path is given by the compact segment $[L(\mathcal{W})/v_{\max}, L(\mathcal{W})/v_{\min}]$. If $\mathcal{U}_i(\mathbf{x}_i)$ consists of a finite set of waypoint paths, then the set of feasible coordination variables given by Equation (1) consists of the union of a set of compact segments on $\mathbb{R}$, as shown in Figure 2. In this paper we will assume that the myopic

Figure 2: The set of feasible coordination variables $\Theta_i(\mathbf{x}_i)$ is the union of a finite set of compact intervals on $\mathbb{R}$.

performance objective J_i is given by a linear combination of threat cost and fuel cost:

$$J_i(\mathbf{x}_i, \mathbf{u}_i) = (1 - \kappa)J_{\text{threat}}(\mathbf{x}_i, \mathbf{u}_i) + \kappa J_{\text{fuel}}(\mathbf{x}_i, \mathbf{u}_i),$$

where $\kappa \in [0, 1]$ gives the designer flexibility to emphasize exposure to threats or fuel expenditure depending on the particular mission scenario.

The threat cost model is based on exposure to threat radar sites and is influenced by the proximity of the threat and the length of time exposed. The signal reflected to the threat radar is assumed to be uniform in all directions and its strength is proportional to $1/d^4$ where d is the distance from the UAV to the threat.¹⁰ The fuel cost for traversing an edge is calculated based on the assumption that fuel usage rate is proportional to the aerodynamic drag force which is proportional to velocity squared.

Next consider the problem of constructing a pseudo-inverse for f_i . The objective is to construct a $\mathbf{u}_i \in \mathcal{U}_i(\mathbf{x}_i)$ from a given $\mathbf{x}_i$ and $\vartheta \in \Theta_i(\mathbf{x}_i)$. As a first step in constructing $f^\dagger$, note that for a given $\mathbf{x}_i$, each $\mathbf{u}_i \in \mathcal{U}_i(\mathbf{x}_i)$ results in both a myopic cost $J_i(\mathbf{x}_i, \mathbf{u}_i)$ and a candidate coordination variable $\vartheta = f(\mathbf{x}_i, \mathbf{u}_i)$. It is interesting to plot the locus of points $\cup_{\mathbf{u}_i \in \mathcal{U}_i(\mathbf{x}_i)} (J_i, \vartheta)$, which is shown in Figure 3. While there are variety of pseudo-inverses that are possible for f , we select $\mathbf{u}_i$ that results in the minimum cost path:

$$\begin{aligned} f^\dagger(\mathbf{x}_i, \vartheta) &= \arg \min_{\mathbf{u} \in \mathcal{U}(\mathbf{x}_i)} J_i(\mathbf{x}_i, \mathbf{u}) \\ \text{subject to : } \quad \vartheta &= f(\mathbf{x}_i, \mathbf{u}). \end{aligned}$$

Figure 3: The locus of points (J_i, θ) over the set $\mathcal{U}(\mathbf{x}_i)$ for a fixed $\mathbf{x}_i$.

The associated coordination function given by Equation (2) is shown in Figure 4. It is important to note that for this problem the coordination function can be conveniently represented by a sequence of (J, θ) pairs that define the straight-line segments represented in Figure 4. Therefore the coordination function for each vehicle is simple to represent and communicate. In this paper, the

Figure 4: The coordination function is based on a pseudo-inverse of f that selects lowest cost path for the candidate coordination variable θ .

team objective function $\mathcal{J}_T$ is simply the sum of myopic objective functions:

$$\mathcal{J}_T(J_1, \dots, J_N) = \sum_{i=1}^N J_i \approx \sum_{i=1}^N \phi_i(\theta).$$

As such, the team objective can easily be expressed as a function of the individual coordination functions. The problem then becomes that of finding the best value of the coordination variable for the team.

3.1 Simultaneous Arrival Constraints

For a team of N vehicles that are constrained to arrive simultaneously at their destinations, the simultaneous arrival constraint can be stated simply as

$$T_1 = T_2 = \dots = T_N = T_s,$$

where $T_i = f(\mathbf{x}_i, \mathbf{u}_i)$ given in Equation (5), and $\theta = T_s$ is the coordination variable. The upper plot of Figure 5 shows coordination functions for a team of three vehicles. The team optimization problem can be visualized as sweeping through the coordination functions while continually monitoring their sum (the team objective). For the timing problems considered here, the coordination functions are piecewise monotonically increasing. Therefore, the team optimum occurs at the left extreme of a piecewise continuous segment of one

of the coordination functions. The resulting optimization is therefore straightforward to carry out. A global search through the left extreme points of each of the coordination functions is all that is required.

Figure 5: Coordination variable determination.

3.2 Tight Sequencing Constraints

Tight sequencing is characterized by enforcing specified intervals between the arrival times of each of the vehicles composing the team. The middle plot of Figure 5 shows coordination functions for a team of three vehicles with the vertical lines indicating the spacing in arrival times. Tight sequencing constraints for a team of vehicles can be formulated as

$$\begin{aligned} T_1 &= T_s \\ T_i &= T_s + \Delta_i \quad i = 2, \dots, N, \end{aligned}$$

where Δ_i represents the interval between the arrival of the first and i^{th} vehicles, and $\theta = T_s$ is the coordination variable. The mapping $f_i(\mathbf{x}_i, \mathbf{w}_i)$ given in Equation (5) must therefore be modified to

$$f_i(\mathbf{x}_i, \mathbf{u}_i) = L(\mathcal{W}_i)/v_i - \Delta_i.$$

As indicated in Figure 5, the team optimization problem can be formulated as sweeping through the set $\bigcap_{i=1}^N \Theta_i(\mathbf{x}_i)$ and examining the critical points where the vertical timing line of each vehicle intersects the left extremes of its coordination function segments. Arrival times for other vehicles on the team are determined by the specified arrival intervals.

3.3 Loose Sequencing Constraints

Loose sequencing can be described as having desired arrival time windows for each vehicle on the team. Figure 5 depicts coordination functions with the vertical bars indicating acceptable time windows for each vehicle on the team. Loose sequencing constraints can be stated as

$$\begin{aligned} T_s &\leq T_1 \leq T_s + \tau_1 \\ T_s + \Delta_i &\leq T_i \leq T_s + \Delta_i + \tau_i \quad i = 2, \dots, N, \end{aligned}$$

where Δ_i is the time interval between the opening of the first time window and the opening of the i^{th} time window and τ_i indicates the duration of the i^{th} time window. The coordination variable is given by $\theta = T_s$ and f_i in Equation (5) must be modified to

$$f_i(\mathbf{x}_i, \mathbf{u}_i) = L(\mathcal{W}_i)/v_i - \Delta_i - \sigma_i,$$

where $\sigma_i \in [0, \tau_i]$ is a slack variable. In this case, the team optimal arrival time for one of the vehicles will occur when the right side of its time window intersects the left extreme of a piecewise continuous segment its coordination function. Team optimal times for the other vehicles will either occur at the left side of their windows or at discontinuities in their coordination functions inside their time windows. Searching through these options to find the optimum is straightforward and fast.

In general, timing constraints for simultaneous arrival, tight sequencing, and loose sequencing can be stated in the form

$$T_s + \Delta_i \leq T_j \leq T_s + \Delta_i + \tau_i \quad i = 1, \dots, N,$$

where $\Delta_1 = 0$ and Δ_i and τ_i are specified for each of the N vehicles composing the team. For loose sequencing, Δ_i and τ_i are positive constants. For tight sequencing, Δ_i are positive constants and $\tau_i = 0$. For simultaneous arrival, $\Delta_i = \tau_i = 0$. This formulation for timing constraints is inherently flexible and can accommodate a mixture of simultaneous arrival, tight sequencing, and loose sequencing constraints in the same mission.

4 Simulation Results

Simulation results are presented for a team of three UAVs flying three different missions: simultaneous arrival, tight sequencing, and loose sequencing. In each mission, there is one target and 33 threats distributed over a 5 km square battle area. The objective is to avoid the threats while meeting the timing constraints imposed for the mission. Simulation results for the three types of timing missions are presented in Figure 6. Solid lines indicate the simultaneous arrival paths, dashed lines indicate the tight sequencing paths, and dash-dotted lines indicate the loose sequencing paths.

4.1 Simultaneous Arrival Constraints

The cooperative path plan resulted in a desired arrival time for the team of 349.7 seconds. In Figure 6, the initial jog in the path of UAV 3 (blue) is a low-risk deviation that enables simultaneous arrival at the target with the other team members. Figure 7 shows range-to-target information for each of the vehicles on the team. It can be seen that the UAVs arrived at the target at 350

Figure 6: Paths to target.

Figure 7: Range to target.

seconds. Errors in the arrival time can be attributed to minor tracking errors by the UAVs. Coordination functions for the team are shown in Figure 8. Each line segment represents one possible trajectory for a UAV. The team-optimal ETA is indicated by the dashed vertical line. It can be seen that the optimal ETA occurs at the left extreme of a coordination function segment for UAV 3. From an individual, myopic perspective, this team ETA is optimal for UAV 3, close to optimal for UAV 1 (red), and suboptimal for UAV 2 (green). Considering the entire team, however the indicated ETA is most cost effective.

4.2 Tight Sequencing Constraints

In the tight sequencing simulation, the UAVs are required to arrive at the target at 30 second intervals. Comparing with simultaneous arrival case, it can be seen that UAV 1 and UAV 3 take the same paths, while UAV 2 takes a slightly longer, but less costly route.

The desired arrival times for the team are 350.3, 380.3, and 410.3 seconds. The range data plotted in Figure 7 shows that these times are closely met by the UAVs.

Figure 8: Coordination functions.

Figure 8 shows coordination function information for the UAVs. Vertical dashed lines indicate the desired ETAs for each of the UAVs. From this information, it can be seen that UAV 1 flies virtually the same trajectory as in the simultaneous arrival case, UAV 2 flies a slower, less costly path, and UAV 3 flies the same path at a much slower velocity. The optimum occurs at a left extreme of a coordination function segment for UAV 2 (380.3 seconds). The arrival times for UAV 1 and UAV 3 are offset ± 30 seconds from this point in time.

4.3 Loose Sequencing Constraints

The loose sequencing constraints give the UAVs flexibility in their arrival times through the use of acceptable time windows. For the problem considered, the openings of the arrival time windows are spaced apart at 30 second intervals, while the windows are each 20 seconds wide. Figure 6 shows that UAV 1 and UAV 3 fly the same paths (although at different velocities) as in the other cases, but UAV 2 flies a different path. The additional flexibility provided by the time windows allows it to choose a lower-cost path.

The desired arrival times for the team are 348.2, 365.0, and 388.2 seconds. Figure 7 shows range to target data for the UAVs that confirms arrival at times very close to those desired. Figure 8 shows coordination function information for the UAVs. The time windows for each UAV are indicated by the shaded areas, while the desired arrival times are shown by vertical lines. For UAV 1 the desired arrival time is at the upper limit of its time window, while for UAV 3 the desired arrival time is at the lower limit of its time window. By making their arrival times as close as the windows will allow, the cost to the team is minimized. For UAV 2, the minimum cost

lies on the interior of the time window rather than the lower or upper bound. Clearly, the flexibility provided by time windows in the loose sequencing scenario results in a lower cost solution than the tight sequencing case.

5 Conclusions

A cooperative control strategy based on coordination functions and coordination variables has been applied to cooperative trajectory planning problems involving timing constraints. Simultaneous arrival, tight sequencing, and loose sequencing constraints can each be accommodated using the cooperative control algorithms and constraint formulations developed. The approach results in a distillation of information essential for cooperation and an efficient means for formulation and solution of team-optimal cooperation problems.

Acknowledgments

This work was supported by AFOSR Grant No. F49620-01-1-0091.

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