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Robust estimation of the total unit hydrograph

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Abstract: This paper investigates three techniques for estimating the parameter values for the unit hydrograph: full model optimisation (the CWI and CMD versions of IHACRES were used), simple rainfall scaling approaches for estimating effective rainfall and an inverse filtering approach. The approaches are tested using synthetic streamflow data generated by the catchment moisture deficit (CMD) version of the IHACRES model, based on observed rainfall data for Cotter catchment (near Canberra, Australia) and the Singkarak catchment (Sumatra, Indonesia). The impact of rainfall and streamflow errors are considered by reducing the raingauge density and introducting errors in the rating curve. The inverse filtering apprach performed well for the synthetic streamflow derived from the rainfall data for the Cotter catchment. For the Singkarak catchment data, the differences between the approaches was small, due possibly to either the higher frequency of rainfall in that area, or to the lower correlation between the rainfall gauge locations.

Keywords: effective rainfall; unit hydrograph; transfer functions; calibration

1 Introduction

Spatially-lumped, conceptual hydrological models are often based on two components: a Soil Moisture Accounting (SMA) component which generates "effective rainfall" and a routing component (based in the unit hydrograph) which uses this to generate outflow. In its original form, unit hydrograph (UH) theory addressed the impulse response of runoff, requiring the subtraction of the baseflow component from the observed streamflow. With the application of the transfer function approach; for example, the IHACRES [Jakeman et al., 1990] and Data-Based Mechanistic (DBM) models [Young, 2003], the UH was expanded to include the total impulse response of the catchment (i.e. including the baseflow component). Investigation of the unit hydrograph is of interest for studies looking at changes in catchment response (e.g. due to climate or land use change, or variations in the rainfall intensity between events).

In calibrating and assessing models using observed streamflow, the SMA and UH components are typically treated as a single unit. While calibrating all the parameters simulatenously will give a good fit to the observed flow, the unobserved state variables are not necessarily well estimated. Thus, in some contexts it can be useful to attempt to decouple the two components. This is an example of the use of theory-based hydrological signatures suggested by Gupta et al. [2008]. Decoupling is achieved by estimating the UH from observed streamflow, using approximations and transfer function methods. This can improve the efficiency of model calibration, and can be used to generate detailed diagnostics to guide model development. Of course, one must be aware of the approximations and sensitivities inherent in these methods.

Optimal estimation of the parameters of the UH requires minimal uncertainty in the effective rainfall estimates. Typically, rainfall runoff models use an assumed functional form for estimating the effective rainfall based on estimates of climate inputs (areal rainfall and a measure of potential evaporation) as well as an indicator of the antecedent catchment soil moisture. This results in considerable uncertainty in the estimated effective rainfall due to uncertainty in the model inputs and error in the model structure. This, in turn, leads to considerable uncertainty in the UH parameter values and difficulty in assessing the deficiencies in the UH model structure.

One approach to bypassing structural errors in the SMA model is to use non-parametric estimation of effective rainfall [e.g. Young, 2003]. Indeed, Kirchner [2009] has demonstrated non-parametric estimation of a complete hydrological model, although that approach is quite different in that it does not distinguish effective rainfall, or the unit hydrograph, but rather considers a storage-discharge relationship. An alternative approach has been developed using an iterative technique where the effective rainfall is estimated from the observed streamflow using an assumed UH, with the constraint that the effective rainfall is less than the observed rainfall (an inverse filtering approach). The effective rainfall sequence is then used to update the UH parameters. The process is repeated until the UH parameter values stabilise. This approach uses the minimum amount of information from the precipitation time series and doesn't require the estimation of a measure of catchment moisture state. Thus the inverse filtering approach described in this paper limits the impact of uncertainty in the precipitation estimates. The result is an improved estimate of the UH and ability to evaluate the adequacy of the UH model structure.

This study explores three methods for estimating the unit hydrograph: full model calibration; simple rainfall scaling methods; and an inverse filtering approach. The synthetic data was generated using the CMD version of IHACRES, and noise added to the rainfall and streamflow data by varying the rain gauge density and introducing a random perturbation to the streamflow rating curve. Observed rainfall data for 2 catchments with very different climates (Cotter catchment near Canberra, Australia, and the Singkarak catchment on Sumatra, Indonesia) were used to generate the synethic streamflow data.

2 The general Unit Hydrograph model

The Unit Hydrograph routing function translates a time series of effective rainfall into a corresponding time series of streamflow. As such it influences the size of flow peaks and the duration of flow after a peak. A Unit Hydrograph recession curve shows the streamflow response resulting from an impulse of 1 unit of input *effective rainfall*.

The transfer function used here is an ARMAX-like (auto-regressive, moving average, with exogenous inputs) model, where the input series is denoted U and the output Q:

$$Q[t] = a_1 Q[t-1] + \ldots + a_n Q[t-n] + b_0 U[t-\delta] + \ldots + b_m U[t-m-\delta] \qquad (1)$$

The *order* is denoted (n, m), with delay δ. The number of parameters is $n + m + 1$.

In hydrology it is natural to decompose the transfer function into a system of exponentially receding components, which may be in a parallel and/or series configuration. Each component is defined by a recession rate and peak response, or equivalently, a time constant τ and fractional volume v. In this form the parameters are physically interpretable. For further discussion, see for example Young [2003].

3 Unit Hydrograph estimation methods

We tested three types of methods for estimating the Unit Hydrograph from rainfall and streamflow data: full model parameter optimisation, simple rainfall scaling methods, and the inverse filtering approach. Only the first type are normal hydrological models; the other methods make use of observed streamflow data and so can not be used for prediction. Rather, they are methods for estimating the unit hydrograph (response characteristics of the catchment). The direct estimation technique developed by Croke [2006] has not been explored here as that approach is best suited to ephemeral streamflow regimes where flow events are well-separated. The direct estimation technique doesn't require rainfall data, and so higher resolution streamflow data that is typically available can be used. As daily rainfall data has been used to generate synthetic streamflow data, higher resolution streamflow data is not available for this study.

In data checking and model diagnostic contexts, it is often adequate to estimate the response characteristics alone. This can reveal changes in catchment response over time, or allow evaluation of a SMA model performance independently from streamflow routing. Alternatively, if a full hydrological model is required, one could fit a SMA model while keeping the UH fixed.

3.1 Full model parameter optimisation

This approach involves joint estimation of the Soil Moisture Accounting model parameters and the Unit Hydrograph model parameters (in the hydrologically-meaningful exponential components form). This is a complex, but general, non-linear optimisation problem.

In this study we used the Shuffled Complex Evolution (SCE) algorithm [Duan et al., 1992], a popular calibration method in the hydrology literature. The objective function was a linear combination of the *R squared* coefficient of efficiency [Nash and Sutcliffe, 1970] of square-root transformed data, and the relative bias.

Two models were used with this approach: the "true" model, which is the IHACRES CMD (Catchment Moisture Deficit) SMA, defined below; and an alternative model with a very different structure, the IHACRES CWI (Catchment Wetness Index) model of Jakeman and Hornberger [1993].

3.2 Simple rainfall scaling methods

Simple rainfall scaling methods involve a simple transformation of the rainfall data to estimate effective rainfall, which is then used to fit the Unit Hydrograph with specialised transfer function methods – in this case the Simple Refined Instrumental Variable (SRIV) algorithm [Young, 2008]. Such approaches have been recommended for estimating the complexity and parameter values of the UH, applicable to simple models such as IHACRES-type models [Jakeman et al., 1990]. This approach is also typical of the early stages of Data-Based Mechanistic modelling [Young, 2003].

We tested two methods: firstly a *runoff ratio* model, where rainfall was scaled by the runoff coefficient (flow as a fraction of rainfall) as calculated in a moving (triangular-weighted) window of 60 days. For added flexibility, a threshold parameter was included, and no effective rainfall was generated when the runoff ratio fell below the threshold. Secondly we tested a typical model structure used in Data-Based Mechanistic (DBM) modelling of catchment hydrology [Young, 2003]. In this case rainfall is scaled by the observed streamflow on the same time step, raised to a power.

The single parameter in each case was selected (from 20 regular samples) according to the objective function described above. For the runoff ratio model, the *threshold* parameter was varied between 0 and 0.3. For the DBM model, the *power* parameter was varied between 0 and 1. In both of these methods, a constant mass balance term was also included in order to set total effective rainfall equal to total observed streamflow.

3.3 Inverse filtering approach

Inverse filtering refers to a method of *deconvolution* – estimating effective rainfall time series U from an observed time series Q of streamflow volumes. The inverse filter for the standard model can be easily derived from the simulation equation and written as:

$$U[t] = (Q[t] - \sum_{i=1}^{n} a_i Q[t-i] - \sum_{j=1}^{m} b_j U[t-j]) / b_0 \tag{2}$$

Importantly, the effective rainfall U is also constrained to be less than the observed rainfall P on the corresponding time step (actually a 10% excess is allowed).

The estimation method is an iterative algorithm. Effective rainfall U is estimated from Q by inverse filtering, then this is passed on to the Simple Refined Instrumental Variable (SRIV) algorithm [Young, 2008] to re-fit the unit hydrograph parameters. This process is repeated until the sum of total differences between U in successive iterations is less than 1/1000th of total Q.

Initialisation could be done with either an initial parameter set or an initial estimate of U. We chose to start with U estimated as the observed rainfall P, scaled by the runoff ratio in a moving window, i.e. starting with the *runoff ratio* model defined above.

4 Test Simulations

For this study, we chose a simple model as the "true" model, i.e. the model used to simulate the test datasets. For the Soil Moisture Accounting, we use the IHACRES-type CMD (Catchment Moisture Deficit) model of Croke and Jakeman [2004]. For the unit hydrograph we used a transfer function with two exponential stores in parallel ($n = 2, m = 1$ in the notation above, with three parameters). The chosen parameter values were: $d = 200$, $f = 0.7$, $e = 0.2$, $\tau_s = 30$, $\tau_q = 2$, $v_s = 0.5$.

For simulation inputs, we use rain gauge and temperature data from two constrasting catchments. One is in the wet tropics and the other is a relatively dry temperate site. Note, however, that the details of these catchments are not important because we use only simulated outputs, and make no use of observed streamflow.

The Cotter catchment is an inland catchment in the Australian Capital Territory, Australia. Rainfall data from six rain gauges in and around the catchment were used in this study (070310, 070316, 570915, 570946, 570948, 570958). The "true" areal rainfall was constructed as a simple average of these.

The Singkarak lake catchment is an inland, mountainous catchment in West Sumatra, Indonesia. Rainfall data from six rain gauges in and around the catchment were used in this study (100001, 100012, 100013, 100027, 100033, 100035). This is an extremely wet series, with rainfall on about 75% of days, and a mean daily rainfall of 6 mm.

5 Error simulation

The UH estimation methods described above were tested with simulated data to assess their robustness to various types of error.

The accuracy of areal rainfall estimation is fundamentally limited by the spatial coverage of rain gauges relative to the spatial variability of rainfall. This type of error was simulated by taking a sub-sample of the available rain gauges. Samples of $n = 5$ and $n = 3$ gauges were taken (out of a total of 6), and 6 replicates were run for each of these levels. The results are shown in Figure 1.

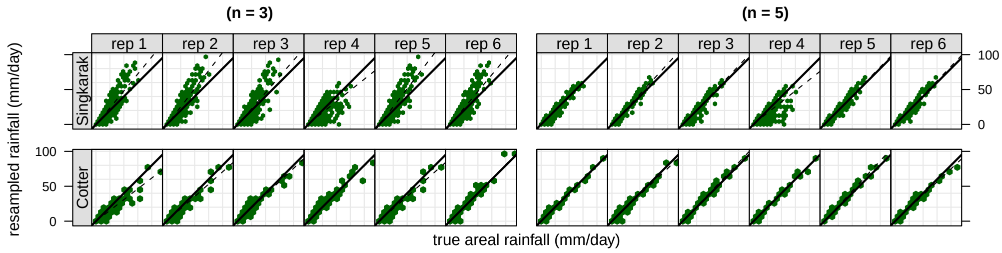

Figure 1: Simulated areal rainfall series with error, plotted against the "true" areal rainfall. Each group of panels refers to the number of rain gauges in the sub-sample (out of a total of 6 gauges). Regression lines (dashed) are shown along with the 1:1 reference line.

Streamflow gauging is subject to error, particularly in the *rating curve*, the transformation from stage height to flow volume. This is likely to be a systematic error, and to be more severe at high flow levels Croke [2009]. This type of error was simulated by a non-linear transformation applied to the true streamflow Q. Specifically, we define four random scaling factors along the range of Q, interpolated by a cubic spline, and use this as a multiplicative transformation function. The scaling factors were chosen to be within $\pm 5\%$ ($d = 0.05$), $\pm 15\%$ ($d = 0.15$) or $\pm 30\%$ ($d = 0.3$). 5 replicates were run in each case. This approach is similar to the error-in-rating-curve simulation of Croke [2009], but is less structured.

The effect of mis-specification of the UH transfer function is represented by the use of different model *orders* as defined in Equation 1. The "true" UH is a second-order transfer function, and we tried each of the calibration methods with the correct form as well as a simpler first-order transfer function.

6 Results and Discussion

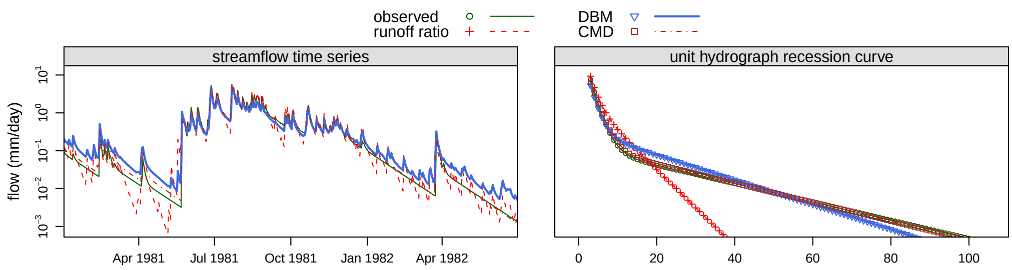

Figure 2: Example of fitted models based on one variant of the Cotter dataset: the streamflow series with error type $d = 0.05$, and rainfall series with error type $n = 5$; taking the first replicate in each case.

The aim of this study was to investigate the performance of methods for fitting the unit hydrograph. As the unit hydrograph is convolved with the modelled effective rainfall time series to produce streamflow, effective rainfall is also of interest.

To aid comprehension, results for a single dataset variant are shown in Figure 2. It shows

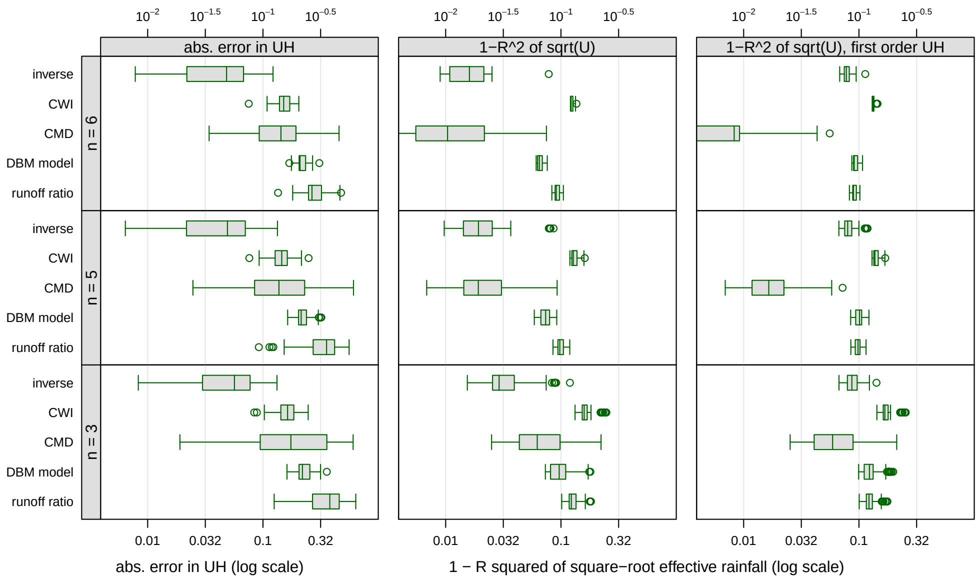

Figure 3: Results for the Cotter dataset, showing accuracy of the modelled unit hydrograph recession curve, and the modelled effective rainfall time series, when fitting by each of the methods shown. In both cases, low values indicate a good fit. The labels $n = 6$, $n = 5$, $n = 3$ indicate the number of rain gauges used, with fewer gauges implying a larger error. The right-most column gives results from fitting a first-order unit hydrograph rather than the true second-order form. Results include 6 replicates of each level of error in rainfall, and for each of these, 5 replicates of each level of error in streamflow.

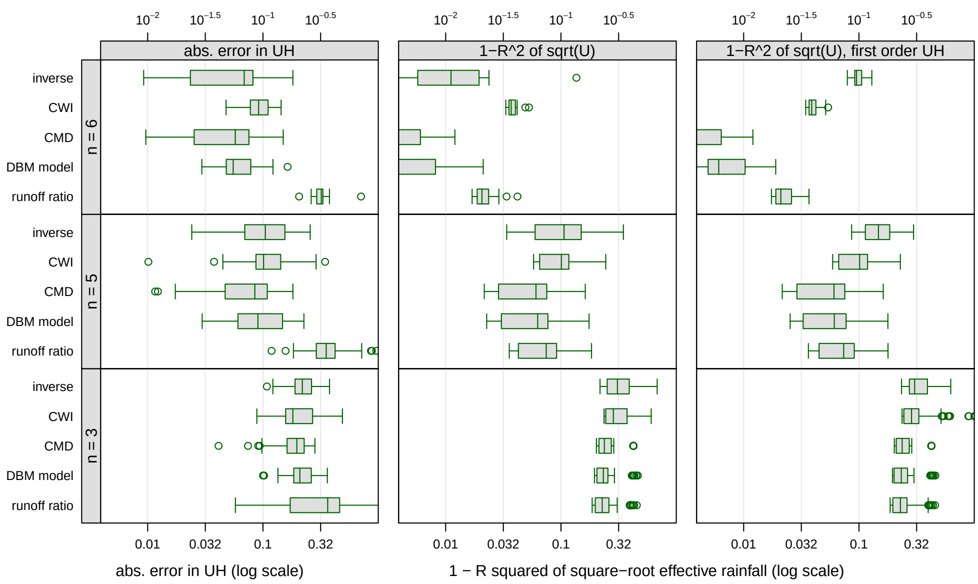

Figure 4: Results for the Singkarak dataset, showing accuracy of the modelled unit hydrograph recession curve, and the modelled effective rainfall time series, when fitting by each of the methods shown. Interpretation is the same as Figure 3.

a section of the fitted streamflow time series, and the fitted unit hydrograph recession curve, from 3 out of the 5 methods. It is clear that, while large streamflow peaks are fitted reasonably well, the recession curve is far too rapid in the runoff ratio method, and a little too rapid in the slow component for the DBM method. The CMD result, which is the "true" model fitted by full optimisation, does reproduce the unit hydrograph very well in this case.

We chose to assess the results from the simulation study in terms of the absolute error in the modelled UH recession curve, and the R Squared [Nash and Sutcliffe, 1970] of square-root transformed effective rainfall. The latter was also calculated from fitting with a reduced-order (first order) unit hydrograph.

The results from fitting with each of the 5 methods, as the level of error in rainfall input data increases, are shown in Figures 3 and 4. These figures show the distribution of results over the three levels of streamflow error (5%, 15%, 30%) and the replications of both rainfall and streamflow error simulation. It was found that the level of error applied to streamflow data made little difference to the results, and that the results were dominated by the level of error in rainfall; for that reason the individual effect of streamflow error is not presented. The effect was noticable only when the rainfall error was low, and particularly for the inverse method.

The results from the Cotter dataset in Figure 3 show that the inverse filtering method generally performed better than the other 4 methods tested. Full optimisation of the true model (CMD) produced good results in terms of effective rainfall, although it was susceptible to errors in the rainfall data. However, the true model performed as poorly as the alternative (CWI) model in reproducing the UH. When a first-order UH transfer function was specified (i.e. one exponential component rather than the true two components), the inverse method gave much worse results, as expected. However, it still gave a better fit to effective rainfall than all methods except the optimised true model. These results confirm that the inverse filtering approach is minimally affected by errors in the rainfall data, and independent of the choice of SMA model.

The results are very different on the Singkarak dataset, shown in Figure 4. All methods were strongly constrained by the level of error in rainfall. For the low error cases ($n = 6$, $n = 5$), the best results were generally from the CMD (true) model and the DBM method. Inverse filtering method did less well. This may be due to either the higher frequency of rainfall in that area, or to the lower correlation between the rainfall gauge locations.

As expected, the performance of full model optimisation was strongly dependent on the choice of SMA model: the true SMA model (CMD) gave much better results than optimisation of an incorrect SMA model (CWI). Of the simple rainfall scaling methods tested, the DBM method consistently gave better results than the runoff ratio method.

7 Conclusions and further work

The inverse filtering method performed well at reconstructing the true unit hydrograph and effective rainfall in the simulations in this paper, except in the case of high frequency rainfall inputs. However, these results are based on reconstructing simulations with the correct model structure of the unit hydrograph transfer function, which in this study was a second-order (two stores) form. The results when fitting a first-order transfer function were much less good. This highlights the need to try different model structures.

Further work should investigate the performance and robustness of methods with different choices for the true unit hydrograph: possibly three or four stores in parallel and/or series, or perhaps with variable partitioning between quick and slow stores, depending on rainfall intensity. Another option is the simulation of groundwater linkages.

Furthermore the effect of the true SMA model should also be investigated. The CMD model used in this study is relatively simple, and other process representations may influence the results to some extent.

Computational details

The results in this paper were obtained using R 2.11.0 [R Development Core Team, 2010] with the packages **hydromad** 0.8–3, **lattice** 0.18–5 [Sarkar, 2008] and **zoo** 1.7–0 [Zeileis and Grothendieck, 2005]. R itself and all packages used are (or will be) available from CRAN at `http://CRAN.R-project.org/`. The R code used to produce the results in this paper is available from `http://www.nfrac.org/felix/papers/2010/iemss/`

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