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Electromagnetic Bias at Off-nadir Incidence Angles
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Abstract.
Nadir and off-nadir measurements of electromagnetic (EM) bias measurements are presented and compared with an off-nadir bias model. Measurements of the bias were made during the BYU Off-Nadir Experiment (Y-ONE) in the months of March and April, 2003. Using radar measurements of the surface and backscattered power, the EM bias was computed at angles from nadir to 17° degrees. Simultaneous surface measurements from a laser rangefinder provide accurate measurements of the long wave surface parameters. An off-nadir bias model incorporating the effects of hydrodynamic modulation of short waves and tilt modulation of long waves is developed and compared to measured off-nadir bias values from the experiment. The magnitude of the measured and predicted EM bias decreases with incidence angle so that the average bias vanishes at approximately 15° for C-band measurements and at 17° for the Ku-band values.

1. Introduction
Past and present satellite altimeters are among the most successful of all earth science space missions. Sea surface height data streams are immensely valuable in commercial and geophysical applications, especially short and long term ocean climate studies. The next great advance for satellite altimetry will be high resolution measurements using a wide-swath approach. With this approach, surface range measurements will be made at off-nadir incidence angles. In order to achieve centimeter-scale surface height accuracy, the variation of the electromagnetic (EM) bias in range measurements as a function of incidence angle must be understood and quantified.

At nadir incidence, numerous experimental and theoretical studies have been made (e.g., Jackson [1979]; Srokosz [1986]; Arnold et al. [1995]; Elfouhaily et al. [2000, 2001]; Millet et al. [2003]; Gommenginger et al. [2003]). Limited off-nadir measurements incidental to a nadir experimental campaign involving aircraft-mounted radar instruments were reported by Walsh et al. [1991] and Hevizi et al. [1993]. Fluctuations in the attitude of the aircraft led to pointing angles spread within a few degrees of nadir. From these measurements, the magnitude of the EM bias was shown to have a decreasing linear trend as a function of angle, −0.186 and −0.100 in the pitch and roll directions, over incidence angles up to approximately 2.5 degrees.

In this paper, we present EM bias measurements at nadir and off-nadir incidence angles collected during a recent experimental campaign, the BYU Off-Nadir Experiment (Y-ONE), located in the Gulf of Mexico in March and April, 2003. C-band and Ku-band altimeters were deployed on a rotating mount with an angle range of nadir to 17°. Descriptions of the collection procedures, processing, and final data are presented.

An EM bias model for off-nadir incidence angles is developed and compared to experimental measurements. The model is derived using the physical optics (PO) scattering approximation and includes the effects of hydrodynamic modulation of short waves and non-Gaussian long wave statistics. The experimental measurements and theoretical model agree in predicting a relatively rapid decrease in the magnitude of the EM bias with incidence angle.

2. Experiment Description
The Y-ONE experiment took place on the Brazos A-19 natural gas platform operated by the Shell Exploration and Production Company in the Gulf of Mexico from March 16 to April 30, 2003. The platform is located south of Houston, Texas at 28°10′N and 95°35′W with a minimum fetch of 58 km to the north. The depth of water at the Brazos A-19 platform is 40 m.

The Brazos A-19 platform complex consists of three rectangular platforms, designated B, C, and D, that are each 20 m by 50 m. Walkways of 50 m and 60 m connect platforms B and D and platforms B and C, respectively, so that the platform complex has an L shape as seen in Figure 1.

The Y-ONE radar and laser systems were deployed in the middle of the 60 m walkway between platforms B and C, approximately 18 m above the ocean surface. A photograph of the walkway where the instruments were deployed is shown in Figure 2. Ocean surface measurements were made using continuous wave Doppler radar systems with center frequencies at 5.2 and 14 GHz. Antennas for the 5.2 GHz system were dish antennas with a 60 cm diameter, corresponding to a spot size of approximately 1.8 m in diameter. The 14 GHz system employed horn antennas that were 20 cm on a side corresponding to a spot size approximately 1.7 m in diameter. The radar systems were rotated through an angular range from −3° to 17° with measurements taken at −3°, −2°, −1°, 0°, 1°, 2°, 3°, 5°, 8°, 11°, 14°, and 17°. To correct for a small mount misalignment the precise nadir angle was determined by the angle stop with maximum returned power over the entire experiment.

Simultaneous surface measurements were made with a system of three Optech Sentinel 3100 laser rangefinders. The rangefinders were mounted in an equilateral triangle, 1 m on a side, with one laser footprint co-located inside the footprints of both radar systems. The footprints of the laser rangefinders were approximately 13 cm in diameter, with height measurements accurate to within 2 cm. A diagram of the configuration of the radar and laser systems in shown in the inset of Figure 1, and a photograph of the mounted system including the radar and laser instruments is shown in Figure 3.

Environmental data was collected using a Vantage Pro weather station mounted on the northwest corner of platform C at a height of 44 m. Measured wind speeds were converted to an equivalent wind speed at 10 m by assuming a logarithmic wind velocity profile and a neutral atmosphere. This adjustment reduces measured wind speed values by approximately 15%.
3. Data Processing

Acquired data in the Y-ONE experiment consists of five minute records, including one minute samples of the local wind and weather conditions, distance measurements from each of the laser rangefinders at 8 Hz, and in-phase and quadrature channels of the C and Ku-band radar systems sampled at 3 kHz. The data was processed to obtain surface profiles and statistics and average EM bias for each five minute record.

3.1. EM Bias Calculations

The EM bias is defined as the normalized correlation between the surface height of the ocean, $\zeta$, and the backscatter coefficient profile, $\sigma(\zeta)$,

$$\epsilon = \frac{E[\sigma^2(\zeta)\zeta]}{E[\sigma^2(\zeta)]}.$$  \(1\)

The EM bias as defined here is one component of the sea state bias in altimeter measurements. The sea state bias is the sum of the EM bias, skewness bias due to the effect of nongaussian surface height distribution in shifting the median of the altimeter return pulse away from the mean, and an instrument-dependent tracker bias. In this paper, we consider only the EM bias. Using sampled measurements of the surface profile and backscattered power, the EM bias is computed from Y-ONE measurements as

$$\epsilon = \sum_i \frac{\sigma^2_i \zeta_i}{\sum_i \sigma^2_i}. \quad (2)$$

Corrections to the measured return power, $\sigma^2_i$, were made to account for changes in the spot size and the spreading.

Figure 1. Diagram of the Brazos-A19 oil platform and the experiment configuration.

Figure 2. View of Brazos-A19 from platform D. The 60 m walkway between platforms B and C where the radar and laser instruments are mounted is shown in the upper part of the photograph.

Figure 3. Deployment of the Y-ONE experiment on the walkway between platforms B and C on Brazos A-19.

Figure 4. Typical surface profile from C-band, Ku-band, and laser rangefinder measurements.
of incident and scattered fields. By combining the $R^2$ relationship of spot size to distance and the $R^{-4}$ dependence of backscattered power, the measured backscatter coefficient profile can be expressed as [Arnold et al., 1995]

$$\sigma^2(\zeta) = \frac{K_e (R_0 - \zeta)^2}{R_e^2} \sigma_{m}^2(\zeta)$$  \hspace{1cm} (3)

where $K_e$ is a calibration constant, $R_0 \approx 18$ m is the mean distance to the ocean surface, and $\zeta$ is the surface profile. Backscattered power measurements were also corrected for a system nonlinearity as detailed in the Appendix.

### 3.2. Data Editing

The Y-ONE data set was edited to eliminate records corrupted by instrument malfunction and spurious data values. The principle cause of data loss was temporary failures of the local oscillators or power supplies in the Ku-band system. Interruptions reduced the number of valid records from 11700 to a total of 9245 usable five minute records at C-band and 4291 records at Ku-band. From these records there are 833 and 389 nadir pointing values for the C-band and Ku-band systems, respectively.

### 3.3. Surface Profile Verification

Ocean surface displacement was measured directly using three laser rangefinders and indirectly using radar Doppler shifts. Due to the proximity of the laser and radar systems, the footprints of the three systems overlapped when the radars were at nadir. This allowed for verification of the indirect profiles obtained from radar measurements.

Integrated Doppler surface profiles were computed from phase angles for the radar measurements. After decimating the signal to 300 Hz, the instantaneous phase angle, $\theta_p$ was computed as

$$\theta_p = \tan^{-1} \left( \frac{Q}{2} \right)$$ \hspace{1cm} (4)

and unwrapped to compute the surface profile, $\zeta$, as

$$\zeta = \frac{\lambda_{em}}{2} \frac{\theta}{2\pi}$$ \hspace{1cm} (5)

where $\lambda_{em}$ is the electromagnetic wavelength. We note that the covariance processing technique used by Arnold et al. [1995] and Melville et al. [2004] in previous tower experiments was also implemented and gave nearly identical results. The surface profiles were low pass filtered to eliminate slow phase drift.

Previous EM bias tower experiments have noted that surface profiles computed from Doppler radar systems tend to underestimate the extrema of ocean surface profiles as a result of averaging over the radar footprint [Arnold et al., 1995]. This effect is most notable near the troughs and crests of the surface. At these times, the velocities of different parts of the footprint have different signs, so that the average velocity across the footprint is nearly zero, and the heights of the surface peaks and troughs are underestimated. A comparison of typical surface profiles created from the radar and laser systems can be seen in Figure 4.

Differences in the Doppler and rangefinder measurements of the surface are apparent when computing statistical properties of the surface, such as the significant wave height,$$H = 4h_t,$$ \hspace{1cm} (6)

where $h_t^2$ is the surface height variance. Using a Thorin infrared wave gauge and Doppler measurements of the surface profile, Arnold et al. [1995] showed that the Doppler measurements underestimated significant wave heights in the GME data set by a constant 10 cm [Arnold et al., 1995]. A very similar relationship can be seen for Y-ONE measurements in Figure 5, where the least-squares linear fit between the significant wave height values from the two different systems is described by

$$H_{\text{Laser}} = 1.03H_{\text{Radar}} + 0.09,$$ \hspace{1cm} (7)

in units of meters.

To compensate for the inaccuracies inherent in the Doppler surface measurements, ocean surface statistics presented in this paper were computed from the laser rangefinder profiles. These include significant wave height, RMS wave slope, and surface skewness. Because the EM bias is strongly dependent on the temporal correlation of power and surface profile, the rangefinder profiles cannot be used to obtain EM bias at off-nadir incidence angles, so C-band integrated Doppler profiles were used in computing the bias. For consistency between the SWH and EM bias computations, the Doppler surface profiles were scaled such that the SWH of the Doppler profiles matches the value given by equation 7. This adjustment had only a small effect on the EM bias.

### 4. Nadir Results

The Y-ONE data set was validated by checking for known properties of the bias at nadir incidence, and by comparison with previous nadir measurement campaigns. [Arnold et al., 1995]. Previously, empirical models have been developed from tower data that describe the EM bias as a function of wind speed, significant wave height, $H$, and RMS wave slope, $S$. Theoretical studies have described the EM bias as a function of higher order moments of the surface statistics, such as tilt-slope cross correlation or tilt modulation, $\lambda_{12}$ [Srokosz, 1986; Rodriguez et al., 1992; Elfouhaily et al., 2000]. In this section, we define the long wave parameters, $S$, $\lambda_{12}$, and $\lambda_{30}$, and compare bias measurements as a function of these long wave statistics to a data set from the Gulf of Mexico Experiment (GME) conducted from the same platform in 1992.

#### 4.1. Long Wave Parameter Definitions

RMS long wave slope is defined using the second moment of the surface height power spectral density,

$$S = \left[ \int_0^{k_{sep}} k^2 W(k) \, dk \right]^{1/2}$$ \hspace{1cm} (8)

![Figure 5. Significant wave height values computed from laser rangefinder profiles are almost a constant 10 cm larger than values computed from measurements made with the C-band radar.](image-url)
The upper limit is a wavenumber cutoff which separates long and short wave surface components. In the experiment data processing, an upper cutoff of $k_{sep} = 2.5$ rad/m is used, to match the cutoff used in the previous GME experiment data processing for consistency. As spatially separated samples of the surface are not available in the tower experiments, the RMS slope must be estimated from the temporal surface spectrum $W_T(\omega)$ of a time series of height measurements at a fixed location. Cox and Munk [1956] give the slope variance in terms of the temporal spectrum as

$$S^2 = \int k^2 W_T(\omega) d\omega$$

where the gravity wave dispersion relation

$$k = \frac{\omega^2}{g},$$

is implied. If the temporal spectrum is estimated from time samples of the surface profile using a common DFT-based periodogram estimator, the slope variance is obtained by numerical integration of (9) as

$$S^2 \simeq 2 \sum_{n=1}^{N} k_n^2 \left[ \frac{T_s}{2\pi} W_{T,\text{DFT},n} \right] \Delta\omega_n$$

where $T_s$ is the sampling period and $\Delta\omega_n$ is the spacing between adjacent values of $\omega_n$.

The long wave skewness parameter is defined as the normalized third central moment of the surface profile,

$$\lambda_{30} = \frac{E[c^3]}{E[c^2]^{3/2}},$$

where $c$ is the long wave surface profile. The long wave tilt modulation

$$\lambda_{12} = \frac{E[\xi c]}{E[c^2]^{1/2} E[\xi^2]}$$

measures the correlation between long wave displacement and long wave tilt angle. Due to relative drift between the internal clocks of the laser rangefinders, direct estimates of the tilt modulation are not available for the Y-ONE data set.
4.2. EM Bias at Nadir

Values of the EM bias ranged from −13.2 cm to 0.0 cm for the C-band system and from −8.4 cm to −0.1 cm at Ku-band over the course of the Y-ONE experiment. In Figure 6 the strong linear correlation with significant wave height is clearly seen. A least squares fit of the EM bias to the significant wave height for Y-ONE is

\[
\epsilon_C(\text{cm}) = -4.41H + 1.67 \quad (14)
\]

\[
\epsilon_Ku(\text{cm}) = -2.79H + 0.39 \quad (15)
\]

where the subscripts \(C\) and \(Ku\) refer to the frequency of the scatterometer. Values of the normalized bias, defined as

\[
\beta = \frac{\epsilon}{H}, \quad (16)
\]

ranged from −5.6% to 0.3% of the \(H\) at 5.2 GHz and from −3.9% to −0.2% of \(H\) at 14 GHz.

Empirical relationships between the RMS long wave slope and relative bias have been investigated in a number of in situ and laboratory experiments [Millet et al., 2003; Gommenginger et al., 2003; Melville et al., 2004]. Figure 7 shows the relationship between \(\beta\) and \(S\) from the Y-ONE data where the linear correlation reported in previous experiments can be clearly seen. The empirical relationship is described by

\[
\beta_C(\%H) = -28.19S + 0.03 \quad (17)
\]

\[
\beta_Ku(\%H) = -10.98S - 1.21 \quad (18)
\]

Values of the RMS slope ranged from .04 to .16 over the course of the experiment, with a mean value of 0.09.

While the height skewness \(\lambda_3\) is not a direct influence on the EM bias, simple hydrodynamic models predict a linear relationship between skewness and tilt modulation \(\lambda_{12}\) Jackson [1979]. Because direct measurements of \(\lambda_{12}\) are not available, we show relative bias as a function of long wave skewness in Figure 8. A negative correlation between \(\lambda_3\) and \(\beta\) is evident. The skewness varied from −0.8 to 0.7. One reason for the negative measured long wave skewness may be that the skewness is computed from long wave surface profiles filtered to wavelengths on the order of one meter or larger, and much of the total surface height skewness may be due to small, centimeter-scale waves [Plant, 2003]. In connection with this, it is important to note that it is incorrect to include the total short and long wave surface nonlinearity in an EM bias study without distinguishing between the two components, because the physics of microwave scattering dictates that the effect of nonlinearity in long waves on measured altimeter bias is very different from the effect of short wave skewness.

4.3. GME and Y-ONE Comparison

The GME and Y-ONE data sets were collected from the same location, allowing for a comparison of EM bias and surface statistics. With the Y-ONE experiment deployed during March and April, 2003, and the GME experiment deployed from December to May, 1991-1992, the data also has an overlap with respect to the time of year.

Because the profile statistics from the Y-ONE data are computed using the laser rangefinders rather than integrated radar Doppler profiles, they are not directly comparable to the GME data. To make a direct comparison between bias measurements from the two experiments, values of the significant wave height and RMS slope for the GME experiment must be adjusted. For the significant wave height, the relationship between laser and radar data was addressed in equation (7). Using Y-ONE data, radar and laser measurements of the RMS slope are related by

\[
S_{\text{Laser}} = 1.02S_{\text{Radar}} + 0.024. \quad (19)
\]

A plot shown the relationship is shown in Figure 9.

Using these relationships in equation (7) and equation (19) to modify the GME values of \(S\) and \(H\) allows a direct comparison of the bias as a function of the significant wave height, seen in Figure 10. The relationship of \(\epsilon\) and \(H\) has a similar magnitude and slope in both data sets. The relationship of the relative bias with RMS long wave slope for both data sets is shown in Figure 11.

5. Off-Nadir Bias Model

From the definition in equation (1), the EM bias can be written as

\[
\epsilon = \int \int \sigma^2(\zeta, \theta + \Theta)P(\zeta, \theta) d\zeta d\theta / \int \sigma^2(\zeta, \theta + \Theta)P(\zeta, \theta) d\zeta d\theta, \quad (20)
\]
where \( \theta \) is the incidence angle relative to the mean surface \( \zeta = 0 \) and \( \theta_0 \) is the local long wave tilt angle. This formulation describes the bias due to scattering from long wave facets roughened by small waves, where the joint long wave height-tilt distribution is \( P(\zeta, \theta) \). The facets are much larger than the electromagnetic wavelength, so that their size scale is \( O(1 \text{ m}) \). Multiple scattering between long wave facets is neglected in (20), meaning that this expression is based on a geometrical optics approximation in combining fields scattered from multiple facets, similar to the standard composite model for ocean scattering [Barrick and Peake, 1968; Fang and Chan, 1969; Brown, 1978]. In order to obtain the correct incidence angle dependence of the bias, however, the scattering from small waves on individual facets cannot be modeled using geometrical optics, because the surface roughness on each facet is on the order of the electromagnetic wavelength in size. Because the incidence angle dependence of the EM bias is determined by the behavior of scattering from small waves, previous EM bias models such as those developed by Jackson [1979]; Srokosz [1986]; Elfouhaily et al. [2000, 2001] that are based completely on geometrical optics assumptions for both long and small wave scattering cannot be generalized to off-nadir incidence angles. In [Millet and Warnick, 2004], it has been demonstrated rigorously that the physical optics approximation is accurate for scattering from ocean-like surfaces with height spectra of power-law form. For these reasons, in determining the backscattering profile \( \sigma^2(\zeta, \theta) \), we employ the physical optics model to obtain the scattered fields from individual facets with small wave roughness.

Using weakly nonlinear theory [Longuet-Higgins, 1963], the joint height-slope distribution of the long waves can be described as a Gram-Charlier series [Srokosz, 1986; Elfouhaily et al., 2000],

\[
P(\zeta, \eta_0) = \frac{e^{-\frac{1}{2}(\eta_0^2 + \zeta^2)}}{2\pi h_{1/2}} \times \left[ 1 + \frac{\lambda_{30}}{6} H_{30}(\eta, \eta_0) + \frac{\lambda_{12}}{2} H_{12}(\eta, \eta_0) \right].
\]

(21)

To simplify notation, the PDF is expressed in terms of the normalized height, \( \eta = \zeta/h_1 \), and the normalized surface slope, \( \eta_s = \zeta_s/n_s \), where \( h_1^2 \) and \( s_1^2 \) are surface height variance and surface slope variance, respectively. The symbols \( H_{30}(\eta, \eta_0) \) and \( H_{12}(\eta, \eta_0) \) refer to Hermite polynomials defined by

\[
H_{30}(\eta, \eta_0) = \eta^3 - 3\eta \\
H_{12}(\eta, \eta_0) = \eta(\eta_0^2 - 1).
\]

(22)

(23)

A more sophisticated multidimensional surface PDF could be used, but this model suffices in allowing us to obtain the leading order incidence angle dependence of the EM bias. Moreover, this distribution is used to represent long waves, which tend to be more directional than short waves, so the corrugated model for the long waves is realistic. Short waves are modeled using a two-dimensional distribution.

Scattering by the short ocean waves is modeled using the physical optics approximation

\[
\sigma^2(\psi) = k_{em}^2 \cos^2 \psi \int \int e^{ik_h \cdot r} e^{-\lambda(1-C(x,y))} dxdy \]

(24)

where \( \psi \) is the local incidence angle of the illuminating electromagnetic field, \( k_h = 2k_{em} \sin \psi \) and \( \lambda = (2k_{em} h \cos \psi)^2 \).

The correlation function

\[
C(x,y) = \frac{1}{h_2^2} \int W(k) e^{i k r} dk.
\]

(25)

is the Fourier transform of the isotropic, normalized short wave PSD and \( h_2^2 \) is the small wave height variance.

To include hydrodynamic modulation in the nadir EM bias model, we employ a modulation transfer function (MTF) to relate modulation of small waves amplitudes to the long wave statistics. Physically, the hydrodynamic modulation strength is described as the normalized correlation between the short wave height variance and surface displacement. Using a result of Rodriguez et al. [1992], this correlation can be computed directly from the MTF. The same MTF as in [Rodriguez et al., 1992] is used. Figure 12 shows the resulting values of the modulation strength versus the RMS long wave slope of the surface. These results are obtained numerically by modeling the short and long waves as independent power law spectra. The separation of long and short wave surface spectra allows the model to capture environmental conditions that vary widely from average values described by a surface spectral model.

Rodriguez et al. [1992] also show that the dependence of hydrodynamic modulation on long wave displacement is nearly linear. This is in agreement with experimental measurements such as that of Arnold [1992]. Combining this linear dependence with the MTF results in Figure 12 leads to the model

\[
h_s(\eta) = h_o (1 + \nu S \eta),
\]

(26)

where \( h_o \) is the average short wave height and \( \nu \approx 0.7 \). The connection between short wave modulation strength and RMS slope was first identified by Melville et al. [2004], based on the theory of Longuet-Higgins and Stewart [1960]. By using this relationship and substituting equation (26) into the PO scattering model, (24), the backscatter profile includes the hydrodynamic modulation in the EM bias model as a function of the RMS slope.

The EM bias model is obtained by substituting the joint height slope PDF from equation (21) and the PO approximation from equation (24) into the EM bias definition (20) and integrating over long wave slope. This procedure leads to

\[
\epsilon(\theta) = -H \left[ \gamma(\theta) \nu S + \tau(\theta) \lambda_{12} \right].
\]

(27)
In this expression,
\[
\gamma(\theta) = \frac{1}{2} \int \int \lambda_0(1-C) e^{(2k_{cem} \mu_\theta^2) e^{-\lambda_0(1-C) e^{-\mu^2/2}} dr dy} \\
\tau(\theta) = \frac{1}{8} \int \int e^{(2k_{cem} \mu_\theta^2) e^{-\lambda_0(1-C) e^{-\mu^2/2}} dr dy}
\]
where \( \mu = 2k_{cem}s_1, \lambda_0 = (2k_{cem}h_0)^2, \) and the coordinate dependence of the correlation function is suppressed. The coefficients \( \gamma \) and \( \tau \) contain the incidence angle dependence of the model.

It is important to note that while this bias model depends on a separation wavenumber between long and short waves, this cannot be viewed as a disadvantage relative to previous bias theories. Models such as those of [Jackson, 1979; Srokosz, 1986; Elfouhaily et al., 2000, 2001] that are based on the geometrical optics approximation inherently treat all wave scales as though they were much longer than the electromagnetic wavelength. The use of a separation wavenumber in the present model allows for more accurate treatment of the scattering from small waves on the order of the electromagnetic wavelength in size.

5.1. Model Analysis

In equation (27), the EM bias is described as a function of long wave surface statistics, \( S \) and \( \lambda_{12} \), modified by the small wave coefficients \( \gamma \) and \( \tau \). The small wave coefficients are determined in large part by scattering from the small ocean waves. In order to compute the small wave coefficients, we model the small waves with a surface height spectrum of power law form. We fix the value of the separation wavenumber at \( k_{sep} = 2.5 \text{rad/m} \), as used above.

![Angular dependence of \( \gamma \)](image1)

**Figure 13.** Dependence of the hydrodynamic modulation coefficient \( \gamma \) on incidence angle for \( k_{cem} = 100 \) and minimum wavenumber of \( k_{min} = 2\pi \text{rad/m} \). For a separation wave number of \( k_{sep} = 2\pi \text{rad/m} \), the typical value of \( h_0 \) for an ocean surface is .01 m.

![Angular dependence of \( \tau \)](image2)

**Figure 14.** Dependence of the tilt modulation coefficient \( \tau \) on incidence angle for \( k_{cem} = 100 \).

![Power/Surface Height Cross-Correlation](image3)

**Figure 15.** Correlation of backscattered power and surface height at 0° and 17° incidence angles. The temporal shift in the minimum correlation point at 17° causes a decrease in the magnitude of the EM bias values.

![Normalized Bias vs. Incidence Angle](image4)

**Figure 16.** Measured and estimated relative bias values at off-nadir incidence angles. Values of the RMS slope and tilt modulation used in the theoretical estimates are shown in the legend. Bars of one standard deviation indicate the extent of the measured EM bias values from the Y-ONE data set. An empirical fit to the C-band data described by \( \beta(\theta) = \beta(0) \cos(\theta \pi/\theta_0) \) is also shown, where \( \theta_0 \) is estimated at 15°.
in computing long wave statistics. As discussed above, the rough surface scattering model dictates that the separation wavelength be $O(1 \text{ m})$, and numerical studies show that the impact of small variations in the separation wavenumber around the selected value do not change the modeled bias significantly. With these parameters, the incidence angle dependence of $\gamma$ is shown in Figure 13 for various values of $h_s$. For the same short wave PSDs, the angular dependence of $\tau$ is shown in Figure 14. The RMS long wave slope is taken to be $S = 0.07$.

Both the hydrodynamic modulation and the tilt modulation contributions to the EM bias decrease in magnitude and eventually reverse in sign as the incidence angle increases away from nadir. The angular dependence of $\gamma$, caused by differences in surface roughness at the crests and troughs of long waves, creates the hydrodynamic bias term in equation (27). At large incidence angles the more Lambertian scattering from the wave crests leads to a larger return from the crests than from the troughs, reversing the sign of the bias. This transition from positive to negative values can be seen in Figure 13.

Changes in the tilt modulation bias with angle are caused by a larger concentration of horizontal scattering facets near the troughs than the crests of ocean waves. This results in a larger specular return for nadir pointing instruments. With increasing incidence angle, the steeper crests create a larger backscatter return than the flatter troughs, and the sign of the bias is reversed. It is also of interest to note that with larger small wave heights the angular dependence of the tilt modulation is greatly reduced. This effect results from a more Lambertian EM scattering pattern for larger small waves. For large enough waves the EM scattering becomes so diffuse that the tilt modulation bias vanishes.

6. Off-Nadir Measurements

In this section we study the angular dependence of the EM bias observed in the Y-ONE data set and the off-nadir bias model. From equation (1), the bias is caused by the negative correlation between the backscattered power and surface displacement. This relationship is shown in the upper axis in Figure 15, where the cross-correlation function $\rho$ and $\zeta$ is shown for a typical nadir-pointing Y-ONE data record. At off-nadir incidence angles, a time shift between $\sigma^2$ and $\zeta$ is introduced. This shift can be seen in the lower axis of Figure 15, where the maximum correlation between $\sigma^2$ and $\zeta$ is near 1 sec. The result is an effective decorrelation that reduces the magnitude of the bias for small off-nadir incidence angles. With larger incidence angles, the time shift increases, until the backscattered power and surface displacement are in phase, resulting in a positive correlation and positive bias values.

6.1. Theoretical Estimates

In the development of the off-nadir bias theory, the angular dependence was shown to be a function of the small wave height, $h_s$, through the bias coefficients, $\gamma$ and $\tau$. Examples of the angular dependence of the bias are shown in Figure 16 for different values of $S$ and $\lambda_{12}$. The bias coefficients, $\gamma$ and $\tau$, were computed with a short wave standard deviation of $h_s = .02 \text{ m}$ as a typical value for the ocean surface.

Because $\gamma$ and $\tau$ are primarily dependent on the small wave surface height, the constant value of $h_s$ leads to curves with identical zero-bias intercept angles. This effect can be seen in Figure 16 where the curves show $\beta = 0$ near $\theta = 15^\circ$. For different small wave conditions, the intersect angle can change such that larger values of $h_s$ result in a larger zero-bias intersect angle.

6.2. Y-ONE Measurements

Average relative bias measurements as a function of angle are shown in Figure 16 with error bars indicating one standard deviation of the data in each incidence angle bin. Measured bias values increase from a minimum value at $\theta = 0$, to $\beta \approx 0$ near $15^\circ$. For incidence angle larger than $15^\circ$, the average bias value at C-band is positive. At Ku-band, the average zero-bias intersect angle is approximately $17^\circ$.

To express the average relationship between the bias and incidence angle in a simple form, an empirical fit can be obtained from the Y-ONE measurements. Using the value of the bias at nadir, $\beta(0)$, and the angle $\theta_o$ at which $\beta = 0$, a cosine curve can be fit to the average bias values, such that that

$$\beta(\theta) \approx \beta(0) \cos \left( \frac{\pi \theta}{2 \theta_o} \right)$$

for $\theta \leq 17^\circ$. Using average values from the C-band data, $\beta(0) = -2.58$ and $\theta_o = 15^\circ$, an empirical fit to the Y-ONE data is shown in Figure 16. A similar curve for the Ku-band data with $\beta(0) = -2.28$ and $\theta_o = 17^\circ$ is not shown.

It appears that the angle $\theta_o$ at which the bias vanishes does not correlate significantly with measured environmental parameters. The zero-bias intersect angle does have a weak dependence on the relative bias at nadir, as shown in Figure 17. The relationship between the zero-bias intersect angle, $\theta_o$, and $\beta(0)$ can be described by

$$\theta_o = -2.16\beta(0) + 9.81$$

for the C-band bias. At Ku-band, no significant correlation is apparent, although the correlation may be obscured by the smaller data set and increased noise in the 14 GHz measurements.

7. Summary

This paper presents off-nadir EM bias measurements from the BYU Off-Nadir Experiment (Y-ONE) conducted in the Gulf of Mexico in 2003. Using C-band and Ku-band radar systems and laser altimeters, the EM bias was calculated directly from ocean surface profiles and backscattered energy returns. Y-ONE measurements were compared with data...
from a previous experiment conducted from the same platform. The comparison showed similar values for the bias and surface statistics from the two data sets, including the correlation of the bias with significant wave height and RMS slope.

To provide a theoretical prediction for the angular dependence of the bias, an off-nadir bias model is developed using a nonlinear ocean surface description and the physical optics EM scattering approximation. The resulting model describes the bias in terms of long wave statistics scaled by coefficients that depend primarily on EM scattering from small ocean waves. The angular dependence of the backscattered power from small wave patches leads to a decrease in magnitude of the bias and eventual sign change as a function of incidence angle. Using measurements from the Y-ONE experiment, the angular dependence of the bias is also shown to be related to a time shift between the surface profile and backscattered power. The resulting time shift causes the magnitude of the bias to decrease at small incidence angles.

Estimated bias values are calculated using the off-nadir bias theory and showed good agreement with measurements from the Y-ONE data set. The magnitude of estimated and measured values of the bias were shown to decrease with incidence angles up to approximately $\theta = 15^\circ$, with increasing positive values for larger incidence angles. An empirical fit of the form $\beta(\theta) = \beta(0) \cos(\theta \pi/\theta_2 2)$, where $\theta_2 = 15^\circ$ (C-band) and $\theta_2 = 17^\circ$ (Ku-band), was developed from the measured Y-ONE bias values.

Future work on this topic includes the investigation of methods to use the angular dependence of the bias to improve operational EM bias corrections for future satellite altimeter missions.

8. Acknowledgements

We express appreciation to Shell Exploration and Production, Inc. and its staff for generously donating platform space and logistical support for the BYU Off-Nadir Experiment.

9. Appendix: Power Corrections

![Figure 18. Typical power values for a 5 minute record are plotted against surface displacement for nadir measurements of the surface. The upper limit on the higher power values is likely a result of the saturation of the amplifiers in the receiver. Power levels were adjusted to restore the expected distribution in Figure 18. The white ellipse indicates a typical distribution pattern of the power.](image)

During the acquisition process of the Y-ONE experiment, the backscattered power measurements of many five minute records were distorted, most likely by a nonlinear component in the receiver chain. An example of the recorded power measurements can be seen in Figure 18, in black, where a cutoff for large power signals can be seen near 3 dB. This soft limit was likely caused by saturation of an amplifier in the radar instruments. Because the saturation characteristics of an amplifier are one-to-one, the nonlinearity introduced into the power measurements can be inverted. Inverting the saturation effects was done by fitting the average power histogram at nadir from the Y-ONE data to a similar histogram from the Gulf of Mexico (GME) data set, a previous tower experiment conducted at the same location [Arnold et al., 1995]. Using a simple polynomial fit the power histogram was made to approximate the Rayleigh distribution observed in the GME experiment. The polynomial used to remove the saturation effects is a power series expansion about the peak of the distribution, $p_o = 0.8$ dB on the scale of Figure 18, such that

$$\sigma^o = \begin{cases} 
\sigma^o (1 + 0.45 \delta_p^3 - 0.15 \delta_p^4 + 0.04 \delta_p^5) & p \leq p_o \\
\sigma^o \delta_p^3 & p > p_o 
\end{cases}$$

where $\sigma^o$ is the recorded Y-ONE power measurement and $\delta_p$ is the expansion term ($\sigma^o - p_o$). Corrected and uncorrected power histograms for the Y-ONE data set are shown in Figure 19 with a similar plot from the GME data set. Applying the correction factor, restores the linear dependence of the backscattered power on surface displacement as seen in Figure 18, where the corrected values are shown in gray.
Two important points should be noted about the power correction. First, this nonlinear power scaling is displacement independent, and therefore does not add any displacement-dependent information to the data set. Rather, it merely makes a corrective increase to the effective power correction factor in equation (32), the corrected data had a distribution similar to that seen in the GME data set.

Figure 19. Histograms of backscattered power values. The clipping effect can be seen in the original Y-ONE data as an upper limit of power values. By applying the correction factor in equation (32), the corrected data had a distribution similar to that seen in the GME data set.

References

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