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International Environmental Modelling and Software Society (iEMSs)
2010 International Congress on Environmental Modelling and Software
Modelling for Environment's Sake, Fifth Biennial Meeting, Ottawa, Canada
David A. Swayne, Wanhong Yang, A. A. Voinov, A. Rizzoli, T. Filatova (Eds.)
http://www.iemss.org/iemss2010/index.php?n=Main.Proceedings

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Abstract: The paper suggested in the context of the general mathematical model of forest fires gives a mathematical setting and method of numerical solution of a problem of a large forest fire initiation. The mathematical model a large forest fire is based on an analysis of known experimental data and using concept and methods from reactive media mechanics. The boundary-value problem is solved numerically using the method of splitting according to physical processes. Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained numerically. It allows investigating dynamics of forest fire initiation under influence of various external conditions. Results of numerical calculations are given which imply that the ignition mechanism in this case is the same as for collision catastrophes. A comparison of the limiting dimensions of ignition zones shows good agreement with experimental data.

***Keywords**:* mathematical model, forest fire, discrete analogue, control volume, radiation, ignition.

1. INTRODUCTION

It is established that large technogenic or space catastrophes are, as a rule, accompanied by the occurrence of large-scale forest fires. Since full-scale studies of these problems are impossible, methods of mathematical simulation become very important. The problem was studied within the framework of a three-dimensional formulation and to consider the initial stage of impact of a high-altitude radiant energy source on the Earth's underlying surface covered with sylvan vegetation. The aim of this study is to determine the ignition time and dimensions of the ignition zone and explore the mechanism of the physicochemical processes involved in ignition. A great deal of work has been done on the theoretical problem of how forest fire initiation. For example, the mathematical model for description of large forest fire initiation in the location of Tunguska celestial body fall was given by Grishin et al. [1993]. Later, Grishin et al. [1996] used a two dimensional model to study the problem of initiation of large-scale forest fires induced by technogenic catastrophes. These models have been limited by assumptions of one dimensional, slab or axisymmetric geometry. This work extends the above studies by using two temperatures approach and three dimensional cases. These results calculation of limiting dimensions of ignition zones have been compared to observations.

2. PHYSICAL AND MATHEMATICAL MODEL

It is known that in the case of entering of body in atmosphere with supersonic the powerful ballistic shock wave is arose at the around stagnation point and the gas temperature has high value Goldin [1992]. As a result of this the sublimation of celestial body matter is took place and temperature tension is arose. Therefore the celestial body is destroyed in Earth atmosphere or its remains are fell with formation of crater. During the celestial body flying a fraction of its kinetic energy transformed into radiation and the heating of Earth surface and forest phytocenoces are took place. As a rule, the sizes of celestial bodies are small as compare with radius of Earth and thickness of over terrestrial layer, it may be considered to be a point source of radiation Grishin et al. [1993] and Perminov [1995]. It is supposed, that the celestial body is destroyed as a result of explosion in Earth atmosphere. Let the radiant energy source be at a height of H from the Earth surface at the initial moment (Fig.1). Where R_0 - the distance from source of radiation to vegetation cover, h - the height of forest massif, O - epicenter of explosion - is the center of the Cartesian system of co-ordinates.

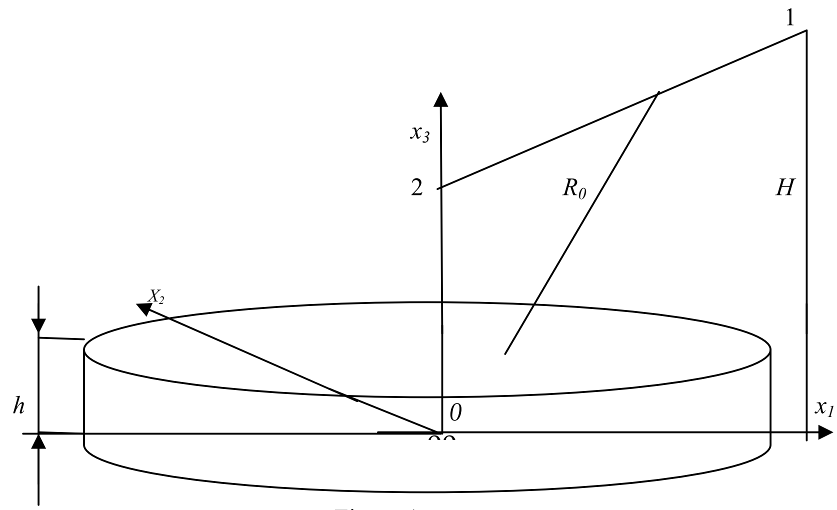

Figure 1.

An upper boundary $x_3=h$ of the forest massif is acted upon by an intensive radiant flux $q_R(R_0,t)$, which defined at the flight stage from Goldin [1992] and Perminov [1995].

$$q_R(R_0,t) = \frac{t_p I Sin L}{4\pi R_0^2},\ I = 0.5 C_H \rho V^2 S_m, \tag{1}$$

where I, V, S_m – the brightness, velocity and midship section square of Tunguska fireball, C_H - the fraction of kinetic energy transformed into radiation; L - angle between radiative heat flux and vegetation cover, ρ - density of atmosphere at a height H. After explosion of celestial body (at moment $t=t_1$) the light flux is defined according to the data Glastone, S. [1962].

$$q_R(R_0,t) = \frac{t_p P_m \sin L}{4\pi R_0^2}\begin{cases}(t-t_1)/t_m, & ,t<t_m\\ \exp(-k_0((t-t_1)/t_m-1)), & t\ge t_m\end{cases}, \tag{2}$$
$$t_m = t_1 + 0.032 W_0^{0.5}, P_m = 1.33 W_0^{0.5}.$$

R_0 - the distance from the source of radiation to forest; t_p - atmospheric transmissivity coefficient; P_m - maximum value of heat radiative impulse at moment $t=t_m$; W_0 - yield, kT/sec; k_0 -empirical coefficient. When the radiant energy reaches vegetation cover, it causes heating forest fuels, evaporations of moisture and subsequent thermal decomposition of solid material, with evaporating pyrolysis products liberation. The last material is burning in the atmosphere and interacting with the oxygen of air. The forest canopy is considered as a homogeneous, two-temperatures, reacting, non-deformed medium Perminov [1995]. Temperatures of condensed (solid) T_s and gaseous T phases are separated out. The first includes a dry organic substance, moisture, condensed pyrolysis products and mineral part of forest fuels. In the gaseous phase we separate out only the components C_α necessary to describe reactions of combustion (α=1 - oxygen, 2 - pyrolysis combustion products of forest fuels (CO and etc.) and the rest components). The solid phase constituting forest fuels has no intrinsic velocity, and its volumetric fraction, as compared to the gaseous phase, can be neglected in appropriate equations. Radiation is the governing mechanism of the energy transfer in this case. The solid phase mainly absorbs, reflects and reradiates. Diffusion approximation is used to describe the transfer in this specific continuous medium. The system of equations for the celestial body is:

$$\frac{dV}{dH} = \frac{C_x \rho V S_T}{2mSin\alpha_0} - \frac{g}{V}; \tag{3}$$

$$\frac{dm}{dH} = 6\frac{C_x \rho V^2 S_T}{Sin\alpha_0} \tag{4}$$

$$\frac{d\alpha}{dH} = \frac{C_Y \rho S_T}{2mSin\alpha_0} + \left(\frac{1}{R_z} - \frac{g}{V^2}\right)ctg\alpha_0, \tag{5}$$

$$\frac{dt}{dH} = -\frac{1}{VSin\alpha_0}, \ \frac{d\ell}{dH} = -\frac{1}{Sin\alpha_0}, \tag{6}$$

$$S_m = \pi R_T^2, R_T = \left(\frac{3m}{4\pi\rho_T}\right)^{\frac{1}{3}}, \sigma_0 = \frac{2Q}{C_x}. \tag{7}$$

where m, R_T , ρ_T- - mass, radius and density of celestial body, C_x, C_y - coefficients of drag and lifting, t - time, α - angle of trajectory inclination, l - the distance along of trajectory, g - constant acceleration, R_z - Earth radius, σ_0 - ablation coefficient, Λ - heat transfer coefficient, Q - celestial body specific energy of ablation. Setting the initial point of considering trajectory H=60 km, the height of explosion - 6.5 km, $\alpha_0=40^0$.The initial values of velocity and mass of celestial body are set up according to the data Goldin [1992]. To describe convective transfer controlled by the wind and gravity in forest canopy, we use Reynolds equations for the description of turbulent flow taking into account diffusion equations for chemical components and equations of energy conservation for gaseous and condensed phases. For the objective of the present studies, wind (velocity) speed was considered to be relatively not high, and the energy was considered mainly to be transferred due to radiation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = \dot{m}, \ j = 1,2,3, \ i = 1,2,3; \tag{8}$$

$$\rho\frac{dv_i}{dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}(-\rho\overline{v_i' v_j'}) - \rho s c_d v_i |\vec{v}| - g_i - \dot{m}v_i; \tag{9}$$

$$\rho c_p\frac{dT}{dt} = \frac{\partial}{\partial x_j}(-\rho c_p\overline{v_j' T'}) + q_5 R_5 - \alpha_v(T - T_s) + k_g(cU_R - 4\sigma T^4); \tag{10}$$

$$\rho\frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j}(-\rho\overline{v_j' c_\alpha'}) + R_{5\alpha} - \dot{m}c_\alpha, \ \alpha = 1,5; \tag{11}$$

$$\frac{\partial}{\partial x_j}\left(\frac{c}{3k}\frac{\partial U_R}{\partial x_j}\right) - kcU_R + 4k_S\sigma T_S^4 + 4k_g\sigma T^4 = 0,$$
$$k = k_g + k_S; \tag{12}$$

$$\sum_{i=1}^{4}\rho_i c_{pi}\varphi_i\frac{\partial T_S}{\partial t} = q_3 R_3 - q_2 R_2 + k_s(cU_R - 4\sigma T_S^4) + \alpha_V(T - T_S); \tag{13}$$

$$\rho_1\frac{\partial \varphi_1}{\partial t} = -R_1, \rho_2\frac{\partial \varphi_2}{\partial t} = -R_2, \rho_3\frac{\partial \varphi_3}{\partial t} = \alpha_C R_1 - \frac{M_C}{M_1}R_3, \ \rho_4\frac{\partial \varphi_4}{\partial t} = 0; \tag{14}$$

$$\sum_{\alpha=1}^{5} c_\alpha = 1, \ p_e = \rho RT\sum_{\alpha=1}^{5}\frac{c_\alpha}{M_\alpha}, \vec{v} = (v_1, v_2, v_3), \vec{g} = (0,0,g)$$

$$\dot{m} = (1 - \alpha_c)R_1 + R_2 + \frac{M_c}{M_1}R_3 + R_{54} + R_{55}.$$

Here and above $\frac{\mathrm{d}}{\mathrm{d}t}$ is the symbol of the total (substantial) derivative; α_V is the coefficient of phase exchange; ρ - density of gas – dispersed phase, t is time; v_i - the velocity components; T, T_S, - temperatures of gas and solid phases, U_R - density of radiation energy, k - coefficient of radiation attenuation, P - pressure; c_p – constant pressure specific heat of the gas phase, c_{pi}, ρ_i, φ_i – specific heat, density and volume of fraction of condensed phase (1 – dry organic substance, 2 – moisture, 3 – condensed pyrolysis products, 4 – mineral part of forest fuel), R_i – the mass rates of chemical reactions, q_i – thermal effects of chemical reactions; k_g , k_S - radiation absorption coefficients for gas and condensed phases; T_e - the ambient temperature; c_α - mass concentrations of α - component of gas - dispersed medium, index α=1,2,...,5, where 1 corresponds to the density of oxygen, 2 - to carbon monoxide CO, 3 - to carbon dioxide and inert components of air, 4 - to particles of black, 5 - to particles of smoke; R – universal gas constant; M_α , M_C, and M molecular mass of α - components of the gas phase, carbon and air mixture; g is the gravity acceleration; c_d is an empirical coefficient of the resistance of the vegetation, s is the specific surface of the forest fuel in the given forest stratum, v_g – mass fraction of gas combustible products of pyrolysis, α_4 and α_5 – empirical constants. To define source terms that characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase, the following formulae were used for the rate of formulation of the gas-dispersed mixture $\dot{m}$, outflow of oxygen R_{51}, changing carbon monoxide R_{52}, generation of black R_{54} and smoke particles R_{55}.

$$R_{51} = -R_3 - \frac{M_1}{2M_2} R_5,\, R_{52} = \nu_g (1-\alpha_c) R_1 - R_5,\, R_{53} = 0,\, R_{54} = \alpha_4 R_1,$$
$$R_{55} = \frac{\alpha_5 v_3}{v_3 + v_{3*}} R_3.$$

Reaction rates of these various contributions (pyrolysis, evaporation, combustion of coke and volatile combustible products of pyrolysis) are approximated by Arrhenius laws whose parameters (pre-exponential constant k_i and activation energy E_i) are evaluated using data for mathematical models Perminov [1995].

$$R_1 = k_1 \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT_s}\right),\, R_2 = k_2 \rho_2 \varphi_2 T_s^{-0.5} \exp\left(-\frac{E_2}{RT_s}\right),$$
$$R_3 = k_3 \rho \varphi_3 s_\sigma c_1 \exp\left(-\frac{E_3}{RT_s}\right),$$
$$R_5 = k_5 M_2 \left(\frac{c_1 M}{M_1}\right)^{0.25} \frac{c_2 M}{M_2} T^{-2.25} \exp\left(-\frac{E_5}{RT}\right).$$

Coefficients of multiphase (gas and solid phase) heat and mass exchange are defined $\alpha_V = \alpha S - \gamma C_P \dot{m}, \quad S = 4\varphi_S / d_S$. Here $\alpha = Nu\lambda/d_S$ – coefficient of heat exchange for sample of forest combustible material (for example needle), Nu – Nusselt number for cylinder, λ – coefficient of heat conductivity for pine needle; γ – parameter, which characterize relation between molecular masses of ambient and inflow gases. The system of equations (8)–(14) must be solved taking into account the initial and boundary conditions:

$$t = 0: v_1 = 0, v_2 = 0, v_3 = 0, T = T_e, c_\alpha = c_{\alpha e}, T_s = T_e, \varphi_1 = \varphi_{1e}; \quad (15)$$

$$x_1 = 0: v_1 = V_e, v_2 = 0,\, v_3 = 0, T = T_e, c_\alpha = c_{\alpha e},$$
$$-\frac{c}{3k}\frac{\partial U_R}{\partial x_1} + cU_R/2 = 0; \quad (16)$$

$$x_1 = x_{1e}: \frac{\partial v_1}{\partial x_1} = 0,\, \frac{\partial v_2}{\partial x_1} = 0, \frac{\partial v_3}{\partial x_1} = 0,\, \frac{\partial c_\alpha}{\partial x_1} = 0, \frac{\partial T}{\partial x_1} = 0, \quad (17)$$
$$\frac{c}{3k}\frac{\partial U_R}{\partial x_1} + \frac{c}{2} U_R = 0\, ;$$

$$x_2 = x_{20}: \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \tag{18}$$

$$-\frac{c}{3k}\frac{\partial U_R}{\partial x_1} + cU_R/2 = 0;$$

$$x_2 = x_{2e}: \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \tag{19}$$

$$\frac{c}{3k}\frac{\partial U_R}{\partial x_2} + \frac{c}{2}U_R = 0;$$

$$x_3 = 0: \frac{\partial v_1}{\partial x_3} = 0, \frac{\partial v_2}{\partial x_3} = 0, \frac{\partial v_3}{\partial x_3} = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, \frac{\partial T}{\partial x_3} = 0, \tag{20}$$

$$\frac{c}{3k}\frac{\partial U_R}{\partial x_3} + \frac{c}{2}U_R = 0;$$

$$x_3 = x_{3e}: \frac{\partial v_1}{\partial x_3} = 0, \frac{\partial v_2}{\partial x_3} = 0, \frac{\partial v_3}{\partial x_3} = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, \frac{\partial T}{\partial x_3} = 0, \tag{21}$$

$$\frac{c}{3k}\frac{\partial U_R}{\partial x_3} + \frac{c}{2}U_R = 2q_R.$$

It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is "grey"), and the so-called diffusion approximation for radiation flux density were used for a mathematical description of radiation transport during forest fires. The components of the tensor of turbulent stresses, as well as the turbulent fluxes of heat and mass are written in terms of the gradients of the average flow properties Perminov [1995]. It should be noted that this system of equations describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy, the forest canopy and the space above it, while the appropriate components of the data base are used to calculate the specific properties of the various forest strata and the near-ground layer of atmosphere. This approach substantially simplifies the technology of solving problems of predicting the state of the medium in the fire zone numerically. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different type of forest; for example, pine forest (Grishin at al. [1996] and Perminov [1995]).

3. NUMERICAL METHOD AND RESULTS

The boundary-value problem (8)–(21) solve numerically using the method of splitting according to physical processes Perminov [1995]. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting was then integrated. To conduct numerical integration, system of equation (8)-(14) was reduced, subject to the initial and boundary conditions (15)-(21), to discrete form using the Patankar-Spaulding method of control volume (Patankar [1981]). The grid equations resulting from the discretization were solved by the SIP method (Perminov [1995]). Correlation of the velocity and pressure fields was performed iteratively by the SIMPLE algorithm (Patankar [1981]). The accuracy of the program was checked by the method of inserted analytical solutions. Analytical expressions for the unknown functions were substituted in (8)–(14) and the closure of the equations was calculated. This was then treated as the source in each equation. Next, with the aid of the algorithm described above, the values of the functions used were inferred with an accuracy of not less than 1%. The effect of the dimensions of the control volumes on the solution was studied by diminishing them. The time interval was selected automatically. The distribution of temperature of gas and condensed phases, velocity, component mass fractions, and volume fractions of phases were obtained numerically at different distances from the source of radiation to forest and different instants of time. The full energy of celestial body (E) is 10^{16} J Goldin [1992] which consist of kinetic energy (K_0) and energy of explosion – E_0. A fraction of the celestial body energy transformed into radiation equals to 0.1.

Figures 2 – 4 illustrate the time dependence of dimensionless temperatures of gas and condensed phases, concentrations of components and relative volume fractions of solid phases at upper boundary $x_3=h$ of the forest for various distances from the epicenter (solid curves — temperature of gas phase; dash curves — temperature of solid phase). Fig. 3 (solid curves — concentration of oxygen; broken curves — concentration of combustible products of pyrolysis(CO)) illustrate the distribution of concentrations of components of the gas phase. At the moment of ignition the CO burns away, and the concentration of oxygen is rapidly reduced. The temperatures of both phases reach a maximum value at the point of ignition.

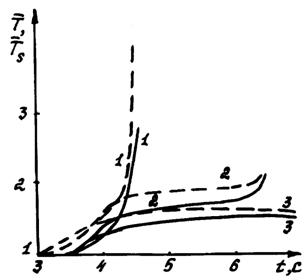

Figure. 2.

1 – x_1, x_2 = 0; 2 – x_1 = -10 km, x_2=0;3 – x_1= - 15 km, x_2 = 0.

$$\overline{T} = T / T_e,\ \overline{T_s} = T_s / T_e, T_e = 300K.$$

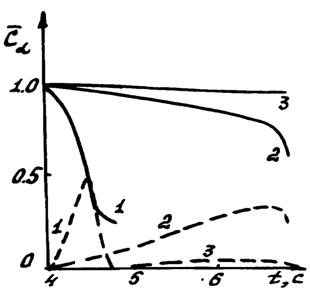

Figure 3.

1 – x_1, x_2 = 0; 2 – x_1 = -10 km, x_2=0; 3 – x_1= - 15 km, x_2 = 0;

$$\bar{c}_\alpha = c_\alpha / c_{1e}, c_{1e} = 0.23$$

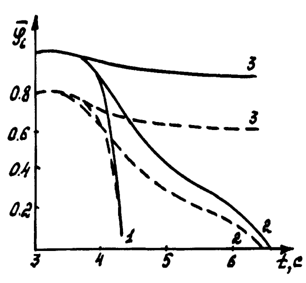

Figure 4.

1 – x_1, x_2 = 0; 2 – x_1 = -10 km, x_2=0; 3 – x_1= - 15 km, x_2 = 0,

$$\overline{\varphi}_1 = \varphi_1 / \varphi_{1e},\ \ \overline{\varphi}_2 = \rho_2 \varphi_2 / \rho_c.$$

The ignition processes is of a gas - phase nature, i.e. initially heating of solid and gaseous phases occurs, moisture is evaporated. Then decomposition process into condensed and volatile pyrolysis products starts, the later being ignited at the upper boundary of the forest canopy. At the ignition zone boundary gaseous fuel products are also generated, but they are not ignited because of not high enough radiant flux power. From the calculation results of forest canopy ignitions (Fig.2 - 4), it is seen that three conditions are realized: the first is factual combustion, the second is so - called normal state of ignition, and third is non - ignition (non - flammability). Within the framework of the problem mentioned above the sizes of the ignition zones were defined. Contours derived for collision catastrophes look like a circle arc in the neighborhood of epicenter of the explosion and take the form of the ellipse extended in the flight trajectory projection direction of Tunguska celestial body (Figures 5). As distinct from collision catastrophes, ignition contours take the form of a circumference as illustrated as the result of numerical experiments for the ignition of a homogeneous vegetation layer by radiation from the air nuclear explosion. Figures 5 present the dynamic of the development of forest fire contours for different types of forest (pine (a) and larch (b)).

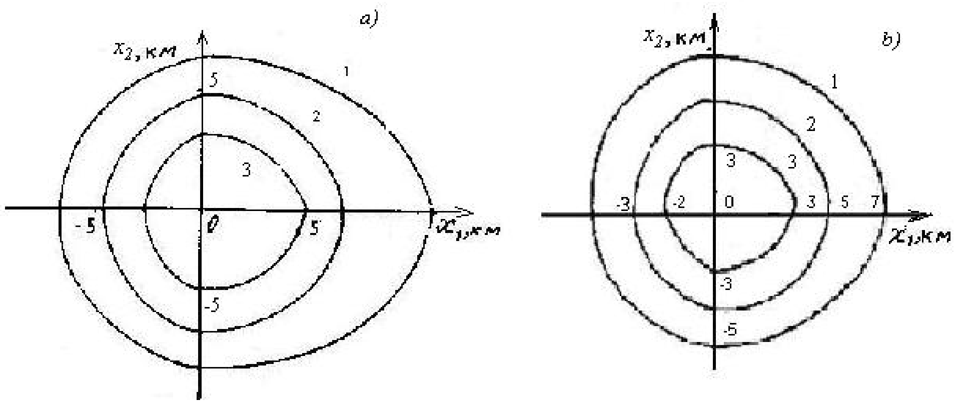

Figure 5.

1 – *t=7.0 sec, 2 – t=5 sec, 3- t=4.3 sec.* (*a* - pine and *b* - larch).

The results obtained agree with the laws of physics and data of observations Goldin [2] (Fig. 6). In this picture the size and contour of the domain of forest combustible materials (FCM) ignition as a result of Tunguska meteorite explosion are qualitatively and quantitatively coincide with numerical calculation results (Figure 6.).

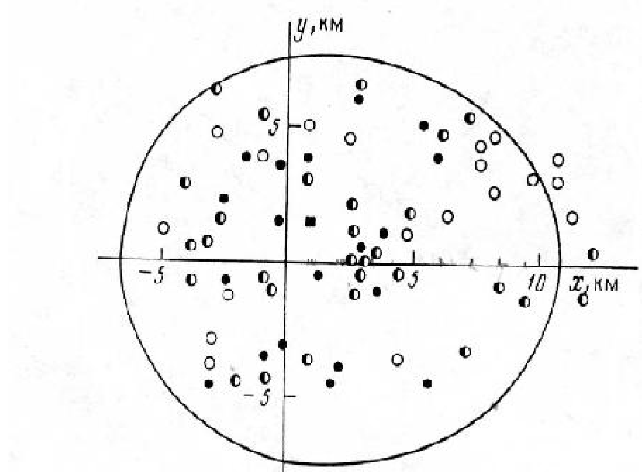

Figure 6.

The domain of burns of forest combustible materials as a result of Tunguska meteorite explosion. (● - strong burn, ◐ - medium burn, ○ - weak burn).

4. CONCLUSION

1. A mathematical model has been developed for the simulation of the problem on the vegetation ignition as meteorites fall down in the Earth's atmosphere. The results obtained agree with the laws of physics and experimental data. Thus, the model can be potentially utilized for the modeling of forest ignition by radiant energy and for the prediction of forest fire contours.
2. There exist two regimes of gas-phase ignition at the explosion of the Tunguska meteorite: the degenerate (high- temperature) regime of the FCM ignition in the explosion epicenter and the normal (low-temperature) regime due to oxidation of the gaseous products of FCM pyrolysis at temperatures less than the combustion temperature.
3. With the energy release of 10^{16} J, which corresponds to the explosion of the Tunguska meteorite, the ground cover ignites at a 12-16-km distance, depending on the other conditions, and the tops of trees ignite within 6-9 km from the epicenter. This is in agreement with the known data of observations.
4. Since the observed and calculated data are in agreement for the nonnuclear explosion model (the radiation fraction is 10% of the explosion energy according to Korobeinikov at al. [1991]), it may speak of a nonnuclear nature of the explosion of the Tunguska meteorite. To construct a more accurate model of explosion of the Tunguska meteorite and it is need additional complex investigations.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Base Research (Grant 10-01-98000 p_c_sibir).

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