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Numerical simulation of the three-dimensional flow field in a reservoir using energy equation

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Abstract: We have studied the three-dimensional flow field of a reservoir using a numerical simulation on an energy equation. Firstly, we used a boundary-fitted-coordinate system applied to a curved computational grid for a reservoir. Secondly, the curved computational grid was made into a fine-mesh by subdividing it into layers/stripes (horizontal and stripes vertical). Lastly, the flow field can be calculated as one-dimensional, plane two-dimensional and vertical two-dimensional using an energy equation. As the grid can be freely tessellated, the calculation accuracy of the flow field can be controlled effectively. The whole process of simulation is easy to be programmed and can be applied to other flow fields.

Keywords: reservoir; numerical simulation; element flow; boundary-fitted-coordinate; flow field

1. INTRODUCTION

A constructed reservoir is essential for the development of water conservation and hydropower. They are generally used in flood control, power generation, irrigation, fish and water supply in regions lacking of water resources. Constructed reservoirs may also cause a number of variations on local environment such as climate, water and organisms.

As a result, there are lots of relative researches focused on reservoirs (Guo et al., 2007; Liang et al., 2009; Ma et al., 2007; Zhang et al., 2006; Huang et al., 2006; Jean et al., 2004; Dong et al., 2006). In the process of studying 3-D numerical simulation of a reservoir, it usually refers to flow field calculation. So, it is necessary to use numerical methods to study the flow field in reservoirs. Ma Fangkai built a flow and water temperature model based on the Navier-Stokes equations of three-dimensional steady flow, and simulated the 3-D flow and temperature field of the area from Miaohe to Three Gorges Dam (Ma et al., 2007). Dong Yanchao found the flow distribution regularities of the primary spillway for Dahuofang reservoir using the RNGk-ε model (Dong et al., 2006). The simulated results tally well with physical experimental results. The flow velocity and distribution of the stepped spillway in Yu Beishan reservoir was calculated by Chen Qun using a three-dimensional turbulent flow simulation based on a k-ε double equation (Chen et al., 2002). From the foregoing, 3-D numerical simulations of the flow velocity in a reservoir must solve 2-D and/or 3-D equations, which makes the modeling process very complex. In this article, the reservoir grid was divided by layers/stripes (horizontal and stripes vertical) on the basis of an energy equation of element flow and the velocity of flow in the reservoir was calculated on these partitions.

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2. THEORETICAL FOUNDATIONS

2.1 Boundary-fitted-Coordinate Transformation

A boundary-fitted-coordinate system is a curvilinear co-ordinate system whose reference axes coincide with a physical plan transformed to a calculating plan (Figure 1). The transformation of boundary-fitted-coordinates is common in two-dimensional numerical simulations but was rarely seen in three-dimensional numerical simulations (Mao et al., 2008; Jie et al., 2004).

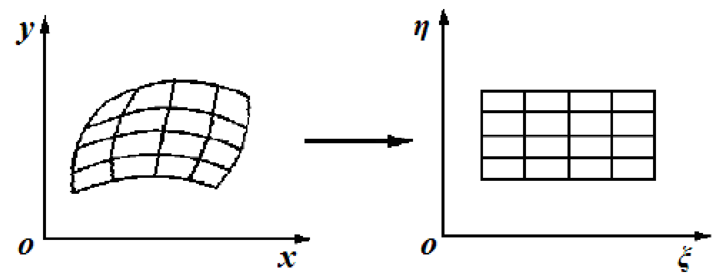


Fig.1 Transformation of the BFC method

In order to use the boundary-fitted-coordinate transformation, we set x, y, z as Cartesian coordinates for an arbitrary point to indicate physical space x^j; set ξ, η, ζ as arbitrary curved coordinates to indicate computation space ξ_i. The following equation can be found; $\vec{r}$ stands for radius vector.

$$\vec{r}=\vec{r}(\xi、\eta、\zeta)=\vec{r}(\xi^i)=\vec{r}(x、y、z)=\vec{r}(x^j) \tag{1}$$

Curvilinear coordinate ξ, η and ζ were taken as independent variables in this article. The value of points (x, y, z) on isolines was calculated from Poisson's equation:

$$\nabla^2\varphi=\frac{1}{g}\frac{\partial}{\partial\xi^i}(\sqrt{g}g^{ij}\frac{\partial\varphi}{\partial\xi^i})=0 \tag{2}$$

We got equations as follow:

$$\begin{cases} g^{ij}\dfrac{\partial^2 x^k}{\partial\xi^i\partial\xi^j}+F^j\dfrac{\partial x^k}{\partial\xi^j}=Q^k \\ F^j=\dfrac{1}{\sqrt{g}}\dfrac{\partial}{\partial\xi^i}(\sqrt{g}g^{ij}) \\ \sqrt{g}=\left|g^{ij}\right| \end{cases} \tag{3}$$

The g^{ij} is a contravariant metric tensor:

$$g^{ij}=\alpha_i^k\alpha_j^k \tag{4}$$

$$\begin{cases} g_{11} = x_\xi^2 + y_\xi^2 + z_\xi^2 \\ g_{22} = x_\eta^2 + y_\eta^2 + z_\eta^2 \\ g_{12} = g_{21} = x_\xi x_\eta + y_\xi y_\eta + z_\xi z_\eta, \quad \dots \ \dots \end{cases} \tag{5}$$

$$g^{ij} = \left(g_{ij}\right)^{-1} \tag{6}$$

Equations 5 were used in numerical calculation and we can derive equation 6. The transformation relation between point (x, y, z) in the original physical space and the curvilinear space point (*ξ, η, ζ*) was found by this method.

2.2 Energy equation of element flow

We adopt an equation of stream tube (7), which is also called Bernoulli's equation.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s} + g\frac{\partial z}{\partial s} + g \cdot J_s = 0 \tag{7}$$

Equation (7) is integrated to obtain:

$$z_1 + \frac{u_1^2}{2g} = (z_2 + \frac{u_2^2}{2g}) + h_w \tag{7'}$$

In this equation (7) and (7)': u= local velocity of 3-D (point flow velocity); s= the distance of the flow; J_s =energy slope. Total energy loss of stream tube h_w =is calculated using the equation, where:

$$h_w = h_f + h_i \tag{8}$$

In this equation (8): h_f indicates the frictional head loss and h_i indicates the inertia head loss. We know the section flow Q, and calculate the mean gradient J_s on every section by the equation (9). The mean gradient of a section for a subregion is:

$$Js = \frac{Q^2}{A^2C^2R} \tag{9}$$

In this equation (9): Q=flow of section; A=area of section; C=Chezy coefficient; R=hydraulic radius. Changes of J_s on different sections indicate the energy loss between sections.

The relationship between section flow Q and peak flow rate u_m can be deduced from:

$$\begin{cases} h = z - zb \\ B = b + b \\ q = u \cdot h \\ Q = \sum_{j=1}^{n} q_j \cdot \Delta b_j \\ vertical \qquad u = u_m (\frac{z}{h})^{0.142} \\ plan \begin{pmatrix} u = u_m (\frac{y}{b})^{0.142} & (y < b) \\ u = u_m (\frac{B-y}{b})^{0.142} & (y > b) \end{pmatrix} (\text{width direction}) \end{cases} \tag{10}$$

In the equation (10): B=river width of section; b=half-breadth of section; z=water potential; zb= reservoir bed elevation; y=Distance divided from boundary on section; h=depth of water; u= local velocity of 3-D (point flow velocity); u_m=maximum velocity of section; q=elemental flow of section; Δb_j =increment of half-breadth of section (Figure 2). The flow rate of a specific point on a section can be determined after the element flow of the section is given.

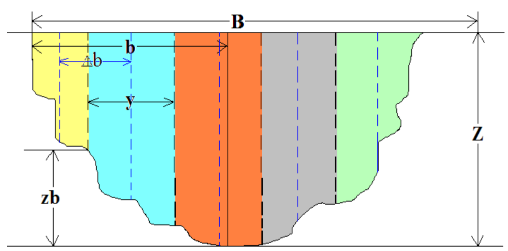


Fig.2 The schematic of section of flow calculation

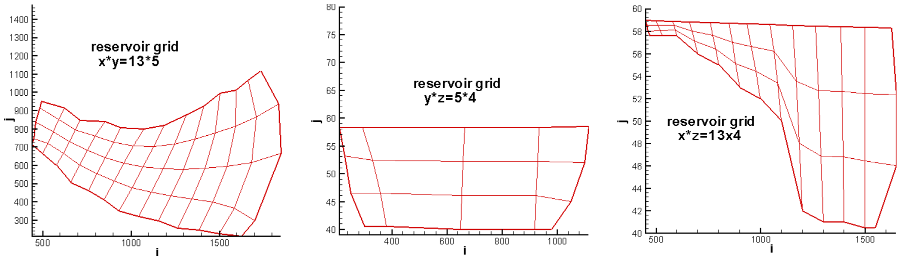
3. THREE-DIMENSIONAL FLOW FIELD NUMERICAL SIMULATION

The transformation into boundary-fitted-coordinate system builds reservoir grids. Furthermore, the curved computational grid was made into a fine-mesh by subdividing it into layers/stripes (horizontal and stripes vertical) and the flow field of the reservoir can be then calculated as one-dimensional, plane two-dimensional or vertical two-dimensional.

We select an assumed reservoir as the simulation object of this paper. The element flow Q=300m^3/s in the reservoir. The reservoir can be divided into a grid with 13×5×4 points. Given grid node coordinates on the grid boundary of the reservoir area.

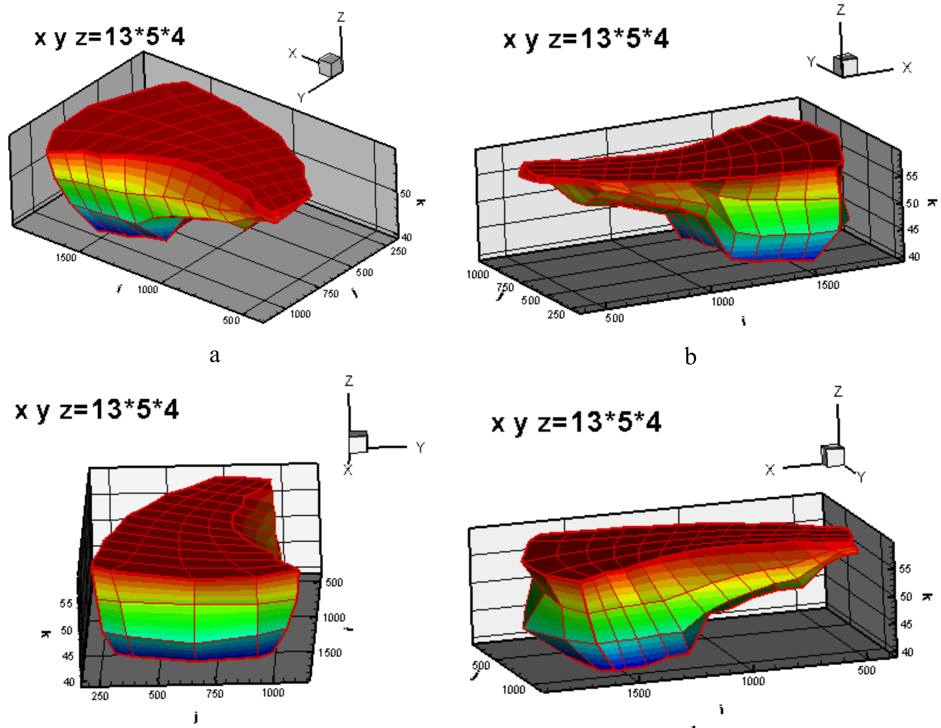
3.1 The meshing results of the reservoir area

The transformation into boundary-fitted-coordinates and meshing by the subdividing process above builds a reservoir trellis on the grid node coordinates. The meshing results for the reservoir are shown in Fig3 and 4.



a b c

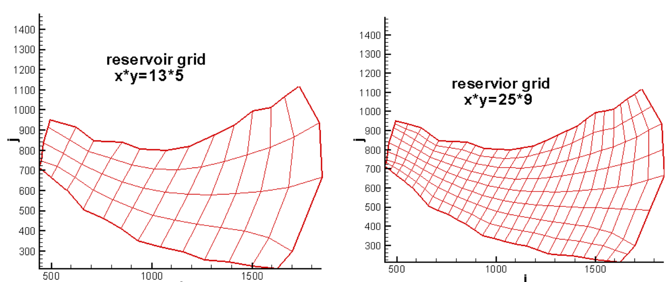
Fig.3 The grid section of reservoir area (a. indicate plan view of reservoir area; b. indicate elevation view; c indicate side view.)



a b c d

Fig.4 The map of reservoir area model (a, b, c, d indicate reservoir simulation diagram of different perspectives on the space.)

The combination of arbitrary grid refinement and subdivision into a fine mesh to calculate increases the accuracy of the calculated velocity as figure 5 shows.



a b

Fig.5 The map of grid refinement (a. before the grid refinement; b. after the grid refinement)

3.2 The simulated results of flow filed

The 3D flow field distribution of a reservoir was found in three steps. First, the relationship between Q and maximum velocity u_m was established according to the exponential distribution of time- turbulent velocity (Equation 9) after the element flow of reservoir was given.

$$\begin{cases} u = u_m (y/b)^{1/n} \\ u = u_m (z/h)^{1/n} \\ Re = 10^n \end{cases} \tag{11}$$

In equation (11)：*Re*—Reynolds number; *n*—order power.

Second, the curved computational grid of the reservoir was divided by layers/stripes (horizontal and stripes vertical) to convert the three-dimensional computational grid into several two-dimensional grids, and then into series of one-dimensional computational grids. Finally, the velocity of each grid node was counted individually based on the energy equation to obtain the 3D velocity distribution of the reservoir. Energy loss from the element flow was represented by mean gradient (*Js*) in the simulation. The energy equation can be used in elements.

The simulated flow field is represented in Fig 6. a、b、c (the axes dimensions is meter.) as horizontal maps of the velocity distribution at 3 depths (from reservoir surface to reservoir bottom). The figures show that velocity gradually becomes smaller along the x direction (axis i in the figure 6) and with depth. The velocity at each depth decreases from the middle to both sides. The predicted velocity distribution of the reservoir area is reasonable.

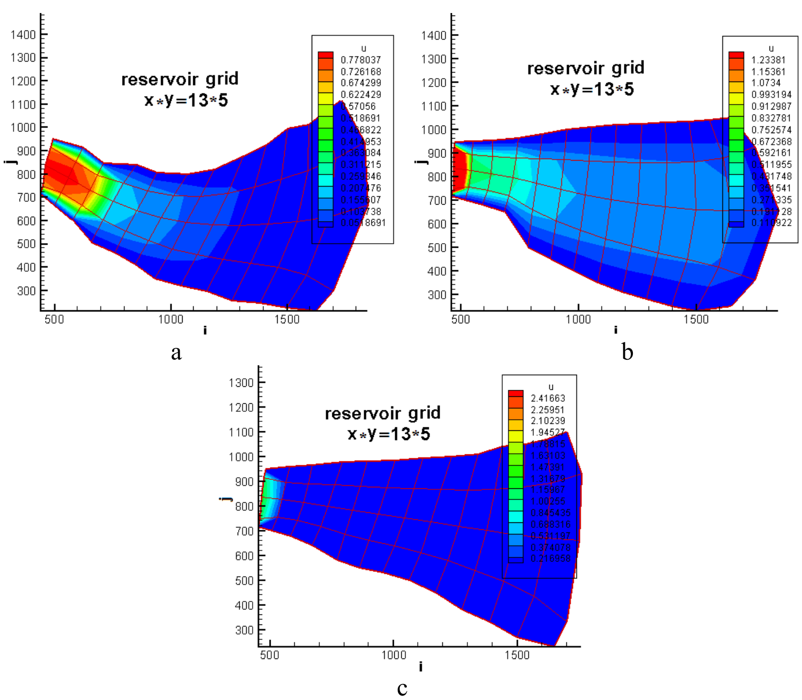


Fig.6 Maps of the velocity distribution of the reservoir area: (a) the reservoir surface; (b) at mid-depth; (c) at the reservoir bottom

4. CONCLUSION

(1) In this paper, first we systematically analyzed the theoretical foundations of the research needed, and then calculated three-dimensional flow field of a reservoir based on an energy equation by transformation of boundary-fitted-coordinates and subdividing the domain into layers/stripes (horizontal and stripes vertical).

(2) The meshing method as applied to the problem of a numerical simulation of the three-dimensional flow field of a reservoir based on an energy equation can also be used to simulate the 3D temperature field, water quality, sediment transportation and density current within a reservoir.

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