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Developing a model for detecting growth pulses in the observations and scenarios of CO_2 emissions

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Abstract: Sustainability of a pathway for world development is currently appraised on the basis of CO_2 emissions presumed by the pathway. Numerous scenarios of emissions growth presume that it cannot grow infinitely: there are certain limits that may not or must not be exceeded. The purpose of my work is to develop a model for analyzing the limits to CO_2 emissions growth and to consider how they may change in the future. Applying the theory of pulsing logistic growth, I assume that CO_2 emissions grow in agreement with the logistic equation where carrying capacity, K (or limit to growth) is a function of time, K(t). In the classic theory of pulsing logistic growth K (t) is supposed to be a stepwise function. In contrast, I suppose that it should be a continuous function and derive it from observations. In order to find changing limits to growth (that is, K(t)), I solve inverse Cauchy problem. Results allow us to detect two pulses of logistic growth. The first pulse culminated in 1930s. The second pulse recently entered the culmination phase. Unexpected acceleration of emissions growth observed in 2000-2004 may indicate to the start of the third pulse, which carrying capacity is even higher. Obviously, constructing a scenario of emissions growth one should pay attention to the fact that limits to their growth does not remain constant. They were and would be changing because of technological advances. Software, developed in course of this study, open wide opportunity for using the theory of pulsing growth in evaluating scenarios of CO_2 emissions growth.

1 INTRODUCTION

Carbon dioxide emissions are a heavily discussed topic since 1970s. It is well-recognized that their growth, associated with industrial development, may have negative consequences, and should be put under control [Heimann and Reichstein, 2008]. The scenarios of emissions stabilization presume that it cannot grow infinitely: there are certain limits that may not or must not be exceeded. The emissions growth rate since 2000, however, was greater than those scenarios would suggest. This fact poses the question of what could be the real limits to their growth.

Raupach and co-authors analyze [2007] accelerated trends in emissions using Kaya identy, which expresses emissions (E) as the product of four inputs: population (P), GDP per capita (G), energy use per unit of GDP (E), carbon emissions per unit of energy consumed:

$$F = P * \left(\frac{G}{P}\right) * \left(\frac{E}{G}\right) * \left(\frac{F}{E}\right)$$

Since this model may not give an estimate of the limits to growth, the purpose of my work is to develop a model based on the theory of logistic growth with time-varying carrying capacity.

2 METHODS

The theory of logistic growth was originally developed for modelling population growth. It assumes that growth rate is proportional to the population size and the amount of available resources. Applying this theory to the study of human population, Meyer [1994] and Ausubel [1996] assumed that carrying capacity of a human system is limited by the level of technology, which is subject to change. Based on this assumption they explained the pulses in the growth of the population of England and Japan. For this purpose they developed the bi-logistic growth model that was found later to be relevant for analyzing growth of US universities, US nuclear weapons tests, US installed electric generating capacity, cumulative number of published works and so on.

3 RESULTS

Applying the logistic growth theory with time-varying carrying capacity to the growth of CO_2 emissions (y), I came to the model

(1) $$\frac{dy}{dt} = my(1 - \frac{1}{K(t)})$$

(2) $$y(t_0) = y_0$$

(3) $$K(t) = \frac{1}{a_0 + \sum_{n=1}^{N} A_n(t) + B_n(t)}$$

(4) $$A_n(t) = a_n Cos\left(\frac{n\pi}{L}(t - t_0)\right)$$

(5) $$B_n(t) = b_n Sin\left(\frac{n\pi}{L}(t - t_0)\right)$$

K(t) is a term, that in economics and other fields where this theory is used, known as a carrying capacity. In mathematic language K(t) will be a time-varying limit of growth from for this differential equation, which shows a rate of growth. *L* is a constant which shows length of time period (in this case it will be 155); *m* is a relative increment for *y* when *y(t)<< K(t); N* is length of series used to approximate K(t).

Then, I found the solution of this problem, noticing that equation (1) is a differential equation of Bernoulli.

(6) $$y = \frac{1}{\left(\frac{1}{y_0} - \frac{1}{K(t_0)}\right) e^{-mt} + a_0 + \sum_{n=1}^{N} \tilde{A}_n(t) + \tilde{B}_n(t)}$$

(7) $$\tilde{A}_n(t) = a_n \frac{Lm(Lm \cos(\frac{\pi n(t - t_0)}{L}) + \pi n \sin(\frac{\pi n(t - t_0)}{L}))}{L^2 m^2 + \pi^2 n^2}$$

(8) $$\tilde{B}_n(t) = a_n \frac{Lm(Lm \sin\left(\frac{\pi n(t - t_0)}{L}\right) - \pi n \cos(\frac{\pi n(t - t_0)}{L}))}{L^2 m^2 + \pi^2 n^2}$$

As it can be seen from Figure 1 the model is able to fit the data on CO_2 emissions and reveal the trends in carrying capacity (I set N=2, and got a_0= -0.000258168, a_1= -

0.0000440057, a_2= 0.000302174, b_1= 0.000951037, b_2= 0.000375412.) Thus obtained curve of changing carrying capacity allows us to detect two pulses. The first pulse, with the carrying capacity of 2±0.8 GtC/y, culminated in 1930s. The second pulse, with the carrying capacity of 12±2 GtC/y, recently entered the culmination phase. Unexpected acceleration of emissions growth observed in 2000-2004 [Raupach et al., 2007] may indicate to the start of the third pulse, which carrying capacity is even higher.

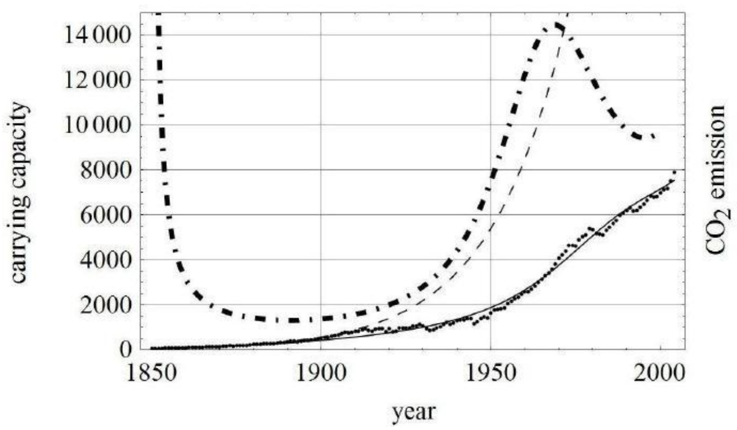

Figure 1. CO_2 emissions and their limits to growth (carrying capacity). Units: MtC/y. Legend: thick dot-dashed line - carrying capacity, dots – CO2 emissions data [Marland et al., 2008], solid line – solution of the Equation (1), dashed line – exponential growth (m=0.045).

4 DISCUSSION

Equations 1-2 represent the well known model of the logistic growth with time-varying capacity. The novelty is the form of K(t) given by Equations 3-5. This allows to find analytical solution (Equations 6-8) that simplifies identification of K(t).

Identification of K(t) seemingly posed a serious problem. Although the logistic model with time-varying carrying capacity was invented a long time ago, nobody applied it to data analysis. The bi-logistic model looks like a model with a time-varying carrying capacity but it isn't. It was developed to detect changes in carrying capacity, and often used for this purpose, but it could not give us K(t).

The bi-logistic growth function is the sum of two logistic functions with different carrying capacity.

$$y(t) = \frac{K_1}{1 + e^{-\frac{\ln(81)}{\Delta t_1}(t - t_{m1})}} + \frac{K_2}{1 + e^{-\frac{\ln(81)}{\Delta t_2}(t - t_{m2})}}$$

Each term of the sum represents a pulse of logistic growth. Time-series data are split in two parts to identify the two levels of carrying capacity.

Thus identified K_1 and K_2 may give us a stepwise function approximating K(t). Such a stepwise function reflects some important features of K(t), but not all of them. As it can be seen from the Figure 1, K(t) does not simply shift from lower level to higher level. It may swing between them.

This quite speculative hypothesis leads to very interesting scenarios of the future CO_2 emissions growth that can be constructed by means of K(t) extrapolation. Assuming that the recent increase in CO_2 emissions is short-term, one may suggest that the peak of carrying capacity was achieved in 1970s, extrapolate the descending curve of K(t), and conclude that K(t) will be as low as 2 GtC/yr by the end of this century. Or, instead, one may extrapolate the 1954-2004 wave, and conclude that K(t) will have next peak around 2030.

5 CONCLUSION

In constructing scenarios of CO_2 emissions it is important to pay attention to the limits of growth. Stabilization trajectories imply that climate regulations impose the limits that will be harder than those imposed by the availability of fossil fuels. In the middle of the 20th century, technological advances reduced the pressure of the limits imposed by availability of fossil fuels and lifted carrying capacity of carbon-based energy system from 2 GtC/y to 14 GtC/y. In the end of the 20th century, the carrying capacity decreased to 10 GtC/y. This downward trend may be attributed either to the pressure of environmental regulations that have been introduced starting from 1970s or to exhaustion of easily available fossil fuel deposits. In the beginning of 21st century, the carrying capacity shows the tendency to increase. Is this the sign that recent technological advances increase availability of fossil fuel to the extent that the pressure of climate regulations cannot compensate for? The model I developed cannot answer to this question. It is designed for discovering the temporal pattern of carrying capacity of the carbon-based energy system staying behind the time series of global CO_2 emissions. The temporal pattern of carrying capacity reflects the temporal pattern of trade off between climate regulations that reduce consumption of fossil fuels and technological advances that expand their availability. Therefore, the model presented in this paper may serve as a tool for monitoring efficiency of climate regulations and for extrapolating the growth of CO_2 emissions proceeding from the theory of logistic growth with time-varying carrying capacity.

REFERENCES

Ausubel, J.H. (1996), "Can Technology Spare The Earth", American Scientist, 84(2):166-178.

Heimann, M. and M. Reichstein (2008), Terrestrial ecosystem carbon dynamics and climate feedbacks, Nature, 451, 289-292.

Marland, G., T.A. Boden, and R.J. Andres (2008), Global, Regional, and National Fossil Fuel CO2 Emissions, in Trends: A Compendium of Data on Global Change, Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn., U.S.A.

Meyer, P. (1994), Bi-logistic growth. Technological Forecasting and Social Change, 47:89-102.

Raupach, M.R., G. Marland, P. Ciais, C. Le Quere, J.G. Canadell, G. Klepper, C.B. Field (2007), Global and regional drivers of accelerating CO2 emissions, PNAS, 104: 10288-10293.