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The Necessity of a Sinless Messiah

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INTRODUCTION

Christianity intrinsically relates to the earthly mission and atoning sacrifice of Jesus Christ. It relates to the remarkable necessity of a single special individual born into mortality to save all of mankind, this individual to be the first-begotten of the Father in the spirit world and the only-begotten of the Father in this mortal world. Furthermore, and even more remarkable, the saving plan of the Father would entail great suffering to be experienced by this special individual who, among all of the Father's children, was the only one who had been perfectly obedient from the beginning.

This requirement of supreme sacrifice must have been due to persuasive and righteous reasons; otherwise it would not have been part of God's plan. But, even so, one wonders why Christ's atoning mission in just this form was necessary. Alma says:

> And now, the plan of mercy could not be brought about except an atonement be made; therefore God himself atoneth for the sins of the world, to bring about the plan of mercy, to appease the demands of justice, that God might be a perfect, just God, and a merciful God also.
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> [Alma 42:15]

Alma does not just simply state that an atonement must occur; he also says that it is necessary if God is to be merciful without violating justice (see also Alma 34:9–13). Thus, Alma refers to an *implied relationship* between these principles and the necessity of the Atonement.

Alma's statement is a simple example of what is formally known in logic and mathematics as an *axiomatization*. In a religious context, this would mean the analysis of a set of eternal concepts and principles to show that the only way they can all be satisfied is for certain doctrines to be fulfilled or, conversely, to show that if certain doctrines are not satisfied, then at least one of these principles must be violated.

Ronald A. Heiner is an associate professor of economics at Brigham Young University.

Such a demonstration of relationship between spiritual principles and implied doctrines, however, does *not* "prove" the doctrines so that faith is no longer needed. It rather shows that there is an implied relationship *between* such principles and the associated doctrines; the validity of the principles necessarily remains outside the scope of the demonstration. Nonetheless, elucidating a necessary relationship between eternal principles and implied doctrines may significantly enhance one's understanding and appreciation of those doctrines, thereby increasing one's faith in them. This, then, is my purpose: to increase faith, understanding, and appreciation by showing that *certain eternal principles imply the unavoidable necessity of Christ's atoning mission.*

The principles involved are expressed in the Book of Mormon and Doctrine and Covenants, and not only demonstrate an overall unity and depth in these two books of scripture but also validate the latter-day restoration of the gospel of Jesus Christ, because temple work for the dead (a doctrine peculiar to the restored gospel) contains additional key principles sufficient to imply the necessity of Christ's atonement, which is the core doctrine defining the very meaning of Christianity.

I. THE SINLESS MESSIAH IMPLICATION

This section presents four principles about justice, vicarious substitution, and death. It then shows how Christ's atonement is implied by them.

1. *Justice Alone Not Sufficient*

Considerations about justice are a focal point in many discussions of the Atonement. A good example is Boyd K. Packer's essay entitled *The Mediator*.[1] Elder Packer uses a parable about monetary debt to emphasize that payment must occur or punishment for sins must occur where sins are symbolically represented by debt. This principle may be stated as follows:

> *PJ (Punishment Justice)*. For an individual to receive eternal life, punishment is necessary for any sins committed by him.

Principle PJ implies that the only way an insolvent debtor (one unable to pay his debts) can escape punishment is for someone else to

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[1]Boyd K. Packer, *The Mediator* (Salt Lake City: Deseret Book, 1978); see also Boyd K. Packer, "The Mediator," *Ensign* 7 (May 1977): 54–56.

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satisfy justice by paying the debt for him. In the parable, there is a friend who reconciles or mediates the division between the insolvent debtor and the unpaid lender. By so doing, the debtor receives mercy because he escapes punishment, yet justice is satisfied because the debt is paid by the friend. Thus, an implied relationship exists between the principle of punishment justice and the necessity of a mediator.

This implication is important in understanding the Atonement. Note that it does not imply that the mediator must be some special individual who has never sinned[2] or, as in the parable, never been a debtor himself.[3] Principle PJ says only that punishment must occur for everyone's sins but does not put any restriction on *who* might suffer these punishments. Thus, those that have sinned could receive mercy by having others who, in addition to suffering for their own sins, suffer punishment for those sinners. In this case, the reconciliation of justice and mercy would be achieved entirely *within* the group of sinners.

Therefore, while punishment justice is a necessary component in understanding the Atonement, it is not sufficient by itself to imply that a special, sinless individual must be part of God's plan.

2. Two Substitution Principles Implied by Temple Work for the Dead

Both the Atonement and temple work are examples of *vicarious substitution*.[4] One example concerns substituting to receive the punishments of another; the other concerns substituting to perform spiritual ordinances for another.

In contrast to punishment substitution, temple ordinance substitution does not require that those doing the substitution be sinless, simply that they must have forsaken past sins so as to be eligible to enter the temple.

This fact provides an important clue to the principles needed to imply that the mediator be sinless. The basic issue is that once one

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[2]For reference to the fact that the Messiah must be sinless, see Hebrews 4:14–15, 1 Peter 2:21–25, D&C 20:22, Mosiah 15:5; see also James E. Talmage, *Jesus the Christ* (Salt Lake City: The Church of Jesus Christ of Latter-day Saints), p. 21.

[3]It is often implicitly assumed that once a sin has been committed a person is not capable of repaying the sin and thus is incapable of repaying anyone else's sins. This assumption is inconsistent with the analogy of monetary debt, where the dominant purpose of borrowing is precisely to generate greater returns than the initial amount borrowed (thus generating a surplus to help others pay their debts). Far more basic is the issue of why it is legitimate for one who has sinned to vicariously substitute in performing someone else's temple ordinances but not legitimate to vicariously substitute in satisfying another person's punishments.

[4]Bruce R. McConkie, *Mormon Doctrine*, 2d ed. (Salt Lake City: Bookcraft, 1966), p. 822.

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realizes *the necessity of vicarious substitution he must then understand the conditions under which such substitution is valid*. Concerning the Atonement, this means that one must satisfy the appropriate *substitution principles* so that justice is not violated in extending mercy. These principles will allow persons who have sinned to do temple work for the dead; yet when these principles are combined with other principles, they will prevent these persons from substituting for others' punishments. Such general principles are implied by the revealed rules of ordinance work.

First, these rules dictate that a person must be baptized or endowed for the dead in the *same* way he, as a living person, had to be baptized or endowed for himself. For example, he cannot be sprinkled with water for another person if for his own baptism he had to be fully immersed. Generalizing this condition to all types of requirements gives the following substitution principle:

> *SE (Substitution Equivalence)*. If a group of persons are all subject to the same requirement, then members of the group can help another in the group satisfy that requirement only by doing the same things that individual would have to do to accomplish the requirement for himself.

Substitution equivalence holds that identical requirements imply identical actions to satisfy those requirements.

Second, in addition, one's own baptism cannot simultaneously count as baptism for someone else, and, similarly, his own temple ordinances cannot simultaneously count for another's temple ordinances. A person's own ordinances, needed to satisfy his own ordinance requirements, must be separate from ordinances he performs for other persons. Generalizing these conditions gives a second substitution principle:

> *SR (Substitution Responsibility)*. In order to substitute for someone else's requirements, one's own requirements must also be satisfied; and actions necessary to self-accomplish one's own requirements must be separate from those actions used to help satisfy another person's requirements.

Substitution responsibility holds that in order for one to help others one's own requirements must be satisfied; and, to help others, one must perform actions separate from those already needed to accomplish one's own requirements.

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These general principles do not limit substitution possibilities to only those who are sinless. Applied to punishment substitution, these principles imply no special restriction that only a sinless person can pay for the sins of others.

3. Death, Immortality, and Eternal Life

Eternal life cannot occur unless one has become immortal with spirit and body inseparably united (see Alma 11:45). However, if a person pays for his own sins without help from anyone else, a permanent death must occur and body and spirit cannot then be reunited (2 Nephi 9:7; 10:25; Mosiah 16:7). If body and spirit are unable to be reunited, death obviously cannot recur. And, since the spirit is forever without a body, the opportunities for eternal life are also nullified (D&C 93:33–34).

Summarizing these conclusions gives the following principle:[5]

> *ED (Eternal Death).* All sin requires payment which, if self-accomplished—that is, without the help of others—implies an endless death that cannot be reversed, which nonreversal implies death cannot be repeated and also nullifies the opportunity for eternal life.

With this condition, sin becomes a serious matter, because unless some way can be found to avoid a person's paying for his own sins, he will be denied eternal life. This is the dilemma facing a merciful God who desires that eternal life not be denied those who have sinned. But how is the Father to exend mercy to his children without violating justice? Is there any way they can share or substitute for one another to pay for their sins?

This is the fundamental question which must be resolved. Its answer explains why the Father's plan of mercy required the atoning sacrifice of his beloved firstborn.

4. Proof of the Sinless Messiah Implication

The basic implication about Christ's messianic role is now derived from the previous four principles.[6] To help follow the chain of reasoning used in the proof, the principles are restated together.

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[5]The only property of death and resurrection assumed is that they are each the reversal of the other (resurrection restores the union of spirit and body which death separates). No assumption about the physical or spiritual nature of body and spirit, nor of the process by which they are united or divided, is used. Hence, other than the reversal relationship, the sinless atonement implication to be obtained is independent of any particular physical or spiritual interpretation of these concepts.

[6]The precise demonstration using logic and set theory tools is presented in the Appendix to this article.

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PJ (Punishment Justice). For an individual to receive eternal life, punishment is necessary for any sins committed by him.

ED (Eternal Death). All sin requires payment which if self-accomplished—that is, without the help of others—implies an endless death that cannot be reversed, which nonreversal implies death cannot be repeated and also nullifies the opportunity for eternal life.

SE (Substitution Equivalence). If a group of persons are all subject to the same requirement, then members of the group can help another in the group satisfy that requirement only by doing the same things that individual would have to do to accomplish the requirement for himself.

SR (Substitution Responsibility). In order to substitute for someone else's requirements, one's own requirements must also be satisfied; actions necessary to self-accomplish one's own requirements must be separate from those actions used to help satisfy another person's requirements.

Assume that all individuals are grouped into a single room before the judgment bar of God. Could *anyone* receive eternal life if *everyone* in the room has sinned?

Since all have sinned, then punishment justice (PJ) and eternal death (ED) imply that every person, if he self-pays for his own sins, must die and not be resurrected. But death without resurrection precludes the possibility of eternal life. Hence, each person will be denied eternal life unless someone other than the person himself helps to pay for his sins.

Conceivably there are many ways a person's sins could be paid for by others. The simplest would be for someone to step in and single-handedly pay for another person's sins. Far more elaborate possibilities might involve the team effort of a number of persons, each of whom pays for a portion of another person's sins. However, since all have sinned, then payment for sin is a requirement for each person in the room. Thus, by substitution equivalence (SE), any joint effort to pay for another person's sins must satisfy the same requirements that this person would have to satisfy if he self-paid for his own sins. Since self-payment requires both death and nonresurrection, then these same requirements must also be satisfied by other persons in the room.

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Thus, two possibilities exist: someone substitutes for the nonresurrection requirement of another person (whom we will call the first person), thus enabling the first person, if he dies, to be resurrected; or someone substitutes for the death requirement of the first person, eliminating the need for him to be resurrected since his spirit and body need not be divided.

Consider the first possibility, that of a second person's substituting for the nonresurrection requirement of a first person. Since person 2 would also have to be not resurrected if he self-paid for his own sins, then substitution responsibility (SR) would require that his nonresurrection restriction cannot also apply to person 1; yet SR also requires that a nonresurrection must be satisfied for 2 in order for 2's nonresurrection to satisfy nonresurrection for 1. The only way out is for there to be a third person whose nonresurrection applies to person 2, so that 2's nonresurrection can substitute for 1's nonresurrection requirement. Applying the same argument recursively implies an endless sequence of further substitutions, but this will eventually deplete all persons in the room yet still require individuals to satisfy these substitutions. Hence, an impasse is reached if eternal life were to be given to any person in the room through nonresurrection substitution.

The other possibility involves a second person's substituting for the death requirement of the first person. Here the situation is more complicated because a person can only be *not* resurrected once, while in contrast he can repeat the occurrence of death many times so long as he does not have to satisfy a nonresurrection requirement himself. Thus, it might appear that someone could experience repeated deaths for others, all of which are separate from the death he experiences for his own sins. Let us see if this is possible.

Remember that person 2 would also have to die if he self-paid his own sins, and thus by substitution responsibility (SR), a single occurrence of death for 2 cannot also apply to 1. Thus, either a third person must substitute for 2's death requirement, or 2 must himself die twice—once for himself and once for person 1. Suppose initially that 2 does experience these two deaths.

Now person 2 would also have to satisfy a nonresurrection requirement if he pays for his own sins, which requirement, by substitution responsibility, must still be satisfied by someone in order for 2 to suffer a death which substitutes for person 1's death requirement. But nonresurrection for 2 would prevent him from dying a second time—for person 1. Hence, in order for 2 to die a second time, for 1, there must be still a third person who satisfies person 2's

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nonresurrection requirement. But we have already shown that non-resurrection substitution implies a never-ending sequence of further nonresurrection substitutions, hence leading to an impasse.

Thus, the only remaining possibility is that person 2 satisfy his own nonresurrection restriction, which prevents him from experiencing his own death plus a second death for person 1. But since 2 would have to die if he self-paid for his own sins, then substitution responsibility (SR) implies that a single death by 2 cannot both apply to himself and to person 1; and since SR also implies that 2's death requirement must still be satisfied for his single death to substitute for 1, then there must still be a third person besides 1 or 2 who dies for person 2. Now, applying the same argument recursively again implies a never-ending sequence of further death substitutions.

Hence, in either the case of death or of nonresurrection substitution, an impasse would be reached if eternal life were to be given.

Therefore, *unless someone in the room is sinless, no one can receive eternal life*. Thus, assume that someone has never sinned. However, the mere presence of such a sinless person does not itself automatically imply that punishment substitution could not be accomplished by the remainder of individuals who have sinned. So consider the possibility that someone's sins are paid for by those who have sinned, that is, without the help of any sinless person. This is equivalent to isolating those who have sinned into a separate room and requiring all substitution activities be limited to just those persons in that room. Then the same reasoning used previously implies that an impasse would be reached if eternal life were to be given to one who has sinned, because an endless sequence of substitutions is again implied. Therefore, *no person whose sins are paid for by those who have sinned can obtain eternal life*.

Since punishment justice principle (PJ) implies that punishment for sins must be paid, *the only remaining possibility for anyone who has sinned to achieve eternal life is for a sinless person to suffer punishment to help pay for the sins of others*. This is possible because a sinless person need not satisfy death or nonresurrection for himself, and thus an endless sequence of further substitutions never arises. Without such a sinless person, no pattern of sharing or substitution (no matter how complicated or how many people might jointly contribute) would enable anyone who has sinned to receive eternal life.

Therefore, the substitution, justice, and death principles imply fundamental restrictions on the possibility for sinful persons' receiving eternal life. These restrictions characterize the atoning

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mission of Jesus Christ. They are summarized in the following basic theorem:

Theorem (The Sinless Messiah). Principles PJ, ED, SE, and SR imply that unless someone has never sinned, then no one can receive eternal life; and no person whose sins are paid by those who have sinned can obtain eternal life. Therefore, the only way anyone who has sinned can achieve eternal life is for there to be a sinless person who suffers punishment to help pay for the sins of others.

Recall the anguished words of Christ in the Garden of Gethsemane, as recorded in Matthew 26:39, ''O my Father, if it be *possible*, let this cup pass from me: nevertheless, not as I will but as thou wilt'' (emphasis added). This is a remarkable plea because it asks for the bitter cup to be withdrawn if it is somehow possible to do so. Yet the Father chose not to withdraw it.

The reason implied is that had the Father done so there would have been no way to save the rest of his children without violating some eternal principle. *Hence, in order to be just, there was no other way the Father could be merciful to those children who had sinned.*

Given this realization, the second half of Christ's plea is equally poignant, and also fundamental to the Father's plan. Since Christ had to be truly sinless for mercy to be possible, it surely would have been unjust to force him into suffering the atoning sacrifice. Only if Christ chose of his own free will to do as the Father hoped could the plan of mercy be implemented. Otherwise, even the Father would have been unable to redeem His sinful children.

Perhaps nowhere is the power of love more powerfully manifest than in this supreme and voluntary act of obedience, which enabled the Father's eternal plan of mercy to be fulfilled.

II. PRINCIPLES PJ AND ED IMPLIED FROM MORE BASIC PRINCIPLES

An axiomatic investigation of ideas or doctrines provides a basis for further analysis of those ideas by showing that the principles which imply them are themselves implied by still more basic principles. This enables ideas or doctrines to be traced back to successively more basic premises.

Accordingly, this section derives the Sinless Messiah Implication from more basic premises by showing that principles PJ and ED are implied by still other principles which interrelate justice and mercy with the nature of sin.

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1. *Justice's Allowance of Mercy Subject to Repentance*

The notion of justice as requiring punishment for sin (principle PJ) is often the focal point in discussions about the Atonement. That is why principle PJ was the first one introduced. However, it is not truly fundamental, but rather is the implication of still other antecedent principles. There are two key reasons for this.

The most common explanation of principle PJ is that justice by its intrinsic nature requires that punishment for sins must occur. In the most extreme version this means that regardless of how minor a sin no amount of subsequent righteousness and self-sacrifice is sufficient without punishment to satisfy justice. However, this unyielding requirement of justice is not stated in the scriptures, even when very strong language is used that "God would cease to be God" if justice were violated. For example:

> Therefore, according to justice, the plan of redemption could not be brought about, only on conditions of repentance of men in this probationary state, yea, this preparatory state; for except it were for these conditions, mercy could not take effect except it should destroy the work of justice. Now the work of justice could not be destroyed; if so, God would cease to be God. [Alma 42:13]

Alma does not say that justice *automatically* demands payment for sin, *but rather that repentance must be satisfied in order to grant mercy without violating justice*. Similarly, Alma 42:22 says that mercy has claim over the repentant; but *otherwise*, justice requires punishment be inflicted. Thus, the requirement of justice stated in the scriptures is that payment for sin must occur except under conditions of repentance. Justice and mercy do not intrinsically conflict but rather are consistent with each other so long as repentance is satisfied (see also Alma 42:24–25; 34:15–16).

Therefore, for God to be merciful without violating justice requires that a strict and impartial judgment of repentance must occur. Accordingly, many scriptures emphasize the necessity of such a judgment, which is also one of Christ's key responsibilities in addition to the Atonement (see, for example, Mosiah 2:38–39; Alma 34:35; Helaman 5:10–11; Alma 12:13–18; Jacob 6:8–10). Though the language is strong, these scriptures are still limited to stating forcefully the consequences of *nonrepentance*, rather than saying a penalty must occur independent of whether repentance is satisfied or not.

In relation to this, consider also one of Christ's most poignant parables, the prodigal son (Luke 15:11–24). Recall the scene as he returns home deeply ashamed of his sins and of the suffering thereby caused to his father—so ashamed that the son hopes only to become a

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servant, having already consumed his birthright in riotous living. However, the father responds in wonderful fashion by running to kiss his son. Had the father spoken his feelings he might well have said, ''My son, you feel anguished and unworthy, but nevertheless do not sorrow; for my love for you cannot die, and I know that you have seen the error of your deeds. Come unto me that I might restore the honored place in my home always intended for you.''

The purpose of this story is to show that God truly loves even his sinful children and wishes deeply to forgive them (as permitted by justice) if only they will repent and forsake their sins. Indeed, this is itself one of the greatest of God's commandments, that we are to love each other so that even if injured by another we will not demand retribution, but instead will forgive.[7]

In summary, then, two major conclusions exist relating to principle PJ. First, justice allows mercy, subject to the condition of repentance; and second, pure love and mercy are inherent in the Father's nature. Together they imply that punishment for sin is not automatically required by the Father. Rather, principle PJ is due to more basic principles. As will be shown, these also imply the eternal death principle ED.

2. *The Nature of Sin's Bondage*

Recall again Elder Packer's parable about a debtor who owes money to a creditor.[8] In such cases, bankruptcy law does not necessarily require payment must be made, *if the creditor who holds claim to the debt chooses to release it without demanding payment*. That is, justice does not automatically force payment to occur but rather gives the holder of the claim the right to demand payment if he so chooses.[9]

> *CJ (Claim Justice)*. A claim on a person can be justly voided without payment only if voluntarily agreed to by the holder of the claim.

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[7]It is sometimes argued that the reason for punishment is that it is necessary to relieve one's guilt. Otherwise one's feelings of unworthiness would cause one to shrink from the presence of God. However, the scriptures do not say that such incapacitating guilt is the necessary consequence of sin, but rather that such guilt would occur *if one has not repented* (see Mosiah 2:38, 3:25; and Alma 36:14–19). It is significant that in this parable the son feels terribly unworthy, but the father runs to embrace him when the son is ''yet a great way off.'' It is as if the father wishes to run and quickly reassure his son before such guilt feelings might cause the son to turn away in despair (see Luke 15:19–24).

[8]Boyd K. Packer, *The Mediator*.

[9]Secular law also satisfies this basic principle. For example, if a robber is caught in the act of stealing, he will still not be prosecuted if the person who was robbed chooses not to press charges. In general, there must be some aggrieved party that demands punishment for justice to imply retribution must be inflicted on the wrongdoer (see John 8:10–11).

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Thus, just settlement of claims does not automatically require punishment simply because a person is unable to pay his debts. Rather, it is because the creditor who holds claim to the debt chooses to demand payment, which is his right according to justice.

It is here that Satan's activities relate to the sinless atonement doctrine. In contrast to the inherent mercy of the Father, Satan's intrinsic nature is the opposite. He wishes not happiness and love but rather misery for those subject to him (see 2 Nephi 2:17–18 and Alma 34:39). Thus, once he obtains a claim over someone, he will never choose to release that claim without payment.

> *MS (Merciless Satan).* Satan is merciless; if he obtains a claim over someone, he will never choose to release that claim without payment.

The next question concerns how Satan could obtain such a claim that would entitle him to payment in order for that claim to be relinquished. Its answer relates to still other fundamental concepts: *free agency* and *the nature of sin.*

Choosing good indicates voluntarily subjecting oneself to God's will in return for the opportunity of eternal life, while choosing to sin means rejecting God's will by voluntarily accepting Satan's authority in return for the favors he has to offer. Joy and happiness are intended for those who are obedient to God's commandments, while, in contrast, misery is intended for those who choose Satan's authority (see Alma 12:4–6; 30:60; 34:35, 39; and Helaman 7:15–16). Obviously, Satan must somehow deceive persons about his ultimate objective in order to get them to sin voluntarily (see 2 Corinthians 11:3; James 4:17; Moses 4:16; and Moroni 7:12).

Nevertheless, because every responsible individual has not only the ability to discern good from evil (Moroni 7:16) but also the agency (Helaman 14:30–31) to make his own free choice, *choosing to sin implies that one is voluntarily rejecting God's will in favor of subjecting himself to Satan's authority* (see 2 Nephi 2:27; Mosiah 16:3–4; and Alma 5:18–20).

Satan's purpose is to deny God's children their potential happiness. To implement this objective, Satan obtains a claim over the sinner that enables Satan, if the sinner pays on his own, to nullify the sinner's possibilities for eternal life by forever retaining dominion over the sinner in Satan's own kingdom. Since Satan's dominion is only over spirits without bodies (see 2 Nephi 9:7–10), this implies there must be a separation of body and spirit (death) which he will not allow to be reversed.

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Since this claim, if satisfied by the sinner himself, will never be relinquished, then death is permanent and cannot be repeated. And, since the sinner's spirit is forever without a body, the possibility of eternal life is nullified. Thus, Satan's claim enables him, if the claim is self-paid by the sinner, to require that death must occur—death which Satan will not allow to be reversed. This nonreversal implies that death cannot be repeated and also nullifies the opportunity for eternal life. This conclusion is precisely the eternal death principle (ED) introduced previously. *Hence, the implied nature of sin's bondage also explains why this principle must hold.*

This also suggests a basic aspect of Satan's plan. His major strategy is simply to prohibit the realization of certain key activities or opportunities which are needed for the Father's children to achieve full happiness, without which they will be denied possibilities to develop their potential abilities. Access to a body is clearly such a key opportunity (D&C 93:33–34).

Sin's implied bondage to Satan and the nature of that claim obtained by him is summarized in the following principle:

> *SB (Sin's Bondage).* Sin is the voluntary rejection of God's will in favor of Satan's authority, allowing Satan a claim which, if self-paid, enables Satan to require that death must occur—death which he will not allow to be reversed; this nonreversal implies death cannot be repeated and also nullifies the opportunity for eternal life.

3. *Proof That Principles CJ, MS, and SB Imply Principles PJ and ED*
To facilitate understanding, principles CJ, MS, and SB are restated as a group.

> *CJ (Claim Justice).* A claim on a person can be justly voided without payment only if voluntarily agreed to by the holder of the claim.

> *MS (Merciless Satan).* Satan is merciless; if he obtains a claim over someone, he will never choose to release that claim without payment.

> *SB (Sin's Bondage).* Sin is the voluntary rejection of God's will in favor of Satan's authority, allowing Satan a claim which, if self-paid, enables Satan to require that death must occur—death which he will not allow to be reversed; this nonreversal implies death cannot be repeated and also nullifies the opportunity for eternal life.

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Because principle SB implies that an endless death occur which cannot be reversed if one self-pays for one's own sins, thus preventing eternal life, *principle SB implies principle ED*.

Consider the implication of choosing to sin. Because of the Father's inherent love, He is willing to extend mercy subject to the condition of repentance as required by justice. However, by sinning, the child has voluntarily subjected himself to Satan's authority and claim (principle SB), which claim Satan will never choose to relinquish without payment (principle MS). Even though justice allows the Father to grant mercy to a repentant child, He still could not justly do so without satisfying payment to release Satan's claim over the child (principle PJ).[10] Therefore, *in order to justly extend mercy, not only repentance but also payment for sin must occur* (see 1 Corinthians 6:20, 7:23; and 1 Peter 1:18–19).

Summarizing the above two implications gives the next basic theorem:

> *Theorem (Sin's Bondage to Satan)*. SB implies ED, and together CJ, MS, and SB imply PJ. That is, sin's bondage to Satan implies the eternal death principle which nullifies eternal life if sins are self-paid. Furthermore, all three principles together imply that in order to receive eternal life payment must occur for any sins committed.

4. *Application to the Sinless Messiah Implication*

Section II presented four principles—PJ, ED, SE, SR—and then derived the necessity of a sinless atonement from these principles. Given this first implication, Section III has proceeded in the opposite direction to show that principles PJ and ED are explained by three other principles—CJ, MS, SB. These two implications together mean that principles CJ, MS, and SB can be substituted for principles PJ and ED. When combined with substitution principles SE and SR, the basic Sinless Messiah Implication still follows.

Thus, the core doctrine of Christianity has been traced back to five principles about claim justice (CJ), merciless Satan (MS), sin's bondage (SB), substitution equivalence (SE), and substitution

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[10]Some have argued (especially in the Catholic tradition following Anselm) that there is no legitimacy in God's having to pay or in any sense bargain with the devil. However, this argument misses the issue, which concerns the nature of sin.

Justice must be completely impartial. Thus, in deciding the validity of a claim, it can consider only whether the claim was knowledgeably chosen without coercion, irrespective of the identity of those involved. And remember also that sin intrinsically involves the voluntary and knowledgeable rejection of God's own will in favor of Satan's authority. Neither God's righteousness nor Satan's evilness changes this essential fact. Thus, Satan's claim cannot be justly ignored simply because God is good but Satan is evil.

responsibility (SR). This implication is summarized in the following corollary to the above two theorems:

Corollary (Sinless Atonement Necessary to Redeem from Satan). Principles CJ, MS, and SB together with SE and SR imply the only way anyone who has sinned can be justly redeemed from Satan's control in order to receive eternal life is for a sinless person to help pay for the sins of others.

The corollary underscores the significance of a number of related scriptures which interrelate the Atonement with resurrection and the Final Judgment (for example, Alma 42:23; 11:41–45; 34:22; 21:9; and 2 Nephi 2:8). These scriptures indicate that the *direct* effect of the Atonement is the occurrence of resurrection. Resurrection releases sin's bondage of death, enabling the sinner to be brought into God's presence for judgment of his repentance (2 Nephi 9:22–23). This judgment determines the degree of reward, which ranges from outer darkness to celestial glory (see D&C 76).

Thus, the Atonement is necessary for mercy because it justly enables resurrection from an otherwise eternal death to occur, resurrection which then permits God to justly extend mercy according to his judgment of an individual's repentance.

III. CONCLUSION: A PARABLE OF THE ATONEMENT

There was a great king with vast dominion and power. The king was righteous and would not use his power unjustly.

One of the king's sons came to him and asked for his inheritance, and the king did according to his son's will. And the son shortly thereafter left for a faraway land and there wasted his inheritance in riotous living. In the process the son foolishly chose to indebt himself in pursuit of worldly pleasures. The creditor, who was an evil and unmerciful enemy to the king, desired to enslave the king's son through insolvency. Finally, the creditor demanded payment, knowing the law justly permitted him to take an insolvent debtor as a slave to pay for the debt.

Having no more funds and facing enslavement, the son came to himself and said, ''Even the lowest servant in my father's palace is better off than a slave to this evil creditor. If only I could return to my father and say to him, 'Father, I have sinned against heaven and before Thee and am no more worthy to be called Thy son. Make me as the lowest of Thy servants!' ''

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Now, the king had wisely foreseen his son's predicament. And the king still loved his son and desired to be merciful while still honoring justice, which the king could not deny. He thus said, "If my son will repent, I shall receive him back into my palace. Yea, if he is thereafter valiant unto my wishes, all that was to have been his destiny as my son shall be restored unto him."

But as the king had foreseen, even if his son should repent, his son still could not return if the creditor's demand for payment were not satisfied. Although the king had great power sufficient to forcibly reclaim his son without payment, to do so would have been unjust, and the king would not violate justice even to be merciful to his son.

Thus, the king pondered whether someone else could justly pay the son's debts. After considering all possibilities, he realized the only just way was to send another son who had not sinned to pay the debt. And the king asked for a volunteer, and his eldest son came forth and was commissioned to go and make payment to the creditor; and to return and bring word of the younger son's repentance.

The evil creditor also attempted to lure the elder son into debt, but he refused the creditor's every offer, and was able to pay the debt of his younger brother.

And when the elder son came to his erring brother, the brother fell before him and said, "I have sinned against heaven and in my father's sight, and am no more worthy to be called his son. If only I could be as the lowest of my father's servants."

But the elder brother said to him, "I have been sent by my father to pay your debts that you may be released from bondage. But according to justice, you cannot return to my father's palace, except that you repent of your sins. Nevertheless, rejoice, for your father loves you. Yea, he will exercise mercy according to justice, and receive you back into his palace if you will repent. If you are henceforth valiant in obeying my father, all honor and dominion that was to be yours shall be restored unto you. Yea, not as a servant, but as a prince and king can still be your destiny!"

The younger son, upon hearing this message, vowed to obey his father's will and never again to sin. Where there had been despair there was now hope, and he said, "This must be true, for why else should my righteous brother be sent to release me from bondage."

As he journeyed toward his father's palace, he was again offered of the creditor more funds to spend on worldly pleasures. But strengthened by his elder brother's message, he denied the creditor's every offer.

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The elder son watched from a distance and saw his younger brother's steadfast refusal. And, he sent word to the king's guards to admit his younger brother into the palace, and went forth to receive him at the palace gates.

Together they went up to the king's chamber and the eldest son told of his younger brother's steadfast refusal. Upon hearing this, the king greatly rejoiced, and ran to his younger son, and fell on his neck, and kissed him. And the father said, ''Bring forth the best robe, and put it on him; and put a ring on his hand, and shoes on his feet: and bring hither the fatted calf, and kill it; and let us eat, and be merry: for this my son was dead, and is alive again; he was lost, and is found'' (Luke 15:22–24).

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Appendix: Formal Statement

The main body of the article presented and intuitively demonstrated the sinless Messiah implications. However, to do so, a number of conceptual and technical difficulties were ignored. Thus, the intuitive arguments do not rigorously prove these results. I will now use formal methods and set theory to precisely define and prove them.

Given the extensive prior interpretation, only minimal further discussion is provided. However, to facilitate understanding, the above literary versions of the principles and atonement implications are restated along with their formal versions.

NOTATION AND DEFINITIONS

U is a finite set representing the universe of individuals to which the plan of salvation applies. A is the set of all *potentially* achievable or *realizable activities* for individuals in U. The notion of "activities" is interpreted broadly as any mental, physical, or spiritual phenomena potentially capable of being received, accomplished, or experienced by an individual. Elements of A could include, for example, the emotion of love, lust for power, riding a bicycle, tasting or eating some food, seeing or perceiving something, running, jumping, listening, etc.

Since A refers to potential activities, they may or may not be actually realized or experienced by someone. The latter concept of *actually realized activities* is specified by defining, for each $y \in A$, the set of realized occurrences of that potential activity for person i, denoted $f_i(y)$. A particular occurrence of y for person i is denoted $\mathscr{y}_i \in f_i(y)$, so that a script letter always refers to realized occurrences of potential activities, which are correspondingly denoted with the associated nonscript letter. Thus, if $y \in A$ is the potential act of jumping, $\{\mathscr{y}_i, \mathscr{y}_i'\} \subset f_i(y)$ are two specific occurrences of jumping by person i. Then, the set of realized occurrences for all potential activities $y \in A$ is denoted $\mathscr{A}_i = \cup_{y \in A} f_i(y)$.

Now, the basic structure of the analysis concerns the relationship between what is potentially attainable and what is actually realized by an individual. The ultimate objective is the achievement of *eternal life,* and the set of persons who achieve eternal life is denoted by E.

It will also be important to consider the set of potential activities which are *not* realized for person i, denoted $\tilde{A}_i = \{y \in A \mid f_i(y) = \Phi\}$. In general, $\sim$ above a variable will always refer to the nonoccurrence of potential activities. Thus, *nonoccurrence* of a particular potential activity y for person i is denoted $\tilde{y}_i$, and from the definition of $\tilde{A}_i$, $\tilde{y}_i \in \tilde{A}_i$ if and only if $f_i(y) = \Phi$.

Both realized and nonrealized potential activities are explicitly considered because the attainment of eternal life may require that certain activities occur or not occur. For example, one *must* be baptized, and one *must not* commit the unpardonable sin against the Holy Ghost. Another key example relates to Satan's claim over one who sins, as we discussed above. Recall that unless that claim is

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somehow paid for, Satan will both require a separation of body and spirit to occur and not permit its reversal to occur, so that eternal life cannot be achieved.

We thus see that *achieving eternal life crucially relates to the occurrence or non-occurrence of certain potential activities.*

To consider such requirements, we first need to specify those activities which someone has either realized or not realized. Thus, let $\mathscr{A}_K = \cup_{i \in K}\mathscr{A}_i$, and $\tilde{A}_K = \cup_{i \in K}\tilde{A}_i$, for $K \subset U$, be the union of realized and nonrealized activities for persons K. Applied to everyone, they are $\mathscr{A}_U$ and $\tilde{A}_U$, which for notational simplicity are denoted simply as $\mathscr{A}$ and $\tilde{A}$. Elements of these are denoted as $\mathscr{y}$ and $\tilde{y}$ and represent respectively: a realized activity for some person in U, or a nonrealized potential activity for someone in U.

Also, let the union of realized and nonrealized activities be denoted $\bar{A} = \mathscr{A} \cup \tilde{A}$, with $\bar{y} \in \bar{A}$ representing either $\mathscr{y} \in \mathscr{A}$ or $\tilde{y} \in \tilde{A}$.[11] In general, a bar ¯ over a variable will always signify that the variable refers to either realized or nonrealized activities. For example, applied to person i we have $\bar{y}_i \in \bar{A}_i = \mathscr{A}_i \cup \tilde{A}_i$, where $\bar{y}_i$ represents either $\mathscr{y}_i \in \mathscr{A}_i$ or $\tilde{y}_i \in \tilde{A}_i$. With this convention, we can conveniently refer to both realized or nonrealized activities at the same time.

Now, besides knowing what potential activities have been realized or not realized, we also need to know which of these realized or nonrealized actions are necessary for a specific person to achieve eternal life. Thus, define the relations 0 and $\tilde{0}$ which relate the occurrence or nonoccurrence of activities to the achievement of eternal life for a person. That is, $\mathscr{y}0i[\tilde{y}\tilde{0}i] \Leftrightarrow \mathscr{y}[\tilde{y}]$ must occur (not occur) for someone in U if eternal life is realized for person i. To refer to both types of requirements, let $\bar{y}\bar{0}i$ mean either $\mathscr{y}0i$ or $\tilde{y}\tilde{0}i$.

We can then define the set of *necessary occurrences or nonoccurrences* which must be satisfied for person i to achieve eternal life; that is, $\bar{N}_i = \{\bar{y} \mid \bar{y}\bar{0}i\}$. For example, if $b \in A$ is potential baptism, then for person i to receive eternal life, there must be someone who performs a particular instance of baptism, $\mathscr{b} \in \mathscr{A}$, which enables eternal life to be realized for person i (i.e., $\mathscr{b}0i$).

The next step is to realize that such requirements might not automatically happen. Other activities may also have to occur for such requirements to be accomplished. For example, if baptism by immersion must occur, then a baptismal font must be filled with water to use it for baptism, and in addition one's arm must not poke out of the water during the act of immersion.

To do so, define the relations R^K_Z and $\tilde{R}^K_Z$, which relate the satisfying of occurrences or nonoccurrences, Z, to the person for whom Z is satisfied. That is, $\mathscr{y}R^K_Zi[\tilde{y}\tilde{R}^K_Zi] \Leftrightarrow \mathscr{y}[\tilde{y}]$ must occur (not occur) for someone in K, if persons K enable $Z \subset \bar{A}$ to be satisfied for person i; and let $\bar{y}\bar{R}^K_Zi$ mean either $\tilde{y}\tilde{R}^K_Zi$ or $\mathscr{y}R^K_Zi$.

Also assume a monotonicity condition, that activities necessary to satisfy some requirements are also necessary to satisfy those requirements when part of a larger set of requirements. That is, if $X \subset Y$, then $\bar{y}\bar{R}^K_Xi \Rightarrow \bar{y}\bar{R}^K_Yi$.

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[11]Note that different $\bar{y}, \bar{z} \in \bar{A}$ can refer to different persons. For example, $\bar{y}$ could refer to $\bar{y} = \mathscr{y}_l \in \mathscr{A}_l$ and $\bar{z}$ to $\bar{z}_q \in \tilde{A}_q$.

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Now, using relation $\bar{R}^K{}_z$ we can also define the activities which someone must do or not do in order that necessary occurrences or nonoccurrences be satisfied for person i; that is, $\bar{V}_i(X) = \{\bar{y} \mid \bar{y}\bar{R}^U{}_X i\}$, for $X \subset \bar{N}_i$.

The nature of $\bar{V}_i(X)$ is unspecified as to what phenomena or individuals might be involved, other than signifying which individual's requirements would be satisfied. Nevertheless, it is crucial *who* might be involved in accomplishing an individual's requirements. Is it only himself, or can others help or substitute for the individual? Of special importance are those requirements a person accomplishes on his own without the help of others. For example, as we discussed, if a person does not receive help from others to satisfy payment for his sins, then Satan will require both death to occur and resurrection not to occur.

Thus, define those occurrences or nonoccurrences for persons in U which person i must do or not do himself in order to *self-satisfy* necessary requirements for him to achieve eternal life. That is, define $\bar{T}_i(X) = \{\bar{y} \mid \bar{y}\bar{R}^i{}_X i\}$.

We are now ready to present the principles intuitively discussed above. Given recognition that both occurrences and nonoccurrences need to be dealt with, they must also apply to both cases and are so stated.

PRINCIPLES OF SUBSTITUTION

Let the *intersection* of necessary occurrences or nonoccurrences for persons $K \subset U$ be denoted $\bar{N}^K = \cap_{i \in K} \bar{N}_i$, and similarly define the *union* of $\bar{A}_i$ as $\bar{A}_K = \cup_{i \in K} \bar{A}_i$.

Axiom SE (Substitution Equivalence)

If $X \subset \bar{N}^K$ and $i \in K$, then $\bar{V}_i(X) \subset \bar{A}_K \Rightarrow \bar{V}_i(X) = \bar{T}_i(X)$.

If a group of persons are all subject to the same occurrences or nonoccurrences, X, then members of the group can help another in the group satisfy those requirements only by doing or not doing the same things that individual would have to do or not do to satisfy these requirements for himself.

Let $\bar{Q}^{ij}{}_{XY} = \bar{A}_i \cap \bar{T}_i(X) \cap \bar{V}_j(Y)$ denote those realized or nonrealized actions of person i which he would have to do or not do to self-accomplish requirements X for himself and which are also used to help satisfy requirements Y for person j.

Axiom SR (Substitution Responsibility)

If $i \neq j$, $X \subset \bar{N}_i$, $Y \subset \bar{N}_j$, then:

$\bar{Q}^{ij}{}_{XY} \neq \phi \Rightarrow \bar{V}_i(X) \subset \bar{V}_j(Y)$; and $\bar{Q}^{ij}{}_{XY} \cap \bar{V}_i(X) = \phi$.

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Actions or nonactions which would be necessary to self-accomplish requirements X for person i, if realized or not realized, cannot be used to help satisfy requirements Y for person j without actions or nonactions necessary to accomplish X also being satisfied; and such actions or nonactions of i to help satisfy Y for person j must be separate from those actions or nonactions which satisfy X for person i.

Or as we could more intuitively state, such as above in Section I:

In order to substitute for someone else, actions or nonactions necessary to satisfy one's own requirements must be met; and actions or nonactions necessary to self-accomplish one's own requirements must be separate from those actions used to help satisfy another person's requirements.

PRINCIPLES OF JUSTICE, DEATH, AND NON-RESURRECTION

Let $S_i \subset \bar{A}_i$ be sinful actions or nonrealized actions whose omission is sinful,[12] for person i. Also, let $\bar{P}_i(S_i) = \mathscr{P}_i(S_i) \cup \tilde{P}_i(S_i)$ be the punishment or payment for sins S_i, punishment or payment which may either be requirements that certain things be done or restrictions that certain actions not be realized. We note that the intuitive and practical meaning of punishment is often a prohibition from being able to do certain things—hence, the typical notion of punishment as some type of imprisonment.

Axiom PJ (Punishment Justice)

If $S_i \neq \phi$, then $\bar{P}(S_i) \subset \bar{N}_i$.

If any sins are committed by a person, then payment for those sins is necessary for him to achieve eternal life.

Now, let $d \in A$ represent the activity of death which if realized separates a person's body and spirit, and $r \in A$ be the reverse activity of resurrection which reunites them. If both d occurs ($\mathscr{d}_i \in \mathscr{A}_i$) and r does not occur ($\tilde{r}_i \in \tilde{A}_i$), then eternal life, which requires a union of spirit with body, cannot be achieved.

Axiom ED (Eternal Death)

There are $\phi \neq p \subset A$; $d, r \in A$ such that if $S_i \neq \phi$, then $p \subset \bar{P}(S_i)$, $\{\mathscr{d}, \tilde{r}\} \subset \bar{T}_i(p)$ where: $\tilde{r} = \tilde{r}_i \in \tilde{A}_i \Rightarrow \mathscr{d}_i = \mathscr{d}_i'$ for all $\mathscr{d}_i, \mathscr{d}_i' \in \mathscr{A}_i$; and $\{\mathscr{d}_i, \tilde{r}_i\} \subset \bar{A}_i \Rightarrow i \notin E$.

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[12] S_i is not denoted with a bar $\bar{S}_i$ to aid visual readability of the subsequent formulas.

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All sin requires payment which, if self-accomplished—that is, without the help of others—implies an endless death occur that cannot be reversed, which nonreversal implies death cannot be repeated and also nullifies the opportunity for eternal life.

These principles imply the following result about the necessity of there being some person who has not sinned, denoted $U^* = \{i \in U \mid S_i = \phi\}$.

Theorem 1 (The Sinless Messiah)

Axioms PJ, ED, SE, SR imply that unless $U^* \neq \phi$, then $E = \phi$; and for all $i \in U\text{-}U^*$, $\bar{V}_i[\bar{P}(S_i)] \subset \bar{A}_{U\text{-}U^*} \Rightarrow i \notin E$. Furthermore, if $E\text{-}U^* \neq \phi$, then there exists nonempty $\bar{P}^* \subset \bar{A}_{U^*}$ such that for all $i \in E\text{-}U^*$, $\bar{V}_i[\bar{P}(S_i)] \cap \bar{P}^* \neq \phi$.

Axioms PJ, ED, SE, SR imply that unless someone has never sinned, then no one can receive eternal life; and no person whose sins are paid for by those who have sinned can obtain eternal life. Furthermore, if any sinful person achieves eternal life, there must be punishment suffered by one who has never sinned, punishment which applies toward payment of any such person's sins.

PRINCIPLES RELATED TO THE NATURE OF SIN

Let $\bar{C}_j(Y_i) = \{\bar{y} \mid \text{person } j \text{ can require } \bar{y} = \bar{y}_i \in \bar{A}_i \text{ because of } Y_i\}$, for $Y_i \subset \bar{A}_i$, be those occurrences or nonoccurrences person j can require of person i because of claims over i conveyed to j by actions or nonactions Y_i. Punishment or payment are those occurrences or nonoccurrences which someone in U must satisfy to justly relinquish claims person j has on person i. That is, define the following relations: $\mathcal{y}W_z i[\bar{y}\tilde{W}_z i] = \mathcal{y}[\bar{y}]$ must occur (not occur) for someone in U, in order to justly release claims Z on person i, and as before let $\bar{y}\bar{W}_z i = \mathcal{y}W_z i$ or $\bar{y}\tilde{W}_z i$. Then define payment for claims $\bar{C}_j(Y_i)$ as $\bar{P}_j(Y_i) = \bar{P}[\bar{C}_j(Y_i)] = \{\bar{y} \mid \bar{y}\bar{W}_z i \text{ for } Z = \bar{C}_j(Y_i)\}$.

Axiom CJ (Claim Justice)

If $\bar{C}_j(Y_i) \neq \phi$, then j can require $\bar{C}_j(Y_i) \subset \bar{A}_i$, unless $\bar{P}_j(Y_i) \subset \bar{A}$.

Person j can require his claims on person i be fulfilled, unless just payment for those claims is satisfied.

Also, let Satan be denoted by $s \in U$, and recall that sin conveys to s claims over the sinner which enables s to require he realize or not realize certain potential activities d and r.

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Axiom SB (Sin's Bondage)

There are d, r $\in$ A such that if $S_i \neq \phi$, then $\{\mathscr{d}, \bar{r}\} \subset \bar{C}_s(S_i)$; and there is $\phi \neq p \subset A$ such that $p \subset \bar{P}_s(S_i)$, $\{\mathscr{d}, \bar{r}\} \subset \bar{T}_i(p)$ where: $\bar{r} = \bar{r}_i \in \bar{A}_i \Rightarrow \mathscr{d}_i = \mathscr{d}_i'$ for all $\mathscr{d}_i$, $\mathscr{d}_i' \in A_i$; and $\{\mathscr{d}_i, \bar{r}_i\} \subset \bar{A}_i \Rightarrow i \notin E$.

Sin is the voluntary rejection of God's will in favor of Satan's authority, which claim, if self-paid, enables Satan to require death occur and its reversal—resurrection—not occur, which non-resurrection implies death is permanent and cannot be repeated, and also nullifies the opportunity for eternal life.

Axiom MS (Merciless Satan)

If $\bar{C}_s(S_i) \neq \phi$, and if $P_s(S_i) \nsubseteq \bar{A}$, then s will require $\bar{C}_s(S_i) \subset \bar{A}_i$.

If Satan obtains claim over someone through that person's sins, and occurrences or nonoccurrences necessary to pay for those claims are not all satisfied for that person, then Satan will require those claims be fulfilled.

Now, these principles imply the following:

Theorem 2 (Sin's Bondage to Satan)

Principles CJ, SB, MS $\Rightarrow$ PJ, and SB $\Rightarrow$ ED.

Hence, combining theorems 1 and 2 gives the following basic corollary:

Corollary (Sinless Atonement Necessary to Redeem from Satan)

Principles CJ, MS, SB together with SE, SR imply the same conclusions as principles PJ, ED, SE, SR, except that now payment for sins is necessary to prevent s from requiring $\mathscr{d}_i \in \mathscr{A}_i$ and $\bar{r}_i \in \bar{A}_i$ for any person i such that $S_i \neq \phi$.

Principles CJ, MS, SB together with SE, SR imply that the only way anyone who has sinned can be redeemed from Satan's control in order to receive eternal life is for a sinless person to help pay for the sins of others.

PROOFS OF THE THEOREMS

To prove Theorem 1, three lemmas are needed. The first one follows directly from the monotonicity assumption on $\bar{R}^K{}_z$, and its proof is omitted.

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Lemma A If $X \subset Y \subset \bar{N}_i$, then $\bar{V}_i(X) \subset \bar{V}_i(Y)$.

Lemma B If $S_i \neq \phi$ for all $i \in K \subset U$, then $\bar{V}_i[\bar{P}(S_i)] \subset \bar{A}_K$ implies $\{\mathscr{d}_\ell, \tilde{r}_q\} \subset \bar{V}_i(p) \subset \bar{V}_i[\bar{P}(S_i)]$ for some $\{\ell, q\} \subset K$, for d, r, p of Axiom ED.

Proof:

Since $S_i \neq \phi$ for all $i \in K$, axioms ED and PJ imply $p \subset \bar{P}(S_i) \subset \bar{N}_i$ for all $i \in K$. Thus, Lemma A and the hypothesis imply $\bar{V}_i(p) \subset \bar{V}_i[\bar{P}(S_i)] \subset \bar{A}_K$. Also, since $p \subset \bar{N}_i$ for all $i \in K$, then axioms SE and ED imply there is $\{\mathscr{d}, \tilde{r}\} \subset \bar{T}_i(p) = \bar{V}_i(p)$; thus, $\{\mathscr{d}, \tilde{r}\} \subset \bar{A}_K$, which implies there must be $\{\ell, q\} \subset K$ such that $\mathscr{d} = \mathscr{d}_\ell \in \mathscr{A}_\ell$ and $\tilde{r} = \tilde{r}_q \in \bar{A}_q$. Therefore, combining results obtains $\{\mathscr{d}_\ell, \tilde{r}_q\} \subset \bar{V}_i(p) \subset \bar{V}_i[\bar{P}(S_i)]$.

Lemma C If $S_i \neq \phi$ for all $i \in K \subset U$, then $\bar{V}_i[\bar{P}(S_i)] \subset \bar{A}_K$ implies $\{\mathscr{d}_i, \tilde{r}_i\} \subset \bar{A}_i$.

Proof:

Assume the opposite that for someone in K, denoted $1 \in K$, that *(1)*: $\bar{V}_1[\bar{P}(S_1)] \subset \bar{A}_K$, but either there is no $\mathscr{d}_1 \in \mathscr{A}_1$ ($f_1(d) = \phi$), or $\tilde{r}_1 \notin \tilde{A}_1$ ($f_1(r) \neq \phi$). From (1) there are three possible cases, *(2a)*: there is $\mathscr{d}_1 \in \mathscr{A}_1$, and $\tilde{r}_1 \notin \tilde{A}_1$; *(2b)*: there is no $\mathscr{d}_1 \in \mathscr{A}_1$, and $\tilde{r}_1 \notin \tilde{A}_1$; *(2c)*: there is no $\mathscr{d}_1 \in \mathscr{A}_1$, and $\tilde{r}_1 \in \tilde{A}_1$. We first show that (1) is false for cases (2a) and (2b).

By Lemma B, p, d, r of Axiom ED satisfy, *(3)*: $\{\mathscr{d}_\ell, \tilde{r}_q\} \subset \bar{V}_1(p) \subset \bar{V}_1[\bar{P}(S_1)]$, for some $\{\ell, q\} \subset K$. Cases (2a), (2b) both imply $\tilde{r}_1 \notin \tilde{A}_1$, which implies there is $q \in K-\{1\}$ such that $\tilde{r}_q \in \tilde{A}_q$. Reindex q as 2, so we have from (3) that *(4)*: $\tilde{r}_2 \in \tilde{A}_2 \cap \tilde{V}_1(p)$. Since $S_2 \neq \phi$, then Axiom ED and the definition of $\tilde{T}_2(p)$ imply $\tilde{r}_2 \in \tilde{T}_2(p)$, which implies *(5)*: $\tilde{r}_2 \in \tilde{A}_2 \cap \tilde{T}_2(p) \cap \tilde{V}_1(p) = \tilde{Q}^{21}{}_{pp}$. Now (5) implies from Axiom SR that $\tilde{V}_2(p) \subset \tilde{V}_1(p)$, which implies by (3) that $\tilde{V}_2(p) \subset \tilde{V}_1[\bar{P}(S_1)]$. Thus, from (1) we have *(6)*: $\tilde{V}_2(p) \subset \tilde{V}_1(p) \subset \tilde{A}_K$. Finally, from SR we have *(7)*: $\tilde{V}_2(p) \cap \tilde{Q}^{21}{}_{pp} = \phi$.

Now use statements (4) through (7), remembering that 1 and 2 are different persons, to form the following inductive hypothesis. *(8a)*: $\{1,...,n\} \subset K$ are different persons; and for $i = 2,...,n$ we have

$$\tilde{r}_i \in \tilde{Q}^{i,i-1}{}_{pp} = \tilde{A}_i \cap \tilde{T}_i(p) \cap \tilde{V}_{i-1}(p) \tag{8b}.$$

$$\tilde{V}_i(p) \subset \tilde{V}_{i-1}(p) \subset \tilde{A}_K \tag{8c}.$$

$$\tilde{V}_i(p) \cap \tilde{Q}^{i,i-1}{}_{pp} = \phi \tag{8d}.$$

We next show that (8a) through (8d) imply there is still another person in K, but not in $\{1,...,n\}$, who must *not* realize r. By (8c), $\tilde{V}_n(p) \subset \tilde{A}_K$, and since $S_n \neq \phi$, Lemma B implies *(9)*: $\tilde{r}_t \in \tilde{V}_n(p)$ for some $t \in K$. We wish to show that $t \in K-\{1,...,n\}$, and to do so assume the opposite that *(10)*: for some

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$i \in \{1,...,n\}$ that $\tilde{r}_i = \tilde{r}_t \in \tilde{V}_n(p)$. Since both cases (2a), (2b) imply $\tilde{r}_1 \notin \tilde{A}_1$, then $\tilde{r}_t \notin A_1$, and thus $\tilde{r}_t \in \tilde{A}_i$ for some $i \in \{2,...,n\}$. From (10) and (8c), $\tilde{r}_i \in \tilde{V}_i(p)$, and from (8b), $\tilde{r}_i \in \tilde{Q}^{i,i-1}_{PP}$ which implies $\tilde{V}_i(p) \cap \tilde{Q}^{i,i-1}_{PP} \neq \emptyset$. But this contradicts (8d).

Thus, the assumption $t \in \{1,...,n\}$ leads to contradiction, so that there must be still another person, denoted $n+1$, such that $n+1 \in K-\{1,...,n\}$ such that $\tilde{r}_{n+1} \in \tilde{A}_{n+1} \cap \tilde{V}_n(p)$. By analogous argument to steps (4) through (7), it is implied that $\tilde{r}_{n+1} \in \tilde{Q}^{n+1,n}_{PP}$; $\tilde{V}_{n+1}(p) \subset \tilde{V}_n(p) \subset \tilde{A}_K$; and $\tilde{V}_{n+1}(p) \cap \tilde{Q}^{n+1,n}_{PP} = \emptyset$. Hence, cases (2a), (2b) imply assumption (1) leads to contradiction, because an endless sequence of different persons is implied, contradicting the finiteness of $K \subset U$, since U is finite.

Thus, the remaining possibility is case (2c) in which there is no $\mathscr{d}_1 \in \mathscr{A}_1$, and $\tilde{r}_1 \in \tilde{A}_1$. The proof is more complicated because an individual i can experience d more than once so long as $\tilde{r}_i \notin \tilde{A}_i$. In contrast, there is only one $\tilde{r}_i \in \tilde{A}_i$ corresponding to $f_i(r) = \emptyset$. However, the nature of r and d require r to occur for d to be repeated for the same person (a separation of body and spirit cannot recur unless they have been reunited so that another separation is possible). That is, $f_i(r) = \emptyset \Rightarrow f_i(d)$ has at most one element, or, as stated in Axiom ED, (11): $\tilde{r}_i \in \tilde{A}_i \Rightarrow \mathscr{d}_i = \mathscr{d}_i'$ for all $\mathscr{d}_i, \mathscr{d}_i' \in \mathscr{A}_i$. This condition enables a similar contradiction to be shown as for cases (2a), (2b), which contradiction we now show.

Now, recall from (3) that there is $\mathscr{d}_\ell \in \mathscr{V}_1(p) \subset \mathscr{V}_1[\bar{P}(S_1)]$ for some $\ell \in K$. As before, reindex ℓ as 2. Also, denote with superscripts the particular occurrence of d for person i to help satisfy requirements p for person j. Thus, $\mathscr{d}_2^1 \in \mathscr{V}_1(p)$ is the particular realization of d for 2 to satisfy p for 1. Analogous to steps (4) through (7), we can show (12): a) $\mathscr{d}_2^1 \in \mathscr{Q}^{21}_{PP} = \mathscr{A}_2 \cap \mathscr{T}_2(p) \cap \mathscr{V}_1(p)$; b) $\mathscr{V}_2(p) \subset \mathscr{V}_1(p) \subset \mathscr{A}_K$; and c) $\mathscr{V}_2(p) \cap \mathscr{Q}^{21}_{PP} = \emptyset$.

From (12b), $\mathscr{V}_2(p) \subset \mathscr{A}_K$ and since $S_2 \neq \emptyset$ then Lemma B implies there is $\mathscr{d}_t^2 \in \mathscr{V}_2(p)$ for some $t \in K$. If $t = 2$, then $\tilde{r}_2 \in \tilde{A}_2$ implies by (11) that $\mathscr{d}_2^1 = \mathscr{d}_2^2$ which then contradicts (12c) because $\mathscr{V}_2(p) \cap \mathscr{Q}^{21}_{PP} \neq \emptyset$ is implied. Hence, either $t \neq 2$ or $\tilde{r}_2 \notin \tilde{A}_2$ must hold. Three cases are possible, (13a): $t = 2$, and $\tilde{r}_2 \notin \tilde{A}_2$, (13b): $t \neq 2$, and $\tilde{r}_2 \notin \tilde{A}_2$, (13c): $t \neq 2$, and $\tilde{r}_2 \notin \tilde{A}_2$. Noting that $\mathscr{d}_2^1 \in \mathscr{A}_2$ regardless of whether $t=2$ or not means that cases (13a), (13b) are analogous to (2a), except that individual 2 is now involved rather than 1. Argument similar to (4) through (10) will then also show a contradiction is obtained.

Therefore, the remaining case is (13c), which with (2c) together imply (14a): there is no $\mathscr{d}_1 \in \mathscr{A}_1$; (14b): $\tilde{r}_1 \in \tilde{A}_1$, $\tilde{r}_2 \in \tilde{A}_2$; (14c): and $\mathscr{d}_t^2 \in \mathscr{A}_M$, where $M = K-\{1, 2\}$ so that $t \in M$. Let us outline the proof for this case.

First, form an inductive hypothesis from (12a,b,c) analogous to (8a) through (8d), except that realized occurrences of d are involved so that no $\tilde{\ }$ symbol is used, and (8b) now is $\mathscr{d}_i^{i-1} \in \mathscr{Q}^{i,i-1}_{PP} = \mathscr{A}_i \cap \mathscr{T}_i(p) \cap \mathscr{V}_{i-1}(p)$. Let us denote these as (8a)* through (8d)* respectively. In addition, from (14b) we can add the following hypothesis, denoted (8e)*: $\tilde{r}_i \in \tilde{A}_i$ for all $i = 1,...,n$. And, analogous to (9), (10) above, assume (15): for some $i \in \{1,...,n\}$ that there is $\mathscr{d}_t^n = \mathscr{d}_i^n \in \mathscr{V}_n(p)$, so that $t \in \{1,...,n\}$.

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From (14a) we know $\mathscr{A}_i^n \notin \mathscr{A}_1$, which implies from assumption (15) that $\mathscr{A}_i^n \in \mathscr{A}_i$ for some $i \in \{2,...,n\}$. From (11) and (8e)* we have *(16)*: $\mathscr{A}_i^n = \mathscr{A}_i^{i-1}$. Also, assumption (15) with (8c)* implies $\mathscr{A}_i^n \in \mathscr{V}_i(p)$, and from (8b)* $\mathscr{A}_i^{i-1} \in \mathscr{D}^{i,i-1}_{pp}$. But these last two results, with (16), imply $\mathscr{V}_i(p) \cap \mathscr{D}^{i,i-1}_{pp} \neq \phi$, which contradicts (8d)*. This contradiction implies assumption (15) is false and thus $t \in K$ through $\{1,...,n\}$ which extends (8a)* to $n+1$. Argument similar to (4) through (7) also extends (8b)* through (8d)* to $n+1$, and argument similar to (13a,b,c) through (14b) also extends (8e)* to $n+1$. Thus, an endless sequence of different persons is again implied, which again contradicts the finiteness of U.

Hence, all three cases (2a), (2b), (2c) lead to contradiction, so that assumption (1) is false. This proves the lemma.

With Lemma C we can now prove *Theorem 1.*

First assume $S_i \neq \phi$ for all $i \in U$ so that $U^* = \phi$. Since the definition of $\bar{V}_i$ implies $\bar{V}[\bar{P}(S_i)] \subset \bar{A}$, then substituting $U = K$ into Lemma C implies $\{\mathscr{A}_i, \tilde{r}_i\} \subset \bar{A}_i$ for all $i \in U$, which from ED implies $i \notin E$ for all i. Hence, $E = \phi$. Therefore, unless $U^* \neq \phi$, $E = \phi$ is implied. Then, substituting $U-U^*$ for K of Lemma C similarly implies that if $\bar{V}_i[\bar{P}(S_i)] \subset \bar{A}_{U-U^*}$ for $i \in U-U^*$, then $i \notin E$. Thus, if $i \in E-U^*$, it must be that $\bar{V}_i[\bar{P}(S_i)] - \bar{A}_{U-U^*} \neq \phi$, which is denoted P^i. Then, $P^i \subset \bar{A}_{U^*}$ because $\bar{V}_i[\bar{P}(S_i)] \subset \bar{A} = \bar{A}_{U^*} \cup \bar{A}_{U-U^*}$; and let P^* be the union of P^i for all $i \in E-U^*$. Thus, P^* has the requisite properties, and we are done.

Now let us prove *Theorem 2.*

If $S_i \neq \phi$, then by SB, $\{\mathscr{A}, \tilde{r}\} \subset \bar{C}_s(S_i) \neq \phi$, which by CJ and MS implies person s will require $\{\mathscr{A}_i; \tilde{r}_i\} \subset \bar{A}_i$, unless $\bar{P}_s(S_i) \subset \bar{A}$. Since $\{\mathscr{A}_i, \tilde{r}_i\} \subset \bar{A}_i \Rightarrow i \notin E$ by SB, then the definition of $\bar{0}$ implies $\bar{y}\bar{0}i$ for all $\bar{y} \in \bar{P}_s(S_i)$.

Hence, from the definition of $\bar{N}_i$, we have $S_i \neq \phi \Rightarrow \bar{P}_s(S_i) \subset \bar{N}_i$, which is principle ED with $\bar{P}_s(S_i)$ substituted for $\bar{P}(S_i)$ [i.e., s denotes to whom the payment applies]. Similar substitution immediately implies that ED follows from SB.

Finally, the corollary immediately follows from theorems 1 and 2 since SB, CJ, MS imply PJ, ED; and PJ, ED, SE, SR imply the conclusions to Theorem 1. The only difference is one of the meaning of the hypothesis, that a sinless atonement is necessary to prevent s from requiring $\{\mathscr{A}_i, \tilde{r}_i\} \subset \bar{A}_i$ for any $i \in U-U^*$.

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