Cooperative Control of Miniature Air Vehicles

Derek R. Nelson

Brigham Young University - Provo

Follow this and additional works at: https://scholarsarchive.byu.edu/etd

Part of the Mechanical Engineering Commons

BYU ScholarsArchive Citation

https://scholarsarchive.byu.edu/etd/1095
COOPERATIVE CONTROL OF MINIATURE AIR VEHICLES

by

Derek Rich Nelson

A thesis submitted to the faculty of Brigham Young University in partial fulfillment of the requirements for the degree of

Master of Science

Department of Mechanical Engineering

Brigham Young University

December 2005
This thesis has been read by each member of the following graduate committee and by majority vote has been found to be satisfactory.

Date ____________________________  Timothy W. McLain, Chair

Date ____________________________  Randal W. Beard

Date ____________________________  Daryl O. Snyder
As chair of the candidate’s graduate committee, I have read the thesis of Derek Rich Nelson in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

Date

Timothy W. McLain
Chair, Graduate Committee

Accepted for the Department

Matthew R. Jones
Graduate Coordinator

Accepted for the College

Alan R. Parkinson
Dean, Ira A. Fulton College of Engineering and Technology
ABSTRACT

COOPERATIVE CONTROL OF MINIATURE AIR VEHICLES

Derek Rich Nelson

Department of Mechanical Engineering

Master of Science

Cooperative control for miniature air vehicles (MAVs) is currently a highly researched topic. There are many applications for which MAVs are well suited, including fire monitoring, surveillance and reconaissance, and search and rescue missions. All of these applications can be carried out more effectively by a team of MAVs than by a single vehicle. As technologies for microcontrollers and small sensors have improved so have the capabilities of MAVs. This improvement in MAV performance abilities increases the possibility for cooperative missions.

The focus of this research was on cooperative timing missions. The issues faced when dealing with multi-MAV flight include information transfer, real time path planning, and maintenance of a fleet of flight-worthy MAVs. Additional challenges associated with timing missions include path following and velocity control. Two timing scenarios were studied and both of these scenarios were flight tested. The first scenario was a sequenced arrival of the MAVs over a target at a predetermined fly-through heading. The second scenario was a simultaneous arrival of the team of MAVs over a known target location. The...
ideas of coordination functions and coordination variables have been employed to achieve coordination. Experimental results verify the feasibility of real time cooperative control for a team of MAVs.

Initial cooperative timing tests revealed the need for more accurate path following. Accordingly, a new method for path following using vector fields was developed. A vector field of desired ground track headings is calculated and commanded ground track headings are calculated such that ground track heading error and lateral following error decay asymptotically even in the presence of constant wind disturbances. The utilization of ground track heading and ground speed in the path following control, in combination with the vector field methods is what makes this zero-error following possible. Methods for following straight lines and orbits as well as combinations of the lines and circular arcs are presented. The assertions that minimal following errors result when using these methods have been verified experimentally.
ACKNOWLEDGMENTS

I wish to express my gratitude to Professor Tim McLain for all his help in conducting this research and writing this thesis. Dr. McLain played a significant role in the writing of the papers that compose Chapters 2 and 3 of this thesis. Thanks are also due to all the members of the MAGICC Lab that helped so much during this process, especially Professor Randy Beard, Steve Griffiths, Blake Barber, Andrew Eldredge, and Derek Kingston. An extra expression of appreciation also needs to be given to my wife, Jessica, for her patience and understanding.
Contents

Acknowledgments vi
List of Tables ix
List of Figures xi

1 Introduction 1
  1.1 Background ........................................... 1
  1.2 Motivation ........................................... 2
  1.3 Contributions ....................................... 3
  1.4 Document Organization .............................. 4

2 Path Following 5
  2.1 Introduction .......................................... 5
  2.2 Problem Description ................................... 7
  2.3 Technical Approach .................................... 10
    2.3.1 Straight Path Following .......................... 10
    2.3.2 Orbit Following .................................. 17
    2.3.3 Straight Line and Orbit Combination ............... 23
  2.4 Results and Discussion ............................... 25
    2.4.1 Hardware Testbed ................................. 25
    2.4.2 Experimental Demonstration ....................... 25
  2.5 Conclusion ........................................... 28

3 Experimental Cooperative Timing 31
  3.1 Introduction ......................................... 31
3.2 Problem Overview .................................................. 33
3.3 Technical Approach .................................................. 34
3.4 Experimental Setup .................................................. 41
3.5 Results and Discussion .............................................. 42
3.6 Conclusion ............................................................. 46

4 Conclusions and Future Work ........................................ 47
  4.1 Conclusions .......................................................... 47
  4.2 Future Work .......................................................... 48

Bibliography ............................................................. 49
List of Tables

2.1 Variables For Straight Line Following .......................... 11
2.2 Variables For Orbit Following ................................. 19
3.1 Cooperative timing algorithm. ................................. 42
List of Figures

2.1 Vector fields for linear and circular paths. ................................. 8
2.2 Relationship between $V$, $W$, and $S$. ..................................... 9
2.3 Vector field geometry for $y > 0$. ........................................... 12
2.4 Vector field geometry for orbit tracking. ................................. 18
2.5 Line segment and arc combinations. ......................................... 24
2.6 (a) Kestrel autopilot (b) Zagi airframes (c) Ground station components ... 25
2.7 Telemetry plot for flying orbits with radii of 150, 100, 70, and 50 m. .... 26
2.8 Telemetry plot for straight line following. ................................. 27
2.9 Lateral following error during straight line following. .................. 27
2.10 Telemetry plot for equal path length following. ......................... 28
2.11 Lateral following error for combined orbit/straight line following. .... 28
2.12 Combination of equal path length and corner cutting following. ....... 29
2.13 Urban terrain following using straight line following. ................. 29
3.1 Initial coordination functions. .................................................. 37
3.2 Waypoint path example. ....................................................... 38
3.3 Waypoint path example with added waypoints. ......................... 40
3.4 Final coordination functions. .................................................. 40
3.5 (a) Kestrel autopilot (b) Unicorn airframes (c) Ground station components . 41
3.6 Simultaneous arrival - telemetry plot ...................................... 43
3.7 Simultaneous arrival - range to target .................................... 44
3.8 Fly by with 3-second desired arrival intervals ............................ 44
3.9 range to target ................................................................. 45
3.10 Fly-by scenario - path lengthened for MAV 1 ............................ 46
Chapter 1

Introduction

1.1 Background

In recent years, research in the field of unmanned aerial vehicles (UAVs), and consequently, use of them has increased dramatically as the capabilities have improved and costs have dropped. Within the next five years more than 12 billion dollars will be spent by the US military on UAV research in order to bring UAVs to the point where they can be prime components in military operations [1].

UAVs are of particular interest to the military and homeland security for a number of types of missions. One type of mission for which UAVs are particularly well suited is long-duration, surveillance type missions where a human pilot might find the task tedious, tiring, and boring. Another type is dangerous missions where the UAV might have the possibility of being shot down. Scenarios like flying reconnaissance missions over unknown terrain, border patrol, or battle damage assessment fall under these categories. Although not at the point where it is feasible, there are significant resources being placed into combat UAVs. Particularly enticing are the lower costs and lower risks for human life associated with UAVs. Larger UAVs, such as Global Hawk, Predator, and Hunter are currently being used in the Middle East and over United States borders, primarily for surveillance type operations.

For all of these scenarios, there are distinct advantages to having a team of UAVs working together to accomplish the task. Surveillance of an area can clearly be done more quickly and effectively using multiple agents. Missions in risky environments are more likely to have success if multiple agents are involved, spreading enemy resources over
more targets. The more effectively a team of UAVs communicates and works together, the greater the chances are for success.

There are also numerous commercial applications for multiple UAV missions such as forest fire surveillance, civilian search and rescue, environmental monitoring, traffic monitoring, and disaster relief [21]. Again, these applications could all be done more effectively using a team of UAVs. As UAV research continues, especially in the area of cooperative control of a system of UAVs, the capabilities of these systems will expand and the systems will become more reliable. The number of applications for UAVs will also continue to grow.

With the improvements in microcontroller, battery, and sensor technologies seen in recent years, there has been a push to improve the capabilities and effectiveness of miniature and micro air vehicles (MAVs). The low cost and versatility of these MAVs create potential for a dramatic increase in usage. There are several advantages associated with using MAVs. Low cost is an obvious one. Size and weight make them desirable for ease of transport. In general, they are less complex and easier to use than the larger UAV systems. With these advantages come a number of challenges. Reliability of components and robustness of the vehicle are areas of concern for MAVs. Range and power limitations can also be a drawback in some situations. MAVs are also much more susceptible to adverse effects from the wind since wind speeds are typically at least 10 percent of the MAV airspeed and often wind speed can reach 50 percent of the MAV airspeed or higher.

1.2 Motivation

The focus of this research was twofold. The first topic was cooperative timing experiments for MAVs. Included in this were two scenarios. The first scenario was a tightly sequenced arrival over a target where the intervals between arrivals were prespecified. The second scenario was a simultaneous arrival of the team of MAVs over the target. Initial attempts at cooperative timing revealed the necessity for the second area of focus which was to develop a robust path following method for MAVs that would be effective even in windy conditions. Because of the wind, there was a large lateral following error. This following error increased the path length and large timing errors were observed.
Much research has been done in the area of cooperative control in recent years in applications such as area searching [13, 28, 17], formation flying [9, 5], and cooperative strategies [10]. Experimental formation flying has been performed [22, 6], but in general, the research has been theoretical with simulated results. The goal for this research was to illustrate the feasibility of cooperative unmanned missions by actually flight testing the algorithms. Associated with hardware testing are a number of difficult problems including path planning and following, adverse effects on timing due to wind, sensor inaccuracies, communication strategies, maintenance of flight-worthy airframes, and algorithm implementation into the autopilot and ground station software.

Accurate path following was critical to the success of these timing missions. Differences in desired path length and actual path length due to following inaccuracies make it very difficult to arrive at the target at the desired arrival time. Even with improved tracking, it is necessary to adjust MAV velocity in order to combat path following errors and wind in order to achieve cooperation.

1.3 Contributions

The research described in this thesis presents a new method for MAV path following that will allow MAVs to follow straight lines and circular paths. It has been proven that this method will allow MAVs to follow these types of paths with asymptotically decaying errors. These path following algorithms will be beneficial in a number of applications other than cooperative timing missions, including obstacle avoidance, target imaging, and canyon following. Flight test results using coordination function and coordination variable methods to coordinate timing in the two cooperative timing scenarios are also presented. These flight tests illustrate the effectiveness of the path following methods and the feasibility of real-time coordination for a team of MAVs. A method for controlling throttle based on desired arrival times has also been developed. A more reliable and robust testbed for multi-agent MAV testing has also resulted from this research.
1.4 Document Organization

The main body of this thesis is composed of two parts found in Chapters 2 and 3. Chapter 2 describes the development and stability proofs for the path following algorithms along with experimental results demonstrating their effectiveness. Chapter 3 describes the methods used for the cooperative timing missions and presents the experimental results. Both Chapters 2 and 3 are taken from paper drafts being prepared for publication. They are both in a format that can stand alone and independent of the rest of the thesis. The abstracts and references have been moved to be included with those of the thesis.
Chapter 2

Path Following

2.1 Introduction

Unmanned aerial vehicles (UAVs), large and small, have demonstrated their usefulness in military applications. Furthermore, there are numerous potential uses for UAVs in civil and commercial applications and the prospects for broad impact are strong. To extend the usefulness of UAVs beyond their current applications, the capability to plan paths and to follow them precisely is of great importance. Unlike piloted vehicles, which rely on the pilot to navigate over demanding terrain or to avoid obstructions, UAVs rely on automation to provide this functionality. As applications such as urban surveillance and rural search and rescue require UAVs to fly down city streets surrounded by buildings or near the surface of abruptly changing mountainous terrain, the ability to follow pre-planned paths with precision is essential. For missions involving cooperation among a team of UAVs, precise path tracking is often crucial to achieving the cooperation objective.

For miniature aerial vehicles, such as those of primary interest in this work, wind disturbances, dynamic characteristics, and the quality of sensing and control all limit the achievable tracking precision. For MAVs wind speeds are commonly 20 to 60 percent of the desired airspeed. Effective path tracking strategies must overcome the effect of this ever present disturbance. For most fixed-wing MAVs, the minimum turn radius is in the range of 10 to 50 m. This places a fundamental limit on the spatial frequency of paths that can be tracked. Thus, it is important that the path tracking algorithms utilize the full

\(^1\)We consider miniature aerial vehicles to be those with wingspans in the 0.3 m to 2 m range and micro aerial vehicles to be those with wingspans under 0.3 m. Here the abbreviation MAV denotes miniature aerial vehicle.
capability of the MAV. Finally, high-resolution state sensors with high-frequency updates are not typically available for MAVs. Successful tracking approaches must exploit fully those sensors that are readily available.

Several approaches have been proposed for UAV trajectory tracking. An approach for tight tracking of curved trajectories is presented in [22]. For straight-line paths, the approach approximates PD control. For curved paths, an additional anticipatory control element that improves the tracking capability is implemented. The approach accommodates the addition of an adaptive element to account for disturbances such as wind. This approach is validated with flight experiments.

In [15], Kaminer et al. describe an integrated approach for developing guidance and control algorithms for autonomous vehicle trajectory tracking. Their approach builds upon the theory of gain scheduling and produces controllers for tracking trajectories that are defined in an inertial reference frame. The approach is illustrated through simulations of a small UAV.

Implicit in the notion of trajectory tracking is that the vehicle is commanded to be in a particular location at a particular time and that this location typically varies in time, thus causing the vehicle to move in the desired fashion. With fixed-wing MAVs, the desired position is constantly moving (at the desired air speed). The approach of tracking a moving point can result in significant problems for MAVs if disturbances, such as those due to wind, are not accounted for properly. If the MAV is flying into a strong wind (relative to its commanded ground speed), the progression of the trajectory point must be slowed accordingly. Similarly, if the MAV is flying down wind, the speed of the tracking point must be increased to keep the MAV from overrunning the desired position. Given that wind disturbances vary and are often not easily predicted, trajectory tracking can be very challenging in anything other than calm conditions.

Rather than pursuing the trajectory tracking approach, this research explores path following where the objective is to be on the path rather than at a certain point at a particular time. With path following, the time dependence of the problem is removed. In [2, 3], performance limits for reference-tracking and path-following controllers are investigated and the difference between them is highlighted. It is shown that there is not a fundamental
performance limitation for path following for systems with unstable zero dynamics as there is for reference tracking.

Building on the work presented in [14] on maneuver modified trajectory tracking, Encarnação and Pascoal develop an approach that combines the features of trajectory tracking and path following for marine vehicles [12]. Similar to this work is that of Skjetne, et al. [27] which develops an output maneuvering method composed of two tasks: forcing the output to converge to the desired path and then satisfying a desired speed assignment along the path. The method is demonstrated using a marine vessel simulation. Ref. [24] presents a path following method for UAVs that provides a constant line of sight between the UAV and an observation target.

The work presented in this paper builds on the concept of path following through the construction of vector fields surrounding the path to be followed. The vectors of the fields provide heading commands to guide the MAV toward the desired path. As with other path following methods, the objective is not to track a moving point, but to get onto the path while flying at a prescribed airspeed. A unique contribution of this paper is the utilization of ground track heading in the path following control, which in combination with the vector field strategy, guarantees that following errors asymptotically approach zero even in the presence of constant wind disturbances. Implementation of the approach is feasible on small MAVs and experimental results validate the potential value of the approach for MAVs flying in windy conditions.

2.2 Problem Description

The objective of this research is to develop a method for accurate path following for small MAVs in the presence of wind. The method calculates a vector field around the path to be tracked. The vectors in the field are directed toward the path to be followed and in the desired direction of flight. The method is currently applicable to paths composed of straight lines and arcs. This restriction is insignificant for most practical applications. Figure 2.1 shows examples of vector fields for linear and circular paths.

The notion of vector fields is similar to that of potential fields, which have been widely used as a tool for path planning in the robotics community (see e.g., [16]). It has
also been suggested in [26] that potential fields can be used in UAV navigation for obstacle and collision avoidance applications. The method of [26] provides a way for groups of UAVs to use the gradient of a potential field to navigate through heavily populated areas safely while still aggressively approaching their targets. Vector fields are different from potential fields in that they do not necessarily represent the gradient of a potential. Rather, the vector field simply indicates a desired direction of travel.

This paper considers the navigation of a fixed-wing MAV with the assumption that altitude and airspeed ($V$) are held constant (or nearly so) by the control of the longitudinal dynamics. The following is a simple model of the navigational dynamics that will be used to study the path following behavior of the proposed approach:

\[
\dot{x} = V \cos \psi + W_x \\
\dot{y} = V \sin \psi + W_y
\]  

(2.1)  

(2.2)

where $(W_x, W_y)$ represent the $x$ and $y$ components of the wind velocity. Heading ($\psi$) will be controlled by the vector field path following approaches presented in this paper. An alternative representation of these equations can be developed by noting the relationship
between groundspeed ($S$), airspeed ($V$), and wind speed ($W$) as depicted in Figure 2.2:

$$\dot{x} = V_x + W_x = S_x \quad (2.3)$$
$$\dot{y} = V_y + W_y = S_y. \quad (2.4)$$

![Diagram showing the relationship between $V$, $W$, and $S$.](image)

Figure 2.2: Relationship between $V$, $W$, and $S$.

Drawing on Equations (2.3) and (2.4) and the definition of groundtrack heading ($\chi$) shown in Figure 2.2, Equations (2.1) and (2.2) can be expressed as

$$\dot{x} = S \cos \chi \quad (2.5)$$
$$\dot{y} = S \sin \chi. \quad (2.6)$$

The key distinction is that the equations of motion are expressed in terms of groundspeed and ground-track heading and are independent of the wind velocity. When ground-based measurements are used in conjunction with the vector field approach to control the path
of the vehicle, wind-disturbance rejection will be improved dramatically, which is vitally important for small, low-speed MAVs.

In the development and analysis of the vector field approach that follows, two primitive path types are considered: straight lines and circular orbits. Circular arcs are treated similarly to complete orbits. The approach is easily extended to paths composed of multiple segments of arcs, orbits, and straight lines.

2.3 Technical Approach

2.3.1 Straight Path Following

Vector Field Construction

Consider the straight-line path illustrated in the first frame of Figure 2.1 by the solid line segment. In order to follow this path, a vector field of desired ground track headings is constructed. When the MAV is far away from the line (lateral distance greater than 2 to 3 times the minimum turn radius) the objective is to fly toward the path. As the MAV approaches the path, the desired heading transitions from approaching the path to flying along the path. The transition region around the path is indicated by dashed lines which lie at a distance $\tau$ on each side of the path. Outside the transition region, the desired heading or entry angle, $\chi^e$, is constant. Once inside the transition region, the desired heading begins to transition from $\chi^e$ to the heading along the desired path, $\chi^d$. The rate of transition is controlled by the gain, $k$.

A complete list of the variables used for the straight line following algorithm is shown in Table 2.1. The navigational dynamics found in Equations (2.5) and (2.6) were used in the development of the algorithm and in the stability proof for straight line following presented in Section 2.3.1, and an outline of the vector field algorithm can be found in Algorithm 1. The basic idea is to find where the MAV is inside the vector field and then command a heading that will result in the MAV matching the desired heading as defined by the field. The parameters $\tau$, $\chi^e$, and $k$ can be tuned, based on the capabilities of the MAV, in order to achieve the desired performance.
Table 2.1: Variables For Straight Line Following

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^f$</td>
<td>heading from waypoint 1 to 2</td>
</tr>
<tr>
<td>$s^*$</td>
<td>parameter indicating MAV progress along path, $s^* \in [0, 1]$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>lateral tracking error</td>
</tr>
<tr>
<td>$\tau$</td>
<td>threshold distance specifying when convergence begins</td>
</tr>
<tr>
<td>$w_1, w_{1x}, w_{1y}$</td>
<td>waypoint 1 and its north and east components</td>
</tr>
<tr>
<td>$\rho$</td>
<td>the side of the path that the MAV is on, having a value of $\pm 1$</td>
</tr>
<tr>
<td>$z = (x, y)^T$</td>
<td>current location of the MAV</td>
</tr>
<tr>
<td>$\chi_c$</td>
<td>commanded heading</td>
</tr>
<tr>
<td>$\chi_e$</td>
<td>entry heading angle ($0 &lt; \chi_e &lt; \frac{\pi}{2}$)</td>
</tr>
<tr>
<td>$\chi_d$</td>
<td>desired heading angle in vector field</td>
</tr>
<tr>
<td>$k$</td>
<td>transition gain, $k &gt; 1$</td>
</tr>
</tbody>
</table>

Algorithm 1 Vector Field Construction Algorithm (Constant altitude).

1: $\chi^f = \arctan2(w_2y - w_{1y}, w_{2x} - w_{1x})$
   \{ Calculate heading from waypoint 1 to waypoint 2. \}
2: $s^* = \frac{(z-w_1)^T(z-w_1)}{\|w_2-w_1\|^2}$
   \{ Calculate position of MAV along path. \}
3: $\rho = \text{sign}[(w_2 - w_1) \times (z - w_1)]$
   \{ Calculate which side of path MAV is on. \}
4: if $s^* > 1$ then \{MAV is past second waypoint\}
5: $\rho = -\rho$
6: end if
7: $\epsilon = \|z - (s^*(w_2 - w_1) + w_1)\|$
   \{ Calculate distance of MAV from path. \}
8: if $\epsilon > \tau$ then \{Distance from path is greater than threshold.\}
9: $\chi^d = \chi^f - \rho \chi_e$
10: else
11: $\chi^d = \chi^f - \rho \chi_e \left(\frac{\epsilon}{\tau}\right)^k$
12: end if

Stability Analysis

It will now be shown that Algorithm 1 enables a MAV to follow straight-line paths with asymptotically decaying error provided the MAV can produce enough thrust to yield a positive ground speed. It will first be shown that if the MAV is initially outside the transition region that the MAV will enter the the transition region. Then, once inside the transition region, it will be shown that the lateral tracking and ground track heading error will approach zero asymptotically. Lyapunov stability proofs will be used to justify these
claims. In the development of the proofs, the following first-order dynamics for heading rate will be used

\[
\dot{\chi} = \alpha (\chi^e - \chi) \tag{2.7}
\]

Without the loss of generality, consider the scenario in Figure 2.3 where \(y > 0\), \(\rho = 1\), and \(\chi^f = 0\). In this scenario, \(\epsilon = |y|\).

![Figure 2.3: Vector field geometry for \(y > 0\).](image)

**Outside Transition Region** The equation in Algorithm 1 line 9 simplifies to

\[
\chi^d = -\chi^e. \tag{2.8}
\]

Since \(\chi^d\) is a constant, let \(\chi^e = \chi^d\). Substituting this in to Equation (2.7) yields

\[
\dot{\chi} = \alpha (-\chi^e - \chi).
\]
Let
\[ \tilde{\chi} = \chi^d - \chi = -\chi^e - \chi. \]

Taking the derivative gives
\[ \dot{\tilde{\chi}} = -\dot{\chi} = -\alpha \tilde{\chi}, \quad (2.9) \]
meaning that \( \tilde{\chi}(t) = e^{-\alpha t} \tilde{\chi}(t_0) \). The ground track heading will converge exponentially to \(-\chi^e\). If \( 0 \leq \chi \leq \pi \), then \( y(t) \) will initially increase. However, as \( \chi \) exponentially approaches \(-\chi^e\), \( \chi \) will enter \((-\pi, 0)\), at which point it will remain in that set. When \( \chi \in (-\pi, 0), \dot{y} < 0 \). The cross track error will decrease until \( y = \tau \) and the MAV enters the transition region. The proof for \( y < -\tau \) is identical and the generalized expression for commanded heading is
\[ \chi^e = \chi^d = \chi^f - \rho \chi^e. \quad (2.10) \]

This leads to the following theorem:

**Theorem 2.3.1** For a MAV outside the transition region (\(|\epsilon| > \tau\)) with the navigational dynamics defined in Equations (2.5) and (2.6), the heading rate dynamics defined in Equation (2.7), and commanded heading defined in Equation (2.10), if \( \alpha > 0 \), then \( \tilde{\chi} \to 0 \) asymptotically. The MAV will therefore enter the transition region with \( \chi \in (-\pi, 0) \) for \( y > 0 \) (\( \chi \in (0, \pi) \) for \( y < 0 \)).

**Inside Transition Region** Again considering the scenario in Figure 2.3, the equation defining the vector field inside the transition region (Algorithm 1, line 11), where \( y \leq \tau \), can be simplified to
\[ \chi^d = -\chi^e \left( \frac{y}{\tau} \right)^k. \quad (2.11) \]
Inside the transition region it is necessary to show that $\tilde{\chi} \to 0$ and $y \to 0$. It will first be shown that $\tilde{\chi} \to 0$ asymptotically for all $y$ in the transition region. This fact will then be used to show conditions guaranteeing $y \to 0$ are satisfied. A suitable choice for the Lyapunov function candidate for showing $\tilde{\chi} \to 0$ is

$$V(\tilde{\chi}) = \frac{1}{2} \tilde{\chi}^2.$$ (2.12)

Taking the derivative with respect to time yields

$$\dot{V}(\tilde{\chi}) = \tilde{\chi} \dot{\tilde{\chi}}$$
$$= \tilde{\chi} (\dot{\chi}^d - \dot{\tilde{\chi}})$$
$$= \tilde{\chi} \left[ \left( \frac{-k\chi^e S}{\tau_k} \right) y^{k-1} \sin \chi - \alpha (\chi^c - \chi) \right].$$ (2.13)

Let $\chi^c = \chi^d - \nu$ so that Equation (2.13) becomes

$$\dot{V}(\tilde{\chi}) = \tilde{\chi} \left[ \left( \frac{-k\chi^e S}{\alpha \tau_k} \right) y^{k-1} \sin \chi - \alpha \tilde{\chi} + \alpha \nu \right].$$ (2.14)

Choose $\nu = \left( \frac{k\chi^e S}{\alpha \tau_k} \right) y^{k-1} \sin \chi$. Substituting $\nu$ into Equation (2.14) and the expression for $\chi^c$ gives

$$\dot{V}(\tilde{\chi}) = -\alpha \tilde{\chi}^2,$$

and

$$\chi^c = \chi^d - \left( \frac{k\chi^e S}{\alpha \tau_k} \right) y^{k-1} \sin \chi.$$ (2.15)

The derivative of the Lyapunov function is negative definite for any $\alpha > 0$. Therefore, $\tilde{\chi} \to 0$ asymptotically if $\alpha > 0$. 

14
Define a new Lyapunov function including both \( y \) and \( \tilde{\chi} \) to be
\[
V(y, \tilde{\chi}) = \frac{1}{2} y^2 + \frac{1}{2} \tilde{\chi}^2.
\]
The derivative of this new Lyapunov function is
\[
\dot{V}(\tilde{\chi}) = y \dot{y} + \tilde{\chi} \dot{\tilde{\chi}} \\
= y S \sin \chi + \tilde{\chi} \left( \chi^d - \dot{\tilde{\chi}} \right) \\
= y S \sin \left( -\chi^e \left( \frac{y}{\tau} \right)^k - \tilde{\chi} \right) + \tilde{\chi} \left[ \left( \frac{-k \chi^e S}{\tau^k} \right) y^{k-1} \sin \chi - \alpha (\chi^e - \chi) \right].
\] (2.16)

Choosing \( \chi^e \) as in (2.15), Equation (2.16) becomes
\[
\dot{V}(y, \tilde{\chi}) = y S \sin \left( -\chi^e \left( \frac{y}{\tau} \right)^k - \tilde{\chi} \right) - \alpha \tilde{\chi}^2,
\] (2.17)
which will be negative when
\[
|\tilde{\chi}| < \chi^e \left( \frac{y}{\tau} \right)^k.
\] (2.18)

It can be shown that the set of headings, \( R \), that satisfy the condition in (2.18) constitute a positively invariant set. The set can be expressed as
\[
R = \left\{ \chi : -2\chi^e \left( \frac{y}{\tau} \right)^k < \chi < 0, 0 < y \leq \tau \right\}.
\] (2.19)

The heading rate dynamics, substituting \( \chi^e \) in, are
\[
\dot{\chi} = \alpha \left[ -\chi^e \left( \frac{y}{\tau} \right)^k - \frac{\chi^e k S}{\alpha \tau^k} y^{k-1} \sin \chi - \chi \right].
\] (2.20)

Evaluating \( \dot{y} \) and \( \dot{\chi} \) at the boundaries of \( R \):
For \( \chi = -2\chi^e \left( \frac{y}{\tau} \right)^k \):
\[
\dot{y} = S \sin \left[ -2\chi^e \left( \frac{y}{\tau} \right)^k \right] < 0 \\
\dot{\chi} = \alpha \left[ \chi^e \left( \frac{y}{\tau} \right)^k - \frac{\chi^e k S}{\alpha \tau^k} y^{k-1} \sin \left( -2\chi^e \left( \frac{y}{\tau} \right)^k \right) \right] > 0.
\]
For $\chi = 0$:

$$
\dot{y} = S \sin(0) = 0
$$

$$
\dot{\chi} = \alpha \left[ -\chi^e \left( \frac{y}{\tau} \right)^k - \frac{\chi^e k S}{\alpha \tau^k} \frac{y}{k-1} \sin(0) \right] < 0.
$$

The derivative of $y$ is not negative for $\chi = 0$, but since the set is an open set, $\chi \neq 0$ but $\chi < 0$, making $\dot{y} < 0$. Since the derivatives at the boundary point towards the interior of the set, the set is positively invariant. This implies that if the condition in (2.18) is ever satisfied, it will remain satisfied, meaning $\dot{\mathcal{V}}$ (from Equation (2.17)) is negative and remains negative.

It is easy to check that the set $\chi \in (-\pi, 0)$ and $0 < y < \tau$ is also a positively invariant set. This implies that the MAV will remain in the transition region and $\chi$ will eventually enter the set $\mathcal{R}$ since $\tilde{\chi} \to 0$ asymptotically. If the MAV is inside the transition region with $\chi \in (0, \pi)$, there are two possible scenarios. The first is that since $\tilde{\chi} \to 0$ asymptotically and $\chi^d \in (-\pi, 0)$, $\chi$ will enter the positively invariant set $(-\pi, 0)$ before exiting the region. The MAV will remain inside the transition region and $\chi$ will eventually enter the set $\mathcal{R}$. The other scenario is that since $\dot{y} > 0$ with $\chi \in (0, \pi)$, the MAV will leave the transition region. However, Theorem 2.3.1 states that a MAV outside the transition region will enter the transition region with $\chi \in (-\pi, 0)$ for $y > 0$. The MAV will not leave the transition region thereafter and $\chi$ will enter the set $\mathcal{R}$. This guarantees that $\chi$ will eventually enter $\mathcal{R}$. This means that the the condition in (2.18) will eventually be satisfied and $\dot{\mathcal{V}} < 0$ thereafter. This results in asymptotic convergence for both $y$ and $\tilde{\chi}$. The proof is identical for $y < 0$ and generalizing Equation (2.15), for both sides of the path and an arbitrary final heading yields

$$
\chi^e = \chi^d - \rho \left( \frac{k \chi^e S}{\alpha \tau^k} \right) e^{k-1} \sin(\chi - \chi^f).
$$

(2.21)

This leads to the following theorem:
**Theorem 2.3.2** For a MAV inside the transition region ($|\epsilon| \leq \tau$) with the navigational dynamics as defined in Equations (2.5) and (2.6), the heading rate dynamics in Equation (2.7), and commanded heading defined in Equation (2.21), if $\alpha > 0$, then $\tilde{\chi} \to 0$ and $y \to 0$ asymptotically.

This implies that with this algorithm, a MAV will follow a straight line path and that the cross-track error and ground track heading error will go to zero asymptotically if $\alpha > 0$. Note that no limits have been placed on the heading rate dynamics as described in Equation (2.7). It is possible to choose $\tau$, $\psi^e$, and $k$ such that the heading rate required for this path following exceeds the realizable heading rate of the MAV. Adjusting these parameters for the particular airframe being flown is critical to successful path following. Theorem 2.3.2 is valid only as long as the derivative of the vector field is less than the maximum heading rate of the MAV.

### 2.3.2 Orbit Following

**Vector Field Description**

The algorithm for creating vector fields for orbits is performed in a manner similar to the straight-line algorithm. Consider the desired orbit path shown in Figure 2.4. In this discussion, a counter-clockwise orbit will be considered. The development for clockwise orbits is similar with the exception of several sign changes. The desired orbit is assumed to have a known center with coordinates $c_x$, $c_y$ and a known radius $r$. When the distance between the MAV and the center of the orbit, $d$, is greater than $2r$, it is desirable for the MAV to fly along a heading tangent to the orbit to be followed so that transitioning into the orbit can happen with minimal transient behavior. The desired heading for $d > 2r$ is

$$\chi^d = \gamma - \frac{5\pi}{6} \tag{2.22}$$

where $\gamma$ is defined as the heading from the center of the orbit to the MAV as shown in Figure 2.4.

Once inside of $2r$, the desired heading field transitions as $d$ decreases from $2r$ to $r$. At $d = 2r$, the desired heading is $\chi^d = \gamma - \pi + \sin^{-1} \left( \frac{r}{d} \right) = \gamma - \frac{5\pi}{6}$. At $d = r$, $\chi^d = \gamma - \frac{\pi}{2}$.
The desired heading for a counter-clockwise orbit when \( d \leq 2r \) is determined by

\[
\chi^d = \gamma - \frac{\pi}{2} - \rho \left( \frac{\pi}{3} \right) \left( \frac{|d - r|}{r} \right)^k
\]  
(2.23)

where \( k \) is a gain determining the rate of transition and \( \rho \) is \( \pm 1 \) depending on whether the MAV is inside or outside of the desired orbit.

Since circular paths are being followed, it is convenient to change the navigational dynamics to polar coordinates in terms of \( \dot{d} \) and \( \dot{\gamma} \) where the center of the orbit is the origin. From Figure 2.4, \( x = d \cos \gamma \) and \( y = d \sin \gamma \). Taking the derivative and substituting into Equations (2.5) and (2.6) gives

\[
\dot{d} = S \cos(\chi - \gamma)
\]

\[
\dot{\gamma} = \frac{S}{d} \sin(\chi - \gamma)
\]

where the \( S \) and \( \chi \) are the ground track speed and heading, respectively. A table listing the

Figure 2.4: Vector field geometry for orbit tracking.
variables used and a summary of the orbit vector field construction algorithm can be found in Table 2.2 and Algorithm 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>orbit radius</td>
</tr>
<tr>
<td>$z = (x, y)^T$</td>
<td>GPS coordinates for the plane</td>
</tr>
<tr>
<td>$c = (c_x, c_y)^T$</td>
<td>GPS coordinates for the center of the orbit</td>
</tr>
<tr>
<td>$k$</td>
<td>convergence gain, $k &gt; 1$</td>
</tr>
<tr>
<td>$d$</td>
<td>distance from the center of orbit to the MAV</td>
</tr>
<tr>
<td>$\chi^d$</td>
<td>desired heading</td>
</tr>
<tr>
<td>$\rho$</td>
<td>if $d - r &gt; 0 \rho = 1$, else $\rho = -1$</td>
</tr>
</tbody>
</table>

Algorithm 2 Orbit Following Vector Field Algorithm (Counter-Clockwise Direction)

1: Obtain current position $z$.
2: $d = \|z - c\|$ {Calculate distance to center of orbit }  
3: if $d - r > 0$
4: $\rho = 1$
5: else
6: $\rho = -1$
7: end if
8: if $|d| > 2r$ {Distance to center is greater than $2r$}
9: $\chi^d = \gamma - \frac{5\pi}{6}$
10: else $|d| < 2r$ {Distance to center is less than $2r$}
11: $\chi^d = \gamma - \frac{\pi}{2} - \rho \left(\frac{|d - r|}{r}\right)^k$
12: end if

Stability Analysis

Let the control law for the heading rate be

$$\dot{\chi} = \alpha (\chi^c - \chi). \quad (2.26)$$
The objective is to find a $\chi^c$ such that circular paths can be followed provided the UAV can maintain a positive ground speed and derivative of the vector field is never greater than the maximum heading rate for the MAV.

**Outside of Two Radii** In order to devise a $\chi^c$ such that $d(t)$ becomes less than $2r$ in finite time, let

$$\tilde{\chi} = \chi^d - \chi = \gamma - \frac{5\pi}{6} - \chi. $$

The derivative of $\tilde{\chi}$ is

$$\dot{\tilde{\chi}} = \dot{\gamma} - \dot{\chi} = \frac{S}{d} \sin(\chi - \gamma) - \alpha(\chi^c - \chi) = \alpha \left[-\chi^c - \tilde{\chi} + \chi^d + \frac{S}{\alpha d} \sin(\chi - \gamma) \right].$$

Choose

$$\chi^c = \chi^d + \frac{S}{\alpha d} \sin(\chi - \gamma) \quad (2.27)$$

to get

$$\dot{\tilde{\chi}} = -\alpha \tilde{\chi}. $$

This implies that $\tilde{\chi}(t) = e^{-\alpha t} \tilde{\chi}(t_o)$ (i.e. heading error approaches zero exponentially) if $\alpha > 0$. Now, let $\tilde{d} = d - 2r$. The derivative of $\tilde{d}$ is

$$\dot{\tilde{d}} = \dot{d} = S \cos(\chi - \gamma) = S \cos \left(\tilde{\chi} + \gamma - \frac{5\pi}{6} - \gamma\right) = S \cos \left(\tilde{\chi} - \frac{5\pi}{6}\right) = -S \cos \left(\tilde{\chi} + \frac{\pi}{6}\right). $$
If \(-\frac{2\pi}{3} < \tilde{\chi} < \frac{\pi}{3}\), \(\dot{\tilde{d}} < 0\). Since \(\tilde{\chi} \to 0\) exponentially, eventually \(\tilde{\chi} \in (-\frac{2\pi}{3}, \frac{\pi}{3})\), at which point \(\tilde{d}\) decreases until \(d < 2r\). This results in the following theorem:

**Theorem 2.3.3** For a MAV outside the transition region \((d > 2r)\) with the navigational dynamics defined in Equations (2.24) and (2.25), the heading rate control defined in Equation (2.26), and commanded heading defined in Equation (2.27), if \(\alpha > 0\) the MAV will enter the transition region in finite time.

**Inside of Two Radii** Now that it is proven that the MAV will enter the transition region with \(d < 2r\), \(\chi^c\) must be found such that \(\tilde{\chi} \to 0\) and \(\tilde{d} = d - r \to 0\). It will again first be shown \(\tilde{\chi} \to 0\) asymptotically for all \(\tilde{d}\) in the transition region. This fact will then be used to show that conditions guaranteeing \(\tilde{d} \to 0\) will be satisfied. Consider the case where \(d - r > 0\) so that \(\tilde{d} > 0\) and \(\rho = 1\). Again, let \(\tilde{\chi} = \chi^d - \chi\), where \(\chi^d\) is defined in Algorithm 2, Line 11. The derivative is

\[
\dot{\tilde{\chi}} = \chi^d - \dot{\chi} \\
= \dot{\gamma} - \frac{kdS\pi}{3r^k} d^{k-1} \cos (\gamma - \chi) - \alpha (\chi^c - \chi) \\
= -\frac{S}{d} \sin (\chi - \gamma) - \frac{kdS\pi}{3r^k} d^{k-1} \cos (\chi - \gamma) - \alpha (\chi^c - \chi) \tag{2.28}
\]

Now let \(\chi^c = \chi^d - \nu\) so that Equation (2.28) becomes

\[
\dot{\tilde{\chi}} = -\frac{S}{d} \sin (\chi - \gamma) - \frac{kdS\pi}{3r^k} d^{k-1} \cos (\chi - \gamma) - \alpha \tilde{\chi} + \alpha \nu. \tag{2.29}
\]

A Lyapunov function candidate is \(V = \frac{1}{2} \tilde{\chi}^2\) and the derivative is given by \(\dot{V} = \tilde{\chi} \dot{\tilde{\chi}}\). Substituting (2.29) into this derivative yields

\[
\dot{V} = \tilde{\chi} \left[ -\frac{S}{d} \sin (\chi - \gamma) - \frac{kdS\pi}{3r^k} d^{k-1} \cos (\chi - \gamma) - \alpha \tilde{\chi} + \alpha \nu \right] \tag{2.30}
\]
Choose \( \nu = \frac{S}{\alpha d} \sin(\chi - \gamma) + \frac{kS\pi}{3r^k\alpha} \tilde{d}^{k-1} \cos(\chi - \gamma) \) and substitute into Equation (2.30) to get

\[
\dot{\nu} = -\alpha \tilde{\chi}^2
\]

which will be negative definite if \( \alpha > 0 \). Substituting \( \nu \) into the expression for \( \chi^c \) gives the commanded heading of

\[
\chi^c = \chi^d - \frac{S}{\alpha d} \sin(\chi - \gamma) - \frac{kS\pi}{3r^k\alpha} \tilde{d}^{k-1} \cos(\chi - \gamma).
\]  

(2.31)

for \( r \leq d \leq 2r \). Generalized for \( 0 \leq d \leq 2r \),

\[
\chi^c = \chi^d - \frac{S}{\alpha d} \sin(\chi - \gamma) - \frac{kS\pi}{3r^k\alpha} \left| \frac{d}{r} \right|^{k-1} \cos(\chi - \gamma).
\]

(2.32)

To show that both \( \tilde{d} \to 0 \) and \( \tilde{\chi} \to 0 \), a new Lyapunov function is created. Let the new Lyapunov function candidate be \( V(\tilde{d}, \tilde{\chi}) = \frac{1}{2} \tilde{d}^2 + \frac{1}{2} \tilde{\chi}^2 \) with the derivative being

\[
\dot{V} = \tilde{d} \dot{\tilde{d}} + \tilde{\chi} \dot{\tilde{\chi}}.
\]

Substituting in expressions for \( \dot{\tilde{d}} \) and \( \dot{\tilde{\chi}} \) and choosing \( \chi^c \) as in (2.31) gives

\[
\dot{V} = \tilde{d} S \cos(\chi - \gamma) - \alpha \tilde{\chi}^2
\]

\[
= \tilde{d} S \cos \left[ -\frac{\pi}{2} - \frac{\pi}{3} \left( \frac{\tilde{d}}{r} \right)^k - \tilde{\chi} \right] - \alpha \tilde{\chi}^2
\]

\[
= \tilde{d} S \sin \left[ -\frac{\pi}{3} \left( \frac{\tilde{d}}{r} \right)^k - \tilde{\chi} \right] - \alpha \tilde{\chi}^2
\]

which will be negative for

\[
|\tilde{\chi}| < \frac{\pi}{3} \left( \frac{\tilde{d}}{r} \right)^k.
\]

(2.33)

It is easily checked that the set of headings that satisfy the condition in (2.33) is positively invariant using the same methods that were used for the straight line following algorithm. If \( \chi \) is inside this set, \( \tilde{\chi} \to 0 \) and \( \tilde{d} \to 0 \) asymptotically. If \( \chi \) is outside this set, there
are two scenarios. The first scenario is that since \( \tilde{\chi} \to 0 \) asymptotically, \( \chi \) enters this positively invariant set before it leaves the transition region where it remains, meaning the MAV never leaves the transition region and approaches the orbit asymptotically. The other scenario is that the MAV leaves the transition region before it can get into this set. It has been shown that the MAV will reenter the transition region and it can be seen that upon reentry, \(-\gamma - \frac{\pi}{2} < \chi < -\gamma + \frac{\pi}{2}\). With \( \chi^c \) as defined in (2.32), this set, \( \chi \in (-\gamma - \frac{\pi}{2}, -\gamma + \frac{\pi}{2}) \), for \( \tilde{d} \in (0, r) \) is positively invariant and since \( \tilde{\chi} \to 0 \), \( \chi \) will eventually enter the positively invariant set making \( \hat{V} < 0 \). The MAV will therefore follow the orbit with asymptotically decaying error. This result leads to the following theorem:

**Theorem 2.3.4** For a MAV with \( d < 2r \) and the navigational dynamics defined in Equations (2.24) and (2.25), the heading rate dynamics defined in Equation (2.26), and commanded heading defined in Equation (2.32), if \( \alpha > 0 \), \( \tilde{\chi} \to 0 \) and \( \tilde{d} \to 0 \) asymptotically.

This implies that a MAV can follow an orbit with the cross-track error and ground track heading error going to zero asymptotically if \( \alpha > 0 \) and the derivative of the vector field is always less than the maximum heading rate of the MAV.

### 2.3.3 Straight Line and Orbit Combination

Most paths planned for UAV flight can be approximated by combinations of straight-line segments and circular arcs. One example of how line segments and arcs can be combined is presented in [4]. Figure 2.5 shows how combined paths can be utilized with waypoint planning to fly paths that preserve equal path lengths, fly directly over the waypoints, or turn in order to minimize flight time. There are also a number of other situations where a combination would be desirable. For example, following a perimeter with irregular geometry could be done effectively by approximating that geometry with a series of lines and arcs.

When there is a situation where a combination is desirable, the question becomes how to construct the fields. In order to avoid the possibility of multiple sinks, dead zones, and singularities that are inherent in the combination of vector fields, only the vector field for the current segment or arc to be followed is calculated. For a multi-segmented path,
the vector field changes each time it is determined that the MAV has reached the end of a segment or arc. Once the MAV has reached the end of a segment or arc, the entire vector field changes to a field directing the MAV onto the next segment or arc. No two fields are ever combined, thus eliminating any issues related to the combining of fields.

The method for determining when to change the vector field must be specified. There are a number of methods for doing this. One way is to detect when the MAV is within a predetermined distance from the end of the segment or arc. This works well for transitioning out of a straight path segment. For transitioning out of an arc, monitoring angular travel of the MAV has proven successful. Using this approach, the MAV transitions to the next path segment when the angle through which the MAV has flown is equal to the included angle of the arc.

Figure 2.5: Line segment and arc combinations.
2.4 Results and Discussion

2.4.1 Hardware Testbed

BYU has developed a reliable and robust platform for testing unmanned air vehicles [7, 8]. Figure 2.6 shows the key elements of the testbed. The first frame shows BYU’s Kestrel autopilot which is equipped with a Rabbit 3400 29 MHz processor, rate gyros, accelerometers, absolute and differential pressure sensors. The autopilot measures $3.8 \times 5.1 \times 1.9$ cm and weighs 17 grams.

The second frame in Figure 2.6 shows the airframes used for the flight tests reported in this paper. The airframe is a 1.2 meter wingspan Zagi XS EPP foam flying wing, which was selected for its durability, ease of component installation, and flight characteristics. Embedded in the airframe are the Kestrel autopilot, batteries, a 1000 mW, 900 MHz radio modem, a GPS receiver, a video transmitter, and a small analog camera.

The third frame in Figure 2.6 shows the ground station components. A laptop runs the Virtual Cockpit software that interfaces through a communication box to the MAVs. An RC transmitter is used as a stand-by fail-safe mechanism to facilitate safe operations.

![Figure 2.6: (a) Kestrel autopilot (b) Zagi airframes (c) Ground station components](image)

2.4.2 Experimental Demonstration

In order to illustrate the orbit following abilities of the algorithm, the MAV was commanded to fly a series of concentric orbits in a counter clockwise direction with varying radii. The results are shown in Figure 2.7. There was wind from the south of 2 to 3 m/s.
Figure 2.7: Telemetry plot for flying orbits with radii of 150, 100, 70, and 50 m.

which corresponds to approximately 15 to 25 percent of the commanded MAV airspeed. The maximum deviation from the desired path was about 9 m and the average lateral error was approximately 3.4 m.

Figure 2.8 illustrates the ability of the MAV to follow straight line segments with acute angles. Excluding the transient errors from the turns, the mean following error was 0.8 m with a standard deviation of 1.1 m. A plot of the distance from the path can be seen in Figure 2.9. The wind for this flight was out of the west and was again about 2 to 3 m/s.

With the straight line and orbit following algorithms working well, the transition to a combination of the two methods was also tested. The techniques described in Section 2.3.3 were implemented and the results are plotted in Figure 2.10. Winds were 30 to 50 percent of the MAV’s airspeed. The thicker line is the desired path in order to conserve equal path lengths. The maximum deviation from the path was about 19 m and the mean distance from path and standard deviation were 3.4 m and 5.0 m respectively. The path following error is plotted in Figure 2.11. Although the transitions from the straight line to the orbit portions show some lateral following errors, the actual path length flown and the
desired length are very close. Five and a half loops through the path shown in Figure 2.10 and the desired length of the path was 14606 m. The actual distance flown was 17.5 m less than the desired distance which is an error of only 0.12 percent.

To further test the robustness and capabilities of the proposed path following algorithms many other types of paths have been flown. The path shown in Figure 2.12 illustrates both obtuse and acute angles and the decision of the trajectory follower to cut the corners
of the obtuse angles and flare out and around on the acute angles. Figure 2.13 shows a path that might be planned through an urban type terrain. Note that although these are actual flight results, the terrain is synthetic, i.e. there were not really buildings in the area the MAV flew through. The straight line follower was used to follow this path. The wind speed was 20 to 30 percent of the airspeed for both of these flights.

2.5 Conclusion

In this paper, a new method for MAV path following has been studied. The idea of vector fields has been extended to constant altitude path following. It has been shown using
Figure 2.12: Combination of equal path length and corner cutting following.

Figure 2.13: Urban terrain following using straight line following.
Lyapunov stability criteria that controlling heading rate based on ground track heading and speed in a vector field yields asymptotic following for straight line and circular paths.

The effectiveness of the described following methods have been illustrated using a Zagi fixed-wing MAV and the Kestrel autopilot system. The MAV followed the desired paths with asymptotically decaying error. Minimal error was observed once the MAV had converged to the path. Vector field path following also proved effective in following smoothed paths composed of circular and straight line segments. All of the flight tests were performed in moderate (15 to 50 percent of MAV airspeed) wind conditions.

The implementation of vector field following is straightforward and the result is a robust method for accurate path following. Controlling heading rate based on ground track heading and ground speed automatically accounts for wind conditions, providing tight following even in the presence of wind.
Chapter 3

Experimental Cooperative Timing

3.1 Introduction

The effectiveness of unmanned aerial vehicles (UAVs) in recent years has increased dramatically. This increase is due to advances in microcontroller, battery, and sensor technologies. As a result, the use of UAVs, primarily in military applications, has also increased. Larger UAVs, such as Global Hawk, Predator, and Hunter, are currently being used for border patrol, surveillance, and even target prosecution. Smaller microcontrollers and sensor packages have also improved the capabilities of miniature aerial vehicles \(^1\), and significant resources have been allocated toward the further development of these technologies [1]. MAVs have a number of applications for which they are especially well suited because of their size and cost. Some of these applications include reconnaissance, battle damage assessment, and surveillance. Although there are many uses for MAVs in commercial or civilian applications, their reliability and capabilities will have to be improved in order to fully transition into the commercial sector.

In recent years, cooperative control for teams of UAVs has been a heavily researched area. In [10], strategies for dealing with the complexity and coupling inherent in cooperative autonomous UAV teams are explored. They show, by computer simulation, that using network flow and auction algorithms for task assignment yields improved team performance for search missions with varying numbers of team members and tasks to be accomplished.

\(^1\)We consider miniature aerial vehicles to be those with wingspans in the 0.3 m to 2 m range and micro aerial vehicles to be those with wingspans under 0.3 m. Here the abbreviation MAV denotes miniature aerial vehicle.
Much work has also been devoted to path planning for multi-agent teams in high-risk environments. In [13], a model for task assignment and path planning involving both a priori and dynamic information about threats and targets is presented. Simulation results in a three-dimensional environment illustrate the effectiveness of this method. Maza and Ollero present a method for cooperatively searching an area with a heterogeneous team of UAVs taking advantage of the capabilities of the individual agents in the team [17]. The area to be searched is divided into a set of polygons and the assignments for the searching of these polygons is based on which of the UAVs is best suited for the task. In [28], swarming methods are proposed for cooperatively searching an area. The UAVs fly in a formation that utilizes the sensing capabilities of each team member and easily adapts if any of the members is lost during the mission.

Formation flying is another area of cooperative control that has been heavily researched. The benefits for fuel savings have been explored in [9] and [11]. In [5], [23], and [25], different approaches for formation flying and maneuvering are put forth. Experimental results for two UAVs flying a follow-the-leader type scenario have been presented in both [22] and [6].

The work in this paper extends work presented previously in [18] and [19] in the area of cooperative timing missions utilizing coordination variables and functions to make path planning and timing decisions. Coordinating timing over a known target can be advantageous in a number of scenarios. One example is the case where a persistent image of a target from a consistent perspective is desired. If the camera footprint for each MAV in a team is known, arrival intervals for the team can be determined such that persistent image of the target can be obtained. Another scenario is surveillance of a target in a risky environment. A simultaneous arrival of a team of MAVs over the target could be coordinated, thus maximizing the possibility for the team, or at least part of the team, to reach the target. An optimization problem based on minimizing fuel consumption was performed in order to determine paths and velocities for each of the MAVs.

An outline of these two scenarios and a description of the issues involved in achieving cooperation for a team of MAVs will first be put forth. A discussion of the approach taken to address these issues will follow as well as a description of the experimental setup.
The primary contribution of this paper is the experimental results illustrating the feasibility of real-time cooperative timing missions for MAVs.

3.2 Problem Overview

The primary focus of this work is experimental demonstration of cooperative timing for MAVs, thereby providing practical validation of our cooperative control approach. Two specific missions are considered: simultaneous arrival and cooperative fly-by.

For the simultaneous arrival mission, the objective is for the MAVs composing the team to arrive at the target at the same time from different approach directions. The ability to have multiple vehicles arriving at destination simultaneously is useful for a variety of tactical purposes including formation join-up and target attack. In an attacking situation, simultaneous arrival maximizes the element of surprise and increases the intensity of the attack.

For the cooperative fly-by mission, the objective is for the MAVs to approach the target along the same approach path at specified timing intervals. The ability to have multiple vehicles fly over a target at specified intervals is useful for persistent surveillance or for prosecuting targets where a sequence of tasks is required (e.g., identification, attack, battle damage assessment). In the scenario considered, the heading through the target is predetermined to be along the imaging path so that a persistent image of the target from the same viewing angle can be obtained.

Both missions have several common elements to their planning and execution. During the initial phase of the mission, the MAVs are occupied with a secondary task (i.e. monitoring secondary targets, positioning themselves for the cooperative mission, etc.). Upon command from the operator at the ground station, paths are planned to satisfy the timing constraints for the mission and to minimize the cooperation objective (e.g., fuel cost). Paths are planned based on current MAV and target locations and the desired headings through the target. Upon completion of the pass over the target, the MAVs return to the initial secondary tasks. Challenges associated with these missions include real-time cooperative path planning, reliable and timely communication of cooperation information, and accurate path following with significant wind disturbances.
Cooperative timing solutions for the two missions presented were obtained from a cooperative timing algorithm employing coordination variables and coordination functions to represent critical timing information for the MAVs. The experimental results presented involve a team of three MAVs although the approach applies generally to two or more vehicles. This cooperative timing strategy is outlined in the following section along with the path planning and path following approaches that were used.

3.3 Technical Approach

Cooperative Timing Strategy

The cooperative timing strategy employed is based on the utilization of coordination variables and coordination functions. The underlying concept of coordination variables and coordination functions is that the minimal information essential to achieving a cooperation objective should be identified and communicated among agents on the team. Minimizing the amount of information being sent improves communication speed and reliability. It also tends to simplify cooperative control calculations by reducing the problem to its essential elements.

Let $x_i$ define the situation state for the $i^{th}$ vehicle on a cooperative team. For the cooperation problems considered here, the situation state includes information about the current MAV position, the target position, the desired heading through the target, and the time interval between MAV arrivals (zero for the simultaneous arrival scenario). Additional information about the environment (e.g., wind speed and direction) can also be included if available and desired. For a given situation $x_i$, the set of feasible decisions for a vehicle is given by $U_i(x_i)$, where $u_i \in U_i$ is the decision variable for the $i^{th}$ vehicle. The choice of the decision variable by each vehicle on the team affects both the feasibility and the quality of the cooperation achieved. In the cooperative timing scenarios considered, the decision variable vector consists of path waypoint and velocity information.

The process of cooperation among agents can be viewed as having objectives and constraints. For the simultaneous arrival scenario, the cooperation constraint requires the MAVs to arrive over the target at the same time. For the fly-by scenario, the cooperation constraint requires the MAVs to fly over the target with the second vehicle at a specified
time interval behind the first, and the third vehicle arriving at a specified time interval after the second. Cooperation is said to occur if the cooperation constraints are met. In the these scenarios, the cooperation objective is to minimize the battery energy required to complete the mission. The contribution of a vehicle to the team cooperation objective is represented by an influence function, \( \phi_i = J_i(u_i) \). In this case the energy consumption for the \( i \)th vehicle (represented by \( \phi_i \)) is a function of the MAV velocity and the waypoints flown (represented by \( u_i \)).

The coordination variable \( \theta \) represents the minimal amount of information necessary to achieve cooperation. For the simultaneous arrival problem, the coordination variable is the arrival time over the target. For the fly-by problem, the coordination variable is the arrival time of the first MAV and the arrival order for the vehicles. If every vehicle has knowledge of the value of the coordination variable and responds accordingly, cooperative behavior will be achieved by the team. For a vehicle, the value of the coordination variable is related to the decision variable, \( \theta_i = f_i(u_i) \), and defines what the vehicle can do to ensure that cooperation constraints are met. In these scenarios, the choice of path and velocity determines MAV energy consumption and arrival time and therefore influences both the cooperation objective and the cooperation constraint.

Under the assumptions that battery power is proportional to aerodynamic drag and that the MAV flies at constant speed, the battery energy consumption to fly to the target is given by

\[
J_i = c_b v_i L_i
\]

where \( c_b > 0 \) is a constant, \( v_i \) is the MAV velocity, and \( L_i \) is the length of the waypoint path taken.

For the cooperative fly-by scenario with three MAVs, the cooperation constraint can be written as

\[
T_1 = T_s \\
T_2 = T_s + \Delta_2 \\
T_3 = T_s + \Delta_3
\]
where \( T_i \) is the arrival time of the \( i^{th} \) vehicle, \( \Delta_i \) represents the interval between the arrival of the first and \( i^{th} \) vehicles, and \( \theta = T_s \) is the coordination variable. Note that for the simultaneous arrival scenario \( \Delta_2 = \Delta_3 = 0 \).

Critical to the implementation of this approach is the definition of the coordination function. The coordination function models a vehicle’s influence on the cooperation objective in terms of what the agent can do to meet the cooperation constraints. The coordination function, \( \phi_i(\theta_i) \) is derived from the influence function and coordination variable definitions as

\[
\phi_i = J_i(u)
= J_i[f_i^\dagger(\theta_i)]
= \phi_i(\theta_i),
\]

where \( f_i^\dagger \) is the pseudoinverse of \( f_i \). Typically, \( f_i \) is not a one-to-one mapping in that numerous values of the decision variable (waypoint paths, velocities) can result in a single value of the coordination variable (time over target). The pseudoinverse of \( f_i \) is found by taking the value of decision variable that minimizes the influence function \( J_i \), thus creating a one-to-one map between \( \theta_i \) and \( u \). In other words, when multiple options exist to get the same result, the lowest-cost option is chosen.

Using coordination variables and coordination functions, the battery energy required to complete the mission can be minimized by solving the optimization problem

\[
\theta^* = \arg \min_{\theta=T_s} [\phi_1(\theta_1) + \phi_2(\theta_2) + \phi_3(\theta_3)].
\]

subject to the constraint of Equation (3.1).

Once a team optimal value for the coordination variable is found, vehicle decisions can be found from the relationship

\[
u_i = f_i^\dagger(\theta^*).
\]
Figure 3.1 illustrates the coordination functions calculated for a cooperative timing mission based on input information provided to the algorithm. Because the coordination function for each UAV is monotonically decreasing, the team-optimal arrival time for one of the vehicles will always lie at the right extreme of its coordination function. For the simultaneous arrival scenario, the solution is simply determined by finding the largest arrival time that all three MAVs have in common. For the fly-by scenario, determining the minimum-energy arrival time and order for the team involves a simple search through the right extreme (minimum) values of each coordination function with an evaluation of the team objective to determine the best values.

![Initial coordination functions](image)

**Figure 3.1: Initial coordination functions.**

### Path Planning

In order to simplify the path planning for this initial implementation, some assumptions about the environment were made. The first assumption is that there are no threats to be avoided. The next assumption is that both the target location and the desired fly-by headings are known, which eliminates any initial target observation and detection steps prior to the cooperative timing mission. Because there are no threats to avoid, a straightforward
heuristic approach for planning straight-line waypoint paths is utilized. Straight-line paths are easy to plan and simplify both the distance calculations and timing optimizations.

Figure 3.2 shows an example of the waypoint paths used in the experiment. Waypoint 1 for each agent was calculated to be 100 m along the same heading that the agent had when the coordination algorithm was executed. Waypoint 2 was set to be a fixed distance from the known target location along the desired approach path. For the experiments, this distance was between 200 m and 300 m. Allowing this much distance ensured that the transient path following errors associated with transitioning from one straight line path to another would be damped out and that the MAVs would be flying along the desired approach heading when they passed over the target. Waypoint 3 was the target location. Waypoint 4 was collinear with waypoints 2 and 3 on the opposite side of the target from waypoint 2. Waypoints 2, 3, and 4 were the same for each agent in the fly-by scenario and only waypoint 3 was the same in the simultaneous arrival scenario. These three points are referred to as

Figure 3.2: Waypoint path example.
fly-through points. Waypoint 5 was a waypoint to help the agents distance themselves from the target before returning to their final loiter positions, waypoint 6.

In some cases, depending on the initial MAV positions and headings, the coordination functions do not yield a solution that satisfies the cooperative timing constraint. For example, a cooperative fly-by with a spacing interval of 10 s between vehicles could not be accomplished with the coordination functions of Figure 3.1 due to the time gap between the coordination function of UAV 1 and those of UAVs 2 and 3. In these instances, it became necessary to develop some simple approaches for adding length to one or more paths so that the timing constraints could be met.

Based on the coordination functions, the length to be added to a path to ensure constraint feasibility was calculated. If the length that needed to be added was less than 150 m, the location of the first waypoint was adjusted using the law of cosines so that the total distance would be the original length plus the necessary additional length. If the additional length was longer than 150 m, two additional waypoints were added to the path immediately following the first waypoint. The second waypoint was calculated so that its distance from the first waypoint was half of the required additional distance, and the waypoint was placed in a direction directly opposite the target. The third waypoint was placed at the same location as the first, thus effectively adding the required length to the path. The same path shown in Figure 3.2 with these added points is shown in Figure 3.3.

Figure 3.4 shows how the coordination function for UAV 3 was modified by adding length to the original path. With the added length, the coordination function for UAV 3 shifted up and to the right allowing the 10 s interval timing constraint to be satisfied.

**Path Following and Velocity Control**

For timing missions, preserving path length is critical in order to be able to satisfy the timing constraints. Flying extra distance due to poor path following adds unwanted time to the mission. To address this issue, a vector field path following method has been implemented to enable the MAVs to accurately follow the planned waypoint paths. It has been shown in [20] that creating vector fields of desired heading in order to direct the MAV onto the path will result in asymptotically decaying tracking error provided ground
track heading and ground speed are used in the outlined control law. Use of this method decreased path following error, and consequently, overall path length error.

Since wind speed is typically 10 to 50 percent of the airspeed for a MAV, wind can have adverse effects on path following and timing constraints. Typically, throttle is
controlled based on a desired airspeed. This creates problems when trying to fly timing missions in the presence of wind. To address this issue, the throttle was controlled based on ground speed with a check to make sure that the airspeed did not drop below the MAV stall speed. Throttle control based on ground speed also helps reduce the influence of path length error on timing.

3.4 Experimental Setup

Hardware Description

BYU has developed a reliable and robust platform for unmanned aerial vehicle testing [7, 8]. Figure 3.5 shows the key elements of the testbed. The first frame shows BYU’s Kestrel autopilot which is equipped with a Rabbit 3400 29 MHz processor, rate gyros, accelerometers, absolute and differential pressure sensors. The autopilot measures 3.8 × 5.1 × 1.9 cm and weighs 18 g. The second frame in Figure 3.5 shows the airframes used for the flight tests reported in this paper. The airframe is a 1.1 m wingspan Unicorn EPP foam flying wing, which was selected for its durability, ease of component installation, and flight characteristics. Embedded in the airframe are the Kestrel autopilot, batteries, a 1000 mW, 900 MHz radio modem, a GPS receiver, a video transmitter, and a small analog camera. The third frame in Figure 3.5 shows the ground station components. A laptop runs the Virtual Cockpit software that interfaces through a communication box to the MAVs. An RC transmitter is used as a stand-by fail-safe mechanism to facilitate safe operations.

Figure 3.5: (a) Kestrel autopilot (b) Unicorn airframes (c) Ground station components
Cooperative Control Algorithm

The cooperative timing algorithm was implemented as part of the Virtual Cockpit ground station software. The current implementation is centralized, in that the ground station acquires all the information required to do the cooperative timing calculations from the vehicles, performs the calculations, and sends only the paths to be flown and desired arrival times back up to the MAVs. Execution of the algorithm takes approximately 1 s, with a significant portion of the time being consumed by communication between the MAVs and the ground station. Once the MAVs receive the paths and arrival times, the autopilot controls heading according to the vector field method described in [20] and throttle based on ground speed so that cooperative timing can be achieved. The algorithm was implemented using the steps outlined in Table 3.1.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Acquire GPS locations and headings for each MAV</td>
</tr>
<tr>
<td>2</td>
<td>Calculate waypoint paths for each MAV</td>
</tr>
<tr>
<td>3</td>
<td>Compute range of arrival times for each MAV</td>
</tr>
<tr>
<td>4</td>
<td>Compute coordination function for each MAV</td>
</tr>
<tr>
<td>5</td>
<td>Based on coordination functions, choose team optimal coordination variable (arrival time of first MAV and arrival order for fly-by and arrival time for simultaneous arrival)</td>
</tr>
<tr>
<td>6</td>
<td>Calculate desired arrival time for each MAV</td>
</tr>
<tr>
<td>7</td>
<td>Send waypoint paths and desired arrival times to each MAV</td>
</tr>
<tr>
<td>8</td>
<td>MAVs fly waypoint paths controlling velocities to achieve cooperation</td>
</tr>
</tbody>
</table>

3.5 Results and Discussion

Simultaneous Arrival

Consecutive simultaneous arrival runs were flown on a during a 25-minute flight. Telemetry data from one of the runs is shown in Figure 3.6. Wind was again between 30 and 60 percent of the MAV airspeed during four runs. The average time between the arrival of the first MAV at the target and the arrival of the final MAV was 1.6 s. Range-to-target
data is presented in Figure 3.7. This illustrates the near simultaneous arrival of the MAVs at the target enabled by the cooperative timing algorithm.

![Simultaneous arrival - telemetry plot](image)

**Figure 3.6: Simultaneous arrival - telemetry plot**

**Cooperative Fly-by**

The fly by scenario was flown six times during a half-hour flight involving three MAVs. The wind speeds during the flight were between 30 and 60 percent of the MAV airspeed coming from the southwest. The position telemetry from one of the fly-bys is plotted in Figure 3.8. The desired arrival intervals for this run were 3 s for both intervals and the actual arrival intervals were 3 and 4 s, respectively. The average error in arrival time for the six runs, in spite of the high wind conditions, was approximately 0.6 s. The range to the target versus time for each of the MAVs is shown in Figure 3.9. The rise in the range curves results from the overshoot that occurs after the first of the fly-through waypoints.
This illustrates the arrival of the MAVs at approximately 3 s intervals.

Figure 3.7: Simultaneous arrival - range to target

Figure 3.8: Fly by with 3-second desired arrival intervals
Figure 3.9: range to target

Figure 3.10 illustrates a scenario where the length of the path of one of the MAVs had to be lengthened to satisfy the timing constraint. The MAV that began in the bottom of the frame was considerably closer to the target when the fly-by command was issued and its path length had to be increased for cooperation to occur. Two waypoints were added after the first waypoint using the method described in Section 3.3. The first two MAVs arrived at the desired arrival times while the third MAV was delayed slightly due to the strong head wind it faced during most of the run.

Although the timing errors for both simultaneous arrival and cooperative fly-by are relatively small, there are a few possible explanations for the errors that occurred. One of the more obvious reasons was the presence of the high relative wind speeds. If the desired groundspeed was in the upper end of the feasible airspeed envelope (20 m/s) while flying into a headwind, it was difficult for timing constraints to be met. Another issue was synchronizing the timing on each of the autopilots. Each autopilot had its own timer that started when the list of waypoints finished uploading from the ground. This took about 1 s for each of the three agents and if it took longer for one MAV to finish downloading the information than another, the start times were different. Transient behavior involved in matching the desired groundspeed as it changed and GPS error could also have influenced proper timing.
3.6 Conclusion

Cooperative timing missions for miniature aerial vehicles have been explored in this paper. The experimental results presented here show the feasibility of real-time coordination for teams of MAVs. Three MAVs were flown in two different flights and multiple cooperative timing missions were run during each flight. Both coordinated fly-by scenarios and simultaneous arrival scenarios were run where the coordination occurred on the ground station using an algorithm based on coordination functions and coordination variables. These results illustrate the repeatability and reliability of this algorithm even in the presence of high winds relative to MAV airspeed.
Chapter 4

Conclusions and Future Work

4.1 Conclusions

MAV Path Following

Path following methods using vector field methods have proven to be very reliable and robust even in the presence of high winds relative to MAV airspeed. This robustness is a result of the use of ground track heading and ground speed in the control law. Wind effects are automatically incorporated into the algorithm and the result is asymptotically decaying following error.

These methods have been tested using a Kestrel autopilot system in a Unicorn fixed-wing airframe. The MAV followed the desired paths with asymptotically decaying error. Once the MAV had converged to the path, minimal error was observed. These methods were flown effectively on straight line paths, orbits, and paths consisting of a combination of straight line segments and circular segments. All of the flight tests were performed with wind speeds between 20 and 50 percent of the MAV airspeed.

Cooperative Timing Experiments

Coordination function and coordination variable methods have been shown to be effective in cooperative timing missions. In the two scenarios studied in this thesis, these methods were used to choose arrival times for the cooperative missions while minimizing battery consumption. Hardware test were performed for both the simultaneous and sequenced arrival scenarios with encouraging results. Cooperation algorithms were executed on the ground station and the required cooperation information was relayed back up to the
MAVs. These tests were performed with wind velocities between 30 and 60 percent of the MAV airspeed. These results illustrate the feasibility of real-time cooperative timing missions for MAVs.

4.2 Future Work

As encouraging as these results are, there are a number of areas for future research. One of the areas for future efforts is the decentralization of the cooperation algorithms. If communication with the ground station is impossible or limited due to proximity or another reason, a decentralized algorithm must be used in order to still achieve cooperation. In order for decentralization to be an option, methods for MAV-to-MAV communication must be developed.

When information from all of the MAVs is not available to each agent, it is necessary to find another way to determine what the desired arrival times should be. Consensus algorithms present a viable solution to this problem. Consensus algorithms have been implemented in the ground station where limited communication is simulated and initial testing is promising, but MAV-to-MAV communication will be necessary to perform longer range missions where communication with the ground station is unavailable.

The scenarios under which these algorithms were tested are not as complex as most real-world scenarios would be. There were no threats in the environment that needed to be avoided which greatly simplified the path planning. It was assumed that the environment was entirely known a priori. In order for cooperative timing missions to be viable in real-world scenarios, path planning and coordination algorithms for risky and unknown environments must be developed.
Bibliography


