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Life Data Analysis of Repairable Systems: A Case Study on Brigham Young University Media Rooms

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LIFE DATA ANALYSIS OF REPAIRABLE SYSTEMS. A CASE STUDY ON BRIGHAM YOUNG UNIVERSITY MEDIA ROOMS.

by

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GRADUATE COMMITTEE APPROVAL

of a project submitted by

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This project has been read by each member of the following graduate committee and by majority vote has been found to be satisfactory.

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As chair of the candidate's graduate committee, I have read the project of Stephen O. Manortey in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

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ABSTRACT

LIFE DATA ANALYSIS OF REPAIRABLE SYSTEMS. A CASE STUDY ON BRIGHAM YOUNG UNIVERSITY MEDIA ROOMS.

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It is an undisputable fact that most systems, upon consistence usage are bound to fail in the performance of their intended functions at a point in time. When this occurs, various strategies are set in place to restore them back to a satisfactory performance. This may include replacing the failed component with a new one, swapping parts, resetting adjustable parts to mention but a few. Any such system is referred to as a repairable system. There is the need to study these systems and use statistical models to predict their failing time and be able to set modalities in place to repair them at least cost to the operator.

The main objective of this paper is to analyze data collected on the projectors used for teaching and learning activities in some designated rooms at the ***Brigham Young University*** (BYU) under the auspices of the ***Office of Information Technology*** (OIT) and help to detect the failure rate of such systems, predict the optimal replacement time for the parts with the view of maximizing the reliability of the systems and finally formulate a cost model that will be used to estimate the optimal cost involve in servicing a failed projector.

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I wish to express my sincere gratitude to all who have helped in diverse ways to make this project a success. In as much as I cannot mention names of all individuals in this write-up, I wish to acknowledge my appreciation to all and let everyone know I sincerely I owe any praise in this work, to all.

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Chapter 1

Introduction

The advent of technology has brought in its wake several improved teaching and learning facilities in the present day classroom settings as compared to the past few years where the chalkboard was the main object mostly used. It is now very common to find ***systems*** such as projectors, computers, microphones, speakers and VCR/CD/DVD combo players all stock in a single room that go to enhance teaching and learning activities.

A system by definition can be said to be a collection of two or more parts that have been assembled to perform one or more functions [Ascher and Feingold, 1984]. It is very obvious that with the passage of time, most of these systems may fail in the course of duty and will therefore need to be repaired to restore them to their intended functions.

The *Office of Information and Technology* **(OIT)** a unit on most college campuses, Brigham Young University (BYU) inclusive, oversees the performance of the systems in such room designated as Custom Multimedia Room (CMR) and Technology Enhanced Classrooms (TEC) where these facilities are often used. As part of the work they do, this office makes and responds to service calls from these rooms. This helps in monitoring the effective usage and performance of the systems. They also maintain a data base of calls that comes to their office with regard to the specific cause of failure of a system and the measures put in place to restore any of such systems to its normal functions.

Some earlier studies on the reliability of repairable systems show that, it is cost effective to have some kind of preventive maintenances on a system before failure rather than to maintain it after failure. It is in line with this that the OIT- (BYU) has began a maintenance program of replacing items such as bulbs and air filters in the projectors currently in use before they actually fail.

The data for the project was obtained with the help of Mark Hales from the OIT. He extracted it from the service calls received at the office over the past few years that was saved in an ORACLE database and gave it to us as an Excel file. The main fields of the dataset are; the Work Order, Date of Collection, Media Room, Serial Number, Item Code, Item Title, Trouble, and Reasons. Stephen Zobell also helped in getting additional fields which include Notes, Comments and Hours of Failure of some the systems over time.Mark Sullivan also helped in extracting the Number of weekly Logins into the projectors in each of the classrooms.

The main objective of this project is to analyze the data collected by the OIT- (BYU) over time to see if a better maintenance procedure can be suggested to this very important unit on campus to enable them render and improve upon their services which go a long way in augmenting teaching and learning activities.

In the analysis, much attention will be geared in getting the optimal replacement time for the parts in the systems and also focusing on the best strategies that need to be put in place to maximize the systems' reliability.

Chapter 2

Literature Review:

2.1 Repairable Systems:

A system is described to be repairable when after it has failed to perform at least one of its intended functions can be restored to fully satisfactory performance by any method other than replacement of the entire system [Basu and Rigdon, 1997]. The restoration can be done by any action including changing of parts, changes to adjustable settings, swapping of components, or even a sharp blow with a hammer [Ascher and Feingold, 1984]. For example a laptop computer not connect to electrical power may fail to start when the battery is dead. In this case, replacing the battery with a new one may solve the problem. A television set is obviously another example of a repairable system which upon failure may be restored possibly to satisfactory performance by simply replacing either the failed resistor or transformer if that is the cause, or by adjustments to the sweep or synchronization settings.

On the contrary, a non-repairable system is any such system which is discarded immediately after it has failed to perform its desired function. For example a burned-out florescent bulb is always thrown into the dustbin after failure. However with the current automated production process turning out inexpensive products, many products that previously were repaired after failure are now discarded when they fail. For example , a desktop fan bought for less than $10.00 at a discounted rate would probably be discarded when it fails because the cost to fix it is greater than the cost for purchasing a new one[Basu and Rigdon, 1997]. Other examples of nonrepairable include the element in an electric iron, a one time use camera, etc.

In the real world, it is very obvious that most systems, such as automobiles, aircrafts, computers are designed to be repaired rather than replaced upon failure. By this implication, it would appear therefore that most literature on models for reliability of systems will be directed on repairable system. The inverse is rather true. This is not to belittle any such study on nonrepairable system. One always needs to appreciate such laudable efforts in totality which has at least brought some form of recognition on the study of reliability of systems as a whole. This is, because every repairable system by detailed study is seen to be made up of components that are nonrepairable. Therefore any study that can improve the reliability of the nonrepairable components is sure to improve the reliability of the repairable system.

[Tobias and Trindade,1994] in their book gave very detailed and more precise reasons why they think some form of attention now needed to be directed to the study on repairable systems. Thus, understanding repairable system will go along way to help in drawing very timely and effective maintenance schedules for the system before it breaks down and also making provision for all needed spare parts to help restore the system when it fails. Related to this, is the fact that any information obtained in the analysis of a system could be used for reliability improvement on later systems.

Alternatively, in a situation where many copies of a system is available, there will always be the urgent need to have some form of projection on the burn-in effectiveness, a very clear forecast on the repair cost, should a system fail and also establishing preventive schedules. The purpose of any such study may be to estimate the repair rate of the population of systems which will be used for such predictions. There is the urgent need for a service department such as the OIT to always have some form of projection on the time to failure of the facilities it operates in enhancing teaching and learning in the classroom since any such failure will surely have a negative impact in achieving the goal for that time frame.

Time scale is an essential variable in the study of the pattern of failure of any kind of system. Thus, as we study models for the reliability of repairable systems, we must be clear as to what time scale is used to measure the failure time. For a refrigerator that runs constantly, it may be appropriate to measure time as actual elapsed time. For projectors such as those used in the Technology Enhanced Classrooms (TEC) and Custom Multimedia Rooms (CMR) as in BYU, it will be ideal to measure time as the total number of hours each system has run prior to failure. It can also not go without mention that, in reality the time to subsequent failure is generally a function of many variables including the basic system design, the operating environment, and the quality of the repairs (the material used, the competency of the technician and so on) [Tobias and Trindade,1994].

2.2 Probabilistic Modeling of Repairable Systems:

Supposing upon failure, a single-component system is apparently restored to "same-as-new" or a brand new condition by using a component from the same population as the failed one, then a ***Renewal Process*** **(RP)** of the system is said to have began. This therefore leads to the assumption that the times between successive repairs are identical and independently distributed. However, replacement of the failed part by itself, does not necessarily assure a renewal process. [Tobias and Trindade, 1994]. For example, [Usher, 1993], describes a system repaired by the replacement of a component with identical unit from the same population. Yet, because the cooling unit of the system was degrading, the times between consecutive fails became shorter, thereby ruling out a renewal process.

It is however worth noting that in the real world situation, successive parts for repairs may not necessarily come from the same population. For example, spare parts may be purchased from different manufacturers than the supplier of the original system. The renewal process is usually a poor model for a system when one is looking at the reliability growth since most repairs involve the replacement of only a small proportion of the system's parts.

In an attempt to gain insight into a data collected on a repairable system and also to select the most appropriate model that best fit the data, graphical tools are best recommended to check for trend in the times between failures. A simple, but powerful, graphical method is to plot the failure time $(\mathbf{t})$ along the horizontal axis and the cumulative number of failure time $\mathbf{N}\ (\mathbf{t})$, on the vertical axis. Such plots generally indicate which analytical methods are most appropriate. Given that, the plot shows some significant amount of curvature, either concave up or down over some range of time **t**, which implies an improvement or deterioration of the system respectively, then a renewal process is not an adequate model. Alternatively, a linear relationship indicates the system remained stable over the time the data were collected. In this case, the model which can fit the data is the ***Homogeneous Poisson Process*** **(HPP)**. [Basu and Rigdon, 1997]. The only caution here is that, a smaller data size may sometime depict a wrong picture on the kind of trend that really exists in a data.

The basic conditions that justify the use of this model are that the process has *stationary increments*, that is, the number of events that occurs in any interval of time depends only on the length on the interval and not the starting point of the interval. Couple with this is the fact that the inter-arrival times are independently and exponentially distributed. In addition, the process has no memory. [Tobias and Trindade, 1994]. In a simpler term, the HPP can therefore be defined as a renewal process for which the interarrival distribution is exponential.

In a more practical sense, the HPP is a laudable model used to calculate the expected number of spare parts to stock to ensure a system operates during a mission period **t**. Thus, given that the interarrival times X_i are independent and exponentially distributed with failure rate λ will imply the PDF is;

$$f(x) = \lambda e^{-\lambda x},\ 0 \leq x.$$

Therefore the probability of observing exactly *N(t)=k* replacements in the interval (0,t) is;

$$P[N(t) = k] = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, k = 0, 1, 2, ...$$

This implies the expected or the mean value M(t) for *N(t)* is λt and the variance is also λt. Intuitively, the probability of no failure in the Poisson process in the interval (0,t) is;

$$P[N(t) = 0] = \frac{(\lambda t)^0 e^{-\lambda t}}{0!} = e^{-\lambda t}$$

and the probability of at least one failure is $1-e^{-\lambda t}$, which is the same as the CDF for an Exponential distribution with failure rate λ . For instance given that components are assumed to fail with an Exponential distribution having failure rate λ= 0.00030 = 0.030%/hr. with a mission duration of 500hr. A 95% probability of successful mission completion and how many parts needed to be carried for a single component system could be derived by first finding out the expected number of failure within the time frame, which in this case is λt = 0.00030 x 500 = 0.15. This implies the probability of two or more failure will be;

$$1-[\frac{(\lambda t)^0 e^{-\lambda t}}{0!} - \frac{(\lambda t)^1 e^{-\lambda t}}{1!}] = 1-[\frac{(0.15)^0 e^{-0.15}}{0!} - \frac{(0.15)^1 e^{-0.15}}{1!}]$$

$$= 1-[0.861-0.129]$$

$$= 0.010$$

$$or 1\%$$

This indicates carrying one spare part will assure operation during the mission with nearly 99% probability. Thus in all, two parts may be required; the first is the original and the second being the spare.

In a scenario where a plot of the data on a system under observation depict a significant curvature in its trends, will therefore imply the interarrival time of failures are not identically distributed hence a different model other than the **HPP** must be fitted. The **Non-Homogeneous Poisson Process** abbreviated ***NHPP*** is therefore an excellent choice in such cases. [Barlow and Hunter, 1960], [Ascher and Feingold, 1984]. When the reoccurrence rate is a function of time, it is called the intensity function, $\lambda(t)$. It has a mean cumulative function defined as; M(t) = $\int_0^t \lambda(\tau) d\tau$. [Ross, 1993] has shown that;

$$P[N(t+s)-N(t)=n] = e^{[M(t+s)-M(t)]}\frac{[M(t+s)-M(t)]^n}{n!}, n \leq 0$$

This clearly implies that any incremental occurrence within the time interval between

t and **(t+s)** has a Poisson distribution. Intuitively, it will imply the reliability R(s) defined on the probability of zero occurrence in the specified time t to (t+s), is

$$R(s) = e^{[M(t+s)-M(t)]}.$$

2.3 Shortcomings of Probabilistic Modeling:

The general assumption that operating time is the only variable in reliability study appears too restrictive and unrealistic. For instance, the number of on/off cycles of parts as simple as the light bulbs need to be considered when looking out for the appropriate probabilistic model for any system that uses such part [Farewell and Cox, 1979]. Thus, in as much as the existing model may be reasonably accurate in predicting the distribution, it needs mentioning that they may become totally inaccurate when other significant variables which could exert increasing effects on reliability are ignored. It is from this perspective that we now discussed the shortcomings of the probabilistic models mentioned above.

i.)*Replacement parts may not necessarily come from the same population:*

There is a strong likelihood that the parts used for repairs may not necessary come from the same population as assumed. Thus, there is the possibility that a part with the same brand name and model could just be an imitation designed by a different company and not from the original manufacturer as the failed part. When this occurs, then the assumption of identical and independent distributed of the renewal process in highly defeated.

ii.) *Overhaul do not necessarily restore a system to a same-as-new condition:*

The general assumption that by replacing a failed part, a system tends to gain a condition as a brand new is highly disputed by the following evidences; Lavalee et al. (1974) stated,"It appears, therefore, that an aircraft will be statistically less reliable and will require more unscheduled lower level maintenance after depot mendaintenance than before." Again, in his 1975 paper, he reported,"There is considerable evidence that engines wear in mishap, accident, and removal rates generally being higher shortly after overhaul than later in the engine's tour." Other conclusions like this,"A review of the work performed in 16 ships overhauls revealed that 10 out of the remaining 12 equipments that show no degradation were worked on in less than 20% of the overhauls. In contrast, six of the nine equipments that showed degradation were worked on in more than 40% of the overhaul." [Tullier, 1976].

iii.) *There may be incomplete repairs where the real problem is not corrected during the first repairs:*

In an attempt to repair a failed system on a first failure, there could be instances where an incomplete repair will lead to either an improvement or deterioration of the system. This could be incidental, however it may have an effect on the reliability of the system over a given duration.

iv.) *All parts will not be in series:*

In a real world sense, it is very obvious that most systems are not designed with components in series where the failure of one may lead to the complete failure of the whole system. It will therefore become very difficult to assess the importance of either a designed-in redundancy (e.g. two fuel pumps for an aircraft engine) or operational redundancy (multiple spark plugs in an automobile engine) [Ascher and Feingold, 1984].

v.) *Inconsistence stresses may result in seasonal effects:*

Time is often emphasized as a factor that affects the reliability of a repairable system. However, it is worth mentioning that, besides time, other stresses like the number of on/off cycling and environmental effects may turn to have seasonal effects. For example, [Molter, 1979] stated in his paper on a study on air conditioner that failures were significantly due, at least in part, to the summer weather.

vi.) *Repair may be made by adjustment:*

The repair of a system will initially sound like replacing a failed part with another one. However, a repair could be done by just adjusting or a giving a gentle tap on the system without necessarily using any part.

2.4 Alternative Statistical Analysis of Repairable Systems:

It must however be acknowledged, that practically there are too few failures that may occur on any one system, at least during the limited observation intervals usually encountered, to apply such techniques as ***Renewal Process*** or ***HPP*** or ***NHPP***. They basically, consider time as the only possible factor that explains reliability growth if used in analyzing data on repairable systems. Also, in a situation where a large number of system copies exist for analysis, there usually are known differences which may have significant effect on their reliability. Thus, the same type of system may be switched on and off at different times (e.g. computers in the classrooms), the rate at which projector filter accumulates dirt may differ based on their location, thermal events in the systems may not necessarily be the same (i.e. there may be differences based on whether the said room is fully air-conditioned or not). In addition, the skilled level of the maintenance men and the way a system may be configured may all have considerable effects on the system. It may therefore always be beneficial to have an alternative analysis even if any of the aforementioned models is assumed for each system copy.

An earlier method of dealing with any such differences that may have effect on the system was to either *(1)* ignore them or *(2)* to break the data set into two or more group based on major difference [Ascher and Feingold, 1984]. The weakness in the second method is that of having inadequate data in any one group for meaningful analysis when the data is divided into too many groups. Also, the arbitrary segregation of the data ignores the fact that the systems are basically the same - or at least similar.

The only useful or best alternative to be considered here is that of a ***Regression*** type of analysis. The kind of approach which provides a common baseline for all systems and which is suited to the special conditions of reliability analyses such as the presence of censored data and lack of knowledge about the suitable choices for interarrival time distributions. The kind of regression approach depends on the number of variables under study. A ***Simple Linear Regression*** approach is used when there is just a single explanatory factor involved, and a ***Multiple Regression*** is used when the analyst has at least two explanatory factors on hand.

The model proposed by Prentice, Williams and Peterson [PWP, 1981] paved way for reliability analyses using the regression procedure. Their model was an extension of Cox's (1972b) model, where multiple failures of a single system can occur. Thus, in a sense, Cox's regression model may be considered to be a nonparametric method. The model may be written as:

$$h(t),(z_1,z_2,...,z_m) = h_0(t)^* exp(b_1{}^* z_1 + ... + b_m{}^* z_m),$$

where *h(t,...)* denotes the resultant hazard, given the values of the m covariates for the respective case $(z_1, z_2, ..., z_m)$ and the respective survival time (t). The term $h_0(t)$ is called the *baseline hazard*; it is the hazard for the respective individual when all independent variable values are equal to zero. We can linearize this model by dividing both sides of the equation by $h_0(t)$ and then taking the natural logarithm of both sides:

$$log[h[(t),(z...)]/h_0(t)] = b_1{}^* z_1 + ... + b_m{}^* z_m.$$

We now have a fairly "simple" linear model that can be readily estimated. While no assumptions are made about the shape of the underlying hazard function, the model equations shown above do imply two assumptions. First, they specify a multiplicative relationship between the underlying hazard function and the log-linear function of the covariates. This assumption is also called the *proportionality assumption*. In practical terms, it is assumed that, given two observations with different values for the independent variables, the ratio of the hazard functions for those two observations does not depend on time. The second assumption of course, is that there is a log-linear relationship between the independent variables and the underlying hazard function. Another attempt to use the regression method as an alternative approach in a reliability study could be traced to Wolfe's work in 1977 where he considered the analysis of ***NHPP*** with a covariate. He applied his method to small numbers of events observed on only few systems. [Anderson and Gill, 1982], also used an extension of the Cox's (1972b) model on repetitive events. Other writers such as Braun and Hoem (1979) put further innovation into this approach by analyzing the data on the birth interval pattern of a set of Danish women with the assumption that the baseline model for interarrival times has a gamma distribution. [Ascher and Feingold, 1984] also assumed the Weibull distribution as a baseline function to determine how different temperature rates accelerates the failure times of sampled capacitors from a given manufacturer in the fashion of regression.

2.5 Cost Models for Repairable Systems :

A system's reliability can be increased substantially by setting either a preventive or scheduled maintenance policies in place whereby units which are about to enter their wear-out life , or are partially worn out , or aged, or are due for a minor or a major overhaul, are replaced with new units at predetermined periods of operation. These policies when implemented effectively have the advantage of reducing the average failure rate of the equipment, reduce the cost and inconveniences associated with failures, increase the equipment availability and productivity, and if it is a production system, it will invariably decrease the unit cost of production.[Kececioglu,1995]

By deduction, it will be much more expensive to handle failures during operation than preventive maintenance, since any such failing unit has the potential of damaging many other parts adjacent to it or other associated systems. The focus of this paper, as stated earlier is among other things, help formulate a kind of cost model that will assist the OIT to adopt an appropriate maintenance policy on their equipments.

The very kind of stochastic process used in modeling the pattern of failures on a repairable system determines the nature of the cost model that is applicable. It is therefore interesting to note that, there is just a limited literature on cost models for repairable systems since in most cases the usual assumption of a renewal process is really inapplicable in the real world situations [Gertsbakh, 1977]. Another reason being that, only a small portion of a system is replaced during most repairs.

Cost Models for repairable systems are mostly classified into two main categories. Thus, models associated with *deteriorating* or those undergoing *reliability improvements*. The main principle addressed by models in the first class is that of establishing some form of policies that will optimize factors such as maintenance cost, availability, reliability that could make considerable impact on the maintenance policies of the operating system. The kind of optimization a model will focus on will always take into consideration the criterion chosen with respect to the length of the planned replacement interval,(i.e. the cost of minimally repairing the system are traded off against the cost of replacing the system upon failure). [Ascher and Feingold, 1984].

2.5.1 Deteriorating Repairable Systems:

Various kinds of Cost Models have been formulated over the years under situation in which a system is identified to be deteriorating. Notable among them is what has become known as the *Type 1 Model* or *Age-Dependent Replacement Policies.* In this model, the item is replaced either at failure or after a fixed operating time I, whichever occur first. It is assumed that the time to failure distribution is of an increased hazard rate function and the cost of planned replacement is always less than the cost of replacement after failure. The proposed cost model is expressed as:

$$E[C(I)] = C_3E[N_3(t)] + C_2E[N_2(t)],$$

where $N_2(t)$ is the number of planned system replacements within the time (0, t], $N_3(t)$ is the number of system failures within the time (0, t] which cause system replacement, *(I)* is the system planned replacement interval, C_2 is the cost of a scheduled system replacement assumed to be constant and C_3 is the cost of an unscheduled system replacement assumed to be constant [Fox, 1966]. Many other modifications have since been made to the above cost model. This led to models such as *Modified Type I*

Policies, authored by Schaeffer (1971), *Minimal Repair Policies* which gave credit to writers such as Barlow and Hunter (1960), Makabe and Morimura (1963) to mention but a few.

2.5.2 Repairable Systems Under Reliability Improvement

A careful trace on the life cycle of certain systems mostly in production shows some form reliability improvement after having subjected them to successive redesigns based on test, analyze and fix programs. When this happens, such systems will always give lower rate of occurrence of failures. Specific example can be related to a situation when a system undergoes a debugging period. Eventually, as the causes of failure are eliminated, the system will operate in a region of constant or near constant rate of occurrence of failures. Such periods however adds directly to the cost of procurement and need to be traded off against the cost of repairing the system that fails one or more times during deployment. The length of the debugging period is however very significant in making an economic decision on whether to undertake the process or receiving the product in an unreliable state.

Again, only a handful of research work has been done in this field of reliability of repairable systems. One such outstanding cost model was that propounded by Plesser and Field (1977). They assumed a "minimal repair" model for the pattern of successive failures with a strict decreasing rate of occurrence of failures, rather than the one with the bathtub shaped. The proposed model is:

$$C(t_\alpha) = B_1 + B_2 t_\alpha + {C_1}^\alpha E[{N_1}^\alpha] + C_1 E[N_1],$$

where B_1 is the fixed cost of debugging,B_2 is the cost per system copy per hour of the debugging program, and ${C_1}^\alpha$ is the repair cost for a copy which fails during the debugging period.

Chapter 3

Methodology

3.1 Introduction:

The goal for this project is to analyze the data collected on the projectors using various statistical approaches and the data at hand. In addition, a cost model will be formulated to determine the total cost of undertaking both preventive and maintenance services on the projectors. This cost model will then be used to make recommendations to the OIT as to what might be the most appropriate maintenance schedule taking into consideration the cost of replacement and down time of a bulb in a projector.

It was from a very great effort and much sacrifice on the part of individuals such as Stephen Zobell, Mark Sullivan, Mark Hales and Blair Warner, that we were able to obtain the data on projector bulb failures in OIT technology room on campus. Each person helped to retrieve some information on the projectors from the OIT's records and I finally compiled them into a format that will be needed for the analysis. The final fields arrived at included the following;

1. Date of Failure or Censor
2. Room Number
3. Serial Number of Projector
4. Hours to Failure
5. Projector Type

6. Number of Logins

7. Number of Thermal Events

8. Failure Indicator

The Number of Logins refers to the counts of logins into the computer system whenever the projectors were used. This information was taken per room over a year's period with the counts computed on weekly basis. With assistance from Mark Sullivan we were able to track down how many times a system was used with regards to the login before the bulb failed. Just like any other system powered on electricity, the projectors also undergo some form of fluctuation in heat. The field for thermal events therefore is the count of times the heat level of a projector exceed a preset temperature level. When this occurs the system automatically triggers off and that is taken as a count of thermal event. The projectors over time have been monitored to find out if they fail before some specified number of hours of used as a measure to warrant a replacement from the producers. Since some operate below and beyond the specified hours, the Failure Indication field is therefore created to represent if the response variable is censored or an actual failure time.

3.2 Graphical Analysis:

Various graphical tools in SPLIDA will be used to determine the sampling distribution that best fit the time between bulb failures of the data. This could be related to the method used by [Davis ,1952] to analyze the number of miles between successive major failures of a bus engine. In doing so the projectors will be sorted, based on type,and a Mean Cumulative Plot [Nelson,1988] will be made for each of the three types of projectors ***(EPSON 8100, 8200, and 8300)***. If the trend in the Mean Cumulative Function plot (MCF) is a straight line, it indicates that the projector is a renewal process where once a bulb is replaced the system is as good as new. Also, probability plots will be made to determine the distribution of the interarrival times between failures. Some viable distributions for time between failures that are most likely to be considered include the Exponential, Weibull and the Lognormal distributions.

If the process is a renewal process and Exponential distribution fits the inter-arrival times best, then the ***Homogeneous Poisson Process*** *(HPP)* model will then be fitted to the data. If the **MCF** plot is convex rather than a straight line, it would lead to the fitting of the ***Non-Homogeneous Poisson Process*** *(NHPP)* as an alternative model. If the MCF plot is straight but the Exponential distribution looks inappropriate, we will then check for Renewal Process with time between failure on other distribution such as the Weibull, Lognormal, etc.

3.3 Regression Model:

A careful study of the final data, taking into account the various field calls for the usage of a regression model as an alternative way of undertaking a reliability study on the projectors. The purpose of using the regression is to determine if the distribution of time between failures is dependent upon some variable such as Projector Type, Number of Thermal Events and Number of Logins.

Both parametric and non-parametric regression models will be explored to find out which of them offers more viable assessment of the variables under consideration. Since there is more than one kind of projector,an indicator variable will be created to represent each particular type. The response variable will include actual time to failure and time to censor. The proposed Parametric Regression Model using ***SPLIDA*** in **Splus** will be;

$$T = \beta_0 + \tau_i + \beta_1 X_1 + \beta_2 X_2,$$

where T is the response variable (Total Hours to Failure), β_i is the regression coefficients, τ_i is Projector Type, X_1 and X_2 are the Number of Thermal Events and Number of Logins respectively.

An alternative approach will be to run a Non-Parametric Regression model again in **Splus** using ***Surreg***(Survival Regression).

3.4 Proposed Cost Model for the Project:

Holding all prevailing factors referred to in the data constant, it may be proposed that the appropriate type of cost model for the systems will fall directly under the class of *Deteriorating Repairable Systems*, given that the time to failure has an increasing hazard rate, however with some kind of modification. Thus, with the passage of time, there is a strong likelihood that the systems will tend to be less effective in executing their intended functions as a result of conditions such as thermal events, the number of login and logout to mention but a few . Also given the fact that, upon the failure of a unit as simple as a bulb, the teaching and learning process could come to some form a stand-still condition, before any alternative measure is set in place to rectify the situation. This could account for a great loss of time. Besides the cost of man hours needed to replace such units. It is further assumed that after each failure the system is only minimally repaired, hence the rate of occurrence of failure of the system is in effect unchanged by the repair. The purpose of the cost model is to find out how to minimize the total cost by choice of planned bulb replacement time. The Expected cost model therefore could be:

$$E[C(t_r)] = C_3E[N_3(t)] + C_2E[N_2(t)]$$

where $C(t_r)$ is the total cost of repair or replacement given the interval (0,t], t_r is scheduled bulb replacement time, C_3 is the cost of unscheduled bulb replacements times,C_2 is the cost of scheduled bulb replacement times, $N_3(t)$ is number of failures in the interval (0,t] that caused a replacement and $N_2(t)$ is the number of planned bulb replacements in (0.t].

Barlow and Hunter, (1960) showed the optimum replacement time t_r that minimizes $\frac{E[C(t_r)]}{t}$ as $t \to \infty$ is given by;

$$h(t_r) \int_0^{t_r} R(t_r)dt - F(t_r) = \frac{C_2}{C_2 - C_3}$$

Currently the OIT replaces bulbs after $1800hr$ or if failure occurs before that time. The research interest is to find out if this is the optimum replacement time.

Chapter 4

Data Analysis and Findings

4.1 Graphical Analysis

Various tools and techniques have been implored on the data to enable the research question of whether the OIT should maintain the current policy of the 1800 hours replacement time of a failed bulb in the projectors. In undertaking this analysis, collected data was first treated as a whole taking into accounts all the regressor variables on the three types of projectors used in the designated media rooms.

Some basic and well known distributions have been fitted to determine which of them will offer a good fit to the data. Fig.1 below shows a plot of these distributions.

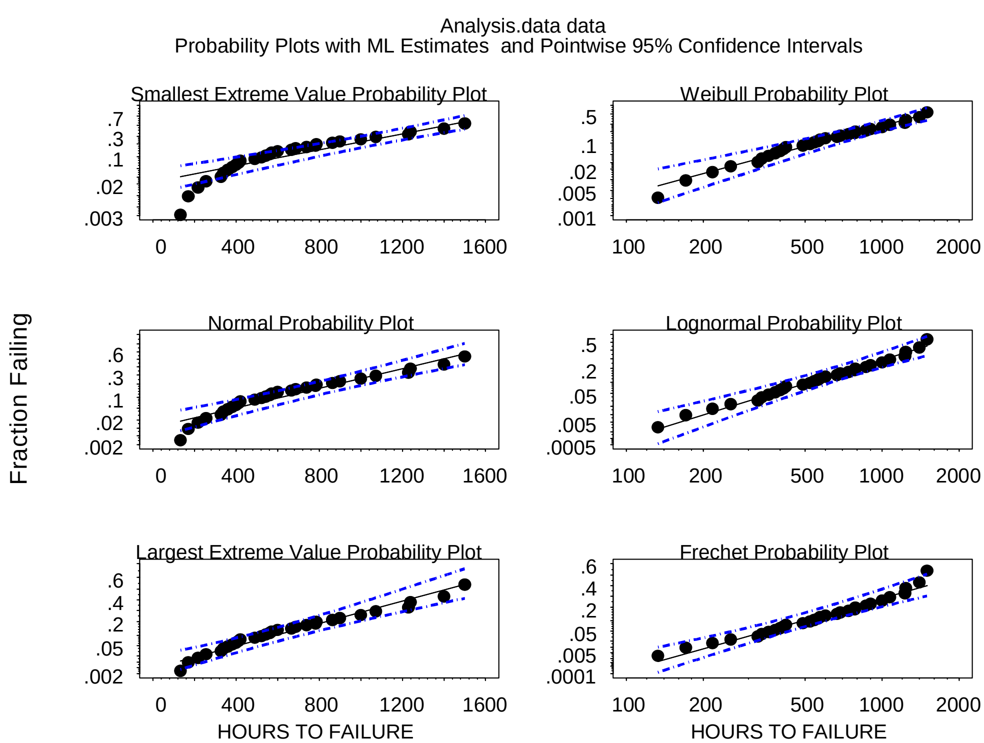


Fig.1 Probability Plots of Interarrival Times

For further study on the distributions, much attention has been paid on their respective log likelihood values which are displayed in the Table 1 below.

DISTRIBUTION	LOG LIKELIHOOD AT MAXIMUM POINT
Exponential Distribution	-292.6
Normal Distribution	-288.5
Frechet Distribution	-285.6
Smallest Extreme Value Distribution	-292.5
Largest Extreme Value Distribution	-286.2
Lognormal Distribution	-284.3
Weibull Distribution	-284.3

Table 1. Log Likelihood for Basic Distributions

Considering the findings from either Fig.1 or Table 1, one will not hesitate to avoid using the Exponential Distribution to fit any model in this study. This will therefore go to rule out the possibility a Homogeneous Poisson Processes since it has been established from the chapter on the Literature Review that we need to have the data to be independent and identically distributed to assume a Renewal Process(RP) and also independent and exponentially distributed to assume the Homogeneous Poisson Process (HPP). Plot on the Exponential Distribution and its Hazard Function is respectively shown in Fig 2 and Fig.3 below.

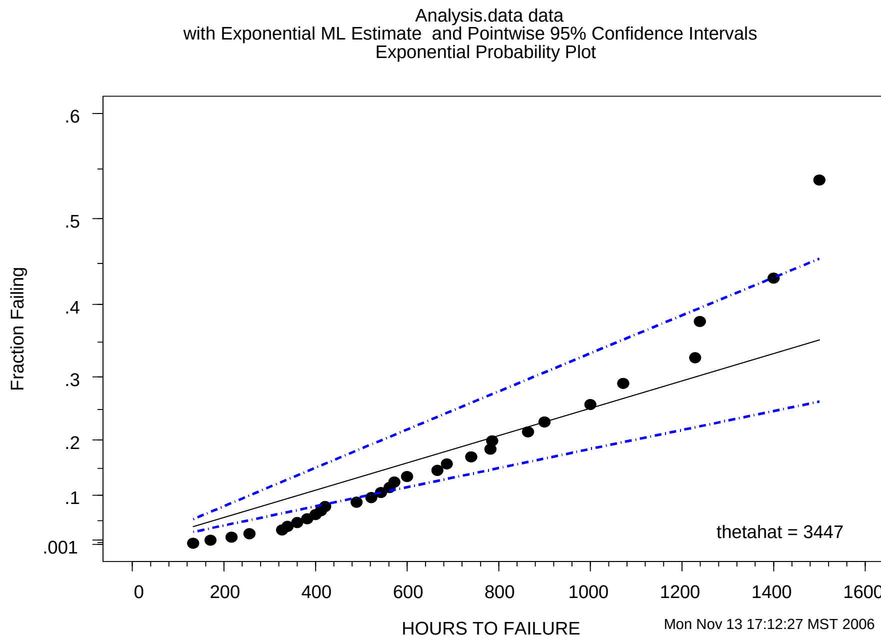


Fig.2 Exponential Probability Plot of Interarrival Times

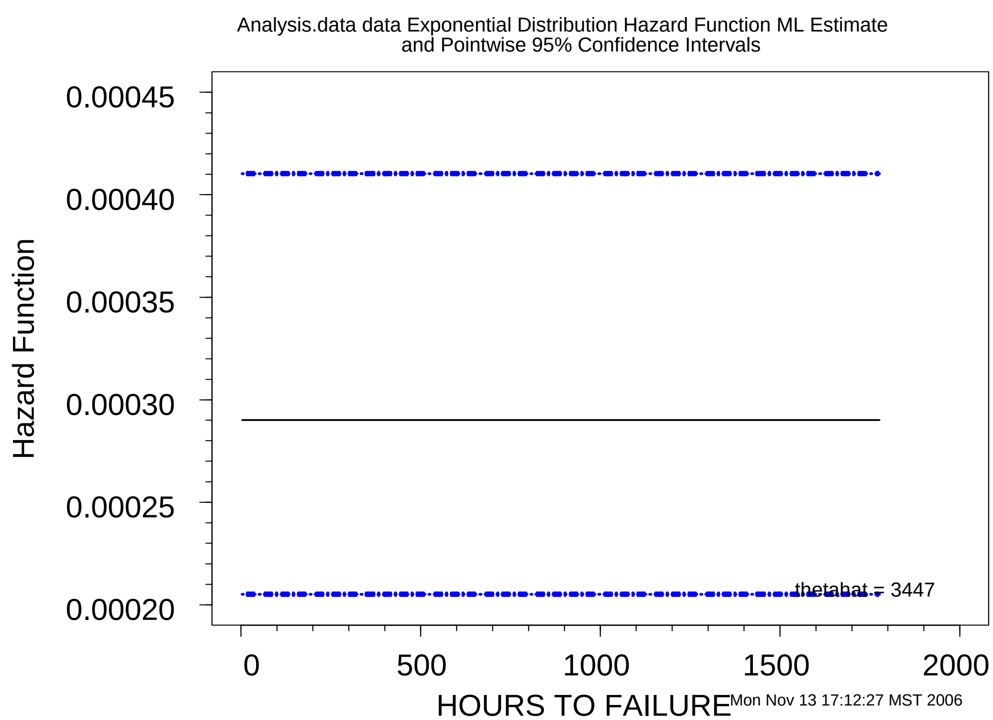


Fig.3 Exponential Hazard Plot

Fig4. through Fig.7 also shows the Lognormal and the Weibull distributions with their respective hazard plots

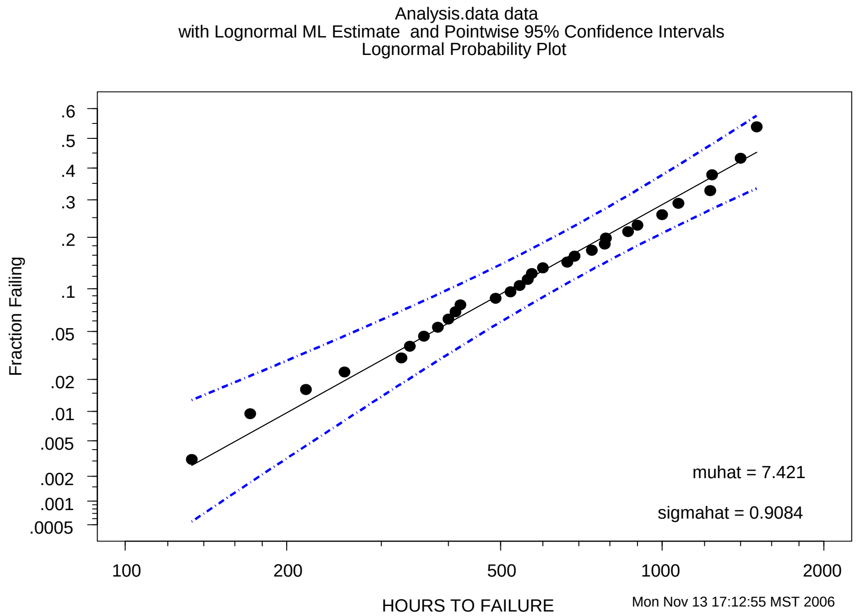


Fig.4 Lognormal Probability Plot of Interarrival Times

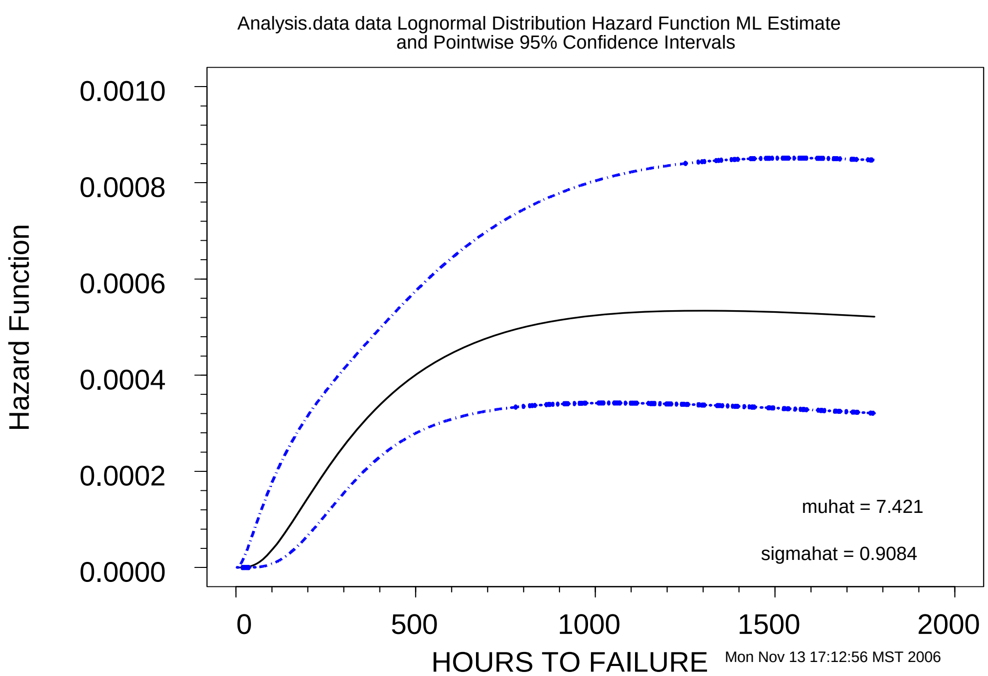


Fig.5 Lognormal Hazard Plot

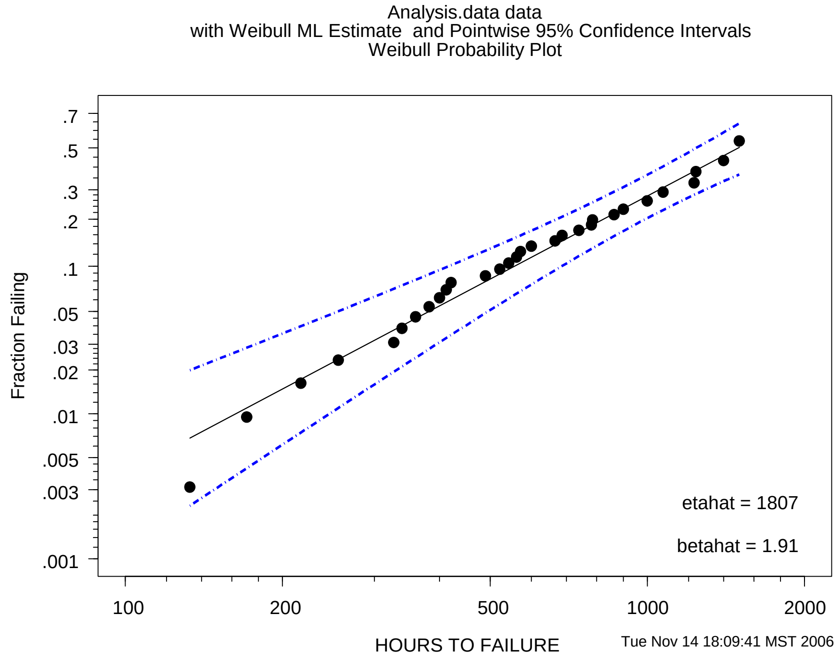


Fig.6 Weibull Probability Plot of Interarrival Times

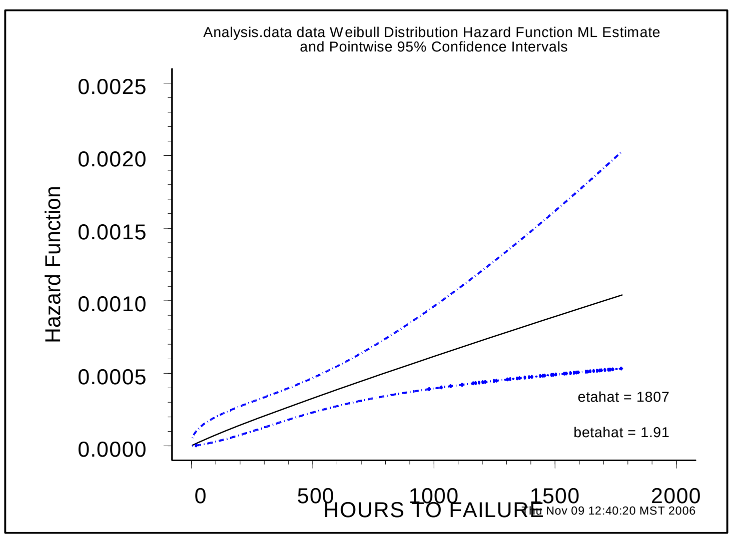


Fig.7 Weibull Lognormal Hazard Plot

The Hazard plots for the Lognormal and the Weibull distributions show clearly there is an increase rate in failure time. However, the Weibull hazard plot shows steadily increasing failure rate while the Lognormal shows an increasing failure rate to a point with a constant failure rate later. Since projector bulbs should not have a constant failure rate in any interval, it was decided that the Weibull distribution is more appropriate.

To further show that, the Exponential distribution is not adequate, a Log Likelihood Ratio Test was used as a diagnostic tool, where T is the test Statistic:

Log Likelihood Ratio Test

$$T = 2 \times (-292.6 - (-284.3)) \quad = 16.6$$

$$\chi^2_{(0.95,1)} \quad = 3.84$$

$$Since \;\; T > \chi^2_{(0.95,1)} \qquad we \; reject \;\; H_o$$

Other diagnostic assessment done on the data was to use the Mean Cumulative Plot to check if the time to failure on the projector really warranty the fit of the Weibull distribution. This was done on the whole data irrespective of the projector type (Fig.8) and also on the specific type of projector (Fig.8 to 11). The plots below show the output.

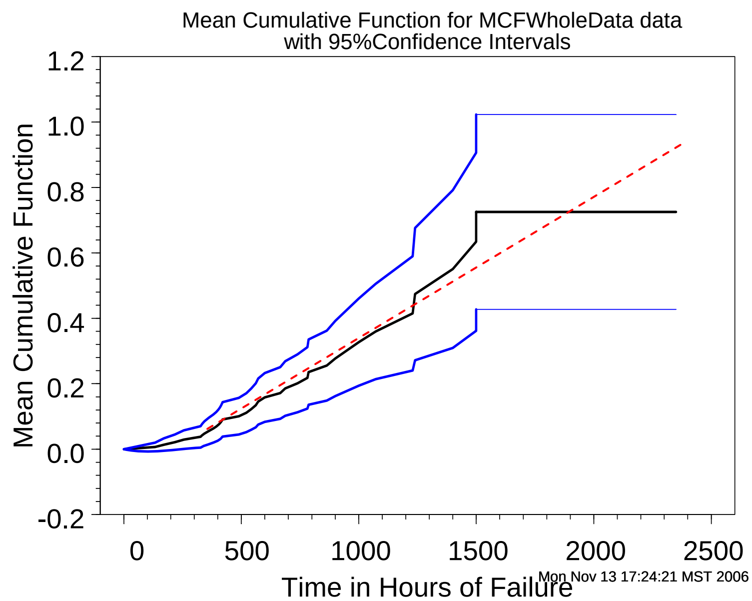


Fig 8.
MCF Plot for Data from all Projectors.

Fig. 9
MCF Plot for Data
from 8100 Projectors

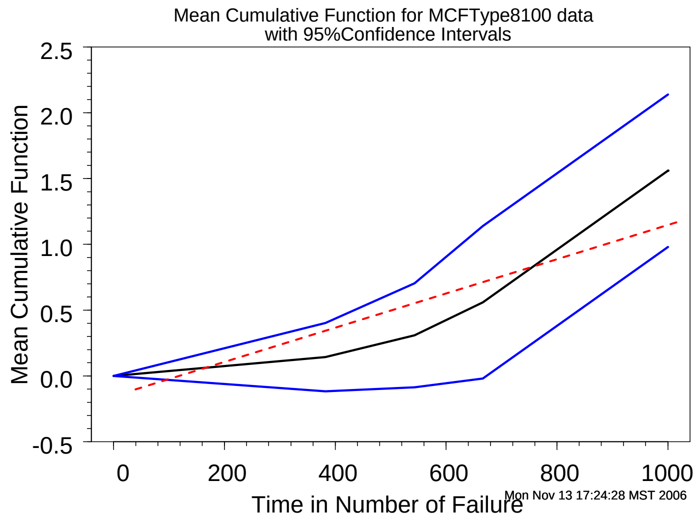


Fig 10.
MCF Plot for Data
from 8200 Projectors

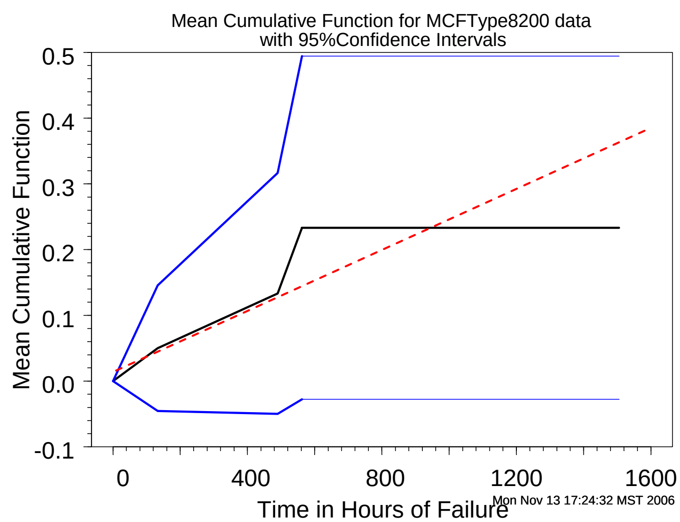
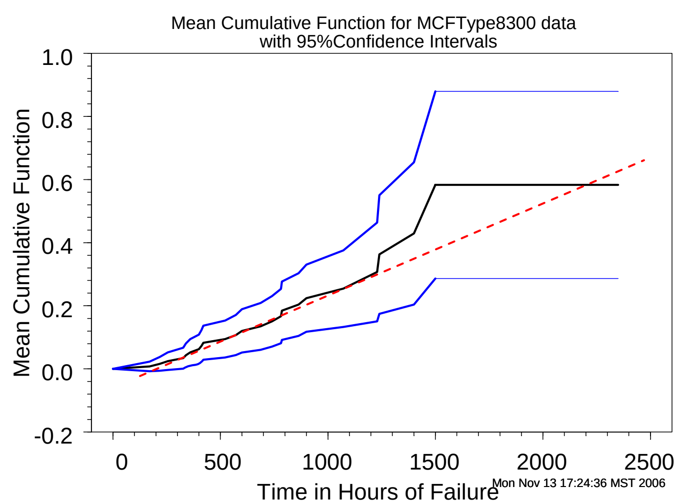


Fig 11.
MCF Plot for Data
from 8300 Projectors



A careful look at Fig.9 shows the data on 8100 type of projectors did not give a good fit which could be acknowledge to the fact that there were not enough data collected on them. The straight line trend in Figure 8, 10 and 11 indicate that a Renewal Process is reasonable.

4.2 Regression Analysis on All Projector Data:

To determine which of the explanatory variables has a significant effect on the hour to failure of the bulbs in the projectors, a multiple regression analysis was run using the Hours recorded at either the failed or censored time as the response variable. The explanatory variables used are the Number of Logins, Number of Thermal Events, and Type of Projector. Indicator variables were created for the Type of Projectors to help distinguish if any type in question has a contributing influence. The output below shows the Number of Logins on the projector and the Type were significant since their respective 95% Lower and Upper confidence intervals do not include zero (0).

Table 2**: REGRESSION Output on the Whole Data**

Variable: Relationship (g)
1 Type8200: Linear
2 Type8300: Linear
3 NO..LOGINS: Linear
4 THERMAL.EVENT: Linear
Model formula:
Location ~ Type8200 + Type8300 + NO..LOGINS + THERMAL.EVENT
Log likelihood at maximum point: -265.2

Parameter			**Approx Conf. Interval**	
	MLE	**Std.Err.**	**95% Lower**	**95% Upper**
(Intercept)	5.675336	0.3461271	4.9969398	6.353733
Type8200	0.600608	0.3694854	-0.1235700	1.324786
Type8300	0.950565	0.2949872	0.3724004	1.528729
NO()LOGINS	0.001698	0.0003653	0.0009822	0.002414
THERMAL.EVENT	0.074458	0.1088100	-0.1388060	0.287721
Sigma	0.453221	0.0610493	0.3480582	0.590157
weibull.beta	2.206431	0.2972087	1.6944649	2.873083

In addition to the output shown above, residual plots were used to check if the model fit and distribution applied was appropriate. Fig.12 is a Weibull Probability Plot of the residuals and it depicts very clearly that, Weibull distribution has a good fit.

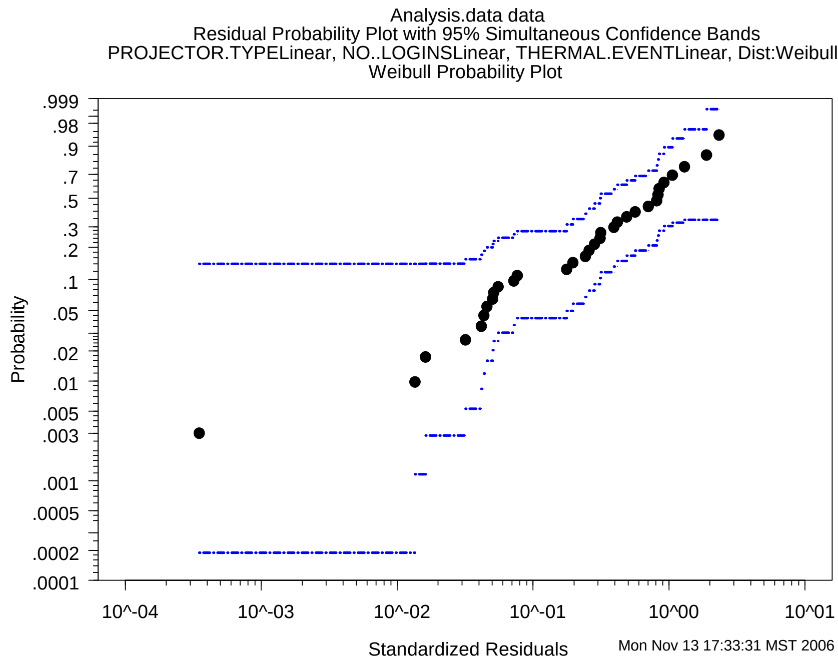


Fig.12 Weibull Probability Plot of Residuals on all Projectors

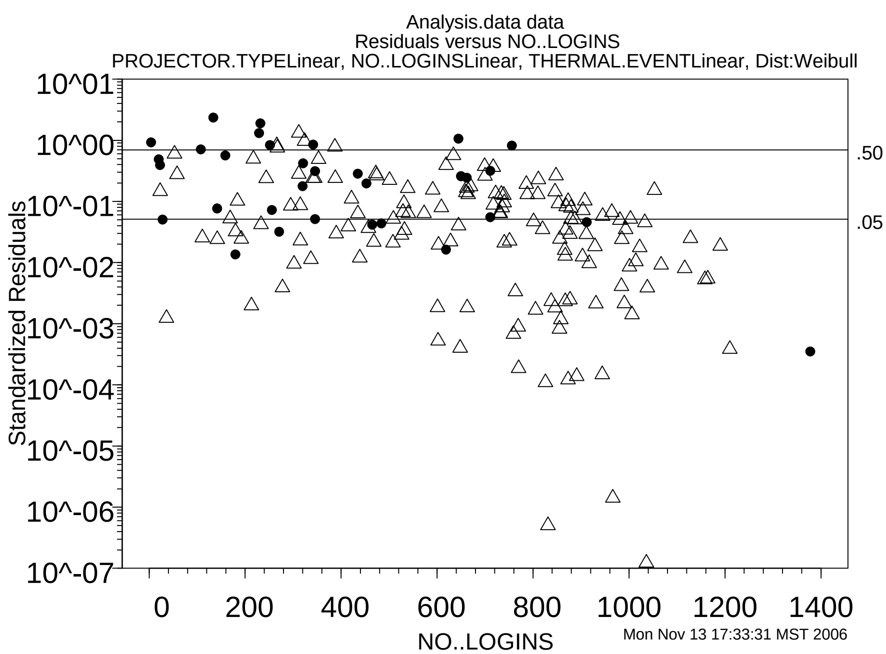


Fig.13 Residual versus Number of Logins on all Projectors.

4.3 Regression Analysis on Type 8300 Projector Data:

In attempt to narrow down the scope of the study, more attention was given to the Type8300 projectors since it has a significant effect on the whole data set. The coefficient of the Number of Logins when the regression was run on the whole dataset turned out to be positive which is the opposite of what was expected. When fitting the regression model to this subset of the data, the average Logins per week was used as the explanatory (rather than the total number of Logins) and the Hours to failure as the response variable. The output below, also confirmed the explanatory variable is significant since the constructed Confidence interval did not capture zero

Table 3: **Regression Output on Type8300 Using Average Logins**

Ty8300AvLg data
Maximum likelihood estimation results:
Response units: Hours to Failure
Weibull Distribution
Variable: Relationship (g)
1 Average.Logins: Linear
Model formula:
Location ~ g(Average.Logins)
Log likelihood at maximum point: **-213.5**

Parameter			**Approx Conf. Interval**	
	MLE	**Std.Err.**	**95% Lower**	**95% Upper**
(Intercept)	8.45704	0.42716	7.61982	9.294260
g.Average.Logins.	-0.04437	0.01790	-0.07945	-0.009293
sigma	0.51008	0.08131	0.37321	0.697139
weibull.beta	1.96048	0.31250	1.43443	2.679444

Some analytical plots such as the Model and Residual plots were again used to assess if this model is appropriate. Fig.14 below show the model plot whilst Fig.15 through Fig. 17 shows residual plots.

Fig.14
Model Plot for 8300 Data Using Average Logins

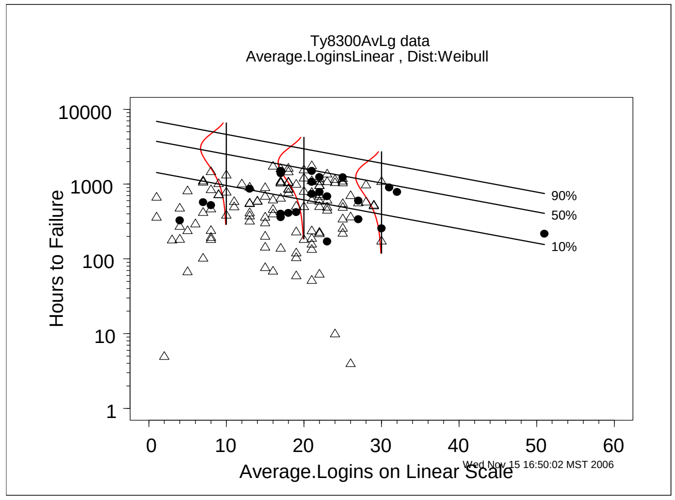


Fig.15
Residual versus Fitted Values on 8300 Data Using Average Logins

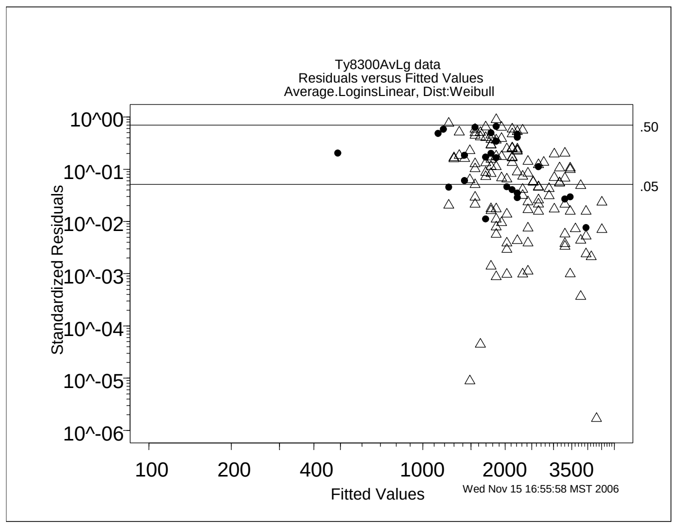
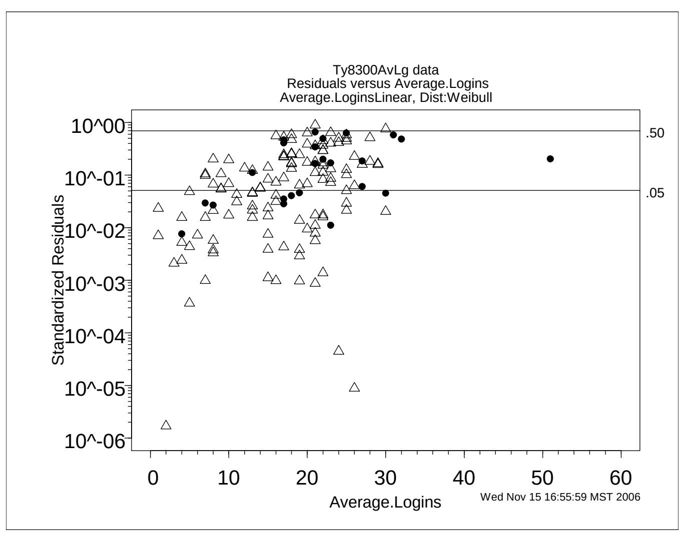


Fig.16
Residual versus Average Logins on 8300 Projector data.



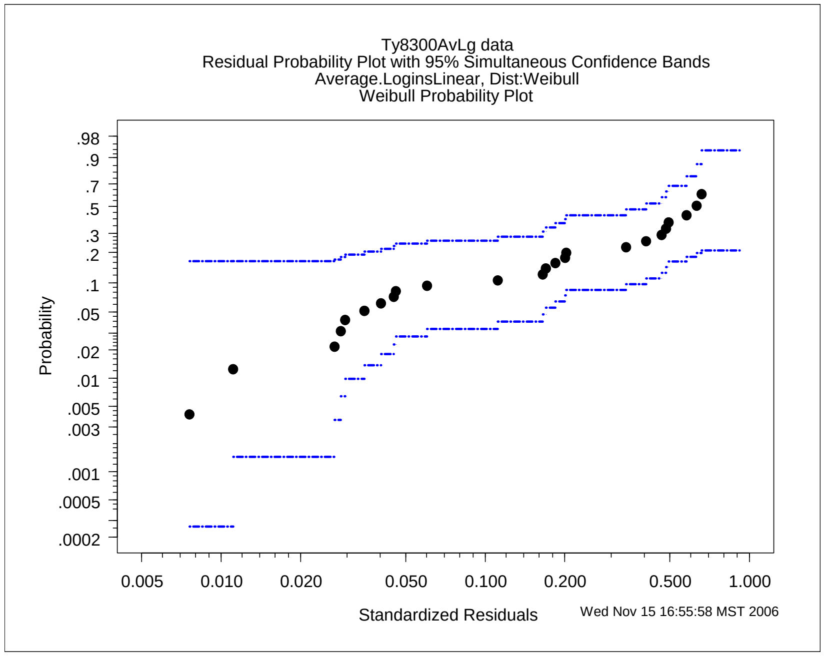


Fig.17 Weibull Probability Model Plot of Residuals on 8300 projectors Using Average Logins

A look at the Box-Cox transformation plot suggested some form of transformation. The figure below shows the plot.

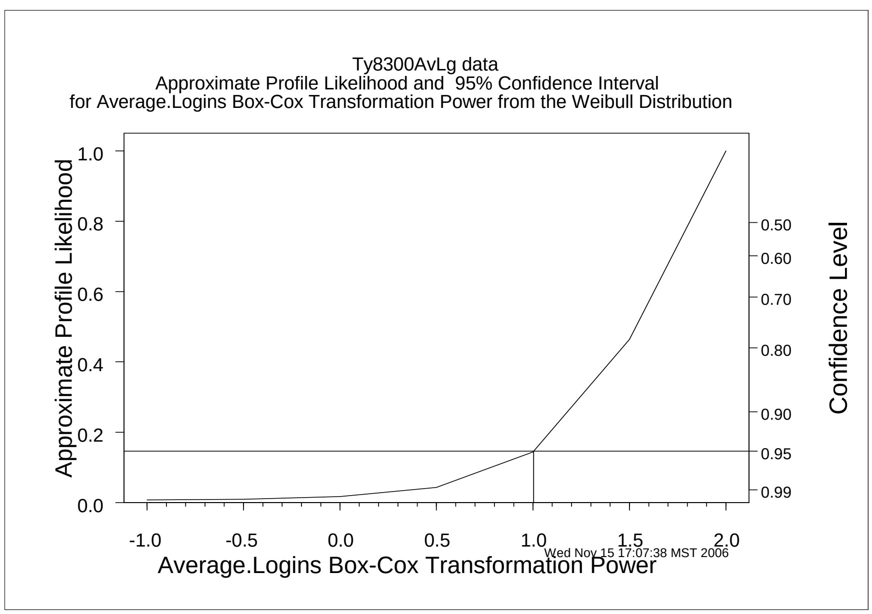


Fig.18 Box Cox Transformation Plot on 8300 projectors

This led to refitting the Model using the square of the average Logins on the Type8300 projectors as the explanatory variable. The output just showed some difference which was noted in the log likelihood value. Thus, instead of -213.5 in the first case, we now had -211.5 which indeed confirm the fact that the transformation yielded a better fit. Below is the output for that analysis and its corresponding Model and Residual plots. As shown in Fig.19

Table 4: **Regression Output on Type8300 Using Average Logins Squared**

Type8300AvLG2 data
Maximum likelihood estimation results:
Response units: Hours to Failure
Weibull Distribution
Variable: Relationship (g)
1 Average.Logins.Sq.: Linear
Model formula:
Location ~ g(Average.Logins.Sq.)
Log likelihood at maximum point: **-211.5**

Parameter			**Approx Conf. Interval**	
	MLE	**Std.Err.**	**95% Lower**	**95% Upper**
(Intercept)	7.9561315	0.1916098	7.580583	8.3316797
g.Avg.Logins.Sq.	-0.0009613	0.0002215	-0.001395	-0.0005272
sigma	0.4792373	0.0760739	0.351100	0.6541390
weibull.beta	2.0866487	0.3312337	1.528727	2.8481884

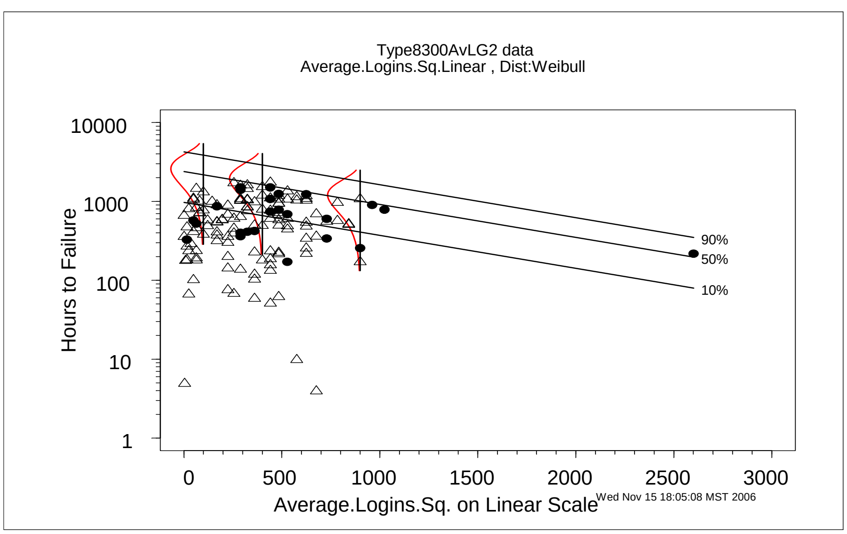


Fig.19 Model Plot for 8300 Data Using Average Logins Squared

Fig. 20

Residual versus Fitted Values on 8300 Data Using Avg. Logins Sq.

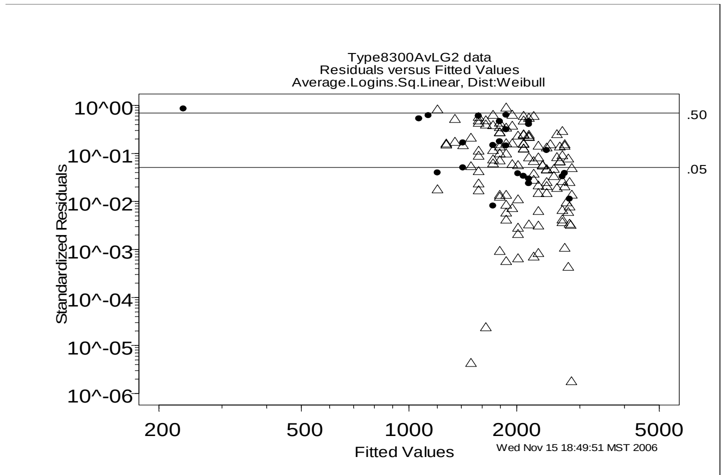


Fig.21

Residual versus Average Logins Sq. on 8300 Projector Data

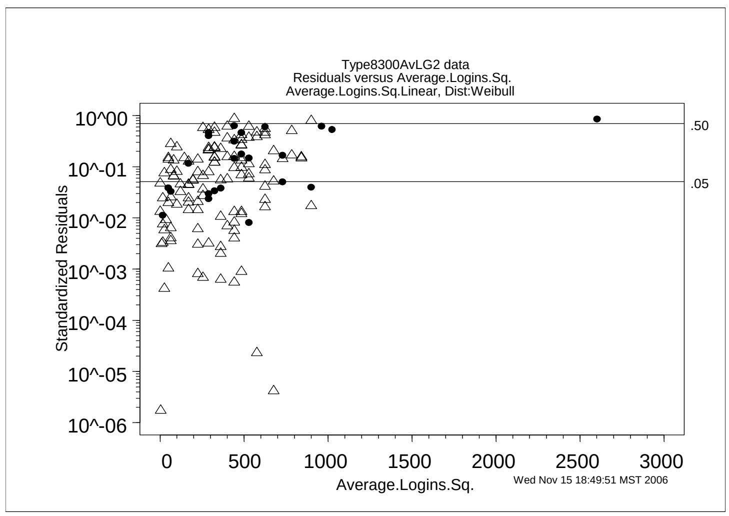
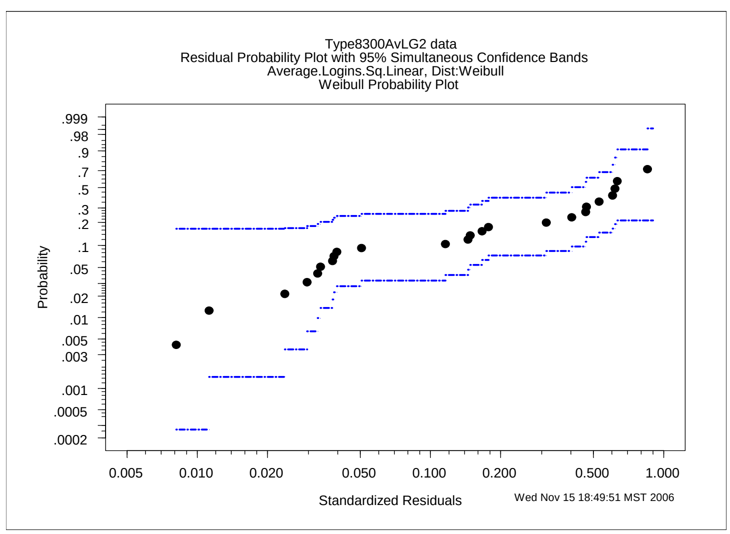


Fig.22

Weibull Prob. Model Plot of Residuals on 8300 Using Avg. Logins Sq.



To confirm the results of the Weibull regression, the Cox Proportional Hazard function was run on the data with average Logins and the average Logins squared as the explanatory variable. The results are shown in Tables 5 and 6 and Fig.23 and 24 below.

Table 5: **<u>Cox Proportional Hazards Output for Type8300 using average Logins</u>**

Call:
coxph(formula = Surv(Hours.to.Failure, Status) ~ Average.Logins, data = Ty8300AvLg, na.action = na.exclude, method = "efron", robust = F) n= 146

	coef	exp(coef)	se(coef)	z	p
Average.Logins	0.0902	1.09	0.0348	2.59	0.0095

	exp(coef)	exp(-coef)	lower .95	upper .95
Average.Logins	1.09	0.914	1.02	1.17

Rsquare= 0.044 (max possible= 0.721)
Likelihood ratio test = 6.58 on 1 df, p=0.0103
Wald test = 6.72 on 1 df, p=0.00952
Score (logrank) test = 6.33 on 1 df, p=0.0119

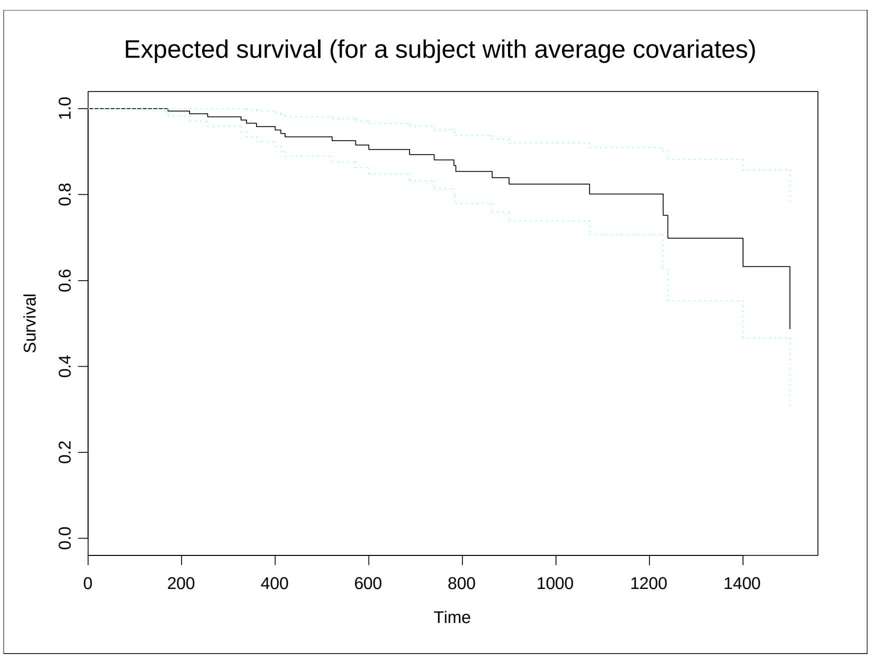


Fig. 23 Survival Curve from Cox Regression with Average Logins as Explanatory Variable

Table 6: **<u>Cox Proportional Hazards Output for Type8300 using (average Logins)²</u>**

Call:
coxph(formula = Surv(Hours.to.Failure, Status) ~ Average.Logins.Sq., data = Type8300AvLG2, na.action = na.exclude, method = "efron", robust = F) n= 146

	coef	exp(coef)	se(coef)	z	p
Average.Logins.Sq.	0.00209	1	0.000546	3.82	0.00013

	exp(coef)	exp(-coef)	lower .95	upper .95
Average.Logins.Sq.	1	0.998	1	1

Rsquare= 0.069 (max possible= 0.721)
Likelihood ratio test = 10.4 on 1 df, p=0.00123
Wald test = 14.6 on 1 df, p=0.000132
Score (logrank) test = 15.5 on 1 df, p=0.0000808

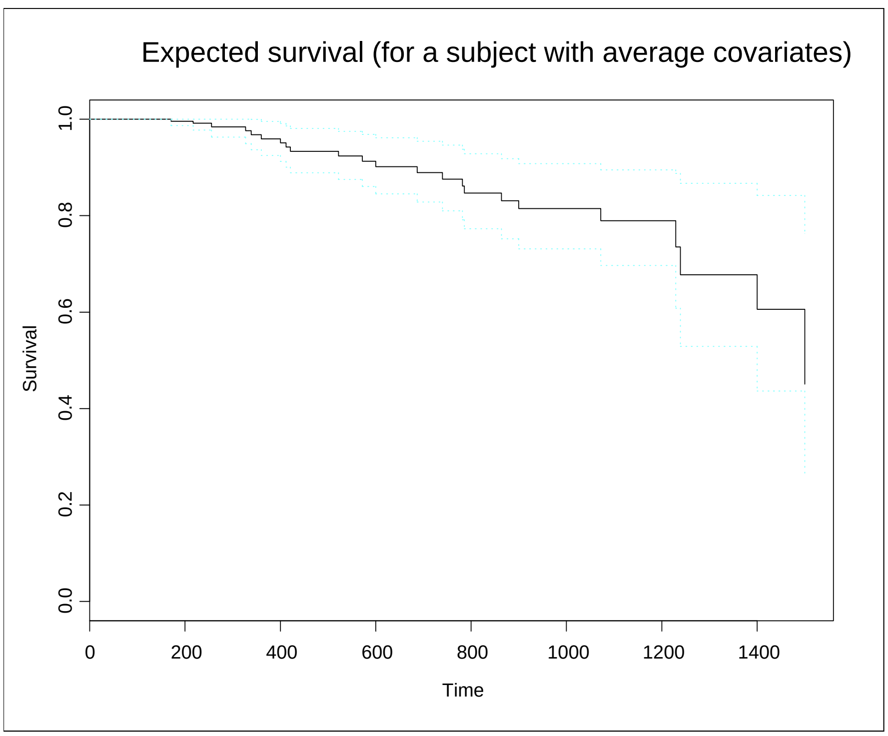


Fig. 24 Survival Curve from Cox Regression with (Average Logins)^2 as Explanatory Variable

The results are essentially the same as the Weibull Regression, with a significant contribution from the explanatory variable and the model with average squared fitting the model better than the model with average Logins.

4.4 Cost Model Based on the Weibull Distribution:

Given that a continuous random variable have a Weibull distribution with parameters, $\beta > 0$ *and* $\eta > 0$, thus, $X \sim Weibull\,(\eta, \beta)$ then its pdf would be expressed in the form;

$$f\,(x;\eta,\beta) = \left(\beta\frac{1}{\eta}\right)^{\beta} x^{\beta-1}\exp\left(-x/\eta\right)^{\beta} \quad , \quad x > 0\,.$$

Also the Hazard rate,

$$h(t) \;=\; \frac{f\,(t)}{R(t)} \;=\; \frac{\beta}{\eta^{\beta}}t^{\beta-1}$$

Relating this to the project scenario, it will imply β and η are the shape parameter and characteristic life respectively and (***x***) can be related to time (***t***) to failure. This will mean the CDF, F(t) can be written as

$$F\,(t\,|\,\eta\,,\beta\,) \quad = \quad 1-\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \;=\; 1-\exp\left[-\exp\left[\frac{\log(\,t\,)-\mu}{\sigma}\right]\right]$$

where $\sigma = \dfrac{1}{\beta}$ and $\mu = \log\eta$. Now fitting a regression model,

$$\log(\,t\,) = \mu + \gamma\, z \,+\, \log\left[-\log\left[1 - p\,\right]\right]$$

based on the Weibull distribution, where (z) is the average number of logins per week in this particular case then,

$$h(t) = \left(\frac{\beta}{\eta}\right)\left(\frac{t_r}{\eta}\right)^{\beta-1} = \left(\frac{1}{\sigma\ \exp(\ \mu + \gamma z}\right)\left[\frac{t_r}{\exp\ (\mu + \gamma z)}\right]^{\left(\frac{1}{\sigma}-1\right)}$$

If we let the cost of an unscheduled replacement $C_3 = MC_2$, where C_2 is the cost of a scheduled replacement in the Cost Model of Section 3.4, then for the Weibull regression model the optimum scheduled time, t_r is the solution of the equation;

$$\left(\frac{1}{\sigma}\right)\left(\frac{1}{\exp(\mu+\gamma z)}\right)^{\frac{2}{\sigma}}\left[(-t_r)^2\right]^{\frac{1}{\sigma}} - \left[1 - \exp\left(-\exp\left[\frac{\ln(t_r) - (\mu+\gamma z)}{\sigma}\right]\right)\right] = \frac{1}{M-1}$$

where μ, γ, and σ are the parameters from the Weibull distribution.

To derive the optimum time for which bulbs in the projectors must be replaced we will therefore have to fix **z** (the average Logins per week), but keep all other variables in the model constant. A value has to be placed on the cost to the multiple, **M** , which in this case will be the cost to the learning activity when a projector failed during class time compared to the cost to replace a bulb as a scheduled maintenance in the evening or over the weekend. Given that, the assumed value, **M,** is placed at ten times.

Using Solver in Excel and the coefficients representing the constants in the equation above, we solve for the t_r ,the optimum time for replacement by varying **z,** the average number of Logins in the case of the first analysis or the that quantity squared in the second case. For example, using $\mu = 7.9561315$, $\gamma = -0.009613$, $\sigma = 0.4792373$, which are the coefficients form either the Regression or the Parametric Survival analysis on the Type8300 with average logins and using **M**=10**, M**=5 or **M**=2. The generated values for t_r are shown in Table 7 and the corresponding plot in Fig.25 below.

Average Logins	Optimum Replacement Times		
z	M=10	M=5	M=2
1	3196.164	3463.158	4272.326
5	2676.394	2899.972	3577.551
12	1961.837	2125.721	2622.396
18	1503.296	1628.877	2009.465
25	1101.938	1193.99	1472.967
36	676.3809	732.883	904.1214
45	453.6953	491.5955	606.4569

Table 7: Optimum time as a function of Average Logins

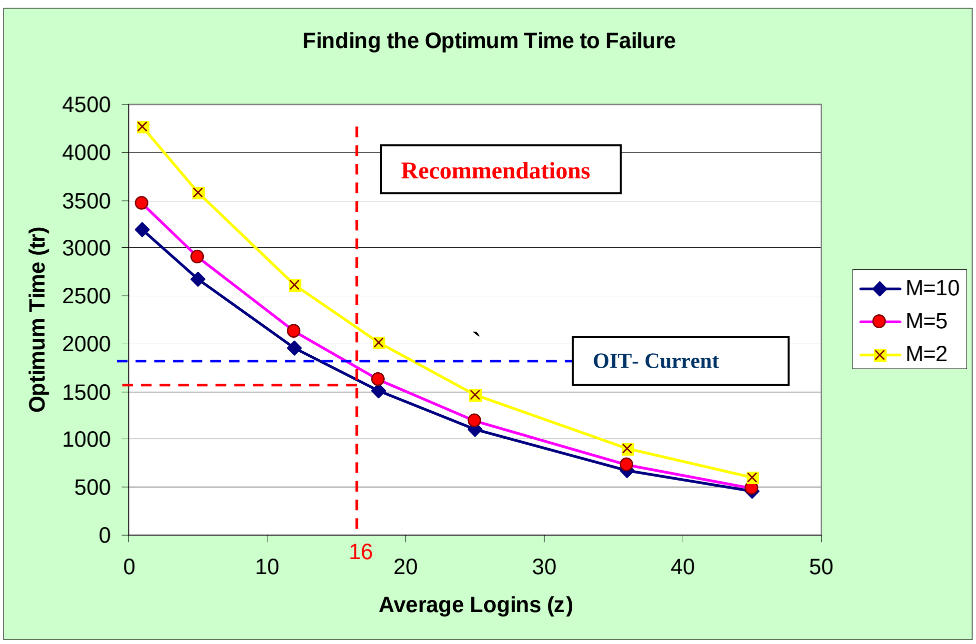


Fig. 25 A plot of Optimum replacement times as a function of Average Logins.

From the graph above, it will be noted that quantifying the lost of time to learning as ten times the cost of the current OIT's scheduled replacement time (i.e. M=10), then it is clear that the office is operating below the average number of Logins per room as accounted for in the data. To improve upon the reliability of the projectors and also avoid the lost of learning hours as a result of a failure of bulb during class hours, we strongly recommend that, the current scheduled time be reduced to about 1550 hr taking since the data accounted for an average 16 logins per room on campus. The graph also shows what might be the situation when M=5 and M=2. It is also clear that for rooms where projector is used more often and the average number of logins per week is greater than 16, the scheduled replacement time should be reduced to less than 1550 hrs as indicated in Table 7 and Fig.25

Chapter 5

Summary and Conclusions

The Office of Information Technology ***(OIT)***, a unit at the Brigham Young University campus has the responsibility among many others of overseeing to the smooth running and maintenance of the projectors used for learning and teaching activities in rooms designated as Media Rooms on the entire university campus. This vital role is played by keeping records on the times and causes of failure of the bulbs in the projectors and other maintenance services on each of the projectors.

The main objective of this paper is to analyze the data so collected and help to detect the failure rate of such systems, predict the optimal replacement time for the bulbs with the goal of maximizing the reliability of the systems and finally formulate a cost model that will be used to estimate the optimal cost involve in servicing a failed projector.

To achieve this goal, several individuals were contacted to help retrieve the needed data set from an ORACLE database used by the office and given out in an Excel spreadsheet. This indeed came with some information which were not needed. However, after various data cleaning procedures, fields such as Date of Failure, Room Number, Serial Number of Projector, Hours to Failure of the Bulbs, Projector Type, Number of Logins, and Number of Thermal Events were used for the analysis.

Findings from the data after trying various probabilistic models, showed that the projector bulbs undergo a renewal process , which goes to imply that, whenever a failed bulb is replaced, the projector operates as same as it was before failure. Also the Weibull distribution gave the best fit to the interarrival times, which led to the acceptance of general renewal process and rejection of the Homogeneous Poisson Process (HPP). Finally, all assessment on the explanatory variables during a regression analysis shows clearly that the Number of Logins into the projector contributes significantly to the failure of the bulbs.

The OIT currently uses 1800hrs as the scheduled replacement time for all replacements. However, modeling the data as a the *Type 1 Model* or *Age-Dependent Replacement Policy* since the systems showed an increasing failure rate, it became very obvious, when assuming that the cost of lost hours of learning is placed at ten times of repairing the system when classes are not in progress, then the scheduled replacement time needs to be reduced drastically taking into consideration the number of average logins per week. Table 7 and Fig.25

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