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ABSTRACT

The NCTM *Principles and Standards for School Mathematics* calls for a balance between conceptual understanding and procedural knowledge. This study reports the results of a survey distributed to AMTE members in order to discover the opinions and practices of mathematics teacher educators regarding this balance. The authors conclude that there is wide disparity of views regarding the meaning of the terms "conceptual" and "procedural" as well as the meaning "balance" between the two, in terms of what constitutes mathematics, the learning and teaching of mathematics, and the assessment of mathematical proficiency.

The State of Balance Between Procedural Knowledge and Conceptual Understanding in Mathematics Teacher Education

The word "balance" has become popular in today's educational climate as educators have become weary of pendulum swings between extreme pedagogical perspectives. Historically, traditional mathematics instruction has been characterized by an extreme commitment to the rote memorization of procedures with little concern for the associated concepts that underlie them. NCTM's *Principles and Standards for School Mathematics* [*PSSM*] (2000) states that balance ought to exist between conceptual and procedural learning in mathematics classrooms. "Developing fluency requires a **balance** and connection between conceptual understanding and computational proficiency." (p. 35)

> On the one hand, computational methods that are over-practiced without understanding are often forgotten or remembered incorrectly ... On the other hand, understanding without fluency can inhibit the problem-solving process ... The point is that students must become fluent in arithmetic computation-they must have efficient and accurate methods that are supported by an understanding of numbers and operations. (p. 35)

Balancing the acquisition of conceptual understanding and procedural proficiency is far from being strictly an American concern. Standards documents and curriculum frameworks from around the world, such as the Australian (Leonelli & Schmitt, 2001) and British frameworks (ATM, 2006) are quite pronounced in their calls for such a balance.

In this introduction, we will demonstrate support for examining the effects of the conceptual/procedural balance upon four concerns: the type of mathematics that should be learned in school, preservice teacher preparation, instructional conceptualization and design, and assessment.

The *PSSM* consistently connects "learning mathematics with understanding" (p. 20) with calls for balance between conceptual and procedural learning and the "... ability to use knowledge flexibly, applying what is learned in one setting appropriately in another." (p. 20) Propounding that mathematics proficiency is dependent upon learning both concepts and procedures, the *PSSM* states,

> One of the most robust findings of research is that conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility. The alliance of factual knowledge, procedural proficiency, **and** conceptual understanding makes all three components usable in powerful ways. Students who memorize facts or procedures without understanding often are not sure when or how to use what they know, and such learning is often quite fragile. (p. 20)

Other mathematics educators call for a similar balance in the learning of concepts and procedures. Ma (1999) describes the development of a "profound understanding of fundamental mathematics" as a well-organized mental package of highly-connected concepts and procedures. Regarding further discussions of an algorithm, Ma states, "'Know how, and also know why.' ... Arithmetic contains various algorithms ... [and] one should also know why the sequence of steps in the computation makes sense." (p. 108)

The *PSSM* contends that a conceptual-procedural balance is fundamental to all of school mathematics, and envisions that throughout their K-12 mathematical experiences "students will reach certain levels of conceptual understanding **and** procedural fluency by certain points in the curriculum." (p. 7; see also p. 30) Such balance is understood as fundamental, leading to the understanding that "Learning ... the mathematics outlined in [all grade bands] requires understanding and being able to apply procedures, concepts, and processes." (p. 20)

The conceptual-procedural balance provides a context for investigating the preparation of preservice teachers. Ma (1999) views teacher preparation as the vehicle to break the "vicious cycle formed by low-quality mathematics education and low-quality teacher knowledge of school mathematics." (p. 149), yet both she and Fosnot and Dolk (2001) decry the ever-widening gap between the needed and actual content understanding of teachers. Simon (1993) investigated gaps in prospective teachers' knowledge of division by examining the connectedness within and between procedural and conceptual knowledge and suggests conceptual areas of emphasis for the mathematical preparation of elementary teachers. Ambrose, Clement, Philipp, and Chauvot (2003) describe seven *beliefs* they hope to engender in their preservice students. The first four are based on the conceptual-procedural balance: (1) Mathematics, including school mathematics, is a web of interrelated concepts and procedures. (2) One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts. (3) Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures. (4) If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

Revised views in what it means to *learn mathematics* have caused a careful examination of the effects of traditional pedagogical paradigms. Marilyn Burns (1999) indicates that this examination has yet to achieve widespread reform. Frequently, a direct instruction model is linked to traditional teaching methodologies (Confrey, 1990) although some have reexamined the role of *telling* in light of both procedural and conceptual learning (Lobato, Clarke & Ellis, 2005).

Many scholars (e.g., Carpenter, Ansell & Levi, 2001; Cobb, Wood & Yackel, 1990; Davis, 1990; Wood, 1999; Wood, 2001) argue from a constructivist perspective for an instructional sequence that begins with the presentation of a meaningful task or problem and continues with an invitation to solve that problem in multiple ways, which leads to the sharing, justifying, and discussing of those problem solving strategies in small or large group discourses. The roles assumed by teacher and student, as well as the environment, associated with this type of teaching are radically different from tradition and are thoroughly described in the *Professional Standards for Teaching Mathematics* (1991). The *PSSM* suggests that this constructivist-oriented sequence can be used to promote both conceptual and procedural learning: "Moreover, in such settings, procedural fluency **and** conceptual understanding can be developed through problem solving, reasoning, and argumentation." (p. 21)

With calls for pedagogical change have come calls for change in assessment practice. Traditional assessment processes tend to reward speed and accuracy in remembering pre-existing facts (Bell, 1995) and communicate a message about the meaning of mathematics that fails to represent its complexity (Galbraith, 1993). The *PSSM* states that assessment tasks communicate what type of mathematical knowledge and performance are valued (p. 22) and that assessment should be aligned with instructional goals (p. 23). Its predecessor, the *Curriculum and Evaluation Standards* (NCTM, 1989) promotes a similar view: "As the curriculum changes, so must the tests. Tests also must change because they are one way of communicating what is important for students to know." (pp. 189-190) All of these statements suggest that if there is to be a balance between conceptual and procedural learning, there should also be a balance in the types of assessment used to capture that learning.

Suggesting that the conceptual-procedural dichotomy is incomplete in defining the meaning of mathematical proficiency, *Adding It Up* (Kilpatrick, Stafford & Findell, 2001) describes five interrelated strands of mathematical proficiency of which procedural fluency and conceptual understanding are but two. The other three are strategic competence, adaptive reasoning, and productive disposition. *Knowing Mathematics for Teaching Algebra Project* and *Teachers for a New Era* projects at Michigan State University, and the *University of Chicago School Mathematics Project* additionally indicate that the dichotomy is incomplete. However, it is possible that this characterization of the incompleteness of the dichotomy may be more a reflection of curricular perspective than a reflection of the nature of mathematics. For example, a careful reading of *Adding It Up* reveals that the first two strands, procedural fluency and conceptual understanding, repeat the conceptual-procedural balance suggested by the *PSSM* Content Standards. The next two strands, strategic competence and adaptive reasoning, are a reiteration of the *PSSM* Process Standards, namely, problem solving, reasoning and proof, communication, connections, and representation. The last strand, productive disposition, reiterates the *PSSM* consistent reference to the affective side of mathematical performance. Thus, while these and other sources may opine that the dichotomy investigated in this project and paper is incomplete, they do not refute the important central question at hand. Indeed, these sources may indicate that the dichotomy between procedural learning and conceptual understanding is more complex than often recognized and warrants both additional research and refinement of the fundamental understanding of the issue.

Altogether, therefore, the literature suggests that the investigation of the opinions and practice of mathematics teacher educators regarding the concept-procedure balance warrants further investigation.

2. Background of the Survey and Respondents

The membership of the Association of Mathematics Teacher Educators (AMTE) primarily consists of preservice and/or inservice mathematics teacher educators employed by institutions of higher education. They typically teach early childhood through secondary mathematics content and/or methods, may work with educators in schools, and may hold various local, state and national leadership positions in the field. Many are engaged in research about the teaching and learning of mathematics and/or mathematics teaching. Thus, the membership of AMTE is, herein, altogether considered expert and its opinions are highly valued within the field.

In preparation for a presentation at the Ninth Annual AMTE Conference in Dallas in January, 2005, the researchers in this study identified some issues related to the balance of conceptual and procedural learning and developed an initial survey to be used at the conference. Subsequent conference discussions and initial survey responses allowed for the refinement of questions associated with those issues and lead to the production of the current survey instrument. This refined survey, which attempts to discern opinions and practice among AMTE members, was developed and distributed via the membership email list. Voluntary responses were returned (N=40) to the researchers for analysis and synthesis into a report. The survey consisted of multiple choice and short answer questions and appears in the appendix. Qualitative and quantitative methods were used to develop the report.

The expertise provided by AMTE member respondents makes generalization of the findings of this study to mathematics education *in toto* tenuous. AMTE member opinions are not considered the general state of mathematics education in the U.S.; rather, their opinions are generally considered among the most well versed opinions in the field. Others in the field may not share some of the opinions of AMTE members. In addition, it is recognized that a small sample can hardly be considered representative of the entire AMTE organization, particularly because the sample was formed strictly from those whole voluntarily completed the survey. However, the significant variance in the professional experience of the respondents (gender; age; current employment (university and/or City & State); number of years at this position; school and year at which highest degree was earned) provides an increased level of confidence that some of the responses can be cautiously generalized to a larger population.

3. Researchers' Presuppositions

The researchers in this study believe that the debate between conceptual understanding and procedural knowledge in mathematics learning can be compared with learning to play a musical instrument. When a child begins to learn to play the trumpet, she may begin with musical theory but must develop the instrumental skills to continue to learn the instrument. Both the development of musical theory and instrumental skill go hand in hand. Conversely, both are limited by the other. Only in the rarest instance can a musician of limited instrumental skill become an excellent theorist or composer. Redirecting this concern to mathematics education, the authors believe that this is the type of conceptual-procedural balance recommended in the NCTM *PSSM*. The authors believe that mathematical skills are learned through procedural knowledge and novel, connected, extended, and applied mathematical ideas are developed through conceptual understanding.

The researchers in this study define the following: a *problem* is a scenario in which, upon initiation, neither the result nor a method for solution is known; an *exercise* is a scenario in which the result is unknown but a method for solution is known. Notably, what may be an exercise for one student may be a problem for another. Furthermore, when students do not know a method to solve a scenario, even though they should, it is a problem until they learn a method for solution; then it becomes an exercise. Within any group of supposed exercises (for instance at the end of sections and chapters in texts), as concepts are further investigated, selected examples may indeed be problems to some students. This is often the case when examples look alike and yet methods for solutions change due to alterations among the examples. Therefore, factoring $90x^2 - 27x - 40$ may be an exercise to a precalculus student and a problem to an algebra I or II student who lacks the necessary foundation. The authors believe that mathematical exercises are solved using procedural knowledge and problems are solved using conceptual understanding. The conceptual understanding required for the latter may lead to, or incorporate, associated procedural knowledge.

4. Initial Relevant Findings

Approximately 63% of respondents believe that students naturally learn mathematics in a sequence that begins with conceptual understanding and leads to procedural knowledge and another 37% believed that students learn from either conceptual understanding or procedural knowledge to the other. No respondents state that procedural knowledge leads to conceptual understanding.

Respondents categorize the focus of the instruction which they provide in undergraduate content courses for preservice K-12 teachers as well blended (50%), mostly conceptual (46%), and all conceptual (4%). Virtually all responses agree with the need to emphasize, or even over emphasize, conceptual understanding over procedural knowledge in order to overcome the perennial focus on procedurally-based instruction in K-12 mathematics education.

Almost all respondents state that they use the NCTM *PSSM* process standards for developing student conceptual understanding, which suggests a high regard for the constructivist instructional sequences discussed previously. Although most respondents claim to use Bloom's Taxonomy to develop conceptual understanding, a significant number of respondents state that since they work in mathematics departments at their universities, Bloom's Taxonomy is not deeply considered. Three respondents admit that they are completely unfamiliar with the taxonomy.

Respondents categorize the skills/knowledge which their mathematics education students demonstrate as they enter their course/program as mostly procedural (90%) and all procedural (10%) and as well blended (66%), mostly procedural (24%), and mostly conceptual (10%) as they exit. Most respondents indicate that they saw a greater need to teach conceptual understanding to preservice teachers than to mathematics students not planning to enter the teaching profession.

Respondents categorize the focus of the assessments which they use in their undergraduate content courses for preservice K-12 teachers as well blended (59%), mostly conceptual (37%) and mostly procedural (4%). They categorize the focus of the assessments which they use in their undergraduate pedagogy courses for preservice K-12 teachers as mostly conceptual (52%), well blended (40%), and mostly procedural (8%). Nearly half of the respondents do not discuss how they taught their undergraduate preservice K-12 teachers to assess their future K-12 students for conceptual understanding.

Approximately 25% of respondents state that they realized that they needed to return to the issue to reconsider their teaching practices. Of these, many note that their self-recognized difficulty in defining and distinguishing between procedural knowledge and conceptual understanding led them to realize their own need to more thoroughly investigate and understand the issues and reinvestigate their teaching and assessment strategies in light of these findings.

5. Digging Deeper

The survey for this study melded multiple choice questions with open ended, short answer questions. Therefore, much more rewarding analysis was possible than could have been accomplished through multiple choice questions alone. The following findings and discussions are supported by these responses. Respondents provided the following descriptors and characteristics for conceptual understanding and procedural knowledge. The numbers in the parentheses indicate the frequency with which the descriptors were found in the responses. It was quite common for a response to include multiple facets.

Respondents opine that conceptual understanding: is adaptable, adjustable, transferable and applicable to other situations (13); is knowing "why"(9) or "how"(4) something works; is born from and develops connections (9); makes math (and the world) more sensible or meaningful (6); is a flexible foundation for long-term retention (5) and understanding; is the essence of mathematical thinking and the only true kind of understanding (3); leads students to see the bigger picture (2); does not rely on memory (2); assists the reconstruction of the idea if details are forgotten (2); is at the heart of "knowing" a topics (2); links facts and procedures (1); leads to new learning (1); and is continually growing and developing (1).

Respondents state that procedural knowledge: produces algorithmic efficiency and accuracy (17); is a set of skills (6); is primarily "how to" do (perform) some operations and employ some properties (4); is learned by rote, repetition, and drill (4); makes "meaning" unnecessary (2); is only used, remembered, and applicable in the short-term (3); only applies to context in which it was first learned (3); saves from rediscovering the wheel each time (2); is worthless by itself (2); is connected to a lack of, or incorrect, memory (1); does not emphasize making connections (1); is a step-by-step sequence (1) automating the routine (1); preserves "harder thinking" for less routine things (1); is "just doing what you are told" (1); allows quick calculations of things that one no longer needs to worry about how it works (1); does not require an understanding of what one is doing (1) and requires no need to think (1); depends on memorization (1) and is easily forgotten (1); is merely superficial learning (1); is useful to communicate concepts (1); is a discrete set of factoids (1); is a stagnant set of rules (1); and is obtained with less personal involvement with the mathematics (1).

Connecting the two concerns, respondents claim that *conceptual understanding and* *procedural knowledge*: work together (5) and are both necessary (5); complement each other (2); form a continuum (1); are both important (1); develop interactively (1) and should not be separated (1); and can both result in identical answers to a question (1).

These responses *in toto* paint a more comprehensive picture of respondents' opinions regarding conceptual understanding and procedural knowledge. This paper reports aspects which are more deeply embedded in the responses.

All characteristics of conceptual understanding provided by responses can be unanimously seen as positive. Procedural knowledge, however, while seen as valuable in producing algorithmic efficiency and accuracy, is much maligned in many ways. The negative tone and context of the majority of the responses overwhelmingly suppresses the minority position, which values procedural knowledge as complementary to conceptual understanding. Only four responses are considered positive characterizations of procedural knowledge: produces algorithmic efficiency and accuracy; saves from rediscovering the wheel each time; automating the routine; is useful to communicate concepts. While another two characterizations are considered neutral, twenty-one other responses are interpreted as negative perceptions of procedural knowledge.

Issues of *memory*, *memorization*, and *retention* deeply connect many comments regarding conceptual understanding and procedural knowledge. Numerous responses denigratingly associate procedural knowledge with a mere exercise of memory and rote practice. *Memory* is recognized as being in opposition to *understanding*. Conversely, many responses indicate that conceptual understanding, via connections and sense-making, leads to greater retention of salient concepts; in such, memory is no longer maligned, but rather applauded. The majority of responses imply that, in respect to learning, understanding should lead to memory, but memory can not lead to understanding. Some responses go further and imply that ideas which are constructed and integrated into a student's schema are independent of memory - that is, *if something is learned, then it is not memorized*. Conversely, others argue that *if something is memorized then it is not learned*. Therefore, together, a false dichotomy is established and promoted from both sides of the argument.

While many respondents associate conceptual understanding with deeper thinking, many responses indicate that procedural knowledge is completely bereft of any real thinking whatsoever, makes "meaning" unnecessary, and does not require an understanding of what one is doing. One respondent insinuates that using procedural knowledge allows for the conservation of thought energy which can later be directed to situations which require "harder thinking". Although these more irregular opinions are less commonplace, their existence further demonstrates the degree to which procedural knowledge is repudiated among some of the respondents. This is particularly deleterious in that it blurs the meaning of thinking. *Thinking*, *understanding*, and *learning* are consistently associated with a student's introduction to novel ideas. Respondents never use the phrase "*learn a skill*."

Respondents recognize the purpose of procedural knowledge in expediting calculations, as a tool which allows students to avoid continually rediscovering the wheel. While devaluing procedural knowledge, respondents highly value the expediency it brings to routine operations. Respondents declare that procedural knowledge cannot be transferable and applicable to other situations and that procedural knowledge is inefficient as a problem-solving tool.

Responses indicate that a contextualized purpose for procedural knowledge is inconsistently understood. The purposes for conceptual understanding and procedural knowledge are only occasionally seen complementarily and only once mentioned contextually. The context of the use of procedural knowledge is continually clouded by the notion of problem-solving. Except in one unique case, respondents indicate no distinction between *problems* and *exercises*. Thus, most respondents do not express value for procedural knowledge to facilitate the solution of exercises. Nor do they indicate any value for the mathematical practice afforded by exercises and the role of practice and exercises for the reinforcement of memory and understanding after concepts are developed.

6. Discussing Balance

A small number of respondents fail to provide cogent definitions which articulate both similarities and difference between procedural knowledge and conceptual understanding. This may have been due to the open-ended nature of the survey question and the respondents unwillingness to write lengthy responses. Many of the descriptors among different respondents for procedural knowledge and conceptual understanding are inconsistent and contradictory. The responses often repeat the notion that procedural knowledge is tantamount to no understanding at all. While a small number of responses delineate distinctions between the two, most respondents devalue procedural knowledge and highly value conceptual understanding.

The imbalance of the relative values attached to conceptual understanding and procedural knowledge within the responses may be best recognized when it is understood that 100% of responses demonstrate a preference that, as students both enter and exit their course/program, students understand mathematical concepts well but lack procedural skills. While this one-sided opinion may be due to the fact that the only other seemingly unsatisfactory option to the question was that respondents preferred students to have strong procedural skills, this result demonstrates the existing imbalance.

As previously noted, the NCTM *PSSM* calls repeatedly for balance between procedural knowledge and conceptual understanding. Most respondents claim to either present a balance of conceptual and procedural instruction and assessment or provide emphasis on mostly conceptual instruction and assessment. Through additional comments on the surveys, those who denote that they use instruction and assessment which are "mostly conceptual" do so primarily for two significant reasons. First, virtually all recognize the need to emphasize, or even over emphasize, conceptual understanding over procedural knowledge in order to overcome the perennial focus on procedurally based instruction in K-12 mathematics education. Second, a number of respondents indicate that overemphasis of conceptual understanding within the mathematics education courses within their program is necessary to counterbalance the overemphasis of procedurally based instruction and assessment in required mathematics courses held in the respective mathematics departments. Thus, respondents often perceive that an overemphasis in respect to conceptual learning is necessary in order to provide both students and the program as a whole a balance between procedural knowledge and conceptual understanding. Unfortunately, even with a purposive imbalance in favor of conceptual instruction, assessment, and understanding, 24% of respondents admit that their students continue to exit these programs with skills and knowledge which are mostly procedural.

Interesting inconsistencies can be found among groups of responses and within individual responses regarding the notion of balance between conceptual understanding and procedural knowledge. As previously noted, the majority of respondents categorized the focus of the instruction and assessment which they provide in undergraduate content courses for preservice K-12 teachers as well blended and 40% categorize the focus of the assessments which they use in their undergraduate pedagogy courses for preservice K-12 teachers as well blended. However, this purported claim toward balance may be philosophically inconsistent with the negative perception which the majority of responses provided regarding procedural knowledge. This naturally leads to questions regarding how so many respondents simultaneously denigrate procedural knowledge and promote balance. If procedural knowledge has as little value as the majority of responses indicate, then it could be argued that it should receive less attention in preservice classrooms. This philosophic inconsistency cannot be answered through the responses provided on this survey and warrants further investigation.

A relatively small number of responses argue that the conceptual/procedural debate poses a false dichotomy. Some responses indicate that the issues reviewed in the survey are outdated, and that these issues have been thoroughly discussed for more than two decades. Unfortunately, the wide diversity of opinion by respondents argue more forcefully that this discussion is far from resolved among the community of professional mathematics educators. Far too much variety of often contradictory opinion resides within responses to indicate common grounding, understanding, and focus on this concern. Thus, the opinions of the AMTE members who were surveyed seem to remain fragmented regarding the balance recommended by NCTM.

7. Connecting to Demographics

The researchers were particularly interested in investigating whether strong correlation could be discovered among various demographic aspects and respondents' opinions regarding the balance between procedural knowledge and conceptual understanding. For instance, among other questions, it was initially wondered if the respondents' number of years in the profession or institution from which they received their last degree could be correlated to one particular opinion over another. A number of factors made this dimension of the investigation of less value. First, as previously mentioned, although great disparity existed among definitions for both conceptual understanding and procedural knowledge among responses, agreement overwhelmingly existed regarding a simultaneous denigration of procedural knowledge and a unified valuation of conceptual understanding. Second, a number of doctoral candidates were respondents. It is not clear to what extent these respondents either mirrored the opinions of their faculty advisors or cautiously responded according to what they believed was the *correct* response. Third, the numerous responses previously provided within this discussion seem well distributed among all respondents. Fourth, the previously mentioned philosophic inconsistency discovered within responses regarding the notions of the need and application of balance between conceptual understanding and procedural knowledge again permeated all demographic distinctions. Altogether, therefore, since all responses shared similarities, the comparison of demographic data to various differing positions was determined to be of little value.

8. Conclusion and Suggestions

While the members of AMTE who participated in this study demonstrate significant inconsistencies in their opinions and practice concerning the balance of procedural knowledge and conceptual understanding in respect to mathematics teacher education, it is assumed that beyond these members the entire field of mathematics and mathematics teacher education would prove to be even more fragmented. Only additional dialog will remedy this fracture.

It is feared that balance between procedural knowledge and conceptual understanding will not be fully met within mathematics education until both are valued and seen as necessarily complementary, albeit each with their primary contextualized focus. Educators must understand that learning new concepts and practicing the skills associated with further unfolding or applying those concepts are interconnected. Learning procedures must be recognized as situated within the process of learning.

Memory must also be seen as a necessary component of learning. Retention of concepts along with the procedures which apply to, and can be employed in expanding upon, those concepts is vital to learning. Memory and retention must not be seen as in opposition to learning; they must be recognized as a necessary and valuable component of learning. One of the problems associated with isolated procedural learning is that learning is quite fragile (NCTM, 2000; Bransford, Brown, & Cocking, 1999), meaning that those procedures are often forgotten quickly, or remembered inappropriately. A balance of learning both concepts and procedures, particularly if the connections between them are made explicit, has been shown to enhance the long term retention of both (Schoenfeld, 1988).

The survey and responses opened many more areas of investigation which this brief paper was unable to address. Many dimensions yet remain fodder for future research, not the least of which is the role of the conceptual-procedural balance in light of No Child Left Behind and high stakes testing. In addition, further investigations into what is meant by "procedural knowledge" such as those discussed by Star (2005) could also prove fruitful. It is hoped that this study will elicit further consideration of this concern.

It seems obvious to us that if the mathematical reforms championed by AMTE, NCTM, MAA, and the like are to achieve widespread implementation, then a greater degree of correspondence should exist among those who champion them particularly with regards to issues as fundamental as the balance between conceptual understanding and procedural knowledge. We believe that the disparity in views held by mathematics teacher educators regarding the meanings of conceptual and procedural learning – and thereby the resulting inconsistent classroom pedagogical methodologies – might, if not rectified, spell trouble for the current mathematics education reform effort.

The New Math Movement of the 1950's and 60's, which had its roots in the same psychological and epistemological perspectives of current reform efforts, suffered from similar fractured pedagogical classroom delivery (Bossé, 1995, 1999; Davis, 1990). While a core of curriculum projects (e.g., P.S.S.C. SCIS, the Madison Project) focused upon reasoning, creativity, understanding, discovery, big ideas, and real world application, other contemporaneous efforts had incongruous foci.

> There was no one "New Math." There were many different ideas offered; some were fundamentally and foundationally different from others. The entire movement was very diverse. The only thing in common within the entire movement was [the perception] that school mathematics education was insufficient as it stood. Therefore, the most historically and intellectually honest definition of the New Math Movement might actually be: All educational movements during the 1950s and 1960s that had an aim of reforming, repairing,

or enhancing mathematics education on the K- I2 level (Bossé, 1995, p. 173).

While many of the New Math reform efforts based on constructivist epistemologies demonstrated significant success, virtually all efforts of that era were collected under the pejorative moniker “New Math”, and dubbed as failures. Therefore, it may be valuable for current mathematics educators to learn from the past, increase dialog regarding epistemological and pedagogical concerns (conceptual understanding versus procedural knowledge), and sufficiently unite in beliefs and practice so as to avoid repeating the mistakes of the past.

Appendix

AMTE Member Voluntary Survey – April, 2005

What is the Meaning of the Term "Balance" as Used by the Principles and Standards Document in Relation to Conceptual and Procedural Learning?

Demographics (Optional):
Gender: Age (or approximate):

Where you currently work/teach (university and/or City & State):

Number of years at this position:

School at which you earned your highest degree: Year:

DIRECTIONS: ***For questions which you select a response, either highlight (electronic) or circle (hardcopy) the appropriate response. For open-ended questions, please limit your responses to no more than 2-3 sentences. These responses can be typed into the electronic document or sent on separate sheets of paper by mail.***

PART 1: Curriculum and Instruction.

1. What is the purpose/value of conceptual understanding?

2. What is the purpose/value of procedural knowledge?

3. What are the similarities between procedural knowledge and conceptual understanding?

4. What distinguishes procedural knowledge from conceptual understanding?

5. As students naturally learn mathematics, what it the natural flow of understanding? (Highlight or circle one choice.)

    conceptual understanding leads to procedural knowledge
    procedural knowledge leads to conceptual understanding
    either leads to either

6. How would you catagorize most K-12 mathematics curricula? (Highlight or circle one choice.)
All Procedural   Mostly Procedural   Well Blended   Mostly Conceptual   All Conceptual

7. How would you categorize most "reformed" K-12 mathematics curricula? (Highlight or circle one choice.)
All Procedural   Mostly Procedural   Well Blended   Mostly Conceptual   All Conceptual

8. How would you catagorize the focus of the training which you received in the program of your highest degree? (Highlight or circle one choice.)
All Procedural   Mostly Procedural   Well Blended   Mostly Conceptual   All Conceptual

9. How would you categorize the focus of the instruction which you provide in your undergraduate content courses for preservice K-12 teachers? (Highlight or circle one choice.)

All Procedural Mostly Procedural Well Blended Mostly Conceptual All Conceptual

10. How would you categorize the focus of the instruction which you use in your undergraduate pedagogy courses for preservice K-12 teachers? (Highlight or circle one choice.)

All Procedural Mostly Procedural Well Blended Mostly Conceptual All Conceptual

11. Many mathematics teacher educators have opined regarding the need to emphasize, or even over emphasize, conceptual understanding over procedural knowledge in order to overcome the perennial focus on procedurally based instruction in K-12 mathematics education. If you responded "mostly conceptual" or "all conceptual" on questions 9 and/or 10 and your reason for doing such is different from the opinion just mentioned, please indicate why you made your selection(s).

12. How would you categorize the textbooks (if applicable) which you most frequently use in your classes? (Highlight or circle one choice.)

All Procedural Mostly Procedural Well Blended Mostly Conceptual All Conceptual

13. Rate the following NCTM process standards in rank order of 1 (highest) to 5 (lowest) in how significantly they can assist students to gain conceptual understanding.

Problem Solving Reasoning & Proof Communication Connections Representations

PART 2: Student Understanding.

14. How would you categorize the skills/knowledge which your mathematics educations students demonstrate as they enter into your course/program? (Highlight or circle one choice.)

All Procedural Mostly Procedural Well Blended Mostly Conceptual All Conceptual

15. How would you categorize the skills/knowledge which your mathematics education students demonstrate as they exit your course/program? (Highlight or circle one choice.)

All Procedural Mostly Procedural Well Blended Mostly Conceptual All Conceptual

16. As a student enters your course/program, if you were REQUIRED to choose ONLY ONE of the following, would you prefer that a student (Highlight or circle one choice.)

can perform mathematics procedures well but lacks deep conceptual understanding of the mathematics

understands mathematical concepts well but lacks procedural skills

17. As a student exits your course/program, if you were REQUIRED to choose ONLY ONE of the following, would you prefer that a student (Highlight or circle one choice.)

can perform mathematics procedures well but lacks deep conceptual understanding of the mathematics

understands mathematical concepts well but lacks procedural skills

18. Please explain your rationale for your selections in Questions 15 and 16. Would your responses be different if you were considering mathematics students who are not planning to become teachers and those who are planning to teach? Why?

19. Could a student succeed through your program if s/he possessed excellent procedural skills, but had limited conceptual understanding? (Highlight or circle one choice.)

Yes Unsure No

20. How have you employed the NCTM process standards to deepen conceptual understanding?

21. How have you employed Bloom's taxonomy of the cognitive domain to deepen conceptual understanding?

PART 3: Assessment.

22. How would you categorize the focus of the assessments which you use in your undergraduate content courses for preservice K-12 teachers? (Highlight or circle one choice.)

All Procedural Mostly Procedural Well Blended Mostly Conceptual All Conceptual

23. How would you categorize the focus of the assessments which you use in your undergraduate pedagogy courses for preservice K-12 teachers? (Highlight or circle one choice.)

All Procedural Mostly Procedural Well Blended Mostly Conceptual All Conceptual

24. How do you assess undergraduate preservice K-12 teachers for conceptual understanding in the content courses you teach? Give some examples. Tell if these evaluations for conceptual understanding are graded.

25. How do you assess undergraduate preservice K-12 teachers for conceptual understanding in the pedagogy courses you teach? Give some examples. Tell if these evaluations for conceptual understanding are graded.

26. How do you teach your undergraduate preservice K-12 teachers to assess their future K-12 students for conceptual understanding? Give some examples.

27. How have you employed the NCTM process standards to assess conceptual understanding in your undergraduate preservice K-12 teaching courses (content and pedagogy)?

28. How have you employed Bloom's taxonomy of the cognitive domain to assess conceptual understanding in your undergraduate preservice K-12 teaching courses (content and pedagogy)?

PART 4: Change in Practice.

29. How has this survey affected your view of your teaching/assessment for conceptual understanding?

30. Which question posed above made you most reflect upon your/your school's/your students' focus on this issue and why?

31. If you completed a previous version of this survey, how did it affect your educational view and practice on this issue?

References

Ambrose, R., Clement, L., Philipp, R., & Chauvot, J. (2004). Assessing prospective elementary school teachers' beliefs about mathematics and mathematics learning: Rationale and development of a constructed-response-format beliefs survey. *School Science and Mathematics, 104(2)*.

Australian Association of Mathematics Teachers (2002). *Standards for Excellence in Teaching Mathematics in Australian Schools*. Adelaide, South Australia: Author.

Bell, K.N. (1995). How assessment impacts attitudes toward mathematics held by prospective elementary teachers. (Doctoral dissertation, Boston University, 1995). *Dissertation Abstracts International,* 56, 09-A.

Bossé, Michael J. (1999). Reforming the NCTM Standards in light of historical perspective: Premature changes. *Journal of Mathematical Behavior*, Vol. 17, No. 3, 317-327.

Bossé, Michael J. (1995). The NCTM Standards in light of the New Math Movement: A warning. *Journal of Mathematical Behavior,* Vol. 14, No. 2, 171-201.

Bransford, C.A., Brown, A.L., & Cocking, R.R., eds. *How people learn: Brain, mind, experience, and school.* Washington, D.C.: National Academy Press.

Burns, M. (1999). *Math: Facing an American phobia.* Sausalito, CA: Marilyn Burns Education Associates.

Carpenter, T.P., Ansell, E. & Levi, L. (2001). An alternative conception of teaching for understanding: Case studies of two first-grade mathematics classes. In T. Wood, B.S. Nelson, & J. Warfield, (Eds.), *Beyond classical pedagogy* (pp. 27-46). Mahwah, NJ: Lawrence Erlbaum Associates.

Cobb, P. Wood, T. & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R.B. Davis, C.A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics.* (pp. 125-146). Reston, VA: National Council of Teachers of Mathematics.

Confrey, J. (1990). What constructivism implies for teaching. In R.B. Davis, C.A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics.* (pp.107-124). Reston, VA: National Council of Teachers of Mathematics.

Davis, R.B. (1990). Discovery learning and constructivism. In R.B. Davis, C.A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics.* (pp.93-106). Reston, VA: National Council of Teachers of Mathematics.

Fosnot, C.T. & Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition, and subtraction*. Portsmouth, NH: Heinemann.

Galbraith, P. (1993). Paradigms, problems and assessment: Some ideological implications. In M. Niss (Ed.), *Investigations into assessment in mathematics education* (pp. 73-86). Boston: Kluwer Academic Publishers.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press. Available http://www.nap.edu/catalog/9822.html.

Leonelli, E. D. & Schmitt, M. J. (2001). Bringing reform to adult numeracy instruction. *Field Notes*, 11(2), 226-246.

Lobato, J., Clarke, D. & Ellis, A.B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for research in mathematics education*, 36, 101-136.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

Schoenfeld, A.H. (1988). When good teaching leads to bad results: The disasters of well taught mathematics classes. *Educational psychologist* 23, 145-166.

Simon, M. (1993). Prospective elementary teachers' knowledge of division. *Journal for research in mathematics education*, 24, 233-254.

Star, R. (2005). Reconceptualizing procedural knowledge. *Journal for research in mathematics education*, 36, 404-411.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for research in mathematics education*, 30, 171-191.

Wood, T. (2001). Teaching differently: Creating opportunities for learning mathematics. *Theory into practice*, 40, 110-124.