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Observations of Piezoresistivity for Polysilicon in Bending that are Unexplained by Linear Models

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Abstract

Compliant piezoresistive MEMS sensors exhibit great promise for improved on-chip sensing. As compliant sensors may experience complex loads, their design and implementation require a greater understanding of the piezoresistive effect of polysilicon in bending and combined loads. This paper presents experimental results showing the piezoresistive effect for these complex loads. Several *n*-type polysilicon test structures, fabricated in MUMPs and SUMMiT processes, were tested. Results show that, while tensile stresses cause a linear decrease in resistance, bending stresses induce a nonlinear rise in resistance, contrary to the effect predicted by linear models. In addition, tensile, compressive, and bending loads combine in their effects on resistance. The experimental data illustrate the inability of linear piezoresistance models to predict the piezoresistive trends of polysilicon in bending and combined loads, indicating the need for more complete nonlinear models appropriate for these loading conditions.

1 Introduction

Since Smith documented the piezoresistive effect in silicon in 1954 [1], piezoresistive MEMS devices have been implemented in a variety of sensing applications including pressure, acceleration, force, and displacement sensing [2–8]. Piezoresistivity—the change in electrical resistance due to applied stress or strain—facilitates the measurement of stress or strain in a silicon member. This enables the use of piezoresistive devices in on-chip and feedback-control applications [9, 10].

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Fig. 1. (a) Traditional cantilever-beam piezoresistive force sensor and (b) integral piezoresistive force sensor.

The literature today contains extensive data, models, and theories describing the piezoresistive effect in tension and compression for both monocrystalline and polycrystalline silicon [11–20]. For most existing sensors, these data and models have been sufficient. For example, a traditional cantilever-beam force sensor, depicted in Fig. 1a, is composed of two basic components. First, a cantilever beam structure experiences the physical phenomenon, such as an applied force. Second, a piezoresistive element diffused on top of the cantilever is thin enough that it experiences nearly purely tensile or purely compressive stress as the cantilever beam deflects [21]. In this configuration, the structure and the sensing element are two distinct objects, with separate fabrication steps. Because the piezoresistive element experiences pure compression or tension during loading, traditional linear models are sufficient to model this behavior.

In contrast, Fig. 1b shows how a compliant u-shaped spring, or U-Spring device, can function as an integral piezoresistive force or displacement sensor. Such a device is fabricated from a single, uniformly-doped layer of polysilicon and allows piezoresistive sensing of the position or force of a MEMS actuator. It represents an elegant way to sense such behavior, because the sensor

is fabricated from the same layer and mask used to create the actuator, and it simultaneously acts as both a sensor and a support spring to guide the desired motion. During motion, the spring compresses or stretches. The resultant change in resistance of the spring is measured and used to calculate the applied force or displacement. Thus, the entire structure is the piezoresistive element, and it experiences bending (rather than pure tension or compression) during motion. An understanding of the piezoresistive effect of polysilicon in bending and combined loads is vital to the design and implementation of this and other innovative piezoresistive sensors. For example, the compliant bistable mechanism with integral piezoresistive state sensing, described in [22], experiences combined bending and compressive loads during motion. A more complete nonlinear model would enhance the design of this and similar devices.

The purpose of this paper is to present the testing of several structures used to explore the piezoresistive property of polysilicon in bending and combined loads. The data are compared to the predictions of a simplified model. The model uses traditional linear representations for piezoresistance derived from Smith’s π -coefficient model. The comparisons show that linear models are inadequate to predict piezoresistive behavior of polysilicon under bending loads. The need for a more complete model of piezoresistance, which accounts for the effect of bending and combined loads on resistance, is thus shown.

2 Background

Although the physical mechanism of piezoresistivity is not completely understood, several important trends of piezoresistance in tension and compression have been documented. For example, it has been shown that the resistance of an n -type (phosphorus-doped) polysilicon member subjected to uniaxial-tensile stress decreases nearly linearly. For the same material, an applied compressive stress causes an approximately linear increase in resistance. The opposite is true in both cases for p -type (boron-doped) polysilicon, with the resistance increasing in tension and decreasing in compression.

Smith’s model of piezoresistance for single-crystal silicon relates the fractional change in electrical resistivity, $\Delta\rho/\rho$, in each crystalline direction to the ap-

plied stress, σ , with a matrix of piezoresistance (π) coefficients as

$$\frac{1}{\rho} \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \Delta\rho_4 \\ \Delta\rho_5 \\ \Delta\rho_6 \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{44} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \quad (1)$$

where the π coefficients are determined experimentally [1, 12, 23, 24]. Note that this model is based on observed behavior and not on fundamental physics. For polysilicon structures, which have grains arranged in many different crystal orientations, Smith's single-crystal model is often simplified to give [25, 26]

$$\frac{\Delta\rho}{\rho} = \pi_l\sigma_l + \pi_t\sigma_t \quad (2)$$

where π_l is the piezoresistance coefficient along the length of a resistor, π_t is the piezoresistance coefficient transverse to the length of a resistor, and σ_l and σ_t are the stresses directed along the length of the resistor and transverse to the length of the resistor, respectively. Because beams in tension, compression, or bending experience a much greater stress in the direction along the length of the beam than the stresses in the other two orthogonal directions, we may assume that these lower stresses are insignificant. Therefore, equation (2) simplifies to

$$\frac{\Delta\rho_1}{\rho} = \pi_l\sigma_l. \quad (3)$$

This paper reports research and testing that has been performed to determine the adequacy of this linear model in bending applications. It describes the collection and evaluation of data on the piezoresistive effect in bending and combined loads. Several test structures are presented and their corresponding piezoresistive behavior is compared to that predicted by linear piezoresistance models.

3 Test Devices and Experimental Setup

Several test structures were designed and tested to explore the piezoresistive effects of tension, bending, and combined loads. Devices were fabricated using the MUMPs [27] and SUMMiT [28] processes which use n -type polysilicon as the fabrication material. For both processes, the polysilicon resistivity is between 1.5×10^{-2} and $3 \times 10^{-2} \Omega\cdot\text{cm}$. The test devices are summarized in Table 1. As shown in Fig. 2, diverse structures were designed in an attempt to capture the probable loading conditions an integral piezoresistive sensor would experience. Scanning electron micrographs of the structures are shown in Fig. 3. Dimensions and measured resistance for each design are included in Tables 2 and 3. Device names such as L150p1p2 represent a tensile structure with length 150 μm and made of poly 1 (p1) and poly 2 (p2) structural layers.

First, the tension structures were tested and compared to the linear piezoresistance model to characterize any differences in piezoresistive behavior due to

Table 1

Piezoresistance test structures.

Device	Fabrication Process	Desired Testing Information
Tension Bars	MUMPs/SUMMiT	Tensile Model Validation; Influence of Fabrication Method, Polysilicon Layer and Beam Length
Folded-Beam	MUMPs	Effects of Bending Loads
U-Spring	SUMMiT	Effects of Bending Loads
S-Curl	MUMPs	Combined Loads
Snake	MUMPs	Combined Loads

Table 2

Nominal dimensions and measured resistance of piezoresistive tensile and bending structures, given in μm and Ω .

Device Name	Process	Length, L_x	Width, w	Thickness, t	Resistance
L100	SUMMiT	100	1	2.25	862
L150p1p2	MUMPs	150	3	3.5	1,432
L50p1	MUMPs	50	3	2	286
L50p2	MUMPs	50	3	1.5	1,274
L50p1p2	MUMPs	50	3	3.5	549
Folded-Beam	MUMPs	150	3	3.5	1,649
U-Spring	SUMMiT	90	2.75	4.5	336

Fig. 2. Test structures fabricated for the characterization of the piezoresistive effect of silicon.

Fig. 3. SEM images of test structures fabricated for the characterization of the piezoresistive effect of silicon.

Table 3

Dimensions and measured resistance of MUMPs-fabricated piezoresistance combined load structures, given in μm and Ω .

Device	Centerline Radius, R	Width, w	Thickness, t	Resistance
S-Curl	4.5	3	2	265
Snake	4.5	3	2	1,149

Fig. 4. Each test structure included a micro probe guide (top) to reduce off-axis forces and deflections and a linear force gauge (bottom) to allow force measurement.

the fabrication process, fabrication layers, and tensile member length. Next, piezoresistivity in bending was explored using the Folded-Beam and the U-Spring devices shown in Figs. 2c and 2d. Finally, piezoresistance in combined loads (bending and tension or compression) was investigated with the S-Curl and Snake structures.

Each test structure was fabricated with an attached micro-probe guide, force gauge [29], and optical vernier, as shown in Fig. 4. These structures allowed for force application and measurement. The probe guide provided off-axis stability for the microprobe used to apply the desired forces to the test structure. With its relatively low stiffness ($k=9.2 \mu\text{N}/\mu\text{m}$) it provided little resistance to motion in the test direction. The MUMPs-fabricated probe guide was designed with the same geometry and dimensions as the folded-beam structure depicted in Fig. 2 (refer to Table 2 for dimensions).

Forces were calculated using large deflection equations [30] for the relative deflection of the folded-beam force gauge in Fig. 4. For the MUMPs-fabricated devices, with the exception of the folded-beam device, the force gauge had a

linear spring constant of approximately $k=147 \mu\text{N}/\mu\text{m}$. Because of the smaller stiffness of the folded-beam device, its force gauge was designed to have a much lower stiffness of $9.1 \mu\text{N}/\mu\text{m}$. A similar force gauge was attached to the SUMMiT-fabricated devices.

The amount the force gauge was compressed or stretched was measured with an optical vernier. Each row of vernier ‘teeth’ was offset from the facing row in such a way that the alignment of each sequential set of teeth represented a displacement of $0.5 \mu\text{m}$. With this displacement resolution, a resolution of applied force of approximately $73.5 \mu\text{N}$ was obtained for most of the MUMPs devices, and a force resolution of $4.6 \mu\text{N}$ for the folded-beam device. The force resolution for the SUMMiT-fabricated U-Spring was $21.1 \mu\text{N}$.

Once fabricated, mechanically released, and wire-bonded, the piezoresistive behavior of each device was measured using a three step process. First, the nominal electrical resistance was measured. Second, a set of specific force steps were applied, with the resistance being recorded at each step. Third, the undeflected, or nominal, resistance was measured again in order to observe any thermal drift or plastic deformation. Most devices were tested with a progressive sweep of applied forces.

4 Experimental Results

The following subsections present the experimental results for each test device and provide a summary of the significance of each data set, including tensile, bending, and combined loading conditions.

4.1 Tensile Loads

The goals of tensile testing were to observe the variation in piezoresistive behavior due to the fabrication process and to compare test results to the linear piezoresistive model. The results of both SUMMiT- and MUMPs-fabricated tensile elements of various geometries illustrate how the electrical resistance decreased linearly as the applied tensile stress increased, as shown in Fig. 5. This piezoresistive behavior in tension followed the linear trend described in the literature for *n*-type polysilicon [1, 15].

For the SUMMiT tensile device, the piezoresistive sensitivity (as approximated by the slope of the linear region) is 0.011% decrease in resistance per MPa. A similar linear decrease in resistance was evident in the MUMPs-fabricated devices, with a sensitivity ranging from 0.010 – 0.012% change in resistance per

Fig. 5. Comparison of piezoresistance in tensile devices for MUMPs- and SUMMiT-fabricated devices of varying length and fabrication layer.

Fig. 6. Piezoresistance behavior of SUMMiT-fabricated U-Spring device and of Folded-Beam device fabricated with the POLY1 and POLY2 layers of the MUMPs process.

MPa. The variation of piezoresistive behavior among the MUMPs tensile elements was explained by the differences in element length and polysilicon layer, with a slightly higher sensitivity (steeper slope) corresponding to longer tensile elements fabricated in POLY2 (the second structural layer deposited during MUMPs fabrication). It should be noted, however, that the measured differences in piezoresistive sensitivity among polysilicon layers was quite small.

4.2 Bending Loads

The U-Spring and Folded-Beam devices were designed to characterize the piezoresistance effect for bending loads. The resistance change was induced by deflection caused by an applied force, much like the tensile devices.

Fig. 7. Piezoresistance behavior of MUMPs-fabricated S-Curl device.

Figure 6 presents the data for the SUMMiT-fabricated U-Spring device and the MUMPs-fabricated Folded-Beam device. The results for both mechanisms reveal that the electrical resistance increased nonlinearly as the applied force increased. This non-linear increase in resistance was observed regardless of the force's direction. The fractional change in resistance for a beam in bending is small at low deflections and becomes more sensitive as deflection increases.

4.3 Combined Loads

The S-Curl and Snake devices were designed and tested to explore the effects of combined tension, compression and bending loads on piezoresistive behavior. Acquiring numerous data points at high applied forces was inhibited by the out-of-plane instability of the combined loading devices. Consequently, only data for relatively low applied forces are presented here.

The results of the S-Curl tests, shown in Fig. 7, illustrate a rise in resistance when the device was compressed. This increase in resistance became slightly steeper, i.e., more sensitive, at higher applied forces, much like the U-Spring devices. This behavior could be attributed to the combined or additive effect of the compressive and bending stresses on the device, both of which induce an increase in resistance.

When the S-curl was put in tension and bending, its resistance initially decreased rapidly and then leveled off at higher applied forces. Initially the tensile stress in the S-curl dominated, resulting in a linear decrease in resistance. As the applied force continued to rise, the effect of bending stresses began to dominate, counteracting the decrease in resistance caused by the tensile stress.

Fig. 8. Piezoresistive behavior for the Snake mechanism.

The piezoresistance effect in combined loads was further shown with the Snake mechanism. As Fig. 8 shows, the resistance of the Snake device initially decreased sharply, began to level off at approximately $800 \mu\text{N}$, and then increased nonlinearly. Just as with the S-Curl device, the Snake device experienced both tension and bending loads. The trends seen in the data can be explained by the hypothesis that, when initially stretched, the dominant stress state was tension. This resulted in a decrease of resistance. As the deflection increased, causing higher stress levels, the bending stresses and resultant rise in resistance became dominant over the resistance drop due to tension.

4.4 Summary of Results

The experimental data show tensile stress induces a nearly linear decrease in resistance with a piezoresistive sensitivity similar to published empirical values for n -type polysilicon [31]. The piezoresistive sensitivity has been described in terms of a unitless piezoresistive gauge factor, G , calculated as

$$G = \left(\frac{1}{\varepsilon}\right) \frac{\Delta R}{R} = \pi_l E + 1 + 2\nu \quad (4)$$

for which ε is the applied axial strain, π_l is the longitudinal piezoresistance coefficient, E is the modulus of elasticity, and ν is Poisson's ratio. As summarized in Table 4, the experimental gauge factors calculated from the tensile stress measurements of Fig. 5 show reasonable agreement with previous empirical data for polycrystalline silicon [31]. Since the gauge factor is expected to drop for highly-doped silicon, this agreement is entirely reasonable for MUMPs- and SUMMiT-fabricated devices, which are both heavily doped.

Table 4

Published and measured piezoresistance gauge factors for tensile stress, unitless. The published comparison data is reported in [31].

Published Values	SUMMiT	MUMPs		
	3,4*	1*	2*	1,2*
-16 to -22	-14.8	-16.5	-18.1	-16.4

*Polysilicon fabrication layer(s)

Fig. 9. Comparison of piezoresistive behavior in tension, bending and combined loads.

Figure 9 compares the resistance drop for tensile members with the nonlinear rise in resistance due to simple bending loads and the additive effect of combined loads on piezoresistive behavior. The figure's x -axis shows applied force to allow comparison in terms of a sample measurand for many applications. Although the piezoresistive sensitivity in bending is slightly inferior at lower force, the effect may be advantageous for piezoresistive devices which operate—and demand high piezoresistive sensitivity—at higher bending stresses or deflections. Furthermore, the bending and combined loading devices would be better suited for applications requiring significant displacement or deformation of the sensors during operation. Such deformation is not feasible with simple tensile members.

Figure 10 compares the resistance changes for the tensile and bending devices as a function of average stress magnitude. Note that, for tensile bars, stress is uniformly distributed, but bending stress varies throughout the beam. The average stress magnitude was found by averaging the absolute value of stress throughout the beam. The figure shows that the average stress magnitudes in each type of device are comparable. Hence, differences in stress magnitude are not responsible for the observed differences in resistance change trends.

Fig. 10. Comparison of resistance change in tensile and bending devices vs. average stress

Fig. 11. End-loaded cantilever beam for resistance change model.

5 Comparison to Linear Model

5.1 Modeling of the Piezoresistive Effect in Bending

The previous section demonstrated that beams loaded in pure tension showed a linear change in resistance, as predicted by Eq. (3). However, the stress state is more complex for beams in bending. In order to make a comparison, this section develops a simple model of the resistance change for a beam in bending, assuming that Eq. (3) is satisfied. This simple model is based on the development in [25], pp.478–479.

Consider the cantilever beam loaded by a force F at its end, shown in Fig. 11. By assuming that all current flows in the x -direction (or that y -direction current flows are negligible), we may approximate the resistance along this beam as

$$R = \int_0^L \rho_l(x) dx \quad (5)$$

where $\rho_l(x)$ is the resistance per unit length, which is assumed to vary along the beam. We can approximate the conductance per unit length g_l by integrating the conductance across the width w of the beam, giving

$$g_l = \frac{1}{\rho_l} = \int_{-w/2}^{w/2} \frac{t dy}{\rho_e(x, y)} \quad (6)$$

where t is the thickness of the beam (out of the plane shown in Fig. 11), and $\rho_e(x, y)$ is the electrical resistivity. As the stress will depend on both x and y , ρ_e also changes with both x and y according to Eq. (3). Hence,

$$\rho_e(x, y) = \rho_0[1 + \pi_l \sigma_l(x, y)] = \rho_0 \left[1 + \pi_l \frac{M(x)y}{I} \right] \quad (7)$$

where ρ_0 is the unstressed electrical resistivity, $M(x)$ is the internal moment along the beam (given by $M(x) = -F(L - x)$), and I is the second moment of area of the beam's cross-section, which is equal to $w^3 t / 12$ for a rectangular cross section (assumed here). Substitution of Eq. (7) into Eq. (6) gives

$$g_l = \frac{1}{\rho_l} = -\frac{tw}{2\rho_0\beta(L-x)} \ln \left(\frac{1 - \beta(L-x)}{1 + \beta(L-x)} \right) \quad (8)$$

where β is given by

$$\beta = \frac{6\pi_l F}{w^2 t} \quad (9)$$

Finally, substituting Eq. (8) into Eq. (5) and making the variable substitution $u = L - x$ gives

$$R = \frac{2\rho_0\beta}{wt} \int_L^0 u \left[\ln \left(\frac{1 - \beta u}{1 + \beta u} \right) \right]^{-1} du \quad (10)$$

This equation can be evaluated using numerical integration to find the beam's resistance as a function of applied force.

5.2 Model Validation

The simple model represented by Eq. (10) is based on several assumptions, such as the negligence of y -direction stresses or currents. To test these assumptions, the model was validated against a nonlinear commercial finite element

Fig. 12. Comparison of finite element simulations to predictions of the simple model for three cantilever beams.

code (ANSYS). The beam was modeled using 3-D solid226 elements, which model both the structural and piezoresistive behavior. The element allows the user to input π coefficients from Smith's model directly into a matrix used in the algorithm. Therefore, the changes in resistivity in the three orthogonal directions and three cross directions from Eq. (1) were all represented. The piezoresistive coefficients used for validation, shown in Table 5, were approximated from the literature assuming a single-crystal silicon beam phosphorus-doped to $1 \times 10^{20} \text{ cm}^{-3}$ [32]. For the simple model, π_l was set equal to π_{11} from Table 5. Note that by comparing the full single-crystal silicon model with the simple model, we test the assumptions required to reduce Eq. (1) to Eq. (3) as well as the assumptions made in deriving Eq. (10).

For validation, three beams were modeled and compared. Each had a width of 3 μm and a thickness of 3.5 μm . The lengths of the beams were 50, 75, and 100 μm . Figure 12 shows the predicted resistance change from each beam computed by the simple model and by ANSYS for a 5 μm deflection of each beam. As the figure shows, the simple model agrees with the computationally intensive finite element model very well, demonstrating that the assumptions made in the simple model are appropriate.

Table 5
Piezoresistive (π) coefficients used for FEM, from [32].

Coeff.	Value	Units
π_{11}	-39×10^{-5}	MPa^{-1}
π_{12}	28×10^{-5}	MPa^{-1}
π_{44}	-16×10^{-5}	MPa^{-1}

Fig. 13. Prediction of the simple model compared to experimental data for the folded-beam device.

5.3 Comparison of Linear Model with Experimental Data

After it was validated, the simple model was compared to experimental data for a beam in bending. Specifically, the folded-beam suspension shown in Figs. 2 and 3 was modeled by treating it as a total of 16 cantilever beams, each with half of the length of a single fixed-guided beam. Since the beams are arranged in both series and parallel with each other, the total device resistance is four times the resistance of a single cantilever, and the total device displacement is four times the displacement of a cantilever. Figure 13 shows the measured change in resistance compared to the change predicted by the simple model. For the simple model, π_l was taken to be $-10.9 \times 10^{-5} \text{ MPa}^{-1}$, which is the value determined from the experimental tension data for the polysilicon layers used for the folded-beam device. The comparison demonstrates that the model predicts resistance change inaccurately, with both the magnitude and the trend (decreasing versus increasing resistance) of the model being incorrect. We conclude that the simple model, based on linearity of piezoresistance, is not sufficient to model the piezoresistance behavior of a device in bending. The next section examines this conclusion in more detail.

5.4 Further Exploration of the Linear Model

All of the data for bending of *n*-doped polysilicon beams show that the resistance rises with increasing deflection or force. However, the mathematical nature of the linear piezoresistive model precludes an upward trend in resistance for a flexure in bending. This inadequacy can be seen in a simple demonstration. Figure 14 shows a flexure being pressed downward by a force.

Fig. 14. Unstressed cantilever and resistor schematic.

Fig. 15. Stressed Cantilever and resistor schematic.

In the figure, the beam is represented electrically by two resistors in parallel. The top resistor represents the part of the beam that will be placed in tension during bending; the bottom resistor represents material in compression. The unstressed resistance of the resistors in parallel (R_T) is given by

$$R_{T0} = \left(\frac{1}{R} + \frac{1}{R} \right)^{-1} = \frac{R}{2} \quad (11)$$

As the flexure is bent, the top half of the flexure is put in tension and the bottom half is put in compression, with the stress magnitudes in each half being the same. Representing the tension and compression in the flexure by the relative deformation of the resistors shown in Fig. 15, the linear piezoresistive model predicts that the resistance of the top half will drop by an amount ΔR , while the resistance of the top half will rise by ΔR . The total resistance is then given by

$$R_T = \left(\frac{1}{R - \Delta R} + \frac{1}{R + \Delta R} \right)^{-1} = R_{T0} - \frac{\Delta R^2}{2R} \quad (12)$$

Hence, the model predicts that the total resistance will always drop for any bending deformation, contrary to the experimental data.

As evident from these experimental results, the linear piezoresistance model failed to capture the trend and the magnitude of the piezoresistive effect for the simple bending device. In fact, regardless of the sign or magnitude of

the piezoresistance coefficients used in the analyses, the model predicts a decrease in resistance due to bending stresses, while every bending test shows an increase in resistance. Consequently, a more complete nonlinear model of piezoresistance is needed to predict the effects of bending and combined loads on piezoresistance. Such a model could be based on previously-published data demonstrating small nonlinearities in the piezoresistive effect in silicon [33]. Such a model would facilitate the design, optimization and implementation of integral piezoresistive sensors and may provide greater insights into the physical phenomenon of piezoresistivity.

6 Conclusion

This paper described an experimental investigation of the piezoresistive effect of polysilicon in bending and combined loads. The design and experimental setup of several n -type polysilicon test structures was provided, as well as the results from experimentation. Tensile or compressive stresses cause a linear change in resistance, but bending stresses induce a nonlinear rise in resistance for n -type polysilicon. A combination of tension, compression and bending stresses add together, with the tensile and compressive stresses dominating at lower applied force levels. The experimental results demonstrate the failure of linear piezoresistance models to predict the piezoresistive effect of polysilicon in bending. This represents a motivation for the development of a more complete nonlinear model of piezoresistance which accurately predicts piezoresistance in bending. Such a model may shed greater light on the physical phenomenon of piezoresistance and will facilitate the design and optimization of integral piezoresistive sensors.

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