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Time reversal of continuous-wave, steady-state signals in elastic media

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Experimental observations of spatial focusing of continuous-wave, steady-state elastic waves in a reverberant elastic cavity using time reversal are reported here. Spatially localized focusing is achieved when multiple channels are employed, while a single channel does not yield such focusing. The amplitude of the energy at the focal location increases as the square of the number of channels used, while the amplitude elsewhere in the medium increases proportionally with the number of channels used. The observation is important in the context of imaging in solid laboratory samples as well as problems involving continuous-wave signals in Earth. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3097811]

Time reversal (TR) is a method of focusing sound energy onto a specific location.[1,2] In the forward propagation step of the TR process, one or more detectors, the channels of a TR mirror (TRM), are used to record signals from an unknown source. In the backward propagation step, these signals are reversed in time and broadcast from their respective detectors, now acting as sources, producing a focusing of energy back onto the source. TR may be used to locate an unknown source of wave energy with application to earthquake localization,[3] ultrasonic medical imaging,[4] underwater acoustic imaging of targets,[5] and for locating cracks nondestructively.[6] It may also be used to focus wave energy onto a user defined location for applications such as acoustic lithotripsy[4] and for ultrasonic therapy, where focusing continuous-wave (CW), steady-state signals would be particularly useful. There are a number of important problems in Earth that involve long duration, CW-like signals as well such as localizing glacial tremors, and volcanic and nonvolcanic tremors. A demonstration of TR for steady state signals would then provide the ultimate test of the ability to focus cw or CW-like long duration signals.

TR almost always employs pulsed source signals of large bandwidth. TR of CW, steady-state signals in elastic or fluid media has not yet been demonstrated. Fink,[7] a respected expert on TR, has argued that a single channel TRM would not focus monochromatic steady-state signals. He reasoned that a single channel TRM operating at a single frequency would broadcast a diverging wave that would not produce a TR focus. Subsequently, Yon *et al.*[8] used TR to focus long duration signals in a closed acoustical cavity. They found that the quality of the spatial focusing degrades as the bandwidth of the signal used for TR decreases. Recently, Ribay *et al.*[9] demonstrated the ability of TR to produce acoustic focusing in a reverberant room of multiple signals, correlated and uncorrelated, Gaussian noise sources of long duration.

In this letter, we demonstrate that in a linear elastic solid, TR can be used to focus CW, steady-state elastic waves when a multiple channel TRM is used. We utilize monochromatic signals, rather than signals of finite bandwidth, to provide the ultimate test of the robustness of TR focusing of long duration signals. If monochromatic, steady-state signals can be focused with TR, then any cw or CW-like signal consisting of multiple frequencies will also focus due to linear superposition. The experiments presented here generate spatially localized focusing of shear waves. Thus, due to elastic mode conversion, the longitudinal motion input by the transducers results in a focus of shear wave energy.[10]

One might suspect that if a system is driven monochromatically to steady state from source position s that the TR process would not produce a spatially unique focus due to the nature of a steady state system response. In the case of a single channel TRM, the steady state response y_i is detected at a receiver location r_i. When the steady state is reached (a steady state condition implies that the system has been driven by a CW excitation and the transient response has decayed), the spatial structure of the wave field in the system in this case is determined by its modal response. The response y_i is a monochromatic signal at the frequency of the source but with a different phase. When y_i is reversed in time and broadcast into the system from r_i, the steady state response will again be determined by the system's modal response resulting in no spatial focusing at s, i.e., the amplitude at s will not necessarily be larger than that at any other point in the system.

A well known consequence of the TR process is that the back propagation response at the focal point from one TRM channel is in phase with all of the other TRM channels. Thus, when a multiple channel TRM is used, the energy from each TRM element will arrive at s in phase even for a CW, monochromatic source excitation. Despite the lack of spatial focusing at s for a single channel TRM, spatial focusing is observable for a multiple channel TRM since the broadcast from each of the elements will arrive at s in phase and produce spatially localized focusing (larger amplitude at s than elsewhere in the system). Furthermore, the system must be driven at a frequency where the modal density is relatively high in order to observe focusing. Imagine if only a single mode is excited by the forward propagation, then the TR backward propagation will also only excite only that mode; however, if there are multiple modes excited then the response at locations away from s during the back propagation due to each of the elements in the TRM will result in constructive and destructive interference, whereas at s they will only constructively interfere. Essentially, for a multimode system driven to steady state, the signal at each spatial location is made up of a superposition of many modes.

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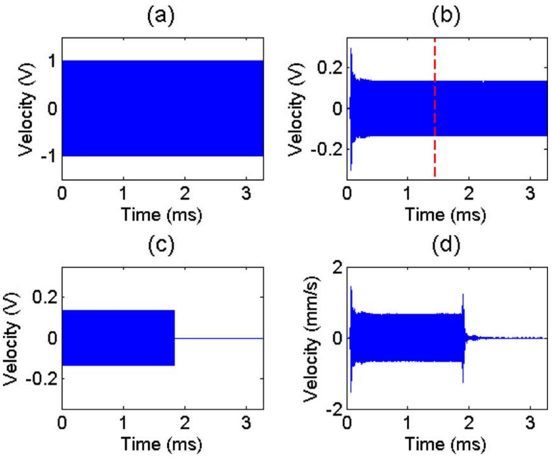

FIG. 1. (Color online) Typical time domain signals of a TR experiment used in this study. (a) Continuous wave sinusoid signal. (b) Typical received signal at a detector during the forward propagation. (c) Windowed, time reversed signal used as the source of the backward propagation [the red dashed line in (b) indicates where the window begins in order to eliminate the transient portion of the received signal]. (d) Time reversed focal signal.

The experiments are conducted in a rectangular block of Berkeley Blue granite (Elberton, Georgia, USA),[11] $22.3 \times 20.9 \times 18.6\ \text{cm}^3$, that was supported by rubber cylinders at the four corners of the bottom face. Berkeley Blue granite has a mass density of $2.61\ \text{g/cm}^3$, a shear wave speed of 3100 m/s, and a quality factor Q of about 200. 23 piezoelectric ceramic transducers were bonded with 5 Minute® epoxy onto five of the block faces. The transducers used are 1 MHz compression resonance piezoelectric ceramic disks, measuring 2 mm in thickness and 12.7 mm in diameter. These transducers are most efficient when operating around their first radial resonance frequencies of 200 kHz. A strip of reflective tape was placed on the sixth side of the block to facilitate measurement of velocity signals with an out-of-plane, scanning laser vibrometer.

For each successive experiment, a different transducer emits a monochromatic 200 kHz signal [see Fig. 1(a)]. The laser vibrometer is positioned at a selected position on the reflective tape and the velocity response is recorded using time averaging. As it takes of order Q cycles for the transient response to die away leaving the steady state response [see Fig. 1(b)], the transient portion of the received signal is windowed out [the signal to the left of the red dashed line in Fig. 1(b) is set to zero] and the remaining part of the signal is reversed in time [see Fig. 1(c)]. The time reversed signal is then broadcast from the source transducer that broadcast the original signal. In this manner, a virtual source[6,12] is created at the laser position s. During the broadcast of the TR signal from the transducer, the out-of-plane velocity of the surface is measured, at a set of uniformly spaced positions along a line that includes s, with a laser vibrometer [see Fig. 1(d)]. The steady state portion of the signals acquired by the scanning laser during the TR broadcast is $u_1(x,t)$.

This single channel/transducer protocol is repeated for each of the 23 transducers, utilizing the same s location, producing the signals $u_1(x,t)\ldots u_{23}(x,t)$. With these signals, we are able to study the dependence of TR focusing on the number of channels used. To do this, we form the quantity, $E_{\text{TR}}(x,N)$, by summing N of the $u_i(x,t)$, and computing the time average of their square, i.e.,

$$E_{\text{TR}}(x,N) = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2}\left[\sum_{i=1}^{N} u_i(x,t)\right]^2 dt. \tag{1}$$

The function $E_{\text{TR}}(x,N)$ is plotted in Fig. 2. Note that, as expected, for a single channel TRM experiment, TR cannot focus monochromatic energy. One may clearly determine from Fig. 2(a) that when multiple TRM channels are used that spatially localized focusing is achieved with TR and that it improves with the number of channels employed. In order to demonstrate that TR is responsible for the localized focusing at the virtual source position and that it is not simply an efficient region for the modal distribution of the sample, the scanning laser is used to record the velocity signals during the first set of transducer broadcasts, i.e., the forward propagation portion of the experiment. The resulting signals can then be used to compute the forward propagation spatial distribution versus N and position, $E_{\text{FP}}(x,N)$ [see Fig. 2(b)]. It is clear from Fig. 2 that only when TR is used is there an apparent localized focusing of monochromatic energy.

According to the theory of TR,[8] the energy at the focal position increases as N^2 and the energy elsewhere (fringes, sometimes referred to as spatial side lobes) in the medium increases as N. In this letter, energylike quantities are computed by squaring the amplitudes. The energy at the focal position as a function of N used is plotted in Fig. 3 along with a second order polynomial least-squares fit (correlation coefficient of 0.9998). Also plotted in Fig. 3 is the average fringe energy as a function of N used along with a first order least-squares fit (correlation coefficient of 0.9983). Thus, as expected for TR focusing experiments, the energy at the fo-

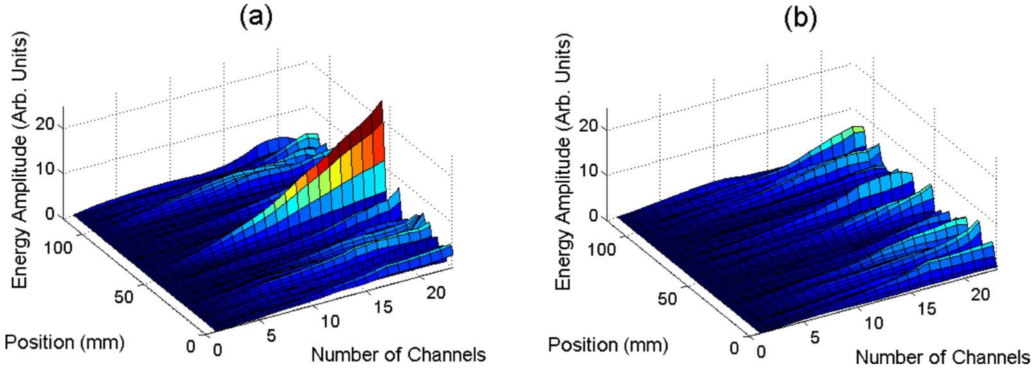

FIG. 2. (Color online) Spatial dependence of the TR focusing of continuous single frequency sine waves vs the number of channels used for TR. (a) TR focusing results (proper phasing for focusing). (b) Forward propagation results (arbitrary phasing).

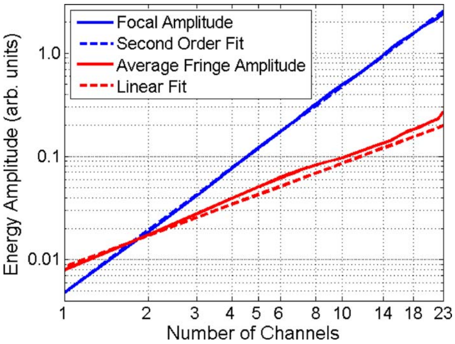

FIG. 3. (Color online) Focusing quality as a function of the number of channels used for TR. The focal energy and second order least-squares fit are displayed in blue (solid and dashed lines, respectively). The average fringe energy and first order least-squares fit are displayed in red (solid and dashed lines respectively).

cal point increases as N^2 while the energy of the fringes increases as N.

We have shown that TR can be used to provide spatially focusing of monochromatic, CW, steady-state elastic wave energy in a reverberant elastic cavity when multiple channels are used. Thus, any cw or CW-like signal can be retrofocused with TR due to linear superposition. The elastic waves predominantly consist of shear waves in this case (previous experiments have focused only acoustic longitudinal waves). The energy at the focal point and elsewhere (fringes) increase according to the expected dependencies on the number of channels used in the TRM. It is postulated that the modal density of the sample must also be relatively high to produce a unique spatially localized focusing result. The importance of this result may find application in the field of medical ultrasound where the use of a long duration monochromatic excitation may be used for lithotripsy or other ultrasonic therapy. Some Earth sources, such as tremor, are long duration CW-like signals, and we are in the process of applying this approach to these problems. Paired with nonlinear techniques used for nondestructive evaluation with industrial[13] and medical[14] applications, the use of focused CW signals could enhance the detection of a nonlinear elastic response thus providing an increased sensitivity to current techniques.

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