2006-07-18

How a Master Teacher Uses Questioning Within a Mathematical Discourse Community

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HOW A MASTER TEACHER USES QUESTIONING WITHIN A

MATHEMATICAL DISCOURSE COMMUNITY

by

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A thesis submitted to the faculty of

Brigham Young University

in partial fulfillment of the requirements for the degree of

Master of Arts

Department of Mathematics Education

Brigham Young University

August 2006

BRIGHAM YOUNG UNIVERSITY

GRADUATE COMMITTEE APPROVAL

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As chair of the candidate‟s graduate committee, I have read the thesis of Omel A. Contreras in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

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ABSTRACT

HOW A MASTER TEACHER USES QUESTIONING WITHIN A MATHEMATICAL DISCOURSE COMMUNITY

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Recent scholarly work in mathematics education has included a focus on learning mathematics with understanding (Hiebert & Carpenter, 1992; Hiebert et al., 1997; Fennema & Romberg, 1999; National Council of Teachers of Mathematics, 2000). Hiebert et al. (1997) discussed two processes that they suggested increase understanding and that are central to this study: reflection and communication. Learning mathematics with understanding requires that the students create a deeper knowledge of mathematics through reflection and communication.

The environment in which such learning can take place must include patterns of behavior, known as social norms that promote deeper thinking. When the social norms encourage reflection and communication among the members of the classroom community, or supports learning with understanding, it becomes what I term a productive discourse community.

The purpose of this study is to find out what a teacher does to create and maintain a productive discourse community where students can reason and learn with understanding. To

accomplish this purpose, this research asks the following question: In what ways does the teacher in the study direct mathematical discourse in order to facilitate understanding?

To answer this research question, data was gathered from eight class periods. The classroom discourse was analyzed and six discourse generating tools were found to be used by the teacher: (1) using lower-order questions to engage students, (2) persisting in eliciting students" reasoning, (3) encouraging as many student participations as possible, (4) encouraging students to analyze and evaluate each other"s comments, (5) encouraging students to share as many strategies as possible and (6) using a focusing discourse pattern. There were also three social norms found to be established in the classroom at the time of the data collection. These norms are: all students are expected to (a) participate (b) share their reasoning when called upon, and (c) listen to, analyze, and evaluate each other"s comments.

Through further analysis, it was found that the six discourse generating tools reinforced the social norms, while the social norms supported the six discourse generating tools. Thus creating an environment where reflection and communication occurred in a way that promoted learning mathematics with understanding.

To my mother

María Ernestina Niño de Contreras

ACKNOWLEDGMENTS

Thank you, Omel C. Contreras and Maria Contreras, for showing me that I can. Thank you Sarah for the long hours without a husband or father to our children; without you I could not have done it. Thank you, Dr. Williams, for the patience and constant guidance throughout my whole graduate experience. Thank you, Dr. Peterson and Dr. Leatham, for timely suggestions which guided the research, analysis and editing. Thank you, Dr. Janet Walter for the resources on questioning. Thank you, Matthew Webb for being a sounding board and for the help with the English language.

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Learning with Understanding

Recent scholarly work in mathematics education has included a focus on learning mathematics with understanding (Hiebert & Carpenter, 1992; Hiebert et al., 1997; Fennema & Romberg, 1999; National Council of Teachers of Mathematics [NCTM], 2000). Such a focus has emerged as studies have shown that students can memorize facts and procedures without knowing when or why to use them (Erlwanger, 1973; Sowder, 1988). Thus there has been increasing interest in helping students not only learn facts and procedures, but how those pieces of information are related to one another and to situations in which they can be applied. Hiebert and Carpenter (1992) described this as knowledge that is rich in connections, with the number and strength of connections giving a measure of understanding.

It has also been increasingly recognized that learning with understanding is a complex endeavor. Knowledge is not transferred from teacher to student, but rather built or constructed by the student with the help of the teacher (von Glasersfeld, 1995). Because of this, ideas can be understood at different levels and in different ways by different students or at different times by the same student. Learning with understanding is a gradual process in which students‟ understandings emerge or develop, rather than a destination at which a student has either arrived or has not (Fennema & Romberg, 1999).

Helping Students Learn with Understanding

Many aspects of students‟ lives contribute to their mathematical understanding. Some are related to their socio-economic status and home life, while others are related to classroom activities and environment. While a teacher has no control over the former, he has great control over the direction of the latter. Some of the components of a classroom that help students learn with understanding include the type of curriculum chosen by the teacher (Fennema & Romberg, 1999; NCTM, 2000), the type of tasks chosen with which to explore mathematics (Hiebert et al., 1997; NCTM, 1991), and the types of practices that emerge in the classroom (Fennema & Romberg, 1999; Cobb, Wood & Yackel, 1993). These normative practices or *norms* in the classroom affect the mathematical discourse and thus influence the learning opportunities that arise for the students and teacher alike (McClain & Cobb, 2001).

This study focused on the norms that are encouraged and supported by one teacher‟s actions. The teacher promoted specific practices in the classroom that helped students learn with understanding. Some of these practices include students‟ reflections of the mathematics, students‟ participations through explication of their thinking to encourage reflection, and students‟ contributions by sharing as many ways of thinking about a problem as possible. One way that the students were encouraged to reflect was by the teacher not placing the authority of mathematical truth solely upon himself, but by sharing it with the students. Other ways in which the teacher encouraged student reflection was by having students listen to and reflect upon other students‟ strategies and reasoning, by using questions to engage students, and by using questions to help students focus their thinking. These practices of the teacher will be discussed in more detail in chapter 4.

Personal Interest

I recently graduated with a Bachelor of Arts with an emphasis in mathematics education. I did my student teaching at a Utah public high school during the winter semester of 2002, during which I had three cooperating teachers who helped me to learn three different styles of teaching. I was particularly impressed with the teaching style of one of the teachers who had had many years of teaching experience. He had chosen an NCTM Standards-based curriculum for the class we taught together. I was very interested in this curriculum, because throughout all my schooling years I had been taught from traditional curricula, and I felt that this reform curriculum was very effective.

After three weeks of observing him teach, I began teaching. I quickly realized that what he did with much ease was very difficult for me to recreate. He had a great ability to teach the students through questioning and classroom discussions that seemed to lead the students to deeper mathematical understanding.

We had a preparation period right after the class we taught. During that time, we began to discuss what had happened during the class discussion, and what needed to happen in the next one. He then began to instruct me in the kinds of questions I should ask. At first he would provide the questions, but with time, I was able to formulate some good questions by myself. I was encouraged and felt I could continue to create such meaningful discussion in my classrooms through thoughtful questioning.

The following school year, I was hired to teach a couple of classes at this same school with this same curriculum. This time, even though the teacher and I prepared together for our lessons, I noticed that my instruction techniques fell back into a more traditional style of teaching. I also noticed that I was not able to promote a good mathematical discussion within the classroom, because I could not engage the students as I had the previous year. I decided to observe the teacher‟s class to see what he was doing differently. It became apparent to me that although our curriculum was the same and we were preparing our lessons together, we were teaching very differently.

I began to wonder what, in his teaching, helped him to engage the students so well. So, among several other things, I began asking the same questions he was asking. I noticed that my students began responding positively to the questions. By no means did my newly borrowed questioning technique solve all of my problems. But it helped my students to stay more on task, and from what I could gather, to understand the concepts better.

Due to this experience I realized that I wanted to learn how he created an environment, through the establishment and reinforcement of social norms that allowed him to use his questioning skills to facilitate students in learning mathematical concepts with understanding. In particular, I wanted to learn how those social norms were related to his questioning techniques. Thus, he became the subject of this research study.

Purpose of the Study

In order to provide students with the tools necessary to deal with mathematical problems effectively, teachers must help students learn mathematics with understanding. Learning mathematics with understanding requires that the students create a deeper, more connected knowledge of mathematics. The environment in which such learning can take place must include practices or patterns of behavior that will promote deeper thinking. This study does not look at whether or not participating students" understandings are deep and meaningful. Instead, the purpose of this study is to find out what a teacher does to establish or create and maintain an environment where students can reason and learn with understanding. In chapter 2, I discuss some of the things that are known about what an environment that promotes learning with understanding looks like, and how a teacher establishes such an environment.

Research Question

In an attempt to accomplish the purpose of this study, I have formulated the following research question: *In what ways does the teacher in the study direct mathematical discourse in order to facilitate understanding?*

I have collected data from eight classroom periods of a specific unit in a Geometry class taught by the teacher in the study. I analyzed the data paying particular attention to the discourse patterns that arose in the classroom as well as the normative behaviors that were present. Through this analysis I was able to identify definite practices that the teacher used to create an environment that promoted learning with understanding. I discuss these practices of the teacher in chapter 4.

Chapter II: Conceptual Context

Introduction

The conceptual context underlying this study includes discussions of the importance of students learning mathematics with understanding, what type of classroom environment promotes such learning, the norms that are established and maintained in such and environment as well as the types of discourse patterns that are utilized, and how such an environment is developed.

Learning Mathematics with Understanding

Recent research and policy statements in mathematics education have focused on the importance of learning mathematics *with understanding.* A good working definition of understanding was provided by Hiebert et al (1997): "We understand something if we see how it is related or connected to other things we know" (p. 4). Thus students who understand mathematics are able to connect it to various problem situations, other mathematics that they have learned, and so forth. Hiebert and Carpenter (1992) suggested several advantages of a focus on understanding. First, understanding makes future learning easier since there are richer connections to be made. Second, it promotes remembering and reduces the amount that must be remembered. Third, it increases the chances that mathematical knowledge will be used in appropriate situations. Finally, understanding helps students develop positive beliefs about mathematics.

Many classrooms characteristics are important in supporting learning with understanding (Fennema and Romberg, 1999; Hiebert et al., 1997; NCTM, 2000), including carefully designed tasks, the social culture of the classroom, the use of mathematical and technological tools, and the curriculum. Hiebert et al. (1997) discussed two processes that they suggested increase understanding and that are central to this study: reflection and communication. Reflection is the process of thinking about experiences. As students reflect, they make connections and establish relationships between what they learn and other experiences, which helps to increase understanding. Communication includes talking, writing, listening, and other forms of interaction among people. This causes deeper thinking, and brings to the front the need to justify thinking to others. According to Hiebert et al., "students who reflect on what they do and communicate with others about it are in a position to build useful connections in mathematics" (p. 6).

Carpenter and Lehrer (1999) also discussed the importance of reflection and communication. After arguing that reflection is central to learning with understanding, they stated:

> The question is: How do we encourage this type of reflection? Providing explicit guidelines for encouraging reflection is difficult, but a critical factor is that teachers recognize and value reflection. When that is the case, teachers establish classroom norms that support reflection. A specific norm that plays a critical role in supporting reflection… is the expectation that students articulate their thinking. Asking students why their solutions work, why a given solution is like another solution, how they decided to solve the problem as they did, and the like, not only helps to develop students' ability to articulate their thinking, it encourages them to reflect. (p. 28)

Carpenter and Lehrer recognize the importance of encouraging reflection and communication through social interaction about mathematics. For them, reflection and communication are a natural extension of social engagement in discourse about mathematics:

> A specific class norm that supports this conception of learning is that students regularly discuss alternative strategies (which they have generated to solve a given problem) with

> the teacher, with other students, and within the context of whole-class discussion. It is not enough to have an answer to a problem; students are expected to be able to articulate the strategy they used to solve the problem and explain why it works. (p. 29)

Thus the expectation that students communicate mathematical ideas with the teacher and with one another is central to supporting their learning with understanding. This study focuses on two aspects of a particular classroom that supported these practices: the development and maintenance of classroom norms and the ways that teacher engaged the students in discourse. I explain each of these below.

Classroom Norms

Wood (1998) described social norms as "an interlocking system of obligations and expectations, established by both the teacher and the students and underlying the manner in which members of the classroom interact, [and which] forms the smooth functioning of the class" (p. 175). The social culture of the classroom is developed through these social norms. The explicit aspects of these norms can be expressed through the establishment of rules. However, there are implicit aspects which are less obvious. These do not emerge from explicit rules or regulations, but come out of the everyday tug and pull of implicit expectations and obligations. Norms, then, are the often unspoken ways of behaving and interacting that constitute "business as usual" in a classroom.

Norms may be widely accepted practices such as the teacher standing in the front of the class and the students sitting in desks facing her. Other norms may be specific to classrooms, such as particular ways of handing in assignments. The norms that are of importance for this study are those that support learning with understanding. As mentioned above, these include ways that students are expected to share mathematical ideas or explain their thinking. Such norms may begin as explicit expectations given by the teacher, and become more normative as students come to accept and adopt such practices.

Classroom Discourse

Discourse Patterns

A good deal of research has been done on discourse patterns in classrooms, and there are many discourse patterns involving teacher questioning. Of those, this study focuses on three: The Initiation-Reply-Evaluation (IRE) Pattern discussed by Cazden (2001) and Mehan (1979), and the Funneling and Focusing Patterns discussed by Bauersfeld (1988), Herbel-Eisenmann and Breyfogle (2005), and Wood (1998).

The IRE pattern. The recitation or IRE pattern is identified by the sequence of teacher-student interaction where the teacher initiates a discussion with a question, the students reply and the teacher then provides evaluation to the students" comments. For example the teacher might initiate an interaction by asking the students what two angles are called when they add up to 90°. The students might then reply that they are called complimentary angles, to which the teacher would evaluate the answer as being correct.

This pattern has been studied at great length and has been the most common form of interaction in all grade levels (Cazden, 2001; Stodolsky, 1988). However, there are alternative patterns of communication in the mathematics classroom that are more effective in helping students to explore, investigate, reason, and communicate about their ideas, thus allowing them to gain a greater understanding of mathematics (Wood, 1998).

The funneling pattern. In this pattern, teachers guide their students through a procedure or toward an answer in a way predetermined by the teacher or the textbook. The questions often move from general to specific in order to "narrow" the discourse to the desired end (Bauersfeld, 1988). However, the teacher is the one who engages in cognitive mathematical activity by choosing the sequence of questions where the "student is merely answering the questions to arrive at an answer, often without seeing the connection among the questions" (Herbel-Eisenmann & Breyfogle, 2005, p. 485). In discussing the effectiveness of learning from the funneling pattern both Wood (1998) and Lundgren (1977) noted that this pattern may give the false impression that students are learning when they really are not. Thus funneling likely does not lead to reflection or to learning with understanding.

The focusing pattern. In this pattern, teachers" questions are based not on a predetermined procedure or answer, but on students" own thinking. Focusing is an attempt to help students articulate and clarify their thinking, thus focusing the discussion for the student and for the rest of the class (Herbel-Eisenmann & Breyfogle, 2005). Herbel-Eisenmann and Breyfogle further suggested that not only by focusing students" solutions but also by restating what students have said can teachers help students make sense of each other"s strategies and reasoning. Focusing thus becomes one way to help students engage in mathematical discourse in ways that promote reflection.

Teacher Questioning

Another important aspect of the discourse patterns that are present in the classroom is the type of questions that are asked. Through questions, members of the discourse community can present ideas or concepts for discussions, compare and clarify their thinking, and direct the path of the conversation. In addition, questions can become a powerful tool for teachers to engage students and to draw out and challenge students" reasoning.

The power of questions to support learning with understanding depends in part upon their ability to stimulate thought and reflection. For this reason, scholars find it important to articulate the cognitive level of questions. Many question hierarchies have been developed in the last 50 years. The most prominent hierarchy is Bloom‟s taxonomy (1956), which distinguishes among six different levels of questions. A much simpler type of hierarchy is a two-level system that differentiates between a higher-order question and a lower-order one. According to Barden (1995) this simpler question hierarchy is the only system that is consistent enough to discriminate among the different types of questions (p. 423). Barden continued by defining what she meant by lower and higher-order questions:

> Within the two-level system, lower-order questions are defined as those that require responses either recalled directly from memory or cited explicitly in text. Higher-order questions, on the other hand, are defined as those that require more than simple recall to produce an answer. (p.423)

For this report, lower-order questions are also defined as questions which do not require explanations or justifications. Examples of this definition of lower-order questions are inquiries that can be answered with “yes” or “no” responses, and inquiries where the teacher is collecting votes on the agreement or disagreement of a statement or concept. Higher-order questions, in contrast, are defined as those that require explanations or justifications of the students‟ reasoning.

Although studies of higher-order questioning seem to disagree on just how much higher-order questioning affects student achievement (Dillon, 1982; Ryan, 1974; Samson, Strykowski, Weinstein, &Walberg, 1987; Winne, 1979), it is nevertheless clear that engaging students in higher-order thinking through higher-order questioning is valuable to the students‟ cognitive progression (NCTM Principles and Standards for School Mathematics, 2000; Hiebert et al., 1997).

Productive Discourse Communities

A community is a group of people who share similar values and have similar goals (Hiebert et al., 1997). Together they create norms or establish patterns of behavior which determine the interactions among its members. More specifically, classroom communities are made up of the teacher and students in any given class. Their day-to-day interaction is the means by which they structure the norms of conduct regarding all aspects of their class. If we look even closer at a component of such a classroom structure, we can see that the discourse in a classroom is also governed by such norms of practice.

Any classroom along with its member and norms of practice creates a basic discourse community. The norms in such a classroom may or may not support learning with understanding. However, in this study we are interested in the type of environment or discourse community that will support learning with understanding. The norms in such a community will maintain the types of behaviors described by Carpenter and Lehrer (1999) above: students will be expected to articulate their thinking by regularly discussing alternative strategies with the teacher, with other students, and within the context of whole-class discussion. In addition, students will be expected to articulate the strategy they used to solve a problem and explain why it works. Such normative practices support the reflection and communication that in turn encourage learning with understanding.

When the norms of practice discussed above, which encourage reflection and communication among the members of the classroom community, are present in a classroom discourse community, it becomes what I term a *productive discourse community*. Productive discourse communities, then, are those that support learning with understanding.

Developing a Productive Discourse Community

According to the NCTM *Professional Standards for Teaching Mathematics* (1991), there are several aspects of a teacher‟s role in facilitating classroom discourse. One of these aspects is provoking students‟ reasoning about mathematics through the tasks teachers implement and the questions they pose. A second is encouraging and expecting students to talk, model, and explain their reasoning. A third is monitoring and organizing students‟ participation, which includes committing to engaging every student in contributing to the class discussions. Finally, although the following facet is discussed as part of the students‟ role in discourse, it is nevertheless the teacher‟s responsibility to "promote classroom discourse in which students listen to, respond to, and question the teacher and one another" (p.45). This last facet is also found in the *Principles and Standards for School Mathematics* (2000), which suggests teachers should encourage students to "analyze and evaluate the mathematical thinking and strategies of others" (p.62). For future reference, these aspects of the teachers‟ role are illustrated in Figure 1.

1. Provoke students' reasoning about mathematics through tasks and questions
2. Encourage and expect students to talk, model, and explain their reasoning
3. Monitor and organize students' participation
4. Encourage students to analyze and evaluate the mathematical thinking and strategies of others

Figure 1. Facets of teacher‟s role in the classroom discourse.

One way the teacher can meet the responsibility of encouraging students to analyze and evaluate the mathematical thinking and strategies of others is by sharing the authority of mathematical truth with the students. By so doing, the teacher allows students to formulate their own opinions regarding the mathematical comments presented before the discourse community.

Summary

Current research in mathematics education suggests that students learn mathematics with understanding through reflection and communication with others about mathematical ideas. Reflection is facilitated both by encouraging students to share their thinking with one another and by direct questioning by the teacher and other students. The study of patterns of discourse thus becomes important. Teachers are able to affect classroom discourse by helping to establish norms for communication and by the way they themselves engage in discourse with students. This research demonstrates how the teacher in the study was able to use what I call *discourse generating tools* in order to fulfill his role in encouraging reflection and communication, thus facilitating learning with understanding.

Chapter III: Research Design and Methodology

Subjects and Research Site

This study used discourse analysis to examine the classroom instruction of the teacher mentioned in chapter 1, Mr. H[1]. He has taught for over 25 years and has received numerous teaching awards. These awards include the Utah Teacher of The Year Award as well as the Presidential Award for Excellence in Mathematics and Science Teaching for his state. He has been the mathematics specialist for his district as well as his school. During the summers, he attends and provides workshops for mathematics teaching development. He was chosen to be the subject of this discourse study due to his experience, merit, constant drive to improve his teaching, and interest in teaching with NCTM standards-based curricula.

A Geometry class was chosen because the teacher utilized the Integrated Mathematics Program (IMP) (Fendel, Resek, Alper, & Fraser, 1997), which is an NCTM Standards-based curriculum. This curriculum provided the teacher with the flexibility necessary to create a classroom environment that fostered understanding.

There were 35 students in the classroom. The students were given a parental consent form for them and their parents to sign. Of those 35 students, 32 obtained signatures and agreed to participate in the study. The students in the class were nearly equally distributed with respect to gender. Most of the students were in 10th grade, and none of them had previously taken a class from the teacher. The school serves a mostly Caucasian, fairly affluent suburban community. Parents are highly involved in their children‟s scholastic activities, and seem to be very concerned with their educational future. Although data were not collected on students‟ mathematical backgrounds, it is likely that almost all of the students came from classrooms were a traditional method of teaching was implemented.

[1] All the names used in this study are pseudonyms.

Data Collection

Qualitative data collection methods were utilized, which allowed for the acquisition of detail-rich data. These methods include classroom observations and field notes, video recordings of the teacher‟s classroom instruction, and interviews with the teacher. Some homework assignments were also collected to provide clarifying illustrative examples used in chapter 4.

At the teacher‟s request, data was collected from a particular unit beginning in the latter part of November and ending in the middle of December. The teacher felt that this specific unit would be the best unit in which to study the discourse in his classroom. The unit spanned eight 80-minute class periods. From those eight class periods, two were chosen to be fully coded because they were rich in questions, and questions were an important early focus on analysis. These two sessions were later found to contain a high concentration of examples of the tools the teacher used in generating classroom discourse. The other six periods were later examined to verify that they also contained the same tools. However, those six periods did not contain as many instances of such tools.

Observations and Field Notes

Observations of the classroom instructions were conducted and field notes were taken at the beginning of the school year. This was done to become acquainted with the teacher‟s mode of instruction and to become familiar with the types of expectations established at the beginning of the year. These observations provided an important backdrop for understanding the instructional unit that was the focus of this study. However, none of the data collected at the beginning of the year were formally analyzed. Observations of seven of the eight classroom lessons[2] were also conducted and field notes were taken as videotape data were gathered. This

[2] By way of clarification, the eighth class period was not observed because of a personal conflict with the researcher‟s schedule.

practice allowed preparation for occasional semi-structured debriefing interviews following the videotaped classroom instructions (discussed below).

Videotaping

Video recordings yielded rich data, which allowed a detailed analysis of the types of questioning techniques and other discourse tools that were employed by the teacher. The recordings also helped to see how the social norms were reinforced and supported by the teacher‟s actions. The semi-structured interviews were also videotaped in order to further analyze the teacher‟s comments.

Teacher Interviews

Three types of interviews were conducted with Mr. H: (a) at the beginning of the unit, (b) after three of the class periods, and (c) after analyzing the data from the unit for the final time. These interviews provided a context for understanding the classroom environment, its social norms, and the teacher‟s efforts in establishing and maintaining such norms. They also provided a way to verify the findings with the teacher. However, the results described in chapter 4, emerged mainly from analysis of classroom interaction rather than from these interviews.

Initial teacher interview. The initial teacher interview lasted approximately 45 minutes. Its purposes were to (a) learn which rules were established at the beginning of the school year, (b) determine what method of implementation the teacher used to develop the norms and rules, (c) determine the teacher‟s views of how these norms helped to establish a discourse community that would support student understanding, and (d) determine the teacher‟s practices on developing questions for the discussions. The questions from the interview are listed in Appendix A.

Semi-structured teacher interviews. As the data were collected during each class period there were times when the teacher did some things that were contrary to what the teacher"s customary practices were, or that were unexpected in the researcher"s opinion. I refer to these contradictions and unexpected events as discrepancies. As the discrepancies would arise, a note was made in the field notes as a reminder to discuss the unexpected events with the teacher during debriefing interviews following the class periods. There were three of these semi-structured interviews, on the first day of filming, the sixth day, and on the seventh day. The first one lasted approximately 5 minutes, the second one lasted approximately 3 minutes, and the third one lasted approximately 10 minutes.

At times teachers do things that go against the grain of what they feel or believe is effective for the progression of the students" understandings. However, this may not be a typical behavior. Some of the reasons for departing from their typical practice may be that sometimes they have outside constraints, influences and expectations, as well as personal conflicting objectives. These constraints, such as time, sometimes cause the teacher to change their course of desired action in order to accomplish a greater objective. Perhaps time constraints are nothing more than poor management of time; however, even if that is the case, all teachers must face conflict in opposing objectives. Lampert (1985) suggests that the teacher, as the autonomous figure in the classroom, must be trusted in making decisions which will best help their students to learn.

Among the data collected, there were instances where the teacher behaved in a manner inconsistent with his typical behavior. Through subsequent data collection I was able to determine that such instances were examples of conflicting goals, and that the teacher made a rational decision to meet a higher educational objective.

Exit interview. An exit interview was conducted at the end of the analysis phase of the study. This interview served as a member check to make sure the findings were in accordance with the teacher‟s views on his teaching practices. For a list of questions see Appendix B.

Data Analysis

The analytic procedures followed were generally consistent with the research tradition of *discourse analysis*. One reason that such an approach is appropriate in this study is that discourse analysis "embodies a „strong‟ social constructivist view of the social world" (Phillips & Hardy, p. 5) and is thus consistent with the view of learning and teaching taken in this study. It is also appropriate because it seeks to analyze the meanings of classroom interactions within a larger context, in this case, the normative practices of the classroom and the larger discourse of learning mathematics with understanding.

The data were analyzed in four distinct phases. The first phase was the daily informal analysis of field notes and review of teacher questions and comments and students‟ responses. Those that reinforced appropriate norms or seemed likely to lead to student reflection or understanding were noted, as were situations and teacher decisions or actions that needed clarification in a post-observation interview. On the basis of this initial analysis, six of the eight class periods were chosen to be transcribed for further analysis. The last two class periods were atypical compared to the first six in that they contained far fewer instances of verbal interaction and thus were omitted from the transcription process because they were not likely to provide rich data.

In the second phase of analysis, two of the six transcribed class periods were coded. Only two of the periods were coded because these were replete with different types of questions the teacher used, thus allowing a better analysis of the teacher‟s questions and their affects on the learning environment. Although the codes developed in this phase were a crucial aspect of the data analysis and helped in recognizing the broader discourse patterns and tools used by the teacher, these codes were not the main tools utilized in the final analysis of data and are not provided in this work.

During the third phase, as sections of coded data were examined, certain patterns became apparent in the discourse. These patterns were studied and a number of discourse generating tools were identified. These tools were used by the teacher to build and support the discourse norms of the classroom in ways that supported student reflection, communication and therefore, their understanding. In the final phase of analysis, the list of discourse generating tools were refined and all transcribed lessons were examined in order to ensure that the list of tools was representative of the teacher‟s actions in all class periods, and that the list was comprehensive. These discourse generating tools are described in detail in the next chapter.

Chapter IV: Analysis and Results

Introduction

Recall the definition of social norms provided by Wood (1998) where the teacher and students establish a system of interlocking obligations and expectations, which dictates the manner in which members of the classroom interact and provides for the smooth functioning of the class. As I analyzed the data gathered during the unit, I noticed that there were certain obligations and expectations or social norms (Cobb, Wood & Yackel, 1993) that appeared to have been established in the classroom at the time of the study. This was supported by the fact that when the teacher produced an expectation the students would comply. There were times when the students did not want to meet the expectations. However, the teacher continued to uphold the expectation until the students acted in accordance.

In the final teacher interview I asked the teacher about three specific norms and asked if he believed they were present at the time of the study (see Appendix B). This is what he said:

> I think they were becoming more present. As I recall Shadows unit was still kind of early in the year. And yet that is a big goal of mine, to do those three things. And so, I was constantly working at it, and I think we were getting there.

Thus, through my observation of the data and the teacher"s comments I was able to see that those expectations and obligations were indeed social norms present in the classroom at the time of the data collection.

The following are the social norms: all students were expected to (a) participate (b) share their reasoning when called upon, and (c) listen to, analyze, and evaluate each other"s comments. For future reference, these social norms are illustrated in Figure 2.

a. All students were expected to participate.

b. All students were expected to share their reasoning when called upon.

c. All students were expected to listen to, analyze, and evaluate each other’s comments.

Figure 2. Social norms present in the classroom at the time of the data collection.

In conjunction with these norms, the teacher used six discourse tools in helping generate and maintain a high-level of cognitive discussion in the classroom: (1) using lower-order questions to engage students, (2) persisting in eliciting students‟ reasoning, (3) encouraging as many student participations as possible, (4) encouraging students to analyze and evaluate each other‟s comments, (5) encouraging students to share as many strategies as possible and (6) using a focusing discourse pattern.

The preceding tools will be closely analyzed in this chapter. By presenting descriptions of each discourse generating tool and providing examples that illustrate how the tools play out in the discourse setting, I will demonstrate how the teacher utilized these tools to reinforce the above mentioned norms. I will also identify the way in which the norms support the discourse generating tools utilized by the teacher, thus creating an interrelation between the discourse generating tools and the classroom social norms. I illustrate this relation in Figure 3.

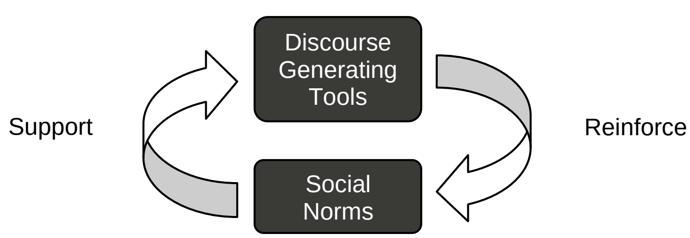


Figure 3. Interrelation of discourse generating tools and social norms.

Six Discourse Generating Tools Used by the Teacher

Discourse Generating Tool 1 - Using Lower-Order Questions to Engage Students

Most of Mr. H’s questions were higher-order questions; however, occasionally he used a lower-order question, such as a yes-no question, to facilitate the discourse. I refer to some of these lower-order questions as *engager questions* because they were usually directed toward the whole class to elicit a response from at least one student. Once a student had responded, the student or students were “engaged” and Mr. H would then follow the lower-order question with a higher-order question. The following examples demonstrate this discourse generating tool. In both of the first two examples, Mr. H was developing the idea of a counter-example:

Example 1:

Mr. H: [He began by writing “If a number is odd, then it is prime.” on the board] Mathematicians often write statements that they’re trying to consider or think about, in if-then form. They often think about, if this is true can we make this conclusion, and they write that as an if-then statement. So I’ve written a statement on the board, “If a number is odd, then it is prime.” Is that a true statement or false statement?

Students: [Some students said “true” and others said “false.”]

Mr. H: Okay, I have some of you [that] are saying it’s true, and I have some of you saying it’s false. So, we’re not necessarily convinced either way. Some of us believe one thing. Some of us believe another thing. So, how can we convince someone? So, those of you who said it’s false, how can you convince those who said it was true, that it is really a false statement? Lisa?

Example 2:

Mr. H: So, I was asking you to try and convince the other people that this statement is false. So, is this statement by itself, “2 is even and prime,” would that convince someone?

Female Student: No.

Lisa: No. I guess not.

Mr. H: Okay, why not? [He directed this question to Lisa.]

Lisa: Because, you didn‟t clarify that all odd ... that all prime numbers are odd.

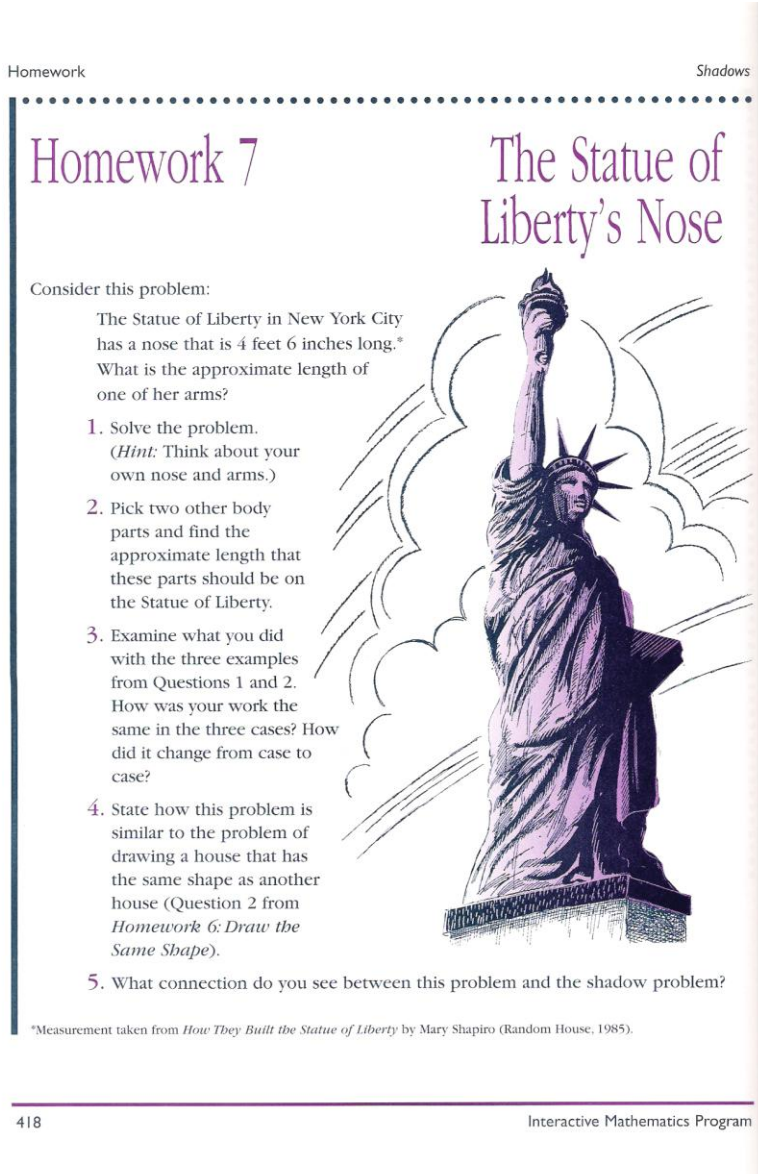
Students: [Several students gave different unintelligible comments.[3]]

Mr. H: Sean. Why not?

Notice how Mr. H asked lower-order questions in lines 6, 14-15 (abbreviated L.6, 14-15) but then followed them with higher-order questions (L.10-12, 18, 21) that elicited justification for the students‟ thinking. Also notice how once any one of the students had responded to a lower-order question, Mr. H had “engaged” the students and then proceeded to ask higher-order questions.

In the next example, the lower-order question (L.22) was not the engaging question; however it was followed by a higher-order question (L.24), which became the engaging question. The discussion for the following example is related to an IMP (Fendel et al., 1997) homework assignment (see Figure 4).

[3] All the comments labeled as unintelligible in the excerpts were unintelligible to the researcher and not necessarily to the teacher.


Homework

Shadows

Homework 7

The Statue of Liberty's Nose

Consider this problem:

The Statue of Liberty in New York City has a nose that is 4 feet 6 inches long.* What is the approximate length of one of her arms?

1. Solve the problem. (*Hint:* Think about your own nose and arms.)
2. Pick two other body parts and find the approximate length that these parts should be on the Statue of Liberty.
3. Examine what you did with the three examples from Questions 1 and 2. How was your work the same in the three cases? How did it change from case to case?
4. State how this problem is similar to the problem of drawing a house that has the same shape as another house (Question 2 from *Homework 6: Draw the Same Shape*).
5. What connection do you see between this problem and the shadow problem?

*Measurement taken from *How They Built the Statue of Liberty* by Mary Shapiro (Random House, 1985).

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Figure 4. IMP homework assignment 7 (p. 418).

Example 3:

Mr. H: Okay, so did everybody hear Sally‟s strategy?

Male Student: It‟s amazing. [Mr. H waited 3 seconds to ask the next question.]

Mr. H: So, what was different about Sally‟s strategy than Olivia and Barbara‟s?

[He waited about 2 seconds before a student answered.]

Male Student: She compared her... the two noses, where…

Mr. H: She compared the two noses, where as Olivia and Barbara compared nose to arm in their body and then used that to compare nose to arm in [the] Statue of Liberty's.

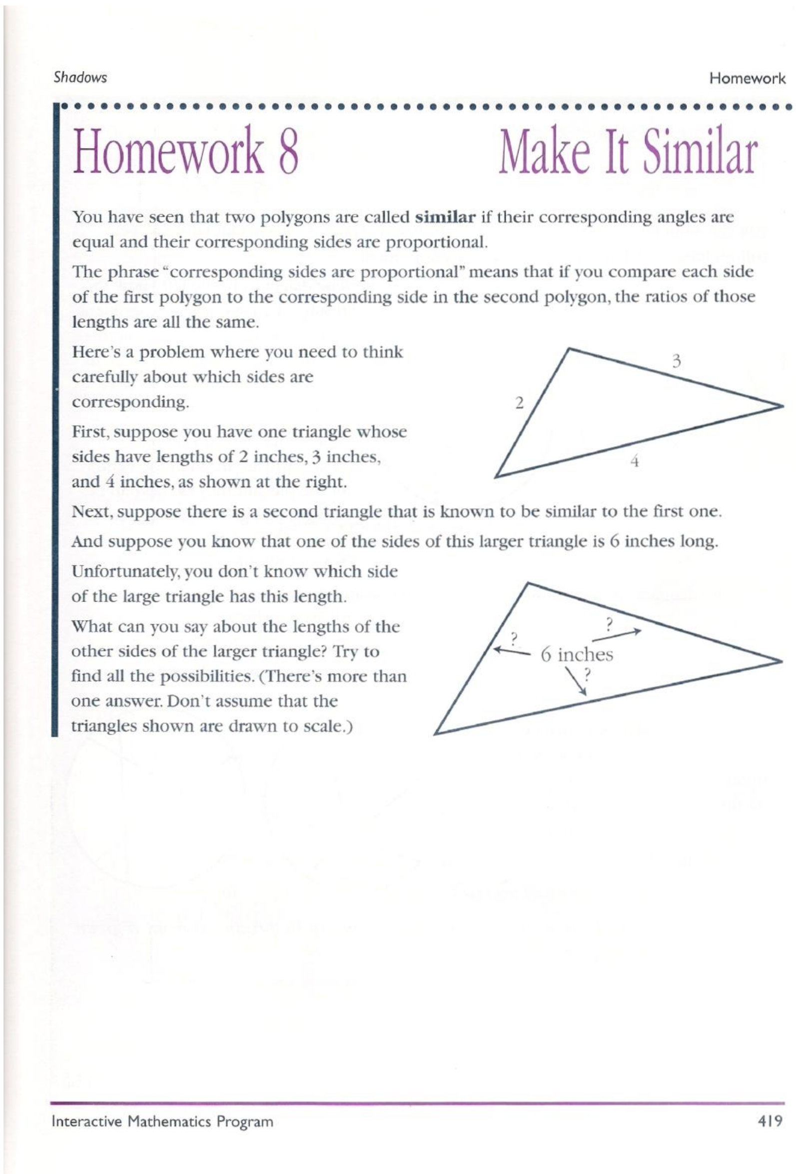
Mr. H's first questions (L.22), which was a lower order question, did not really engage the student in line 23. However, he followed that lower-order question with a higher-order question (L.24), which then engaged a student in the conversation (L.26). In all three examples, a lower-order question was followed by a higher-order question and the lower-order question was usually the engaging question that allowed Mr. H to develop the discourse in the direction he envisioned.

This tool helped to accomplish two purposes: first, it helped increase student reflection by engaging the students in the class discussion, and second, even if the contributions from the students were not the best possible, it reinforced the social norm that students were expected to participate (see Figure 2). Thus, this tool helped create an environment where learning with understanding could take place.

Discourse Generating Tool 2 – Persisting in Eliciting Students' Reasoning

Typically, when Mr. H asked questions of his students, he expected to elicit descriptions of thinking, strategies and so forth. There were times when this expectation was not met. For example, sometimes when Mr. H would question his students they would give responses that seemed to signify their unwillingness to engage in the discussion. At these times, Mr. H would not allow the students to withdraw, but would continued to ask them questions to keep them involved.

The following examples are illustrative of Mr. H's typical responses in these situations. The discussion for the next two examples is related to an IMP (Fendel et al., 1997) homework assignment (see Figure 5).



Shadows Homework

Homework 8 Make It Similar

You have seen that two polygons are called **similar** if their corresponding angles are equal and their corresponding sides are proportional.

The phrase "corresponding sides are proportional" means that if you compare each side of the first polygon to the corresponding side in the second polygon, the ratios of those lengths are all the same.

Here's a problem where you need to think carefully about which sides are corresponding.

First, suppose you have one triangle whose sides have lengths of 2 inches, 3 inches, and 4 inches, as shown at the right.

Next, suppose there is a second triangle that is known to be similar to the first one.

And suppose you know that one of the sides of this larger triangle is 6 inches long.

Unfortunately, you don't know which side of the large triangle has this length.

What can you say about the lengths of the other sides of the larger triangle? Try to find all the possibilities. (There's more than one answer. Don't assume that the triangles shown are drawn to scale.)

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Figure 5. IMP homework assignment 8 (p. 419).

Example 4:

Mr. H: Okay. Uhm, suppose we know this is 6 inches long, how could we make the enlargement? [He chose a card from a stack of cards with students‟ names.[4]] Adam, tell us what the other two sides would be. Tell us how you…

Adam: I don‟t know.

Mr. H: Okay, could you use a nose to nose strategy, or would you like to use an arm to nose strategy?

Adam: Uh, nose to nose.

Mr. H: Nose to nose. Okay, what would that mean in terms of this picture?

Adam: I don‟t know. [He laughed and other students laughed as well.]

Mr. H: What are the correct… when we say nose to nose it‟s taking us back to that metaphor that‟s looking at the Statue of Liberty‟s nose and then looking at my nose. So, we have the same corresponding things going on here. That‟s why that‟s the Statue of Liberty‟s nose... [He underlined the side labeled 6 on the enlarged triangle.] and that‟s my nose [He underlined the corresponding side on the smaller triangle]. So how would you use that nose to nose comparison to help you find the other sides?

Adam: Doubled. So...

Mr. H: Okay, that one got doubled. So, what [would] the other sides be?

Adam: Wouldn‟t they all be doubled? Like. So it‟d be 4 and then 8.

Mr. H: Okay. And that‟s a strategy people were using with the Statue of Liberty. When Olivia described her strategy, she said once she found that ratio she

[4] I discuss this practice of choosing students to answer questions, using 3 by 5 cards, in the third discourse generating tool, *Encouraging as Many Student Participations as Possible*.

could multiply by that number to increase the sides. 51

Notice when the student said “I don‟t know” (L.33), indicating his intention to withdraw from the discussion, Mr. H changed his question type to a more simple question (L.34-35) in order to engage him. Once the student was engaged (L.36), Mr. H followed with a higher-order question (L.37). Observe how even when the student tried to withdraw from the discourse a second time (L.38), Mr. H did not give up on the student, but summarized the concept the student needed (L.39-44) and continued to pursue his involvement by asking a higher-order engaging question (L. 44-45).

In the next example the student seemed to be engaged. However, she did not feel she could make a good contribution to the discussion and wanted to withdraw from the conversation as well.

Example 5:

Mr. H: Okay. What about the last one? What if this is 6 inches? [He chose a 52
card from the stack of name cards.] Rachel? 53

Rachel: I did it wrong. 54

Mr. H: Okay... could you do it right, now? 55

Rachel: Uhm... [Some students around her started giving her suggestions. Mr. H. 56
waited 9 seconds before asking the next question.] 57

Mr. H: Is there any side that would be easy for you to think about? [He waited for 58
about 20 seconds before she answers the question.] 59

Rachel: Uhm... Is the left side 4? 60

Mr. H: And how did you decide the left side would be 4? 61

Rachel: Uhm... cause the one on the bottom... the 4 on the bottom is... you just 62 add 2 to get 6, and so you must add 2 to the other sides.

Mr. H: Okay, so you added 2 to get 6, and added 2 to get 4. So, what do you think goes here? [He pointed to the third side of the triangle.]

Rachel: Five.

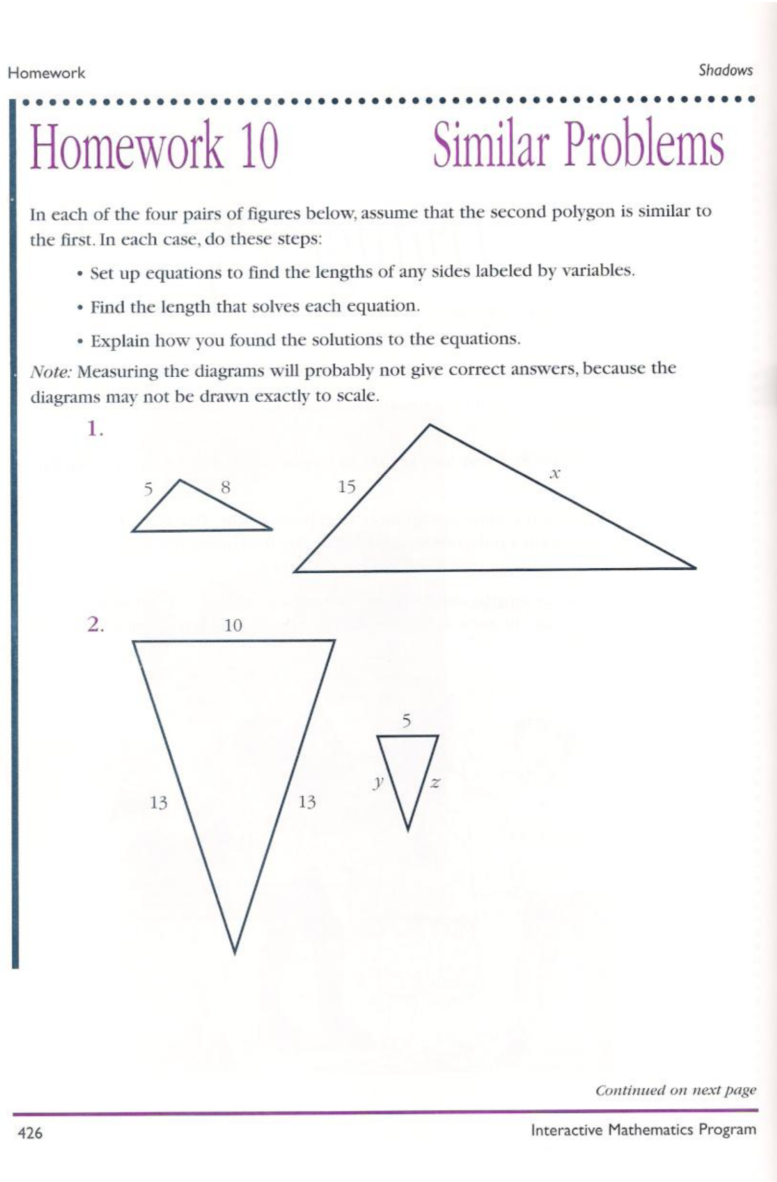
Mr. H: Okay, what do you think about that? [He directed this question to the whole class.]

In this case, the student seemed to have evaluated her own thinking and realized that it was incorrect, so she did not feel like she could add to the discussion (L.54). However, by asking if she could correctly do it now (L.55), Mr. H did not allow her to withdraw from the discussion. Also notice the amount of wait-time after his questions. In one instance, he waited approximately 20 seconds (L.58-59) before the student answered. This demonstrated the willingness of Mr. H to wait in order for his students to meet his expectation of remaining engaged in the discussion.

In both of these examples, Mr. H expected his students to not only remain engaged in the discussion, but also to contribute to the discussion. His students seemed to accept this and they both provided responses. His behavior and expectations supported the underlying social norms that each student is expected to share their reasoning when called upon (see Figure 2).

In addition to being persistent in eliciting students‟ reasoning, notice how Mr. H changed his question type to more simple questions (L.55, 58), but then followed with a higher-order question (L.61), thus, using lower order questions to engage his students as well. Also notice how the student gave an incorrect contribution by adding the same amount to all sides of the triangle to enlarge it (L.60, 62-63, 66). Mr. H did not evaluate her response, but asked her how she decided on her response (L.61) and then asked the class what they thought about her input (L.67-68). This practice of turning to the class to find out what they think about a student‟s comment is an example of the fourth discourse encouraging tool, Encouraging Students to Analyze and Evaluate Each Other‟s Comments. I will expound on this tool later in this section.

In the following example Mr. H seemed to give up on a student who was reluctant to participate in the discussion, but as we will see he actually did not. The discussion is related to an IMP (Fendel et al., 1997) homework assignment (see Figure 6).


Homework Shadows

Homework 10 Similar Problems

In each of the four pairs of figures below, assume that the second polygon is similar to the first. In each case, do these steps:

- Set up equations to find the lengths of any sides labeled by variables.
- Find the length that solves each equation.
- Explain how you found the solutions to the equations.

Note: Measuring the diagrams will probably not give correct answers, because the diagrams may not be drawn exactly to scale.

1.

2.

Continued on next page

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Figure 6. IMP homework assignment 10 (p. 426).

Example 6:

Mr. H: Did everybody... I guess no one did the arm to nose ratio, then. 69

Larry: I did a dumb one. But... 70

Mr. H: What did you do? 71

Larry: I did 15 divided by 5/8. I don‟t know why I did it. 72

Mr. H: Okay, where if... 15 divided by 5/8... Can you tell us why you did that? 73

Larry: I have no idea. It worked so... [He double-checked his work and said…] 74

Yes, it works. 75

Mr. H: Okay, that‟s interesting. So, you decided 15 divided by 5/8 would do 76

what? 77

Larry: Give you twenty-four. 78

Mr. H: Would give you the number you‟re looking for, which is 24. That‟s 79

interesting. Okay, we‟ll keep that up there. [Mr. H pointed to the place on the 80

white board where he wrote Larry‟s contribution.] 81

Notice how Mr. H tried to elicit Larry‟s reasoning on lines 71, 73 and on lines 76-77, but Larry could not justify his reasoning (L.72,74). At that point, Mr. H decided to continue to the next question. However, he made the choice to leave Larry‟s work up on the board (L.80,81). Also, notice how Larry seemed to accept the social norm that students were expected to share their reasoning when called upon (see Figure 2) when he said, "I did 15 divided by 5/8. I don‟t know why I did it" in line 72. Mr. H had not asked him to provide his reasoning. However, Larry felt that even if he could not provide his reasoning, he had to explain that he did not have a justification.

At that point, they moved on to discuss another problem. While they were discussing the other problem, Larry raised his hand to make another contribution. However, Mr. H did not get to him until after about a minute or so (L.82).

Example 6 (continued):

Mr. H: Okay, number 3. Oh, Larry you had something you wanted to say. 82

Larry: Oh... doesn‟t matter. 83

Mr. H: Go ahead and say it.

Larry: Uh... 5‟s divided by 10/13.

Mr. H: So, you‟re taking... You did the same kind of thing up here? [He pointed to the place on the white board where Larry‟s contribution was written, from a few minutes earlier.]

Larry: [Nodded in affirmation.]

Mr. H: Okay. So you‟re taking the side we know over here and dividing it by a ratio formed by both sides. [On the board he wrote "$5 \quad \frac{10}{13}$"] And that gave you "y." [He then wrote " y" next to "$5 \quad \frac{10}{13}$" to get "$5 \quad \frac{10}{13} \quad y$"]

And how did you think about doing that?

Larry: Uh, me?

Mr. H: Yeah.

Larry: What did you say? [Some students laughed].

Mr. H: Why are you doing that? Where is that coming from?

Larry: I don‟t know. Just... I made it up.

Mr. H: Why does it... Why does it work?

Larry: Uhm ... I don‟t know. Well... I originally... uhm... never mind.

Notice how Larry signaled to withdraw by saying that his intended comment did not matter (L.83). Mr. H did not allow him to withdraw by requesting him to share his comment (L.84). Once Larry shared his comment, Mr. H realized that his contribution was similar to the one he made several minutes earlier (L.86-88). He then proceeded to re-voice what Larry had shared in an attempt to help him clarify his thinking (L.90-93), and asked for his reasoning once again (L.94). Larry did not seem to be paying attention, possibly attempting to signal his withdrawal from the discussion (L.95,97). However, Mr. H did not allow him to withdraw from the conversation (L.96) and restated the question is several ways to maintain Larry‟s engagement (L.98,100). Larry gave a final attempt to express his reasoning but decided not to continue (L.101).

At that point there was a distinct tension in the classroom, similar to the tension portrayed by Bauersfeld (1998) in describing a funneling questioning pattern. In such a funneling pattern, Bauersfeld (1998) suggests:

> Continued deviant answering on the student‟s side meets on the teacher‟s side a growing concentration on the stimulation of the "adequate" answer through more precise, that is, narrower, questions. Thus the standard for "adequateness" deteriorates, the quality of the discussion decreases. (p. 36)

Note how Mr. H had been relentless in seeking Larry‟ logic to no avail, thus creating that tension between the teacher and student. However, Mr. H did not revert to narrower questioning, which could have decreased the quality of the discussion. Instead, realizing that Larry was not providing the mathematical stimulation to continue the discussion, he decided to turn the question to the whole class (L.102). This pattern, where the teacher uses the students‟ comments to direct the discourse is discussed in greater detail in the sixth discourse generating tool, Using a Focusing Discourse Pattern.

Let us now continue to look at example 6 to see what the teacher did to direct the discourse.

Example 6 (continued):

Mr. H: Ah. I think it‟s interesting. You guys see a reason why it works? [He waited about 9 seconds before Sally responded].

Sally: I think so. Cause you‟re just doing 5 times 13 over 10, which is the original... [She trailed off and said something unintelligible] So... you just...

Mr. H: Now you said something different. You said you‟re just doing 5 times 13 over 10. [On the board he wrote “ 5 $\frac{13}{10}$ ”] Where did that come from?

Sally: Well. When you divide the fractions, you can invert it, right?

Mr. H: Oh, so you just took his division problem, and turned it into a multiplication problem? Okay, I see. Does everybody see that she‟s rewritten Larry‟ 5 divided by 10/13 as a multiplication problem, 5 times 13/10?

Larry: Oh yeah cause you switch the 13/10 when you divide right? Isn‟t that the same as multiplying...?

Male Student: Yeah, pretty much.

Mr. H: Okay, that‟s what you were told in elementary school, to invert and multiply [to] divide by fractions?

Larry: Uh... I didn‟t learn that in elementary school. [Some students laughed].

Male Student: Yeah, I didn‟t either.

Larry: I learned that last year.

Mr. H: Oh. [Mr. H and some students laughed.] Somewhere somebody taught

you that. So why 5 times 13/10? Why does that make sense?
Sally: Cause 13/10 is the ratio. The first one [She said something unintelligible and some students laughed.].
Mr. H: Oh, okay. So we‟re back to this arm to nose ratio over here. Thirteen tenths is the ratio of these. And we‟re trying to make sure that we get that same ratio over here. Okay, interesting.

After asking the whole class, notice how he waited about 9 seconds (L.102-103) before another student answered the question. Thus Mr. H demonstrated his willingness to allow the students to reason and his persistence in eliciting the students‟ thinking. When a student finally gave a possible explanation to the response, Mr. H then had a student engaged and the discussion was able to continue. And even though Larry was not able to provide the substance for the discussion, originally, he was able to join in the conversation (L.115-116) after the other student gave her explanation. Thus Larry also remained engaged.

In the end Mr. H was able to connect Sally‟s contribution to what they were talking about, and brought resolution to the tension build-up between teacher and students. He was finally resigned to the fact that Larry was not able to provide the mathematical substance to generate discourse enough to deepen the students understanding. However, he did not give-up on the concept or idea. He continued by inquiring of the whole class, until they were able to discuss it in more detail.

We can see how his persistence helped students, who would otherwise withdraw, to remain engaged and to make contributions to the discussion. The use of this tool also promoted student reflection of the mathematical concepts the class was discussing and helped the students to communicate their mathematical ideas. Thus, the teacher was able to create an environment that reinforced not only the social norm that students were expected to participate, but also the social norm that students were expected to share their reasoning when called upon (see Figure 2).

Discourse Generating Tool 3 – Encouraging as Many Student Participations as Possible

When asking a question to provide material for the discourse, Mr. H would often ask the question to the whole class. Very rarely did he pick a student to respond before he had asked the question. Occasionally, when he picked a student to respond, he did so by picking their name from a pile of 3 by 5 cards. At the beginning of the school year, during the first week, Mr. H handed out 3 by 5 cards for the students to write their names and other information about themselves. He then used those cards to pick a student to respond to questions during specific situations. He also used the cards as a tool to generate discourse through students‟ contributions even from those who would not otherwise share their reasoning. However, in the final interview I learned that he would not use the cards unless the students had had a chance to prepare a reasonable response (see Appendix B).

Through this tool, Mr. H was able to increase student involvement by first asking the question and then drawing a card from the pile of cards and calling on that student. According to Cangelosi‟s (1993) suggestions on questioning sessions, teachers should "avoid directing a question to a particular student before articulating the question," (p. 174) because students may not listen to the question if they know it is not directed towards them. Mr. H followed this pattern in Example 4 (L.31-32) and in Example 5 (L.52-53).

The following example is also indicative of Mr. H‟s attempt to involve as many students as possible in the thought process of each question.

Example 7:

Mr. H: Okay. I‟d like to hear some of your strategies for doing the homework, or for doing the quiz if you didn‟t do the homework. So, question number one. If the Statue of Liberty‟s nose is 4 feet 6 inches long, how long, approximately, is one of her arms? And how did you... uh... what was your strategy for thinking about that? Uh... [He chose a card from the stack of name cards.] Olivia?

Mr. H tried to engage as many students as possible by asking a question of the whole class, and then picking a name card (L.134-135). In lines 130-131, we can also see that the students had had a chance to work on the homework or the quiz, thus Mr. H used the cards in a non-threatening way. In conjunction with Mr. H‟s persistence in eliciting students‟ thinking, these cards became very powerful in generating material for the discourse.

Once again, by encouraging as many student participations as possible, Mr. H was able to promote student reflection and communication of their ideas. It is also interesting to note that, when called upon, the students did not seem to mind. That behavior seemed to indicate that, at that point, the expectations to participate and to share their reasoning, when called upon, had been well entrenched in the classroom community (see Figure 2).

Discourse Generating Tool 4 - Encouraging Students to Analyze and Evaluate Each Other’s Comments

Recall Facet 4 of the teacher‟s role in the classroom discourse (see Figure 1): Encourage students to analyze and evaluate the mathematical thinking and strategies of others. One of the ways in which Mr. H encouraged this standard was by not evaluating students‟ responses. Instead, he would turn to the class and asked what they thought about the responses. This practice helped students to create a deeper understanding of mathematics because it did not shut down the students‟ thinking, but encouraged them to think about each other‟s contributions, particularly when those contributions were incorrect. For instance, in Example 5, the student gave an answer that suggested incorrect thinking, so Mr. H asked the whole class “Okay, what do you think about that?” (L.67-68). Notice that this is not the typical IRE pattern used in many classrooms (Mehan, 1979), because Mr. H did not provide evaluation. Instead, by turning back the responsibility of evaluating each other‟s comments, Mr. H allowed students to analyze each other‟s thinking while reflecting on their own reasoning. This tool also allowed the students to develop deeper understandings, thus enhancing their mathematical learning.

On the other hand, at times a student provided great insight on a problem. Mr. H sometimes gave an evaluation of these contributions by giving praise to the student. Previous to the following example the class had been discussing the concept of enlarging a geometric shape by finding a ratio and multiplying the sides by that ratio. They had also established the fact that they could not add the same amount to all the sides to end up with a proportional shape. Sally, however, found a way to add some amounts to the sides and still keep the resulting shape proportional. The discussion came from an IMP (Fendel et al., 1997) homework assignment (see Figure 6).

Example 8:

Mr. H: Oh, I didn‟t ask. Did anybody do any other strategy back here? [Sally raised her hand.] Sally?

Sally: I think I used Melissa‟s but I also did it a little bit different.

Mr. H: Okay. What else could you do?

Sally: Well. It is not much harder. It‟s just, uhm ... Yeah. Five is 5/10 of 10.

So...

Mr. H: Okay.

Sally: ... 5/10 of 10... Oh no. I did it uhm... an arm to nose one. When... uhm... 13 is 3 times 4 plus and 3 times the 5 is [Unintelligible].

Mr. H: Come write it another time. I‟m not sure that... [He signaled to Sally to come up to the board.]

Larry: You must do 5 divided by 10/13.

Sally: I wrote it really long, but it‟s actually correct.

Mr. H: Okay, you want to tell us about what that is?

Sally: Yeah. Uh... basically you just, since you just add 3 to 10 to get 13. Uhm... 3/10 of 10 is... so it‟s 3/10 of 10 that you‟re adding, so 3/10 of 5 is three 5‟s. So you‟re just adding... [Unintelligible.].

Mr. H: Okay. Did everybody... The other day we talked about, can we add the same amount to both pieces? But Sally didn‟t add the same amount to both pieces. But she did add something to both pieces to get the enlargement. [Turning to the whole class he asked...] What [do] you think about that? [He waited about 5 seconds before a student answered.].

Male Student: That‟s a lot harder than the other way.

Mr. H: Okay. It‟s harder but I think it brings out some interesting things to think about. That Sally is recognizing how much she‟s scaled up. She said she was doing an arm to nose ratio. She‟s looking at the same pieces... two pieces in the same figure, and saying that this is a scaled up version of this. And then scaling that up by the same proportion.

I think that‟s an interesting strategy. Thank you for sharing that. That... That‟s really interesting, because it brings up some things that maybe will help us. For those who thought they could just add the same amount, we‟re not adding the same thing here, and here. But we‟re adding the same proportional amount in both places.

Sally gave a very interesting solution to the problem, because even though they had established the fact that they could not add the same amount to all the sides to enlarge the shape, she found out how to add a proportional amount to all sides and still keep the same shape. In that case Mr. H gave positive feedback or praise to the student (L.164-166) for sharing an innovative strategy. Notice though, how he praised Sally‟s contribution rather than her as a person and made the praise as specific as possible (L.153-156, 159-166).

This type of praise is exactly the kind of praise Kohn (1999) suggested as most appropriate: “Not only should we focus on the act or product, but we should do so by calling attention to the specific aspects that strike us as especially innovative or otherwise worthy of notice” (p.108-109). Finally, notice how Mr. H gave praise only after he had asked the students what they thought about the strategy (L.156-157). Also, the praise he provided was not a direct evaluation of the correctness of the solution. Instead he commended the student for the innovative aspect of her solution. Thus, he allowed the students to continue thinking about the correctness of the solution, and their own ideas about the problem.

Other instances where Mr. H had his students analyze each other‟s responses include Example 3 (L.22, 24) and Example 6 (L.102), where he had them figure out how a student got a particular solution. We can see again that by using this tool, Mr. H encouraged his students to reflect upon the mathematical ideas that emerged from other students‟ comments, as well as reinforcing the expectation not only to listen to each other‟s comments, but also to evaluate those comments (see Figure 2).

Discourse Generating Tool 5 – Encouraging Students to Share as Many Strategies as Possible

Under the *Problem Solving* standard of the NCTM *Principles & Standards for School Mathematics* (2000-2004) we find the suggestion that students should "apply and adapt a variety of appropriate strategies to solve problems." This standard suggests that

> Students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. These strategies need instructional attention if students are to learn them. However, exposure to problem-solving strategies should be embedded across the curriculum. Students also need to learn to monitor and adjust the strategies they are using as they solve a problem. (¶ 4)

It is crucial that the teacher creates an environment where students are encouraged to "explore, take risks, share failures and successes, and question one another" (NCTM, 2000-2004, ¶ 5). Only through such an environment can "students develop the confidence they need to explore problems and the ability to make adjustments in their problem-solving strategies" (NCTM, 2000-2004, ¶ 5).

Mr. H often encouraged his students to share different strategies. In Example 8, when Mr. H said "Oh, I didn‟t ask. Did anybody... do any other strategy back here?" (L.136) he showed his commitment to developing problem solving skills within his students by having them share as many strategies as possible. The following questions also indicate such a commitment: "Did anybody do a strategy different than Olivia?" (L.262-263), "Okay, did anybody do it any differently?", and "What was your strategy for thinking about that?"[5] At one point, he even explicitly asked his students to share as many strategies as possible:

Example 9:

Mr. H: Okay, I‟m going to call on some of you to come up, and [I‟d] like you to share your strategies for thinking about... Ultimately, I‟d like the rest of you to think about, if you have any other strategy for thinking about the problem. I‟d like to see if we can get as many different strategies up here for how you are approaching [unintelligible]... as possible. So, if you have a different way, even if you‟re not really quite sure if it‟s different or if it looks just a little bit different than what they did. Uh... maybe you ought to be willing to share it.

It is interesting to observe how Mr. H and his students interacted in the classroom. He seemed to have established an environment as the one described above, where students are able to explore mathematical concepts, take risks by sharing different strategies, and share failures and successes in the active process of learning mathematics. And the students seemed to take those risks willingly.

Part of what shaped these social norms within the classroom was this tool, of encouraging students to share as many strategies as possible, along with the tools spoken of above: his persistence in seeking his students‟ thinking and his expectation of them analyzing and evaluating each other‟s comments. Thus, we can see how the "interlocking system of obligations and expectations, established by both the teacher and the students and underlying the manner in

[5] The last two quoted lines were not taken from any of the excerpts in this report, but came from the transcribed data.

which members of the classroom interact, forms the smooth functioning of the class" (Wood, 1998, p. 175).

Once more, notice how this tool promoted reflection and communication by encouraging students to share as many strategies as possible, as well as reinforcing the norms where students were expected to participate and to share their reasoning when called upon (see Figure 2).

Discourse Generating Tool 6 – Using a Focusing Discourse Pattern

Recall how Herbel-Eisenmann and Breyfogle (2005) distinguished between the funneling and the focusing patterns of discourse. In Example 1, we can see how Mr. H allowed the students' responses (L.7) to guide his next comments and question (L.8-12). Below I consider the whole episode[6] to see how Mr. H focused the discussion by helping students articulate and clarify their thinking. In this discussion Mr. H was developing the idea of a counter example.

Example 10:

Mr. H: [He began by writing "If a number is odd, then it is prime." on the board] Mathematicians often write statements that they're trying to consider or think about, in if-then form. They often think about, if this is true can we make this conclusion, and they write that as an if-then statement. So I've written a statement on the board, "If a number is odd, then it is prime." Is that a true statement or false statement?

Students: [Some students said "true" and others said "false."]

Mr. H: Okay, I have some of you [that] are saying it's true, and I have some of you saying it's false. So, we're not necessarily convinced either way. Some of us believe one thing. Some of us believe another thing. So, how can we

[6] Although I have already used part of this episode in Example 1 and Example 2, the whole episode provides a clear example of the focusing pattern.

convince someone? So, those of you who said it‟s false, how can you convince those who said it was true, that it is really a false statement? Lisa?

Lisa: Cause 2 is an even number and it‟s prime, and 25 is an odd number and it‟s not.

Mr. H: Okay, so you said 2 is even and prime. And, then what was your other statement? [At that point Lisa was distracted by someone, so she did not immediately respond to Mr. H.] Lisa, what was your other statement? Twenty-five...

Lisa: And 25 is an odd number and it‟s... and it‟s, and it‟s not prime.

Mr. H: [He wrote the statements on the board and continued.] Okay, so Lisa gave me two statements. Let‟s look at them one at a time. Lisa said “2 is even and prime.” Did that help me to know if this statement is false? [Referring to the first statement he wrote on the board.]

Male Student: Sort of.

Mr. H: So, I was asking you to try and convince the other people that this statement is false. So, is this statement by itself, “2 is even and prime,” would that convince someone?

Female Student: No.

Lisa: No. I guess not.

Mr. H: Okay. Why not? [He directed this question to Lisa.]

Lisa: Because, you didn‟t clarify that all odd... that all prime numbers are odd.

Students: [Several students gave different unintelligible comments.]

Mr. H: Sean, why not?

Sean: [Sean said something unintelligible.]

Mr. H: Okay, 2 is prime, and 2 is even. That‟s definitely a true statement. Two is even. Two is prime. Does it prove this is a false statement? Since that‟s what we‟re trying to convince some people. [Bruno raised his hand.] Bruno?

Bruno: Well... you[„re] just saying... like 2 is even and prime, you just said… if the number is odd it is a prime. You didn‟t say if the odd number was a prime. Oh…

Mr. H: Alright. So, that really doesn‟t do it.

Bruno: Yeah.

Mr. H: This statement doesn‟t make a statement about even numbers not being prime. It just makes the statement about odd numbers being prime.

First, Mr. H decided to have those who claimed that the statement, “If a number is odd, then it is prime” was false convince those that said it was true (L.187-188). He could have asked for those who said it was true to convince those who said it was false; however, he seemed to have made this decision to help the discussion focus on the concept that would help the discussion along. In order to convince everyone that the statement was false, Lisa gave two declarations (L.189-190). Mr. H seemed to realize that the first declaration did not disprove the original statement. However, he decided not to evaluate her contribution and instead he allowed the rest of the students to analyze and evaluate her comments (L.196-199). Eventually, through his focused comments and questions (L.201-203, 206, 209, 211-213) they were able to come to the realization that Lisa‟s first statement did not disprove the original statement (L.218-221).

He then continued analyzing Lisa‟s second statement by asking the whole class if it proved that the original statement was false (L.222-223).

Example 10 (continued):

Mr. H: What about this, “25 is odd but is not prime.” Does that prove that this is a false statement?

Adam: No.

Male Student: Yes.

Adam: NO!

Students: [Other students said “YES!”]

Adam: That only proves that one number is not prime.

Male Student: But… [He and Mr. H spoke at the same time; however Mr. H spoke louder so I could not hear the Male Student.]

Mr. H: So is one number enough to disprove this statement?

Students: [Several students said yes.]

Adam: No.

Mr. H: Adam doesn‟t think one number would be enough to disprove it. [He had his hand by the statement “25 is odd is not prime,” and he turned to it as if waiting for something.]

Male Student: You could have said if a number is prime, then it‟s always prime. Then... that would‟ve… [He trailed off as he finished the sentence.]

Mr. H: Okay, mathematicians when they say this... [He underlined the “If “ and the “then” in “If a number is odd, then it is prime”.] they‟re implying the always. That this is a universal statement, that if a number is odd...

Adam: Well, I didn‟t know that.

Mr. H: ...then it is prime. Universal, pick an odd number, it will be a prime number. Is that true or false?

Male Student: That‟s false.

Mr. H: That‟s false. Is 25, is odd but not prime. Is 25 a good proof that it‟s not a true statement?

Students: [Several students said “yes.”]

Mr. H: Okay, when we can find something that satisfies this [he circled the phrase “a number is odd”] is an odd number, and shows that this is not true [he double underlined “is prime”], we call that a counter example. [He wrote “Counter Example” on the board.] And one single counter example is enough to disprove things, because it‟s suppose to be a universal statement. So, if I can find one example that‟s counter to the statement, a counter example, it disproves it. Now, for Adam, proving it may take a lot more. And we‟ll talk about proving things later. For right now we are going to try to disprove statements.

Instead of telling the students that one counter example was enough to disprove a statement, or even using a funneling pattern to get to that realization, Mr. H began by asking a question to focus the remainder of the discussion (L.222-223). When the question was met by a split response, Mr. H allowed the disagreement to continue. Once Adam said “That only proves that one number is not prime” (L.228) Mr. H decided to focus the discussion on Adam‟s comment by asking if one number was enough to disprove the statement (L.231). Adam was still convinced that one number was not enough to disprove the original statement (L.233), so Mr. H restated that fact as if hoping that someone would address that disagreement (L.234-236). A student finally said that they had to include an "always" in the statement (L.237-238). At that point Mr. H decided to discuss a mathematical convention on the word "Always" (L.239-241, 243-244) and the original statement (congruent with Hiebert et al.‟s (1997) idea of providing relevant information as the role of the teacher), after which he asked again whether the original question was true or false (L.244, 246-247). Several students, now convinced, responded in the affirmative (L.248) and he ended by summarizing the whole discussion (L.249-257).

Herbel-Eisenmann and Breyfogle (2005) noted that a focusing "type of interaction values student thinking and encourages students to contribute in the classroom" (p. 486). In the last episode, several students were able to make contributions to the discussion, and even though not all were correct or helpful to the discussion, Mr. H allowed those comments to surface in order to be discussed.

This discourse generating tool, of Using a Focusing Discourse Pattern, promoted student reflection in two ways: first, it encouraged students to reflect by helping them focus their thinking and their comments, and second, it promoted student reflection by allowing the students to share the authority for mathematical truth. This tool also promoted student communication by drawing students‟ comments through the teacher‟s skillful questioning techniques. Finally, this tool reinforced the social norms where students were expected to (a) participate (b) share their reasoning when called upon, and (c) listen to, analyze, and evaluate each other‟s comments (see Figure 2). Students were encouraged to participate, share their reasoning, and listen to, analyze, and evaluate each other‟s comments through the way in which he asked the questions. Thus, we can see how Mr. H allowed the students to continue thinking, exploring and comparing their reasoning with the rest of the contributions made in the discourse.

How the Teacher Brought It All Together

Novice teachers sometimes have difficulty focusing or working on several aspects of teaching at the same time. However, for a veteran teacher such as Mr. H, applying multiple discourse generating tools seemed to have become second nature. As I pointed out earlier, in Example 4 Mr. H was persistent in eliciting student‟s reasoning (Discourse Generating Tool 2) by using lower-order questions to engage students (Discourse Generating Tool 1) (L.34-35) and then following up with higher-order questions (L.37, 44-45).

In Example 8, not only did he encourage students to analyze and evaluate each other‟s comments (Discourse Generating Tool 4) (L.156-157), but he also began by encouraging students to share as many strategies as possible (Discourse Generating Tool 5) using lower-order questions to engage students (Discourse Generating Tool 1) (L.136). Once a student was engaged in the discussion he followed with higher-order questions (L.139) and a request to clarify the strategy (L.149), which is a form of persisting in eliciting students‟ reasoning (Discourse Generating Tool 2). Finally, he used a focusing discourse pattern (Discourse Generating Tool 6) by asking the student to go up to the board and write down what she was saying (L.145-146) in order to clarify her comments to all. Notice how he continued to focus the discussion by asking her to explain it to everyone (L.149). Then he made a connection with what they had learned previously about proportions (L.153-156) and ended with a summary of the student‟s contribution (L.159-168).

This final example includes Example 3; I have reproduced a larger part of the episode here because it is a great example of how Mr. H utilized multiple tools in conjunction. The discussion comes from the IMP (Fendel et al., 1997) homework assignment about the Statue of Liberty (see Figure 4).

Example 11:

Mr. H: Okay. So when we‟re working with ratios we‟re comparing two things. And both Olivia and Barbara have used the strategy where they have compared the length of their nose to the length of their arm to get a number, to help them use that number, that ratio, to figure out how long the Statue of Liberty‟s nose is. Did anybody do a strategy different than Olivia and...? [Several students raised their hand.] Okay, Sally what did you do?

Sally: Well I compared my nose to the Statue of Liberty‟s nose and then just used that comparison to do this.

Mr. H: Okay, so did everybody hear Sally‟s strategy?

Male Student: It‟s amazing. [Mr. H waited 3 seconds to ask the next question.]

Mr. H: So, what was different about Sally‟s strategy than Olivia and Barbara‟s? [He waited about 2 seconds before a student answered.]

Male Student: She compared her... the two noses, where…

Mr. H: She compared the two noses, where as Olivia and Barbara compared nose to arm in their body and then used that to compare nose to arm in [the] Statue of Liberty‟s. [He pointed to a student in front of the class].

Male Student: I took the nose off of the Statue of Liberty and then... and then put it up to the arm.

Mr. H: To… ?

Male Student: To the picture.

Mr. H: So, you compared the nose of the Statue of Liberty to the arm on the

Statue of Liberty‟s picture. So, he‟s doing ... uh ... Is that similar to Sally‟s or similar to Olivia‟s?

Male Student: To Olivia‟s.

Mr. H: And how is it similar to Olivia‟s?

Male Student: Cause you‟re cal... you‟re measuring the nose comparing it to the arm.

Mr. H: Okay, so basically we have had two different strategies that have been described. One strategy has been to [use] what I‟m going to call a nose to nose ratio. Some people took the nose of the Statue of Liberty and compared that to their nose. So, that was one of the strategies. I‟m going to find out how my nose compares to the Statue of Liberty‟s nose, and get a ratio, and then make sure that that ratio holds true for the arms also. The other strategy was to do what I‟m going to call an arm to nose ratio. Look at my arm, and compare it to my nose. And then make sure that the ratio of the Statue of Liberty is also the same when we compare the Statue of Liberty‟s arm to the Statue of Liberty‟s nose. So, we have two different ways we can be doing this kind of process of comparing these two figures. We can be working with objects that measurements come from both objects and comparing the same parts, nose to nose, or we can be looking at measurements that come from the same object, my arm to my nose and comparing them, those same parts, arm to nose on the Statue of Liberty. So those are two different ways that we work with ratios, and we‟ll talk about that a little bit in a minute.

We can see how he used lower-order questions to engage students (Discourse Generating Tool 1) in lines 262-263, 267 and 280-281. He then followed those with higher-order questions on lines 263-264, 269, and 283, respectively. Mr. H used the lower-order questions to engage at least one of the students in order to commence the discussions or to collect the object of discussion. However, he always followed with higher-order questions which helped the students to reflect upon the mathematical concepts more in depth, thus promoting learning mathematics with understanding.

We see him encouraging students to analyze and evaluate each other‟s comments (Discourse Generating Tool 4) in lines 267, 269. In line 280 he was about to analyze the student‟s comment when he said "So, he‟s doing...", but caught himself and asked the students to analyze it instead (L.280-281, 283). Notice how some of these requests to analyze each other‟s comments are the same engaging questions discussed above. By allowing his students to analyze each other‟s comments and evaluate them he helped them to respect each other and yet be critical of all the participants‟ contributions. This practice also helped the students to juxtapose their own reasoning with everyone‟s thinking, which is beneficial in developing more connections and a deeper understanding of the mathematical content.

Notice how even after he had obtained strategies from two other students (L.259), Mr. H continued to encourage students to share as many strategies as possible (Discourse Generating Tool 5) in lines 262-264. He sought out additional strategies because, through such a practice, he was able to help students "develop the confidence they need[ed] to explore problems and the ability to make adjustments in their problem-solving strategies" (NCTM, 2000-2004, ¶ 5), as well as helped them to build more mathematical connections and in turn a broader understanding of the mathematical concepts.

Finally, observe that by the simple fact of seeking additional strategies (L.262-264), his questioning pattern was not one of funneling. Instead, Mr. H utilized a combination of lower-order and higher-order questions (L.267, 269, 280-281, 283) to use a focusing discourse pattern (Discourse Generating Tool 6) in order to allow access to the problems for his students. He also provided restatements (L.272-274, 279-280) and a summary (L.286-302) to encourage a focus on the important aspects of the conversations as well as to help clarify comments made by the students.

Summary

Through analyzing the data I found that Mr. H used six discourse generating tools. These tools reinforced the social norms (see Figure 2) present at the time of the data collection, thus facilitating reflection and communication of mathematical ideas.

More specifically, by using the Discourse Generating Tool 1: Using lower-order questions to engage students, Mr. H was able to help increase reflection by engaging the students in the class discussion. This tool also helped reinforce the norm that students were expected to participate. Through using the Discourse Generating Tool 2: Persisting in eliciting students‟ reasoning, Mr. H promoted reflection and communication of their ideas by being persistent through questioning. This tool also reinforced the expectations of participation and sharing their reasoning when called upon.

With the Discourse Generating Tool 3: Encouraging as many student participations as possible, Mr. H promoted reflection and communication of mathematical ideas by allowing many students, at once, to think about a question. This tool also reinforced the norms of participation and sharing reasoning when called upon. By using the Discourse Generating Tool 4: Encouraging students to analyze and evaluate each other‟s comments, he encouraged reflection of mathematical ideas of other students" comments. Through this tool Mr. H was also able to reinforce the norm where students were expected to listen to, analyze, and evaluate each other"s comments.

Mr. H was able to promote reflection and communication by using the Discourse Generating Tool 5: Encouraging students to share as many strategies as possible. He was also able to reinforce the norm of participation and sharing reasoning when called upon. Finally, by using the Discourse Generating tool 6: Using a focusing discourse pattern, Mr. H was able to encourage reflection in two ways: directly, by helping students focus their thinking and indirectly, by sharing the authority of mathematical truth with his students. This tool also reinforced all three norms found in Figure 2.

Thus, we can see how the teacher"s actions in using the six discourse generating tools reinforced the norms, that were present at the time of the data collection that all students were expected to (a) participate (b) share their reasoning when called upon, and (c) listen to, analyze, and evaluate each other"s comments. However, we can also see that these norms also support the discourse generating tools utilized by the teacher, thus creating the interrelation between the discourse tools and the classroom social norms (see Figure 3).

Chapter V: Conclusions

Answer to Research Question

In response to the research question: *In what ways does the teacher in the study direct mathematical discourse in order to facilitate understanding?*, I found that the teacher promoted reflection and communication of mathematical ideas. He did this through the use of the following six discourse generating tools: (1) using lower-order questions to engage students, (2) persisting in eliciting students" reasoning, (3) encouraging as many student participations as possible, (4) encouraging students to analyze and evaluate each other"s comments, (5) encouraging students to share as many strategies as possible and (6) using a focusing discourse pattern. The teacher"s use of these tools helped to reinforce the norms that the members of the class had established from the beginning of the school year. The norms that were present at the time of the data collection are the following: all students were expected to (a) participate (b) share their reasoning when called upon, and (c) listen to, analyze, and evaluate each other"s comments.

Through the promotion of reflection and communication of mathematical ideas, as well as through the reinforcement of social norms the teacher was able to create the type of environment or productive discourse community, which promoted learning with understanding. Thus we can see that the discourse generating tools and the social norms discussed above create an interrelationship or a cycle where both components supported each other.

Limitations & Suggestions for Future Research

The contribution of this study does not lie in showing how students learn with understanding. This is a study of a teacher"s classroom discourse, and how he developed a productive discourse community. Also, I am not making the claim that all teachers could teach in the way portrayed in this study. In fact, in a similar study to this one, Nelson (1997) showed the difficulties a teacher had in implementing a reformed way of teaching. However, the teacher in this study is an unusually good teacher, as his credentials show (see chapter 3). Therefore, another teacher may not be able to use the tools found is this study successfully because it takes great skill to use more than one tool at the same time.

Another limitation was that the length of the period when the data was collected was only a snapshot of the whole school year, as well as a snapshot of the whole career of the teacher. So there may be other factors that are involved in creating a productive discourse community which this study did not capture.

A suggestion for future research is to gather data from the whole school year to look at both the teacher and students‟ expectations in conjunction with the discourse generating tools. This further analysis could help examine how the social norms are established and maintained. One can see how all three aspects of the discourse community would affect the social norms, and visa versa. One way this interrelation, between the discourse generating tools used by the teacher, the teacher‟s rules and expectations, the students‟ expectations, and the social norms can be modeled is shown in Figure 7.

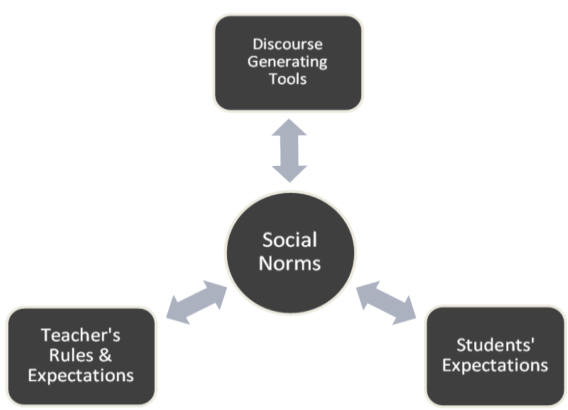

Figure 7. Interrelation between social norms, discourse generating tools used by the teacher, teacher‟s rules and expectations and students‟ expectations in a classroom.

As we have seen in this study, the social norms are well connected with the tools the teacher used. Thus, it would also be beneficial to look at how the social norms are developed, from the beginning of the school year, in order to better understand the discourse generating tools that the teacher used. It would also be interesting to study how the discourse generating tools affect student understanding. Thus, putting measures that help us see student understanding might also be helpful.

Finally, it might also be interesting to see how other teachers are able to implement the discourse generating tools. One aspect we can look at is seeing if other teachers can do what the teacher in the study did in order to create a productive discourse community. We might also look at what problems these teacher encountered.

Implication for Instruction

As mentioned above, this study is proof that a teacher can direct the discourse community in order to support learning mathematics with understanding. This can be done through the patterns of discourse described in the six tools used by the teacher. Thus, the use of these tools should be considered by educators as something to be developed by teachers. Particularly, the findings of this study can be used to enrich the experience of pre-service teachers in university methods classes. The data collected could be used as examples for discussion in those classes.

Another area in which the findings of this study could be used is with in-service teachers. Once again the data collected for this study could also be used in providing examples for discussion during in-service teacher training meetings. Mathematics is too prevalent in our lives to be learned in ways that do not promote understanding. Thus it becomes our duty, as mathematics educators, to create productive discourse communities where students can explore the mathematics, argue about the correctness or usefulness of mathematical tools, and create a deeper more meaningful understanding of mathematics.

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Appendices

Appendix A

Initial Interview Questions

Rules / Norms

1. Did you establish any rules and/or norms at the beginning of the year?
2. Did you use any programs, documents or methods to establish the rules and/or norms?
3. How did you establish the rules and/or norms?
4. What other rules and/or norms have you established since the beginning of the year?
5. Do you think these rules and/or norms have helped you to develop a mathematical discourse community conducive to students" learning and understanding?
6. How do you think they have helped?
7. Are there any changes you would make to the initial establishments of the rules and/or norms? If so, what would they be?

Questioning Skills

1. When preparing to teach any concept, do you think of questions you would like to ask your students?
2. How do you decide what questions to ask your students?
3. What helps you to decide your questioning sequence?
4. While you are teaching, do you ever change the structure of your questions?
5. Do you ever change any of the questions you have prepared? If so, why?

Appendix B

Final Interview Questions

Curriculum

1. How long had it been since you last taught Geometry before this class?
2. How did that affect your instruction this time around?
3. Why did you decide to teach from the IMP curriculum?
4. You mentioned that this curriculum helped the students to talk about the mathematics more. Then, to me, that comment seems like that„s important to you, and if so, why?
5. Why did you choose the Shadows unit for me to collect the data?

Findings

1. It seems to me that you believe that mistakes are sites for learning (Hiebert et al., 1997). Is this true, and if so, why?
2. In several instances, as I went through the data, I found that you continued to ask for different strategies, even when you already had some strategies. Is this a practice you value? If so, why?
3. In a couple of instances, you were very persistent in eliciting students‟ reasoning, even when they did not want to participate, or when they did not feel they could participate. Why were you persistent in eliciting their reasoning?
4. Through out the school year, you used name cards to call upon students. Are there specific times when you use these cards to ask questions, as opposed to the times when you ask the class as a whole?
5. It seems to me that you used the cards when the students had a chance to prepare.

6. I noticed in the examples that I looked at, that you used the cards when the students had had a chance to think about the problems first. Is this true? If so, why?
7. These were some norms I noticed were established in you class at the time of the video recording. They were (a) each student must participate, (b) each student must share their reasoning when called upon, and (c) each student must listen to, analyze and evaluate each other"s comment. Do you feel that these norms were in deed present at that time of the recording, during that Shadows unit?
8. Do you feel that there were other norms present that I did not Pick up on, that you noticed?
9. I showed you the six discourse generating tools. Do you feel they are indicative of your overall teaching?