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Abstract: This work shows an application of a two-phase SPH model for the details of the interaction between wave propagation and coastal structures. The numerical solution is based on SPH approximation of the mixture model of the Herschel-Bulkley-Papanastasiou viscoplastic model for calculating a variable yield stress. We have investigated the dynamics and characteristics of beach profile transport related to the wave breaking process. Further, the numerical model results were validated against the experimental and numerical data found in the literatures and some relatively good agreements were observed. Afterwards, variable characteristics of the waves propagated over different sloped beaches are carried out. The results of this study show that the method provides a useful tool to investigate the wave propagation in term of coastal applications and scour around structures in civil and environmental engineering flows.

***Keywords**: wave-induced erosion, multi-phase simulation, smoothed particle hydrodynamics*

1 Introduction

To assess local damage of sea wave impact on coastal structures for a proper design of coastal defences, some theoretical or numerical models have been developed for generally characterising and measuring(Nicholas C. Kraus 1988, Donald K. Stauble 1993, Thomson 2006, Demir 2007, Nur Yuwono 2009, Mizutani 2010, Hsu 2012, Luo 2014, Niroshinie 2014, Zhang Yang 2015) .The current application is highly complex, involving the two or phases, the large deformation of free surfaces, and the strong interactions between the wave and structure (Shao, S.,2006).

Thereby the present study considers relatively strong ability to incorporate complicated physical effects into the CFD formulations, which makes this problem ideal for the meshless method ---Smoothed Particle Hydrodynamics (SPH). SPH uses smoothed particles as interpolation points to represent materials at discrete locations, it thus can easily trace material interfaces, free surfaces and moving boundaries due to large deformations(Cleary 2007). In recent years, SPH technique has been widely applied to the numerical simulation of breaking waves such as wave processes of propagating, run-up, run-down and backwash over coastal structures(Kim and Ko 2008, Altomare, Crespo et al. 2014, Amin Mahmoudi 2014). In general, these referred applications were mainly utilized in the success of pure water flow problems involving wave propagations, which possesses some limitations with multi-phase problems (e.g. subaqueous sediment souring flows and suspended sediments). (Andrea Amicarelli 2013) and (Altomare, Domínguez et al. 2015) carried out a two-phase numerical solution based on a SPH approximation of the mixture model, which combined the main advantages of wave propagation models and SPH model. Such hybridization both improved accuracy and computational efficiency. (Crespo, Dominguez et al. 2011, G. Fourtakas 2013) adopted the parallel processing power of either CPUs and/or GPUs to accelerate simulations that dramatically decreased the expensive computational cost.

In this paper, we implemented a multi-phase fluid-sedment model based yield criterion and a constitutive viscoplastic equation in SPH to model wave-induced beach profile change, and the results is compared with experimental data. Further, we have investigated the inner-relation of sandy beach deformation with different wave conditions and beach slopes to give new insights in the characterization of beach erosion by using SPH model.

2 Numerical Models

2.1 Governing equations of a multi-phase SPH model

In the SPH method, the fluid domain Ω is discretized in a finite number of particles representing elementary fluid volumes d_V , thereby any function f can be approximated through the convolution sum:

$$\langle f\rangle(\mathrm{r}_i)=\sum_j \frac{m_j}{\rho_j} f\left(r_j\right) W\left(\mathrm{r}_i-\mathrm{r}_j, h\right) \tag{1}$$

where *i* and *j* denote the interpolated particle and its neighbours respectively, *W* is a kernel function represents a weighting of the contribution of particle *j* to the value of represents a weighting of the contribution of particle *j* to the value of $f(r_j)$ at position r_i.
Generally, in SPH, the fluid has been traditionally considered weakly compressible. The reason is that it is easier to calculate the pressure from an equation of the state rather than solving for the pressure. The governing equations of mass and momentum evolution for the multi-phase model are given by

$$\frac{d\rho_i}{dt}=\rho_i \sum_j^N \frac{m_j}{\rho_j} v_{ij}^{\beta} \frac{\partial W_{ij}}{\partial x_i^{\beta}} \tag{2}$$

$$\frac{du_i^{\alpha}}{dt}=\sum_j^N m_j\left(\frac{\sigma_i^{\alpha\beta}+\sigma_j^{\alpha\beta}}{\rho_i \rho_j}\right) \frac{\partial W_{ij}}{\partial x_i^{\beta}}+g_i^{\alpha} \tag{3}$$

where α and β denote the Einstein summation, u_i^{α} is the velocity, $\sigma_i^{\alpha\beta}$ is the total stress tensor, g_i^{α}, m and ρ is the gravitational acceleration ,mass and density respectively.
The total stress tensor term $\sigma_i^{\alpha\beta}$ is given by

$$\sigma_i^{\alpha\beta}=-P_i \delta^{\alpha\beta}+\tau_i^{\alpha\beta} \tag{4}$$

where P_i is the isotopic pressure, $\delta^{\alpha\beta}$ is the Kronecker delta, the viscous stresses are defined as

$$\tau_i^{\alpha\beta}=2\mu \mathrm{d}_i^{\alpha\beta} \tag{5}$$

in which, the deviatoric strain rate $\mathrm{d}_i^{\alpha\beta}=D_i^{\alpha\beta}-\frac{1}{3} D_i^{\gamma\gamma} \delta^{\alpha\beta}$, with $D_i^{\alpha\beta}=\frac{1}{2}\left[\frac{\partial v_i^{\beta}}{\partial x_i^{\alpha}}+\frac{\partial v_i^{\alpha}}{\partial x_i^{\beta}}\right]$.
In the majority of the weakly compressible SPH (WCSPH) approach is adopted using the equation of state (EOS) to describe the relationship between pressure and density

$$P_i=B\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma}-1\right] \tag{6}$$

with

$$B=\frac{c_0^2 \rho_0}{\gamma} \tag{7}$$

where $\gamma=7$ and the reference density $\rho_0=1000\mathrm{kgm}^{-3}$. c_0 is the numerical speed of sound.

2.2 Solid model and sediment

According to several full descriptions of the treatment for sediment phase (Manenti, Sibilla et al. 2011, Ulrich, Leonardi et al. 2013), it follows two assumptions in order to model scouring and suspension. Firstly, the sediment phase is treated as a slightly compressible pseudo-fluid behaviour. Secondly, the sediment is regarded as fully saturated.
Several relevant equations were given for the composition of sediment layer from the interface to the mobile layer and suspended sediment particles.
Eq. (8) firstly gives the Drucker-Prager yield criterion, which is used for describing the maximum shear strength of the soil sediment to the hydrodynamic shear strain at the fluid-soil interface (Fourtakas 2013)

$$2\mu\sqrt{II_D}=\mathrm{a}P_{eff}+k \tag{8}$$

where II_D is the second invariant of the strain rate, P_{eff} is the effective pressure, *a* and *k* represent the material constants of the Coulomb parameters that given as

$$\mathrm{a} = \frac{tan\varphi}{\sqrt{9+12tan^2\varphi}} \tag{9}$$

$$k = \frac{3c}{\sqrt{9+12tan^2\varphi}} \tag{10}$$

where φ and *c* are the repose angle and cohesion angle respectively.

After bringing forth the shear layer between un-yielded and fluid phase, the multi-phase SPH model then adopted the Herschel-Bulkley-Papanastasiou(HBP) model (Papanastasiou 1987) to characterize the rheological yielded region. The HBP model was given by

$$\tau_i^{\alpha\beta} = 2\mu_{pap}D_i^{\alpha\beta} \tag{11}$$

where μ_{pap} is the HBP apparent viscosity as

$$\mu_{pap} = \frac{|\tau_y|}{\sqrt{II_D}}\left[1-e^{-m\sqrt{II_D}}\right] + KD_i^{\alpha\beta^{n-1}} \tag{12}$$

where the critical threshold of shear stress $|\tau_y| = 2\mu\sqrt{II_D}$, *m* is controlling factor for the exponential growth of stress, *n* is the power law index and *K* is the consistency index that reduces to the dynamic viscosity of the material when *n*=1.
As a slightly compressible pseudo-fluid, the pore water pressure used the EOS as

$$\mathrm{p}_{pw,i} = B\left((\frac{\rho_{sat,i}}{\rho_{sat,0}})^\gamma - 1\right) \tag{13}$$

with

$$\mathrm{B} = \frac{c_{w,s0}\rho_w}{\gamma_w} \tag{14}$$

where unit weight $\gamma_w = \rho_w g$; s, w and sat denote the sediment, water and saturated phase respectively.

Extensive research has been conducted to water seeping through the pores of a soil produces drag originating from viscous forces(Bui, Sako et al. 2007). In multi-phase SPH model, the seepage force is added in the momentum equation that reads

$$S_{s,i}^{\alpha} = \sum_{j\epsilon W,Sat}^{N} \frac{m_j}{\rho_i\rho_j} S_{ij}^{\alpha} W_{ij} \tag{15}$$

where S_{ij}^{α} is the seepage force, which, according to Darcy law, can be described as

$$\mathrm{S} = \mathrm{K}(u_w - u_s) \tag{16}$$

with

$$\mathrm{K} = \frac{n_r\gamma_w}{R} \tag{17}$$

Where n_r is the porosity and R is the soil permeability.
Due to the high velocity scouring, some of the sediment particles will be suspended into the fluid flow, it is modelled as a pseudo-Newtonian fluid using (Skillen, Lind et al. 2013) with a suspension viscosity (Vand 1948)

$$\mu_{susp} = \mu_{\mathrm{fluid}} e^{\frac{2.5c_v}{1-\frac{39}{64}c_v}}; \qquad c_v \leq 0.3 \tag{18}$$

where c_v is rate the volume fraction of the mixture in the form of

$$c_v = \frac{\sum_{jsat \in 2h}^{N} \frac{m_j}{\rho_j}}{\sum_{j \in 2h}^{N} \frac{m_j}{\rho_j}} \tag{19}$$

3 Numerical Results

In this part, the numerical model was tested by a 2-D validation case for evolution of fine sandy beach profile, which has been carried out through experimental study (Diane P. Horn 2007). The experiment was performed in a 27.3 m long, 1.4 m wide and 0.8 m deep flume (Figure 1). In the numerical test, the parameters for this simulation was set as recommended in Fourtakas(2013), which demonstrated to approximate a Bingham model as closely as possible. The main parameters included the fluid dynamic viscosity was 0.001 Pa.s and sediment viscosity was set to 150 Pa.s with the HBP, the controlling indexes of *m* and *n* set to 100 and 1.8 respectively. The sinusoidal waves were generated by a regularly periodic moving boundary with time period T=2.5s and wave height H=0.15m.

After illustrated the experimental conditions, we next present the beach profile change against the experimental data and investigate the characteristics of waves with beach profile changes.

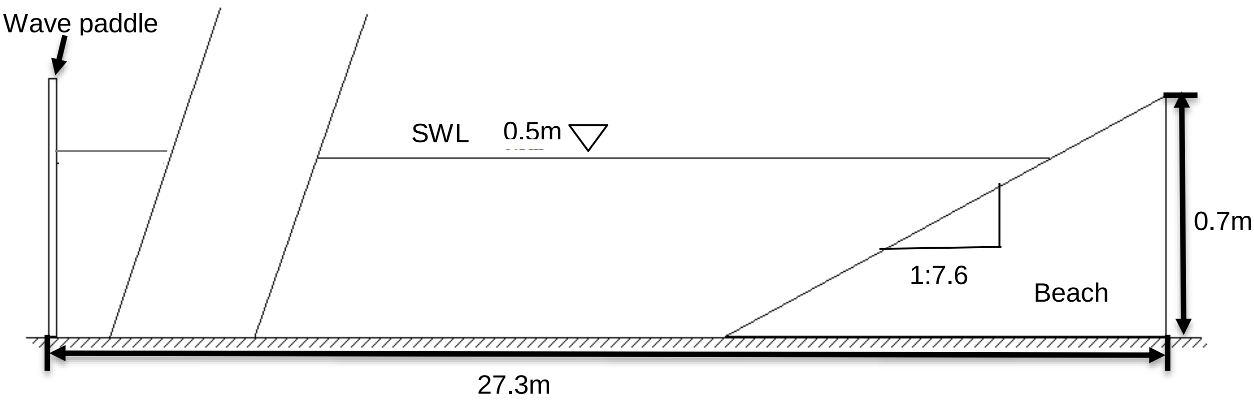


Figure 1. Schematic of the experimental setup.

Herein, the comparison of numerical results with experimental data is shown in Figure 2 after 60 minutes scouring. In this case, sediment transport and accretion were observed at cross section, with a trend of offshore transport, followed the upper beachface eroding and a bar developing.

Figure 2 summarized the characteristics of the beach profiles variations. The shape of the beach deformed following the berm erosion and the bar accretion. It showed that the accretion mainly occurred between x = 1m and 2m. Meanwhile, the berm (0-1m) thus has been eroded by scouring. Figure 3 gave the snapshots of the velocity wave propagation in a time period.

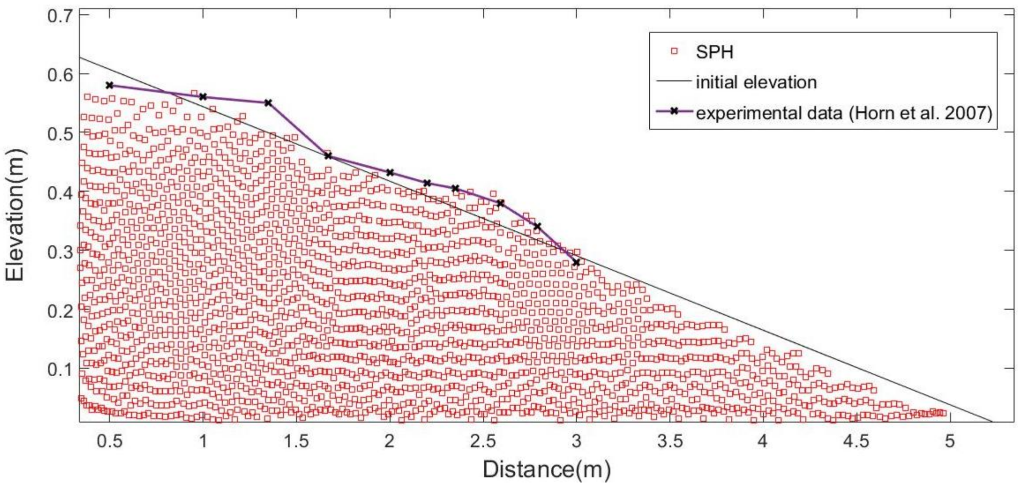


Figure 2. The beach profile: comparison between numerical and SPH model (T=2.5 sec. H=0.15m)

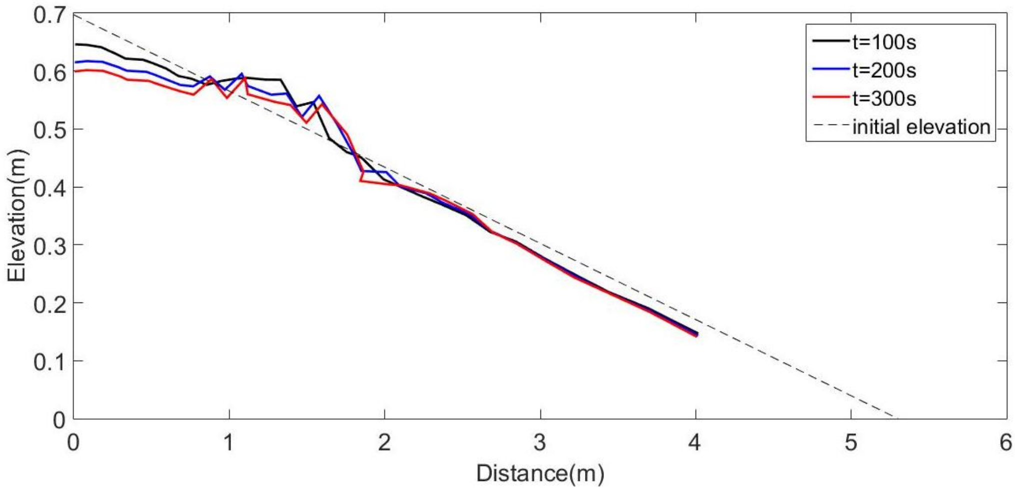
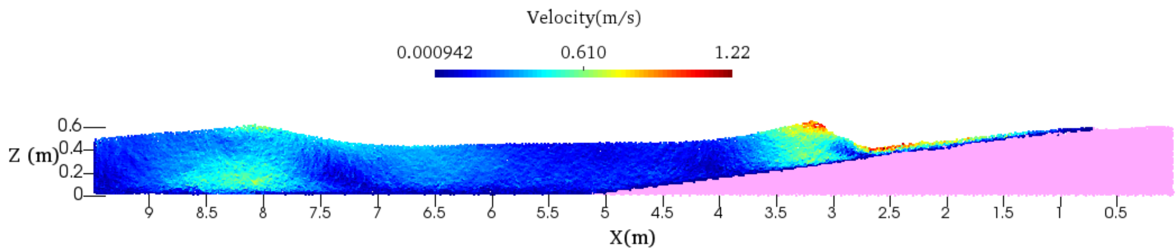
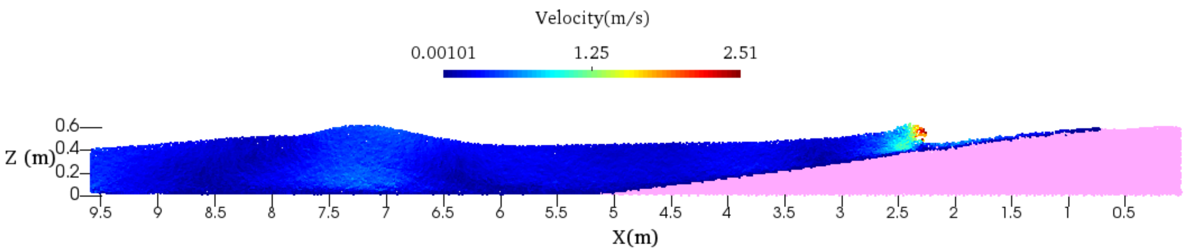


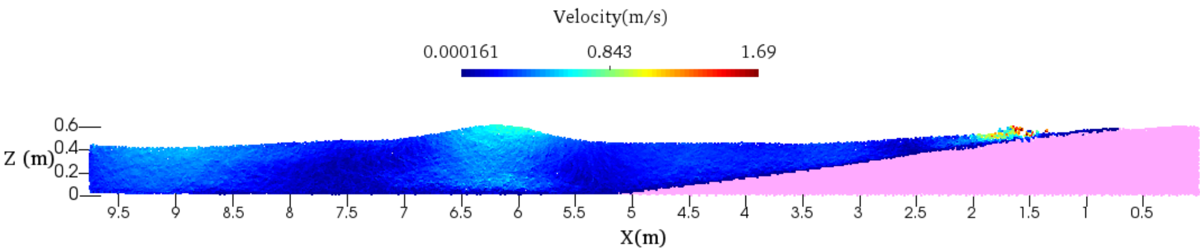
Figure 2. Variations of the beach profiles
(T=2.5 sec. H=0.15m)



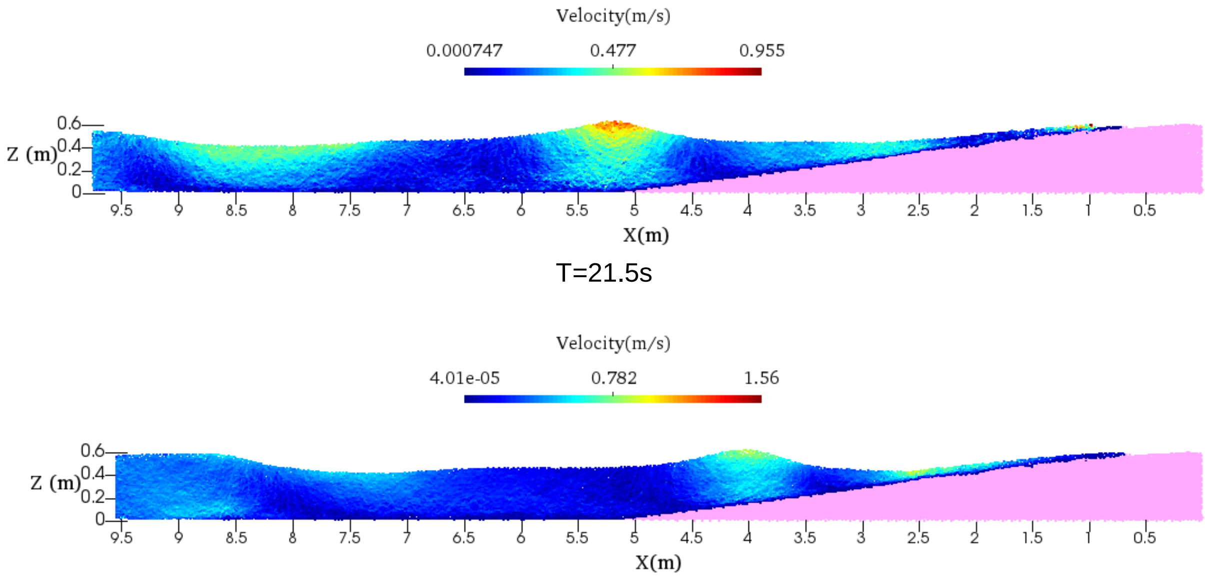
T=20s



T=20.5s



T=21.0s



T=22.0s

Figure 3. Velocity magnitude profile of the waves
(T=2.5 sec. H=0.15m)

The hydrodynamics of the waves inevitably linked to the sediment scour mechanisms. In order to find the relationship, two alterations are used in this work, 1) change the time period, 2) change the slope of the beach.

Under the condition 1), Figure 4 showed the beach profiles variations are similar to the results summarized in Figure 2, but with smaller bar developing for smaller time period and amplitude. Meanwhile trough began to develop at about x=1.5m, but the trough started to develop at about x=2.0 in Figure 2. Besides, we can find that the erosion acting differently at fore-end of the beach. More Shallow and short time period led to more erosion at fore-end, whereas, deeper water and longer time period mainly contribute greater to the dune accumulation.

Under the condition 2), unlike under previous conditions, we can observe a berm developing on the upper beach face, with onshore transport. Meanwhile, it also showed that berm accreting firstly began at the middle part of the beach face, and then formed the berm crest by the sand brought from beach cusps and the top.

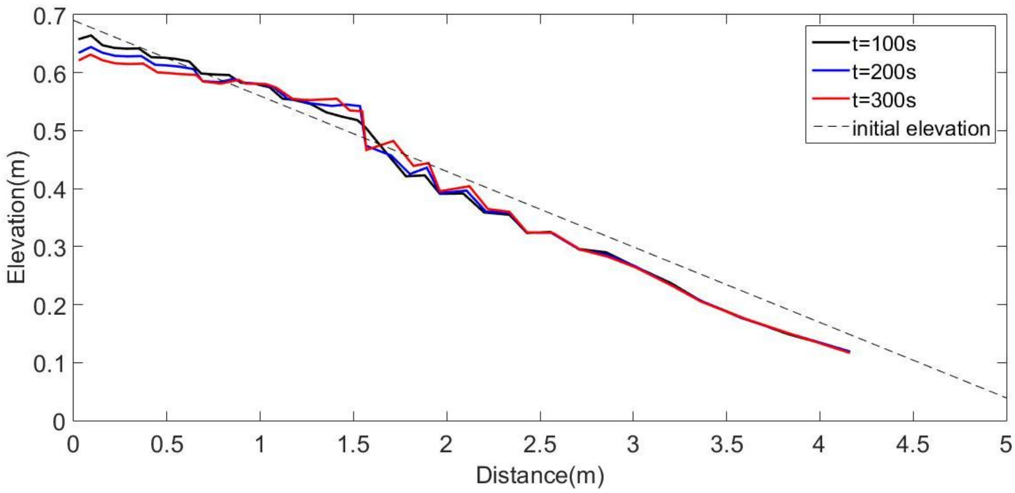


Figure 4. Variations of the beach profiles
(T=1 sec. H=0.1m, slope=1:7.6)

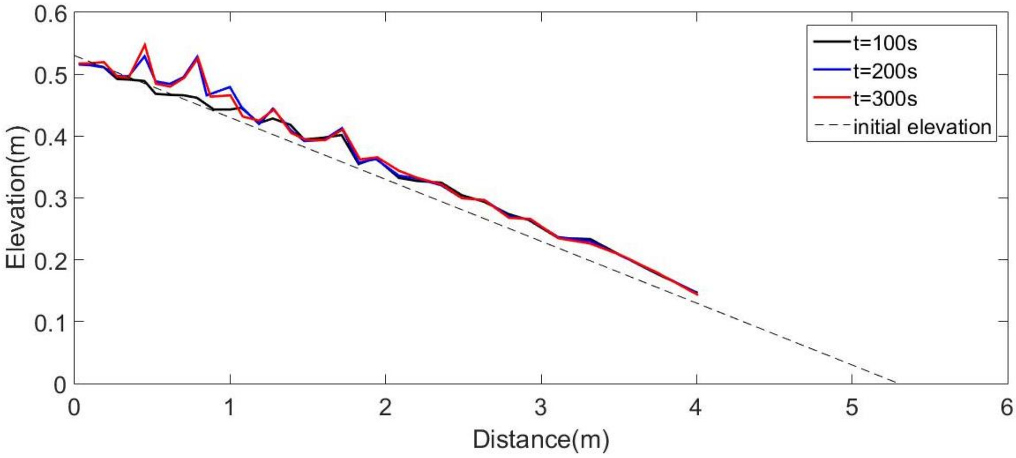


Figure 5. Variations of the beach profiles
(T=2.5 sec. H=0.15m, slope=1:10)

4 Conclusions

In this paper, we report the implementation of multi-phase SPH model for wave propagation over the beach. The comparison between SPH model and experimental data, and they are generally in a good agreement. Two different conditions were also used in this work, and these result in similar consequences with different wave conditions but different erosion levels. Differences could be noticeable if change the beach slope. Our simulations indicate that erosion phenomena and materials load transport over sea beach or riparian areas react to combined actions due to both of the geometrical conditions and the wave conditions. Future works will be devoted to the application of environmental problems, e.g. non-point pollution problems along riparian zones and flood propagations with mud and sand.

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