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An Examination of the Role of Writing in Mathematics Instruction

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AN EXAMINATION OF THE ROLE OF WRITING

IN MATHEMATICS INSTRUCTION

by

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ABSTRACT

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This study uses qualitative methods to investigate the use of writing in a content course for elementary education majors in which writing was considered an important part of mathematical learning. The study differs from previous studies by investigating the role of writing in the everyday instructional activities, rather than investigating writing as a separate mathematical activity. An analysis of the instruction and class discussions that took place in this class reveals that components of writing that were addressed implicitly and explicitly in classroom instruction were developed simultaneously with conceptual understanding, suggesting a much stronger and more integral relationship between writing and learning than the relationship that has been hypothesized by previous research. Furthermore, specific ways in which the class was structured seemed to support the development of students' written explanations. Appropriate explanations of particular concepts were modeled by both teacher and

students, and explanations of mathematical concepts were developed gradually in a relatively consistent progression that paralleled the development of the concepts themselves. The findings of this study contribute to the field of research by helping to describe the relationship between writing and learning and by illuminating some of the ways in which both student learning and student writing are affected by classroom instruction.

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Chapter One — Introduction

For many people, recalling their own school experiences, written language seems out of place in a mathematics classroom. The nature of the beliefs about mathematics that underlie both traditional curriculum and accompanying teaching practices render written explanation unnecessary, and perhaps even antithetical to the purposes of the classroom. The type of understanding required by students in a traditional mathematics classroom has been described by Skemp (1987) as the possession of and the ability to use rules and methods specific to solving particular types of problems. Students manifest this type of understanding by carrying out step-by-step manipulations of symbols that the teacher can ultimately judge as correct or incorrect by looking at the answer produced, and perhaps by checking to see that steps are followed accurately (Romberg & Kaput, 1999). Such an emphasis on procedural knowledge is furthermore embedded within a rather rigid daily structure. The teacher reviews the previous day's material, demonstrates the new procedures that the students are expected to master, and assigns practice in the form of classwork and homework (Stigler & Hiebert, 1999). This context, compounded with the pressure to cover an extensive curriculum in a relatively short period of time, leaves little room or necessity for any student writing beyond paper-and-pencil calculations, and perhaps written definitions of mathematical vocabulary.

It isn't surprising, therefore, that if writing is introduced into the mathematics classroom at all, it tends to take place at the outskirts of the curriculum. Mathematics teachers may believe that good writing is a worthy goal in and of itself, but generally consider writing to be the responsibility of teachers of other subjects, and are hard-pressed to concede a place for it in their goals as a teacher of mathematics (Quinn &

Wilson, 1997). Mathematics teachers who do attempt to introduce writing into their classes often focus initially on tangential topics by requiring students to write biographies of mathematicians or reflections on their individual progress in the classroom (McCann, 2001). These types of writing assignments may help to increase the students' interest in mathematics as a field of study or to focus their attention on academic goals and study skills, but they do not directly contribute to the principal goal of helping students to do and understand mathematics. Thus writing is at best a nice aside, and at worst a distraction.

However, gradual changes in the way that mathematics teaching and learning is conceptualized have created a context for learning in which students' written explanations of mathematical reasoning, concepts, and solution strategies play a much more important role. Within the field of mathematics education, the idea of mathematics as a fixed body of knowledge is increasingly being supplanted by an emphasis on the process behind the mathematics and on mathematics as a human activity (Sfard, 1998). As a result, students are expected to understand mathematics at a deeper level, to be able to solve rich mathematical problems, and to communicate their mathematical knowledge both to the teacher and to each other.

This increased focus on the importance of students' communication stems from perspectives in learning mathematics at both the individual and social levels. From the individual perspective, research and theory has addressed the ways in which students make sense of mathematics as cognizing individuals, and how teachers can guide students as they construct meaningful conceptual structures (von Glasersfeld, 1995). This perspective legitimizes a focus on communication in two ways. From an educator's point of view, an increased focus on student sense-making suggests that it is important not only for students to demonstrate the process by which they solve mathematical problems, but also for them to communicate their reasoning for using these processes and their understanding of the meaning that underlies their solutions and solution strategies. Such reasoning and understanding cannot be thoroughly communicated in traditional step-by-step lists of symbolic manipulations, and so students are under greater demands to justify their reasoning in written and spoken language. Second, from the point of view of the cognizing individual, communication about mathematics can actually contribute to a learner's understanding as their knowledge is transformed through the reflection on prior knowledge and activity that is inherent in the act of communication (Lampert & Cobb, 2003).

From a social perspective, students' ability to do mathematics can be equated with their ability to participate as a member of a community of knowers of mathematics (Forman, 2003). Mathematical knowledge is shaped within the context of the social practices of the community and developed through communication among members of that community. Therefore, an essential part of students' learning in mathematics is their ability to participate in the social practices of the community. In fact, from a social perspective, such participation is not only an important part of learning mathematics, it is inseparable from learning. Learners do not know mathematics unless they are able to participate in the social practices of the mathematics community, including ways of doing mathematics and speaking about mathematics, as well as reading and writing about mathematics.

The increasing value of writing as perceived by the mathematics education community is reflected both in influential documents and in changing classroom practices. The *Principles and Standards* of the National Council of Teachers of Mathematics (NCTM, 2000), for instance, includes writing as a part of its prominent communication standard. This standard encourages teaching students to be able to coherently communicate their mathematical thinking, to analyze and evaluate others' mathematical thinking, and to use the language of mathematics to express their ideas (NCTM, 2000). Writing is included in the standard as one of many aspects of communication to be nurtured, as well as a specific form of mathematical communication to be developed for its own sake. Furthermore, as calls for mathematics reform diffuse into the schools, writing is increasingly being introduced into the classroom as a vehicle for learning and as a means of assessing student understanding (Berenson & Carter, 1995; Philipp, Flores, Sowder, & Shappelle, 1994). Students in classrooms that are closely aligned with *Standards*-based mathematics are typically asked not only to solve rich problems, but also to explain their solution strategies and mathematical reasoning. Writing is a very natural vehicle for these explanations because it allows each student to share his or her thinking and allows the teacher to see each individual student's explanation.

Within the body of empirical research in recent years, student mathematical writing appears primarily in two forms. First, writing is collected as a form of data and analyzed for insight into student thought. Such writing may be part of a larger body of data used to answer research questions pertaining to student learning or it may itself be the specific focus of investigation as researchers seek to understand how writing reflects student understanding or thought processes (Aspinwall & Miller, 2001; Pugalee, 2001). Second, writing is studied as a classroom practice with potential to inform and guide instruction (Miller, 1992) or to promote student learning by helping students to clarify and connect mathematical concepts (Clarke, Waywood & Stephens, 1993; Jurdak & Zein, 1998; Powell & Lopez, 1989; Rudnitsky, et al., 1995). Research questions addressed by studies of the second type typically address whether the use of writing in the classroom can help students better understand mathematics or whether the use of writing can help teachers improve classroom instruction.

One potential shortcoming of both the use and study of writing in mathematics classrooms, however, is that the focus on the mathematics tends to overshadow the focus on writing itself, a shortcoming that may affect the success of teachers trying to use writing as a learning or assessment tool for their students and of researchers trying to analyze writing for insight into student learning. The preference for focusing on mathematics over writing is, of course, understandable given that the goal of mathematics educators is to understand how mathematics, not writing, is taught and learned. However, if teachers and researchers are attempting to evaluate student conceptions through their written explanations, the question of what information can be extrapolated from writing is a crucial one, and ignoring the complexities of writing itself obscures one's ability to answer that question. In fact, there is evidence to suggest that the transfer of student understanding to words on paper is not as direct as might be believed. Shield and Galbraith's (1998) analysis of student-produced texts suggests that exposure to traditional textbook explanations and classroom practices strongly influences students' attempts to produce "elaborate" conceptual explanations by establishing modes of argument that students deem acceptable. Similarly, Morgan (2001) describes the interference of other written genres that students have encountered on their efforts to write mathematical explanations, as well as the difficulty teachers have in recognizing their own intuitive understanding of what constitutes a “good” mathematical explanation. And Draper and Siebert (2005) call into question the assumption that mathematical understanding can be equated with the ability to communicate that understanding, recognizing the insufficiency of sole focus on the underlying mathematics and the subsequent need for explicit instruction on the creation of written explanations.

The likelihood that fundamental characteristics of writing affect the type and means of understanding that take place through, and are reflected in, written mathematical explanations necessitates a closer look at writing and the ways in which writing, mathematics content, and classroom instruction are interrelated. Through my research, I will address this issue by examining a classroom in which writing about mathematics is an integral activity. I will look at the context in which the writing is produced in order to gain insight into the relationship between writing and classroom practices, as well as the classroom structures that may enable students to learn to write good mathematical explanations.

Chapter 2 — Conceptual Framework

Within the body of mathematics education research, student mathematical writing is more frequently a source of data with which to address particular research questions than a direct focus of investigation. Nevertheless, recognition of the potential of writing as a mathematical learning tool has led to the development of a fairly substantial body of literature on writing and its relationship to mathematics learning and teaching. I will situate my own research within the existing body of research by addressing the theoretical justifications for the use of writing, the methods and conclusions of empirical studies that have examined writing in the mathematics classroom, and the complexities involved in writing that suggest the need for new approaches to the study of writing in mathematics classrooms.

The Place of Writing in the Mathematics Classroom

The practical and theoretical justifications for the use of writing in the mathematics classroom are important to address for two reasons. First, as mentioned in the previous chapter, writing has not typically been viewed as an important or worthwhile component of traditional mathematics instruction. And although the use of writing is more natural in classrooms where the instruction has shifted away from traditional procedural skills in favor of rich conceptual understanding, the fact that writing *can* be used does not in itself justify its implementation as an important part of classroom instruction. One of the purposes of this section, therefore, is to examine the relationship between writing and learning mathematics and the potential of writing as a learning tool, and to thereby support its use in mathematics classrooms.

Second, the justifications that support the use of writing in a mathematics class, by providing insight into the reasons why writing is expected to help students understand mathematics, subsequently suggest which types of writing might be useful for learning mathematics. We cannot expect that every writing activity would be equally relevant to the learning that is expected to take place in a mathematics classroom. Examining how writing is thought to be related to learning, and to mathematical learning specifically, will therefore help to define the type of writing that will be considered for the purposes of this study.

In this section, I will begin where researchers in the past have begun, with theories of how writing may affect learning in general, and then address the qualities of writing that make it particularly relevant to mathematics.

Writing and cognition

The hypothesis that students, by engaging in writing, can come to better understand mathematical concepts has its roots in theories on the connection between writing and learning in general. Some of the foundational work on the relationship between learning and writing originated with Vygotsky, who outlined particular differences between writing and speech in order to suggest that writing uniquely transforms thought (Vygotsky, 1962). He argued that, rather than mirroring the progression of speech development, writing "is a separate linguistic function, differing from oral speech in both structure and mode of functioning" (p. 98). Written language, Vygotsky asserted, requires abstraction on two levels: abstraction from the "musical, expressive, intonational qualities of oral speech," and detachment from the context of face-to-face conversation between interlocutors (pp.98-99). Furthermore, the act of writing demands a high level of consciousness of the structure and form of language, as well as an awareness of details of context that are present in oral communication but absent in written communication.

Vygotsky is here referring to the very early development of writing abilities in young children and the subsequent changes in thought that are the result of this transition from speech to literacy. As Sierpinska (1998) points out, the journal entries and descriptions of problem-solving processes that are often promoted in mathematics classrooms today are not likely the use of writing that Vygotsky considered when he developed and described his theory. Therefore, although Vygotsky's theory on the effect of writing on cognition is often used to validate the use of writing to promote student learning in various content areas, his work should not stand alone as a blanket justification for all writing-to-learn activities.

Still, his theory does lend credence to a common belief that writing is inherently different from other uses of language and therefore has great potential to influence cognition and strengthen understanding at any developmental level. Emig (1977), for instance, built upon the theories of Vygotsky and others in her analysis of the unique characteristics of writing that make writing an ideal learning tool. In particular, she pointed out the relationship between writing and Bruner's three forms of representing and dealing with reality, asserting that writers simultaneously deal with enactive, iconic, and symbolic elements of learning. By describing this relationship, Emig was able to link theories of writing to more general theories of learning. This connection between cognitive theories of learning and the process of writing has since served as powerful motivation to use and investigate the potential of writing as a mode of learning.

Writing and learning mathematics

Theories about the potential of writing to affect learning in general have given rise to applications of writing to the discipline-specific goals of various content areas. In particular, mathematics education researchers have noted several distinct characteristics of writing that seem to make it a useful tool for developing mathematical knowledge. These characteristics include the potential of writing to engage students in mathematical learning, to structure thought, and to make the learner's thought process explicit. I will address each of these characteristics individually.

First, writing is thought to be a natural way to engage students directly in the processes involved in thinking and learning about mathematics. When students are asked to write about their solution process in addition to solving a problem, they are required to think about their reasoning and about the underlying mathematical concepts. In addition, the teacher's ability to tailor writing assignments to specific classroom goals can encourage students to engage not just in thinking about mathematics, but in very particular ways of thinking about mathematics. While similar student engagement in mathematical thinking can be achieved through talking about mathematics, writing has the advantage of allowing all students to communicate their individual understanding, and to do so at their own pace.

Mental engagement in learning through writing can also be augmented by emotional engagement. Writing, by providing students with a means for expressing their individual understanding, as well as their questions about mathematics, is frequently assumed to have a positive effect on the learning atmosphere in the classroom and the attitudes of students (Jurdak & Zein, 1998; Swinson, 1993). While the attitude of students is not often the object of explicit study within the literature on writing in mathematics, it is frequently seen as a positive side effect of the introduction of writing into the classroom. Still, as Ackerman (1993) points out, the changes in attitudes that are demonstrated in studies of writing to learn may be tied not to the use of writing specifically but rather to other changes in classroom organization and beliefs about learning that occur naturally in classrooms which writing is introduced.

Second, the structuring of abstract thought inherent in the process of writing suggests that writing can play a role in structuring students' knowledge and understanding. Vygotsky (1962) refers to writing as a "deliberate structuring of the web of meaning" (p. 100), and Emig (1977) emphasizes that "the medium of written verbal language requires the establishment of systematic connections and relationships" (p. 126). This characteristic of writing is particularly appealing to mathematics educators because the precision and structure of writing appears to parallel the precision and structure of rigorous mathematics (Elsholz & Elsholz, 1989; Morgan, 1998). This precision is more difficult to achieve in context-laden verbal exchanges. Morgan (2001) points out that in spoken mathematical arguments, shared understandings and assumptions often remain implicit, whereas written arguments lack face-to-face communication and immediate feedback and therefore require the explication of these assumptions, as well as attention to the logical order of the argument. The assumption that the necessary structuring of writing should transfer to the structuring of mathematical arguments makes sense intuitively as well, as most of us have struggled at one time or another to put our complex and often unrelated thoughts into some logical form when we produce a piece of writing.

Third, from a cognitive perspective, writing is frequently seen as a way for students to be made aware of their own thought processes and to thereby learn from and become able to control those processes. Emig (1977) suggested that “a unique form of feedback, as well as reinforcement, exists with writing, because information from the *process* is immediately and visibly available as that portion of the *product* already written” (p. 125, emphasis in original). Similarly, Morgan (2001) suggested that the concreteness of written records allows students to reflect upon and revise what they have already written, and to bring together texts to be compared and discussed. In this way, when the thought process of a student is written down, it itself becomes the object of thought. Pugalee (2001) suggested that writing is one of the more promising vehicles for providing the experiences necessary to promote metacognitive behaviors in student problem solving. And Clarke, Waywood, and Stephens (1993) based their use of journals in a school-wide mathematics curriculum on the idea that writing both engages and mirrors processes that are essential for learning, thereby bringing such processes out into the open.

Type of writing

The type of writing advocated by such theoretical assumptions is writing about the ideas, concepts, arguments, and logic involved in mathematical understanding. This is important to point out because, as discussed in the previous section, the mathematics involved in some writing tasks used in mathematics classes is peripheral to the task itself. The theoretical assumptions generally used in support of writing, on the other hand, point to mathematics, and particularly mathematical reasoning, as the central focus of the writing. This is consistent with the objectives of a mathematics class that effectively promotes mathematical understanding. The central focus of any mathematical task should be the mathematics (Hiebert, et al., 1997), and the same is therefore true for any writing task meant to encourage mathematical thinking and understanding. This is not to say that other writing tasks, such as students' reflections about their progress, frustrations, feelings towards mathematics, and so on, cannot have an important and worthwhile place in the classroom. Rather, because my focus is on the use of writing to directly aid student understanding of mathematics, I feel that it is important to clarify that the type of writing relevant to my research is writing in which students explain their reasoning and justify their understanding of mathematical concepts.

Studies of Writing in Mathematics Classrooms

A review of the existing research on writing in mathematics shows that researchers have tended to consistently address variations of a single, fundamental question: Does the use of writing in a mathematics classroom help students to better learn mathematics? Interestingly, a review of the research also reveals two apparently contradictory ideas. On one hand, the majority of the studies are motivated to some degree by a general dissatisfaction with empirical support for the benefits of writing in the learning process. But at odds with this dissatisfaction is the fact that nearly every study seems to conclude with favorable outlooks on the use of writing. In order to situate my own research, therefore, I will discuss several prominent, illuminating, and representative studies on writing in mathematics classrooms with particular attention to details of the studies and the results that may contribute to the above-mentioned dissatisfaction. My ultimate goal, which I will address in the concluding section of this chapter, is to suggest that a crucial difficulty with studies of writing in mathematics is not the methodology or conclusions, but rather the possibility that researchers have been asking the wrong questions, and that different research questions and different points of view might be more valuable for understanding the role of writing in the learning of mathematics.

One common methodology in the study of writing in mathematics classrooms has been to study the influence of writing on students' learning by employing writing in the form of mathematics journals or responses to mathematical prompts as a treatment, and subsequently comparing the results of tests of mathematical skills administered either before and after treatment, or to writing and control groups. The results of such studies have usually been positive. Johnson (1990) found that students in a college-level History of Mathematics class who consistently kept journals outperformed their peers who did not. Jurdak and Zein (1998) compared middle school students who engaged in prompted journal writing at the end of class three times a week for twelve weeks with a control group given the same instruction without the journal writing, and administered pretests and posttests intended to measure conceptual understanding, procedural knowledge, problem solving, mathematical communication, and student attitudes towards mathematics. The treatment group scored significantly higher on questions measuring conceptual understanding, procedural knowledge, and mathematical communication, and Jurdak and Zein were able to plausibly explain the lack of significant difference between groups on questions involving problem solving and attitudes towards mathematics as due to the informal nature of the writing in which students engaged.

Similarly, Rudnitsky, et al. (1995) and Johnson, et al. (1998) measured the results of the introduction of writing into mathematics classrooms, this time using structured writing programs rather than less formal journal writing. Rudnitsky, et al. (1995) integrated writing and problem-solving with a carefully designed and implemented method of instruction for elementary-age children. The context of writing in this case was narrowed to very specific types of mathematics problems, namely addition and subtraction word problems, and used a writing method that was intended to directly support students' understanding of problems of that nature and the corresponding solution processes. Students who engaged in the "structure-plus-writing" approach performed better than students in two control groups both on immediate posttests intended to measure improvement and on delayed posttests intended to measure retention. Johnson, et al. (1998) assessed fifth grade students' ability to engage in probabilistic thinking while simultaneously incorporating writing prompts into students' instruction over the course of a unit on probability. Significant differences between pre- and posttest scores were found on tests of both probabilistic thinking and writing level, although there was no significant correlation between the two.

While these studies appear to succeed in demonstrating a positive relationship between writing and learning mathematics, the separation of the treatment and the measurement inherent in the research design makes it difficult to understand *why* the treatment appeared to be successful. The authors of both studies present plausible explanations for their results, and Jurdak and Zein even suggest explanations for the unexpected failure of writing to impact students' problem solving abilities and attitudes towards mathematics. However, these explanations remain only plausible ones because of the inherent invisibility of processes and causes involved in choosing a single variable and a single measure. The focus on a test score rather than on the writing itself obscures the link between writing and understanding and makes it difficult to identify the specific types of learning that can be associated with writing and the specific writing processes that impact learning.

This disconnect certainly does not render the research meaningless. The methods are consistent with the research questions, which focus on whether writing can make a difference in student learning. But because these methods allow only speculation on precisely *how* writing plays a role in student learning, the studies cannot contribute to an understanding of underlying principles that might connect the acts of writing about mathematics and coming to understand mathematics. This in turn makes it difficult to apply writing in settings that are very different from the settings in which these particular studies were carried out. Based on the results of these studies, we are able to say with some confidence that writing made a difference for these particular students in this particular subject under these particular conditions, but cannot say for certain how similar results might be produced in very different circumstances.

Other research has avoided the disconnectedness of treatment-effect studies by directly analyzing student writing (rather than test scores) for evidence of learning. Waywood (1992, 1994) integrated writing into the school mathematics curriculum of an entire secondary school and looked for evidence of questioning and dialogue within the students' written journal entries. He found that the writing in students' journals seemed to become more sophisticated throughout the years that journals were kept, and further noted that the journal entries seemed to reflect student learning styles and individual approaches to participating in mathematics as an activity (Waywood, 1994). Pugalee (2001) analyzed students' writing for use of metacognitive strategies and found that all four metacognitive phases of problem-solving (as adapted from Garafolo and Lester, 1985) were present in at least some portion of student writing. Similiarly, Cooley (2002) collected writing from college Calculus students and identified visible evidence of reflective abstraction about important Calculus concepts, and also noted that a noticeable change in the students' writing over the course of the semester seemed to reflect that students became better at reflecting on and abstracting calculus concepts.

One of the difficulties of examining student writing for evidence of mathematical learning and reflection is the unavoidable question of whether the writing itself is actually aiding the understanding, or whether the writing is merely a reflection of understanding that students arrive at through other means. This issue is addressed directly by Cooley (2002), who states that, "It is not clear, nor could it ever be absolutely clear, if the questions asked of the students or the thinking they do in order to write a response are the catalysts or if the writing assignment is a conduit by which they can demonstrate that the process has occurred" (p. 280). This is a difficult, and possibly unanswerable question. As discussed above, measuring learning through writing by looking at completely separate means of assessing understanding limits our insight into how the writing influenced the learning. However, resolving the problem by directly assessing learning through writing puts unavoidable constraints on our ability to say for certain that the writing was the impetus for the learning.

A second difficulty is that the evidence of student learning is not entirely consistent. Journal entries or written responses that seem to reflect students' thinking processes and increased understanding are used to show that writing *can* impact mathematical learning, but in general only a small portion of the students who are engaged in writing actually do demonstrate this type of learning process. For example, Pugalee (2001), in his research on the evidence of metacognitive activity in students' mathematical writing, found that while all four metacognitive phases of problem solving (orientation, organization, execution, and verification) were present in at least some written responses, very few students exhibited use of all four phases. In fact, the majority of students focused primarily on describing their execution of the problem (the execution phase of problem solving). Even Waywood's (1992) hierarchical classification of writing modes was based largely on 65 out of 150 students who employed one of three modes of journal writing predominantly, which indicates that only a relative handful of students were consistently engaged in the highest mode of journal writing. And Johnson, et. al (1998), who measured changes in writing as well as changes in learning, noted that despite the teacher's encouragement of clear written explanations, some students who exhibited strong probabilistic reasoning produced weak explanations and did not change their writing over the course of instruction.

Apparent successes among some students engaged in journal writing may or may not be negated by other cases in which students do not seem to engage in learning through writing or to be able to clearly express mathematical ideas and processes. It is possible that some students were able to benefit from writing in ways they may not have benefited from other instructional activities. But it is also possible that the characteristics that enabled students to write good mathematical explanations correspond directly to the characteristics that enable good mathematical performance. This would suggest that writing itself does not lead automatically to learning and that other factors need to be taken into account. In fact, there is evidence in research on writing that the complexity of writing may influence the success of using writing in the classroom. This complexity will be addressed in the following section.

The Complexity of Writing

I have already alluded to a general dissatisfaction with the empirical research on writing in the mathematics classrooms. Research studies attempting to add to the body of evidence for the value of writing in mathematics are replete with suggestions that claims for the benefits of writing have been proffered in the past with little empirical support (Jurdak & Zein, 1998; Powell & Lopez, 1989; Pugalee, 2001). The difficulty of determining the role of writing in the treatment-effect studies mentioned in the previous section may be part of this perceived lack of evidence. Studies in which writing is introduced into a mathematics classroom seem to demonstrate that students' understanding improved in conjunction with the writing, but they do not necessarily link writing and learning satisfactorily. In fact, empirical support for writing to learn across all disciplines, not just mathematics, is tenuous at best—studies attempting to link writing and learning are frequently inconclusive or contradictory (Ackerman, 1993).

Given the amount of research on writing to learn, this perceived lack of empirical evidence of its inherent benefits may seem surprising. Anecdotal evidence abounds with practitioners describing how writing has changed their students' thinking and the atmospheres of their classrooms (Countryman, 1992; Meier & Rishel, 1998). Furthermore, the belief that writing can promote learning, as well as proposed theories on precisely how such learning might occur, seem intuitive and speak to our experiences as writers and as learners. In fact, Hill (1994) points out that, even lacking an empirical foundation, the use of writing as a learning tool "is a commonsense notion, not just in the writing community, but in the larger educational community" (p. 6). Therefore, despite dissatisfaction with the empirical foundations for writing, research continues in this area because researchers already believe that it ought to be a fruitful area of investigation, and because the qualitative support, though sparse, is compelling.

Perhaps, then, it could be argued that the important research questions to be addressed in seeking to understand the relationship between writing and learning do not involve whether students are more successful in understanding mathematics when they are required to write about it and why they are or are not more successful, but rather how writing fits into the learning process and what role writing plays in instruction and student learning when writing is an important classroom activity. Such studies would inherently involve examining writing not as an isolated mathematical activity, but as part of all learning events that take place in the classroom, with the potential to influence and be influenced by these events. In fact, the necessity of studying writing within this broader context is supported by research into the complexities that stand in the way of studying writing as a relatively isolated event. These complexities include problems involved in defining writing and learning, the position of writing as part of a much larger context, and students' understanding of what it means to write about mathematics, each of which must be taken into account when investigating the use of writing in mathematics classrooms. I will describe and clarify each of these three issues, and then discuss the implications for research.

Definitions of writing and learning

The first and most apparent setback in most studies of writing to learn is the difficulty of defining basic terms. If we as educators are to use writing to promote learning, what exactly do we mean by writing, and what do we mean by learning? As Tynjäla (2001) argued, both the conception of learning that underlies instruction and students' perception of what it means to learn necessarily impact any investigation or use of writing in a learning context. But as Hill (1994) pointed out, "both *writing* and *learning* are complicated and ill-defined concepts" (p. 4, italics in original). This is not a difficulty that can be overlooked. The conceptions of learning that underlie research involving writing or instruction in which writing is used influence how that learning will be measured and what kind of learning can occur, and the type of writing used in the classroom both influences and is influenced by the type of learning that can take place.

The effects of this ambiguity in the definitions of writing and learning can be seen in the literature on writing in mathematics. For example, Lesnak (1989) demonstrated the effectiveness of writing in mathematics by quantitatively and qualitatively evaluating student success in writing and non-writing mathematics courses. His mathematical focus, however, is very procedural, and his writing assignments concentrate heavily (and explicitly) on recognizing and using steps to solve problems in a "correct" way. While he concludes that his writing group outperformed his non-writing control group on measures of learning, many mathematics educators today would probably question the value of the type of learning in which the students were engaged.

Davis (1993) cautioned that in both traditional and constructivist teaching approaches "one can make use of writing done by students—but according to which view one holds, one will have different goals, and will, in the long run, shape the instruction in quite different ways. What the students will learn will be different in the two different cases" (p. 300). Furthermore, the differences that result from using writing in the context of varying beliefs about learning are not necessarily limited to a traditional-constructivist dichotomy. Learning goals that vary according to the teacher, the students, and the curriculum can influence how writing is used, what it is meant to accomplish, and what sort of influence it has on students' learning.

Situating writing within a larger context

A second issue that is often neglected in studies of writing is the inevitability of the influence of context. Not only is writing influenced by the beliefs about writing and learning that underlie instruction, but it is also embedded within the complexities of the classroom environment. Hill (1994) pointed out that "both writing and learning inevitably take place within some sort of institutional and social context, and there are an almost endless number of ways in which the context can influence the outcome of writing and learning activities" (p. 2). Schumacher and Nash (1991) similarly stated that even a specifically designed writing task can be processed and carried out very differently "depending on the context in which the task is presented or the manner in which the task is interpreted by the subject" (p. 71). Even consistency in writing tasks and learning expectations may not automatically lead to consistent and stable results. In fact, one reason that has been put forth for the incongruity and inconclusiveness of writing-to-learn research is that the contextual complexities of writing stand in the way of obtaining the clear results that might theoretically be expected (Ackerman, 1993). This inevitable complexity suggests that research would benefit from close attention to factors surrounding the use of writing, in particular the classroom structures that contribute to students' use of writing and the nature of their written work.

Students' understanding of mathematical writing

Finally, differences in student writing may be less a result of differing levels of mathematical understanding than a result of different interpretations of what it means to write a mathematical explanation. Morgan (1998, 2001) examined this hypothesis because of her concern over the increased use of writing as a measure of student learning. By studying both student writing and teachers' interpretations of the students' written explanations of mathematical strategies and results, she found that teachers sometimes looked for mathematically correct *ways* of writing when they evaluated students' written work, rather than just mathematically correct solutions and strategies, even though teachers were often not consciously aware of the non-content-related characteristics that comprised good mathematical writing (Morgan, 2001). Furthermore, what students chose to include or not to include in writing often reflected what they believed the teacher, or whoever was evaluating their work, expected of them rather than the mathematics that they understand (Morgan, 1998). If writing is not necessarily a direct reflection of student understanding, then it is important to look not only at students' writing but also at the context in which the writing takes place and how students understand the writing task or how they learn to create mathematical explanations.

Conclusion and Research Questions

The questions and complexities of writing addressed in the preceding sections suggest the need for different directions in research on writing in mathematics classrooms. As discussed above, the designs used in most research on writing in mathematics classrooms typically introduce some form of writing into a mathematics classroom and then evaluate students' learning through various sources, whether the writing itself or some other form of measurement. But these research designs tend to neglect important aspects of the context in which mathematical writing-to-learn activities take place, which makes it difficult to explain the role that writing might play in students' learning. This in turn has led to some dissatisfaction with the research on writing. However, the increasing promotion of written explanations of solution strategies and mathematical concepts found in classrooms, texts, and professional journals suggests that practitioners do not generally question the idea that writing can be an important learning tool within the discipline of mathematics. In fact, from a social perspective, the ability to participate in legitimate mathematical activity, including writing, actually constitutes a student's learning. Therefore, if students' writing is considered to be participation in a legitimate mathematical activity, and if this writing takes place within the complex context of the classroom, then it makes sense to study the role of writing in students' learning by examining the place of writing in everyday classroom instruction and activities.

Research designs that introduce writing into a classroom and measure subsequent changes in learning also tend to give little attention to how writing is developed and understood within the context of the classroom. This creates an incomplete picture of student writing, and tends to emphasize students' individual cognition more than the outside factors that influenced their thinking and ability to create acceptable mathematical writing. Since the arguments in the preceding section suggest that the content and structure of writing is influenced by context, it seems useful to study some of the ways in which written explanations are developed and supported in mathematics classes that use writing.

In my research, therefore, I will focus on the place of writing in the everyday instruction of a mathematics classroom in which writing plays an important role in student learning. I will address two questions in particular:

1) How are aspects of writing incorporated into the instruction that takes place in a classroom in which writing is an important part of learning and evaluation, and what role do these aspects of writing play in students' mathematical learning?

2) How is a classroom in which the teacher values students' written mathematical explanations structured to support the students' ability to write good mathematical explanations?

Chapter Three — Research Methods

In this chapter I will describe the study designed to answer my two research questions. The study was a qualitative analysis of the instruction that took place in a mathematics course for elementary education majors in which students frequently wrote explanations of the mathematical concepts that they were learning. I examined the classroom instruction for evidence of how this emphasis on mathematical explanations influenced the instruction, as well as ways in which the instruction was structured to support students' ability to write such explanations. I will first describe the setting in which this study took place and how my choice of setting was influenced by my research questions. Following my description of setting, I will describe the sources of data that I analyzed and the process by which I analyzed the data.

Subjects and Setting

The data I analyzed were collected in a mathematics course for elementary education majors during the winter semester of 2001. The course, Basic Concepts of Mathematics, is the first in a sequence of two semester-long mathematics classes required by the major. The sequence of courses is offered through the mathematics education department and is intended to involve students in an in-depth study of elementary-level mathematics, and to introduce them to how children think about mathematics and to ways that they might have their students explore and learn mathematics. The content of this particular section of Basic Concepts of Mathematics included fractions, probability, and early geometry.

The majority of students enrolled in this course were Caucasian females between the ages of 19 and 22. All students were elementary education majors in their first semester of the four semester elementary education program. Basic Concepts of Mathematics is a required course during this semester of the program. The students taking this course typically have widely varied mathematical backgrounds. Some students have struggled with mathematics in the past and feel that mathematics does not come easily or naturally to them, while other students have previously found mathematics to be interesting or enjoyable and look forward to learning and teaching the subject. Despite their varied backgrounds and attitudes, however, most of these students have experienced mathematics in more traditional classroom settings and therefore have little experience with the type of conceptual reasoning about mathematics that they experience in their elementary mathematics content courses. I chose to analyze data for this particular class because the nature of student writing and the extent of its use within the classroom made it an ideal setting for studying questions about student writing in the context of learning and the development of students' written explanations through classroom instruction. The learning that took place in class was focused on developing meaning for symbols and operations, and on making sense of mathematical patterns, formulas, and algorithms. When students wrote, they wrote explicitly about these concepts and so their writing was focused entirely on conceptual understanding and making meaning of mathematics.

Also, writing was an important part of the classroom structure and student learning. On all assignments and tests, students were required not only to solve problems, but to explain their reasoning in a way that directly addressed the mathematical concepts underlying their solutions and reasoning processes. Students were given assignments at the end of each class period, and in these assignments they were expected to write detailed explanations of their solutions to at least four problems, and sometimes more.

Students’ ability to explain their reasoning was directly supported in the day-to-day activities that took place in class. Students were given tasks and questions, which they worked on in small groups and then talked about in whole-class discussions. Students presented ideas and solutions to the class and discussed their reasoning with direction from the teacher. Because the writing related directly to the students’ learning and was an important part of both students’ learning and the teacher’s assessment of students’ learning, it made sense to study the influence of written mathematical explanations on classroom instruction in this particular class. Furthermore, because most students were initially unfamiliar with the way that mathematics was taught and understood in this classroom, they had not had experience with this type of mathematical writing. It was therefore reasonable to expect that their understanding of how to write these mathematical explanations emerged from somewhere within the classroom.

Data Sources

Data were collected over the entire course of the Winter 2001 semester. All class sessions were recorded on videotape, and those tapes were later transcribed. The tapes record the whole-class instruction and discussions that occurred, including explanations of concepts and solution methods that students presented for the entire class. In addition, a literacy professor from the school of education observed the class and took fieldnotes describing what took place on each day of instruction. The literacy professor was particularly careful to record and identify literacy events, including writing activities, during instruction. Because my research questions focus on the role of writing in classroom instruction, the detailed accounts of day-to-day classroom instruction provided by the fieldnotes and the transcripts are sufficient for addressing my research questions.

The transcripts of the class sessions can be divided into three segments of instruction. The first and longest segment consists of a unit on fractions, which addressed meanings and representations of fractions, fractional relationships, and operations with fractions. This segment lasted for about half of the approximately four month semester. It was followed by a unit on probability and another unit on early geometry. For the purposes of this study, I chose only to analyze the instruction that took place during the unit on fractions. This yielded data from fourteen days of instruction, or seven weeks in which the class met for 1-1/2 hours twice a week. Analyzing data from a single unit of instruction allowed me to look at the development of writing in a coherent conceptual progression, in which each new concept built clearly on previous concepts, beginning with definitions of fractions and culminating in a conceptual understanding of the division of fractions and the “invert and multiply” rule. My choice to use the unit on fractions instead of the unit on probability or the unit on geometry provided me with a sufficiently large chunk of data (as the unit on fractions took up the first half of the semester). This choice of data also gave me access to data ranging from the very beginning of the semester when students had had no exposure to the type of written explanations that would be required of them, to the middle of the semester when they would be expected to have a relatively clear understanding of how to write appropriate mathematical explanations.

Analytic Methods

I approached my analysis from a grounded theory perspective (Strauss & Corbin, 1998). Charmaz (1995) explains grounded theory as a "set of inductive strategies for analyzing data" (pp. 27-28). That is,

> …you start with individual cases, incidents or experiences and develop progressively more abstract conceptual categories to synthesize, to explain and to understand your data and to identify patterned relationships within it. (p. 28)

I began my own analysis by choosing a set of particular incidents that occurred within the classroom according to criteria that was based on my research questions and which I will further explain later in this chapter. I then developed categories and models to explain the data from the perspective of the development of students' written explanations. I wrote memos to describe my categories as they evolved and their relationship to the data and to my research questions, and revised my categories, descriptions, and theoretical models through a process of introducing new data into my analysis and continually referring back to data that I had previously analyzed. I will use the remainder of this chapter to describe the process in more detail.

Familiarization with the data through fieldnotes

I prefaced my analysis of the classroom transcripts by reading through all of the fieldnotes that had been taken during each class period. This reading served two purposes. First, it gave me an overview of the entire class. I was able to acquaint myself with how instruction typically took place, what mathematical topics were covered in class and how they were covered, and how the teacher and students interacted on a day-to-day basis. The fieldnotes provided me with an outline of what went on in class, broadly summarizing details that I could later look at more closely in the transcripts. Thus I was able to familiarize myself with the data before beginning my analysis.

Second, I read through the fieldnotes for the purpose of determining where I wanted to start my analysis of the classroom transcripts, and for the purpose of thinking about the direction I wanted to take with my analysis. In particular, I identified episodes of instruction that appeared relevant to my research questions. These episodes of instruction ranged in size from short explanations of a particular topic that may have been as little as a few minutes in length, to longer segments of class that could have occupied up to half of the 1 1/2 hour class periods, including the time spent in whole class discussion and the time spent in related group work. These episodes included those in which the teacher referred directly to students' written explanations. Since the fieldnotes had been taken by a literacy researcher who was particularly interested in classroom literacy activities, including writing, these fieldnotes were especially useful for the purpose of tracing events related to writing.

I also noted episodes of instruction that, though they did not necessarily involve direct reference to students' written explanations, nevertheless appeared to be relevant to the way in which students learned to write explanations. These primarily included episodes in which the students' use of language was a topic of discussion, or in which student language seemed to be closely tied to their conceptual learning. Other aspects of instruction in addition to language that at first glance did not appear relevant to writing emerged as relevant in later stages of my analysis.

Initial transcript analysis

I used the episodes of instruction that I identified in the process of reading through the fieldnotes as described above in order to choose particular segments of data with which to begin my analysis. Because the transcripts were organized by class period, I chose several different class periods in which one or more of these episodes of instruction appeared. From these transcripts, I chose one date of instruction in particular that contained both explicit references to writing and typical classroom instructional episodes. This is the date with which I chose to begin my analysis.

I did not begin coding the data immediately. Rather, I read through the entire transcript to familiarize myself in more detail with the events that had taken place in class that day and began to write memos detailing how I was making sense of the relationship between instruction and whole-class discussions, and students' written explanations. I looked for explicit references to student writing that helped to identify characteristics of writing that the teacher valued, as well as whether and how such characteristics appeared in classroom instruction that did not explicitly refer to written explanations. I also looked at how instruction and discussion conveyed messages about how to communicate the mathematical concepts that were being taught. I wrote down my observations and interpretations, and then checked those observations against re-readings of the entire transcript and more focused re-readings of specific parts of the transcript to determine whether my tentative observations about particular events were accurate in the context of other classroom events.

I observed two patterns in the data. First, I observed that there were different types of instructional events that took place. Some of these instructional events took place at the group level, such as activities in which students worked to solve new problems, or times that were set aside for students to discuss previously completed assignments with their group members. Other types of events took place at the whole-class level, including student presentations of solutions to problems, teacher explanations of concepts that students were struggling with, and class discussions about unfamiliar mathematical concepts. This observation was supplemented by what I had observed during my reading of the fieldnotes. I noted that in most of these instructional events, some sort of explanation was taking place. For instance, when students worked in small groups they were explaining concepts to each other. At other times, the teacher explained a concept to the students, and at still other times students presented explanations to the class. I began to categorize these events in which explanations were given because I believed that the nature of explanations given in class and the feedback that students received on these explanations might have bearing on the written explanations that students eventually produced. I also noted that the conceptual context of an explanation seemed to influence the nature of that explanation. For example, the teacher's explanations of new concepts were different from the teacher's explanations of concepts that the class had already talked about. Therefore, as I began to categorize types of explanations, I categorized them according to who was giving the explanation, the apparent purpose of the explanation, and the class's relative familiarity with the concept. It was then natural to order them according to their place in the progression of conceptual instruction, and I developed a tentative progression of instructional events by which students learned mathematical concepts.

Second, by focusing carefully on how writing was addressed in class I noted that writing was sometimes addressed explicitly (that is, it was readily apparent that the class was talking specifically about written explanations) and sometimes implicitly (that is, written explanations were not spoken of directly, but the instruction still addressed ideas that could potentially carry over into student explanations). I further noted that there seemed to be varying levels of explicitness of writing instruction, and created a tentative model of a continuum of explicitness of writing instruction along which instructional events could be placed. On one end of the continuum, writing was addressed very implicitly, almost hidden within conceptual instruction, and at the other end of the continuum, writing was very explicit and central to the instruction. I used specific instructional events from the day I was analyzing as guidelines for developing and describing this continuum. I then simplified the continuum for the sake of describing the data by dividing them into four categories representing four levels of explicitness of writing instruction [Figure 3.1].

Implicit ←→ **Explicit**

1. Writing addressed entirely implicitly	**2. Writing addressed mostly implicitly**	**3. Writing addressed somewhat explicitly**	4. **Writing addressed very explicitly**
Writing instruction (if it occurs at all) is embedded, almost hidden, within conceptual instruction.	Writing instruction is not explicitly addressed, but aspects of writing are central to the conceptual instruction.	Explicit discussion of students' written explanations is interjected in conceptual instruction.	Writing is the explicit focus of an entire episode of instruction.

Figure 3.1
Continuum of Levels of Explicitness of Writing Instruction

Expanding the analysis, refining ideas, and developing codes

At this point in the analysis I began to check both of my initial observations and models against data from other days of instruction. During this segment of analysis I gradually expanded my data set to include about half of the seven weeks of fraction instruction, including days of instruction for each of the major topics (definitions of fractions, ratios, addition and subtraction, multiplication, and division). In order to check and refine my model of varying levels of explicitness, I categorized specific classroom events, which ranged in size and complexity from single utterances to entire instructional activities, according to the categories of explicitness that I had created. To do so required defining what I meant by explicit and implicit reference to writing, as well as defining the aspects of classroom instruction that I considered relevant to writing. At the same time, these definitions themselves emerged as I applied the model to the data.

This process of data analysis, then, was a recursive process. I developed informal definitions for what constituted explicit and implicit writing instruction in order to describe my model, and I identified elements of instruction that were relevant to writing in order to determine the types of information about written explanations that appeared in classroom instruction and discussion. I used this information to identify the types of events that were described by each of the four levels of my model. Then I applied my model to new data and revised the model as necessary to take into account instructional episodes that did not fit into my model of classroom writing instruction as it had been defined. Throughout the process I frequently returned to previously analyzed data in order to check the functionality of my revised models.

At first, because I was mostly concerned with the development of students' ability to write conceptual explanations, I focused primarily on ways that students had access to knowledge about how to write their mathematical explanations. However, as I struggled to develop a model that would fit the data I realized that if I was categorizing some writing instruction as implicit, I had to take into account what the writing was implicit in relation to. I therefore adjusted my continuum to take into account the students' conceptual learning. At first, my new model reflected this idea by suggesting that on one end of the continuum, information about writing was learned implicitly while classroom instruction was focused on conceptual understanding, and that on the other end of the continuum, conceptual understanding became implicit while the instruction centered on student writing [Figure 3.2]. This model implied that there was a range of instructional activity between the two extremes where the relative importance of writing and conceptual instruction were much closer together.

Implicit Writing/ Explicit Concepts ⟷ **Explicit Writing/ Implicit Concepts**

1.	**2.**	**3.**	**4.**
Concepts are the explicit focus of instruction. Writing is addressed implicitly.	Concepts are still the explicit focus of instruction, but writing plays a more important role.	Writing is the explicit focus of instruction, but concepts still play a very important role.	Writing is the explicit focus of instruction. Concepts are addressed implicitly.

Figure 3.2
Modified Continuum of Explicitness of Writing Instruction

But the more I examined classroom events, the more difficult it became to position events in the four categories along the continuum. I finally determined that one of the difficulties lay in assuming that either writing or concepts must necessarily take precedence in a particular instructional episode. I subsequently changed my model from a one-dimensional model to a two-dimensional model that could simultaneously take into account conceptual learning and learning about aspects of written explanations, without requiring one to be more important than the other. I represented this two-dimensional model as a four cell table, with each cell representing different levels of importance of the importance of concepts and writing to instructional episodes [Figure 3.3]. Applying this new model to the data much more clearly illustrated the relationship between conceptual understanding and aspects of writing that were developed over the course of the class.

At the same time, I returned to my observation that in-class explanations were influenced by the instructional context in which they took place. I had already noted a general progression of conceptual instruction, and so situating the development of explanations within that context allowed me to look not just at what aspects of writing were addressed in class, but at how written explanations were developed over the course of instruction. I looked more closely at how explanations that were given in class differed in form and function and determined that some explanations, but not all, served as models for the type of explanations that might be expected in student writing. Based on this observation, I began to code explanations in the data as either explanations that served as models or explanations that served other purposes. In so doing I began to develop a definition of what constituted a model, and to observe the function of modeling in classroom instruction and in giving students access to knowledge about how to write mathematical explanations, and how that function differed according to the instructional context.

CONCEPTS	Explicit focus of instruction	Concepts are the explicit focus of instruction; writing, if addressed at all, is addressed implicitly.	Both concepts and aspects of writing are central to the instruction.
	Not explicit in instruction	Neither concepts or writing are explicit in instruction (generally classroom "business" or tangents from instruction).	Writing is the explicit focus of instruction; mathematical concepts, if addressed at all, are addressed implicitly.
		Not explicit in instruction	Explicit focus of instruction
		WRITING	

Figure 3.3
Two Dimensional Model of Conceptual and Writing Instruction

Further expansion of the analysis and formal coding

During the final stage of my analysis I expanded my analysis to include all eleven class periods that had been transcribed out of fourteen class periods of instruction on fractions. This final stage consisted of defining terms, formally applying codes to the data, and interpreting the patterns that were emerging in the process of coding and analyzing the data. By doing so I was able to formalize and justify my developing theories as to how my research questions were answered by the data.

In order to define different regions of my model according to the importance to classroom instruction of both conceptual understanding and aspects of written explanations, I identified what constituted conceptual instruction, and what types of instruction were relevant to written explanations. In doing so, I was able to identify three specific components of written explanations that were addressed in class, both through explicit writing instruction and day-to-day conceptual instruction. I will discuss these three components in more detail in my results section.

I then applied my two-dimensional model of classroom instruction to the data by identifying the importance of mathematical concepts and the importance of components of written explanations to each instructional episode in the transcripts I had analyzed. I found that explicit reference to these components of writing was not the only indicator of the relative importance of these components to a particular instructional episode. Even in episodes in which writing was never mentioned I found indications that an understanding of these components was often crucial to the development of students' conceptual understanding. Similarly, I found that episodes of instruction that I labeled as focusing explicitly on writing while appearing not to focus explicitly on mathematical concepts were still conceptually-motivated. The results from this portion of my analysis enabled me to answer my first research question about how aspects of writing are incorporated into instruction and the role of these aspects of writing in student learning. I will address these findings in my first chapter of results.

In the context of this emerging understanding of how writing was addressed in various classroom events, I refined my definition of a model explanation. I identified different ways that model explanations emerged in the instruction, and positioned these different types of models within the progression of conceptual instruction that I had identified at the beginning of my analysis. In so doing, I was able to identify a corresponding progression by which explanations of individual mathematical concepts were developed within the class. This answered my second research question about how the classroom was structured to support students' ability to write mathematical explanations. My findings from this portion of my analysis will be the subject of the second chapter of results.

Chapter 4 — Results: The Relationship between Writing and Conceptual Instruction

The primary purpose of my research was to study the place of writing in the everyday instruction of a mathematics classroom in which writing was an important part of student learning. One of the assumptions underlying my analysis, which I addressed in my conceptual framework, was that students' ability to write conceptually oriented mathematical explanations requires more than just an understanding of the concepts. Therefore, while I recognized the centrality of conceptual instruction in the classroom, my analysis was focused not on how students learned mathematical concepts but on the role that particular aspects of writing played in everyday learning activities. I analyzed classroom instruction for specific ways that students were given access to particular components of written explanations that would be necessary for them to understand in order to explain their developing conceptual understanding in writing. In the course of my analysis, three components of written explanations emerged as being important to the creation of written explanations. These components—language, expectations for what needed to be explained, and purpose and audience—could be identified throughout the seven weeks of classroom instruction that I analyzed, and once identified served as a lens through which I could examine classroom events from the perspective of my two research questions.

My first research question was as follows: How are aspects of writing incorporated into the instruction that takes place in a classroom in which writing is an important part of learning and evaluation, and what role do these aspects of writing play in students' mathematical learning? My primary focus, then, was how these aspects of writing that I identified appeared within the conceptual instruction, and I considered the overall development of students' conceptual understanding to be outside the range of my analysis. However, when I tried to understand how these three aspects of writing emerged over the course of the unit of instruction that I studied, I found that an attempt to focus on writing alone was insufficient. My conclusion is that, for this classroom, the development of students' knowledge for written explanations could not be separated from the development of their conceptual understanding, and that learning concepts and learning to explain those concepts in writing were mutually supportive activities. The nature of students' understanding was shaped in essential ways by the fact that students were required to explain their understanding in writing on a regular basis, and the ways that students learned to write their explanations were motivated by the development of their conceptual understanding.

This conclusion directly addresses my research question. Aspects of writing were incorporated into instruction as a natural part of students' conceptual instruction, and in fact played a crucial role in students' learning by virtue of being inseparable from the mathematical concepts themselves. In my first chapter of results, therefore, I will discuss how writing was enmeshed with conceptual instruction within the classroom. This will involve explanations not just of how writing was addressed in the classroom, but also of how writing was motivated by conceptual instruction and how mathematical concepts themselves were shaped by aspects of writing.

I will begin this first chapter of results with a brief overview of the three components of writing that I identified through my analysis. After this overview, I will show how individual instructional events throughout the semester provided students with access to these three components of written explanation in close conjunction with the development of their conceptual understanding. This occurred in two ways. First, in most of the classroom instruction, particular aspects of written explanations emerged from conceptual instruction and were integral to that conceptual instruction. Second, written explanations were themselves addressed explicitly in class, and were driven not just by concerns with student writing but by underlying conceptual concerns. I will address each type of instruction, conceptual instruction and instruction centered on students' written explanations, individually.

Overview of Components of Written Explanations

Language

By language I mean the use of words to describe or refer to particular ideas. On one level, this refers to the development of vocabulary, such as students' association of the words *sharing* and *measurement* with two different processes of whole number and fraction division (1-22)[1]. However, it also refers to the way words, combinations of words, and grammar are used to convey particular meanings. For instance, the description of $a/b \times c/d$ as "a/b times c/d" conveys little about the meaning of the symbols in contrast with other phrases that students used, such as "a/b groups of c/d," "a/b copies of c/d," and "*a*-*b*ths of c/d," all of which convey images of actions on quantities (2-12).

Language is an indispensable component of both understanding mathematics and writing conceptually oriented explanations. In order to understand an idea, students must have a language with which to communicate this understanding. Communication and language were especially important in this class of elementary education majors because they were being prepared not only to understand and use mathematics, but to share this

[1] Excerpts from data are here referenced by month and day, as well as by page number when necessary.

understanding with their own students. Language, therefore, played an important role in classroom instruction. In order to explain concepts, particularly in writing where a great deal of context was necessarily implicit, students needed a vocabulary with which to explain mathematical ideas and relationships and the way they understood these ideas and relationships. They also needed an awareness of how their language would be interpreted by people other than themselves, whether those people were their classmates, their teacher, their future students, or a wider community of mathematicians.

What to Explain

Understanding what to explain means understanding when an explanation is sufficient and being able to identify details and ideas that may need further explanation or clarification. For example, students were expected not only to find a common denominator for fraction addition problems and to be able to explain the meaning of the common denominator in terms of pictures of fractions, but also to explain why it was necessary to find a common denominator in the first place (2-12, p.1). Students who intuitively understood the need for a common denominator in order to add the fractions together may or may not have understood that it was necessary to explain this particular aspect of the process in a written explanation.

A knowledge of what to include in an explanation is especially important for written explanations because of the lack of context and feedback inherent in writing. In spoken conversation the person explaining has the advantage of feedback from the listener to guide what he or she says. If the speaker makes a conceptual leap, the listener can give verbal and non-verbal cues that tell the speaker that the explanation was insufficient or ineffective. In writing, however, writers must judge for themselves both what needs to be explained and what can be taken as shared understanding and left implicit, and must do so without the help of a physically present listener or reader. This is further complicated when the reader of the explanation will use the writing for the purpose of judging the understanding of the writer in some way, as was the case in this class, where student explanations were partly intended to give information to the teacher about their level of understanding. In such a situation, the writer also needs to know what the reader will be expecting to see, and what elements of understanding the writer needs to make clear in order to fully demonstrate his or her knowledge.

Purpose and Audience

The purpose of an explanation refers to more than just the purpose of the explanation as an assignment that will help determine a students' grade; it refers to the reason such explanations are assigned in the first place, and the motivation for students to learn to create this particular type of explanation. This is closely related to the audience of the explanation, or the person or group whom this explanation is intended to address or to convince. For example, the nature of an explanation might be very different for a student who considers the audience to be the teacher and the purpose to be to meet the teacher's requirements and receive a good grade, than it would be for a student who considers the audience to be someone who does not fully understand the concepts and the purpose to be able to clearly explain the mathematical concepts to that person.

An understanding of the purpose and the anticipated audience of an explanation is important because that understanding can guide a student's choice of what to include in an explanation and how to make use of language. Writing is a form of communication and therefore there is always an implied purpose and audience. Whether or not this is explicitly addressed in class, it is always present as a necessary component of writing of which students need to be aware in order to best create an explanation that will be effective for that particular setting. A misunderstanding of the purpose and audience of a mathematical explanation could very well lead to an explanation that is inappropriate to the actual context.

Each of these three components of written explanations appeared in instruction throughout the class's unit on fractions, and will each be addressed in more detail within the subsequent sections of this analysis.

The Emergence of Writing as Integral to Conceptual Instruction

A cursory look at the data from the perspective of how students learned to write conceptually oriented explanations seemed to show that instructional events in the classroom could fall under one of two categories. Either the instructional events clearly used student writing as the primary impetus for instruction or they clearly did not. I will discuss those episodes in which writing was the impetus of instruction in the next section; in this section, I will concentrate on the classroom instruction in which writing was not the clear impetus. I will discuss how, throughout conceptual instruction in which student's writing was not explicitly addressed, the development of the understanding necessary for writing was still supported. In fact, students' understanding of language, what to explain, and purpose and audience was also an essential part of how students learned mathematical concepts.

I will organize this section around the three components of writing that I identified and described in the previous section. For each component, I will discuss first how the component of writing developed through the conceptual instruction, and second how that component was integrally and essentially linked to the development of students' understanding of concepts.

Language

Language issues were addressed by giving students access to new language and helping them to refine existing language use. This occurred in several ways. Students were supported in developing appropriate ways to talk about new and unfamiliar concepts, were introduced to new ways of talking about familiar concepts, and were guided to recognize and refine language that they were already using.

First, appropriate ways of talking about new and unfamiliar concepts emerged throughout the semester because, although the topics covered in class were elementary-level mathematics topics that the students had been exposed to through years of schooling, these topics were approached in nontraditional ways that were unfamiliar to the students. The language that students had used to talk about fractions prior to their enrollment in this course was tied to an algorithmic conception of fractions and fraction operations, and was therefore insufficient for explaining the visually-oriented conceptions that formed the basis of their learning in this class. This necessitated the development of new language in conjunction with the development of new mathematical concepts. For example, when the class discussed the addition and subtraction of fractions, students were faced with the task of explaining common denominators. The images suggested by the symbolic procedure of multiplying the numerators and denominators of fractions by the same number did not necessarily coincide with their developing understanding of fractions, and classroom discussion subsequently focused on making meaning of the procedure in terms of actions on quantities that could be represented meaningfully with pictures.

In the process of coming to understand and explain the operation in terms of these pictures, students needed a language to describe both the process and the relationship between the pictorial process and the symbolic process of creating common denominators. In particular, they needed to be able to describe this process as a process of partitioning. From the beginning of the instruction on addition and subtraction, the denominators (and common denominators) of the fractions were frequently referred to as the number of partitions in a whole (2-5, p. 1; 2-7, p. 16; 2-12, p. 3). And in addition to introducing the language of partitioning into explanations of common denominators, the teacher asked the students questions meant to help them think and talk about the process as one of partitioning. For instance, one episode of instruction was motivated by the teacher's question, "What does multiplying the numerator and denominator do to your fractions?" (2-5, p. 1). The immediate student response, "It just changes the size of your pieces," is representative of the language of piece size and partitioning around which the subsequent student and teacher explanations were centered. The ability to talk about common denominators in terms of partitioning and piece size was essential to students' ability to create conceptual meaning for the algorithmic procedures associated with common denominators, as well as addition and subtraction of fractions.

Second, new language was developed in conjunction with more familiar concepts in ways that allowed students to reflect on their existing conceptual understanding by talking about these concepts in different ways. Development of language in relation to familiar concepts was also important to the instruction because new conceptualizations of fractions and fraction operations built upon concepts that had been discussed earlier in the instruction as well as the understanding with which students entered the class. For example, having been through years of school mathematics, students were able to use and identify fractions effectively given images or quantitative situations when they entered the class. However, because the first half of the course was intended to help students develop richer meanings for fractions and fraction operations, being able to recognize and use fractions was not enough. Students needed to be able to explain why they made particular judgments about fractions (how they knew, for example, that a picture or object that represented 2/3 was actually 2/3). The students' ability to explain fractions at a basic level was necessary both for their own sake, since they would later use that basic understanding to make sense of fraction operations, and for the sake of the children they would eventually teach.

Therefore, in order to help students develop powerful language for describing fractions, the first few class periods were devoted in part to helping students to develop a language with which to talk about their implicit understanding. Students were asked to explain their understanding of the definition of a fraction early on in the context of activities involving representations and comparisons of fractions using Cuisennaire rods. As they did so, the teacher focused on ways of explaining fractions that were most effective, in particular the complementary ideas that a quantity of size $1/n$ could be iterated n times to form a whole (an iterating definition of fractions), and that if a whole were broken into n equal parts, each part would be of size $1/n$ (a partitioning definition of fractions). He focused on these ways of explaining fractions by drawing students' attention to these two ideas when they showed up in student explanations (1-10, p. 6, p.

11; 1-17, p. 7), and also by drawing their attention to the essential differences between different ways of explaining the meaning of a fraction (1-10, pp.11-12). He also gave them opportunities to practice using their developing language in new situations, such as having students explain fractions in terms of sets of objects once they had discussed fractions as parts of a solid whole (1-10, pp.15-21), while encouraging them to be aware of the language they used and how it reflected mathematical conceptions. Some of the mathematical concepts made clear in these descriptions of fractions had already been present in the students' understanding, and through class discussion and instruction they were able to develop a common language with which to talk about these ideas effectively.

Finally, in addition to developing facility with new language in the context of new and familiar concepts, the language that students were already using was discussed and refined. This involved making students aware of how their use of particular words or phrases or ways of speaking about concepts perhaps unintentionally reflected certain conceptions, and sometimes misconceptions, of mathematics. Notice that the process of refining language is related to the process of developing language. In the example of the definition of fractions, for instance, developing new language to talk about familiar concepts involved recognizing why some students' initial language for talking about fractions was insufficient in a mathematical context. A common way for students to explain the meaning of $1/n$ was to say that $1/n$ meant one out of n pieces. The teacher demonstrated several unintended mathematical consequences of this description, such as the impossibility of fractions greater than one. For instance, if $1/n$ means one out of n pieces, then 5/4 means 5 out of 4 pieces, which does not make sense (1-10, p. 14). It was partly this awareness of what was intentionally communicated by certain uses of language that led to the development of very specific new language with which students could more effectively communicate their underlying conceptions of fractions.

Student language that was developed over the course of instruction (as opposed to the language with which students entered the class) also underwent examination and refinement. As students learned to describe the process of creating common denominators as a process of partitioning, they also developed language to explain why their particular choice of a common partition worked for both of the given fractions in an addition or subtraction problems. For example, when adding 5/6 + 3/8, some students found a common partitioning by cutting each sixth into eight pieces and each eighth into six pieces, creating a whole divided into 24 pieces [Figure 4.1].

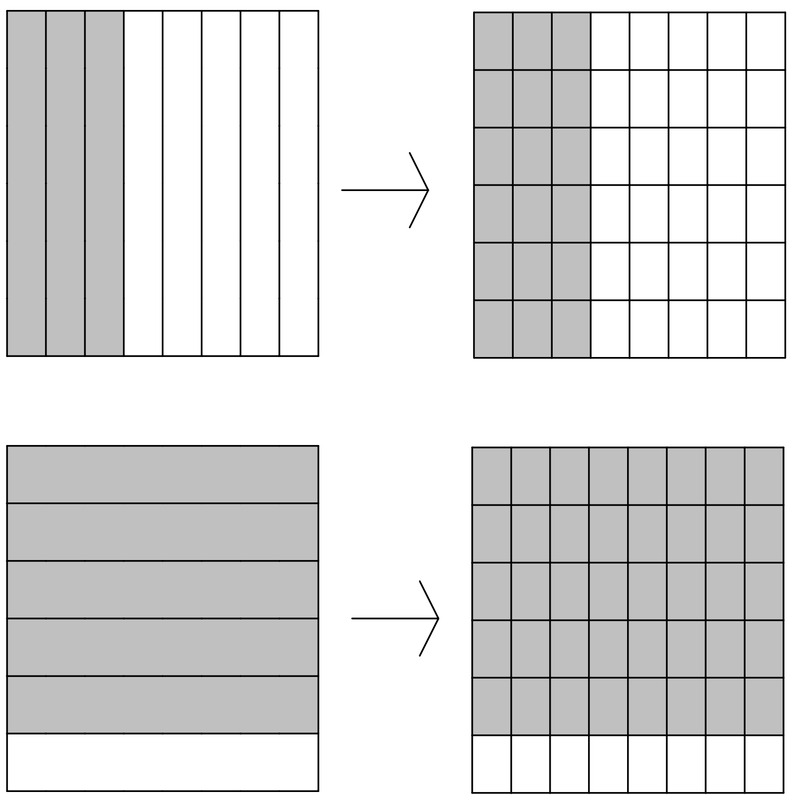


Figure 4.1
Common Partitioning for 3/8 and 5/6.

One explanation that arose in student explanations for why this partitioning worked was that, from the partitioning, one could make “groups of sixths and groups of eighths” (2-12, p.1). This description in particular obscured much of the underlying understanding. The teacher explicitly drew the students’ attention to this difficulty with language. He asked students to think about what they really meant by a “group of sixths,” and one student replied that the “group of sixths” was “actually…1/8 out of the whole” (2-12, p. 3). By explaining that the 24 pieces could be grouped into “sixths” and “eighths” students had intended to describe that the 24 pieces could be grouped into groups of six pieces in order to form eighths, and groups of eight pieces in order to form sixths. The teacher’s concern with students’ level of conceptual understanding, as expressed in their use of language, led to the clarification and refinement of student language as part of the clarification of their understanding.

Although language developed in conjunction with students’ conceptual understanding, the development of language was not just a reflection or natural consequence of the development of students’ conceptual understanding. The language that students developed and used shaped their understanding of the concepts, and was a critical component of that understanding. The language that students used to talk about fractions helped students to formulate their understanding of fraction relationships and operations throughout the semester. Being able to talk about fractions from a partitioning perspective led to students’ understanding of common denominators as a way of partitioning the whole (2-12, p. 6). Being able to talk about fractions from an iterating perspective helped students to be able to see and explain fractions in the context of fraction operations such as addition and subtraction (2-12, p. 8) and division (2-26, p. 4).

Similarly, the discussion of the language that students used to describe and justify the process of creating common denominators allowed them to conceive of the relationship between common denominators and the addition or subtraction of fractions with unlike denominators in terms of pictures and models. In general, the language students used to describe certain ideas and processes influenced how they were able to conceptualize those particular ideas and processes.

What to include in an explanation

Of the three components of writing that I identified, what to include in an explanation was the most readily apparent component of writing in classroom instruction. As the teacher asked students to explain what they meant by certain statements as they presented solutions to problems in front of the class and to justify why they made certain choices in their solution processes, he implicitly communicated to them what they would need to take into account when they created their own written explanations later on. When a student explained his or her thought process to the teacher or to the class, the questions the teacher asked served not only to clarify the students' explanation, but to implicitly inform the students of the details that needed to be included when they wrote their own explanations. And similarly, when the teacher went over the solution processes for particular problems with the class, the details he chose to explain and the questions he directed towards the students also served to inform students of the details that needed to be included in an explanation.

One example of the role of teacher questioning in making students aware of what they needed to be included in an explanation can be found, again, in student explanations of common denominators. The concepts students learned in this class were always grounded in pictures, and so an important part of understanding addition and subtraction of fractions was understanding how to work with picture representations of the fractions in order to find a common denominator, and how to explain why the process used to find the common denominator actually worked. The strategy mentioned in the preceding section of partitioning each fraction in an addition or subtraction problem according to the denominator of the other fraction was a strategy used frequently by students. For instance, for the problem 2 1/3 + 1 1/7, students partitioned each 1/7 into three pieces, and each 1/3 into seven pieces. This then formed pieces of size 1/21 that could be added together.

Simply describing this process was insufficient, however, as evidenced by the teacher's questions following the creating of the 1/21's from thirds and sevenths:

Teacher: Why does this work? How do you know…that making sevenths here in each one of the thirds and making thirds here in each one of the sevenths works? It's two things we're explaining here.

There followed an discussion of the aspects of the process that needed to be further explained, namely that splitting sevenths into three pieces and thirds into seven pieces created an even partitioning that could be grouped into either sevenths or thirds as necessary, and that this even partitioning created equal-sized pieces that could subsequently be added together (2-7, pp. 16-17). The teacher then reemphasized these particular aspects by having students explain them.

Teacher: Somebody else try it…Why does it work? Dividing the seven into three parts and three parts into seven?

Student 1: You make it…into the same size pieces so you can compare them, so that the fractions are the same size pieces.

Teacher: Okay, so that's one part of it. You know that if you do that, you're gonna get equal size pieces on both sides, right? Okay? What's the second part?

Student 2: You can make groups of threes and groups of sevens.

Teacher: That's right. You can get groups of three [pieces] which corresponds to sevenths, and you get groups of seven [pieces] which corresponds to thirds.

(2-7, p. 17)

Such instructional episodes occurred constantly. Either students explained their solutions and the teacher pushed them to explain more if their explanation had been insufficient, or the class discussed a particular problem together and the teacher emphasized particular aspects of the explanation. Such episodes served functions related to both writing and to conceptual understanding. As for writing, the parts of the explanation that occurred in class emphasized to students what needed to be explained when they went to write their own explanations. This was especially the case when particular aspects of the explanation were either explicitly emphasized, as above when the teacher specified, "it's two things we're explaining here" (2-7, p. 16), or repeated from one problem to the next, as when the teacher emphasized throughout the entire unit on fractions that when fractions arose students needed to explain how they could see the fractions using the definitions of fractions that had been developed at the beginning of the semester. Thus, because helping students to understand the concepts necessarily required emphasizing and explaining particular key aspects of the concepts, such explanations closely informed the eventual written products that students produced.

Conversely, students' understanding of what needed to be explained when solving a particular problem was an important part of their conceptual understanding. Students' awareness of which aspects of a solution process were necessary to explain shaped the type of understanding that was expected from students. Because the teacher emphasized continual explanations of the meaning of numbers and operations even after students might have been expected to have become comfortable with such meanings, students' understanding of newer concepts was constantly based solidly on their explicit understanding of previous concepts. And frequent questions on why actions and operations made sense or what numbers and operations meant in terms of the accompanying pictures led students to make connections between ideas that they might not otherwise have made.

Purpose and Audience

If what to explain was the component of writing that was most obvious during classroom instruction, purpose and audience was the least. In the data that I analyzed, students were never explicitly told the purpose of their writing or the audience to whom they should address their explanations. This meant that students' access to an understanding of the purpose and audience of their explanations was almost entirely based on implicit classroom communication. This is not to say, however, that students had no access to an understanding of the purpose and audience of their writing based on what occurred in class. Rather, classroom discussions and everyday instruction did create a distinct context within which students could make sense of the purpose of their explanations and the intended audience for their writing, but there was little if any explicit reference to this component of writing.

My observations on the development of a sense of purpose and audience within the learning context of the classroom reveal a potential conflict that could have influenced students' approach to written explanations. The teacher himself showed awareness of this conflict at one point in class when he verbalized a concern about the way students approached their writing for the class:

> I think there's two mutually conflicting goals at times in this classroom. One is that there is this goal to put down something on the paper that I will recognize as full credit. Ok? And so you're, you constantly feel that pressure. Right? You've gotta word it right, you've gotta put it right so that you can get full credit from me. There's another pressure that I hope you will feel, and you'll feel it much more once you get out of this class and you get in a classroom of your own. And that is a pressure to make sense of the mathematics so that you can present it to your kids or talk to your kids about it, or somehow engage them in a mathematical conversation that's meaningful. (2-12, p. 11)

Here the instructor recognized a conflict that was apparent in the implicit messages students received about the purpose of their written explanations and the audience to whom the explanations were addressed. On the one hand, the immediate purpose of the explanations was for students to demonstrate their understanding of mathematical concepts to the teacher so that he could evaluate their understanding and assign them grades accordingly. Tests and assignments received comments and point values, and while the instructor admitted to the class that "there is also this evaluative aspect of what goes on in this classroom, and if I had my choice, we would get rid of that" (2-12, p. 13), evaluation was still a very real part of the context in which students were writing their explanations. The reality of the students' writing was that the writing was ultimately seen by the teacher and judged by the teacher, and the implication of this reality was that students' audience was the teacher as an evaluator of knowledge and their purpose was to say whatever would satisfy the teacher as evaluator.

On the other hand, a second implied purpose for the writing conflicted with the immediate purpose of the writing. The students in this particular class were being prepared to teach students of their own, and therefore the ability to explain a concept so that it would make sense both to themselves and to someone who did not already have a full, explicit understanding of that concept was a key component of their learning. The instructor frequently interjected explanations with references to "your students," and to children's understanding. For example, when re-emphasizing the importance of using the definition of fractions in fraction operations such as division, the teacher said, "It all goes back once again…to the meanings that we have associated with these fractions…I keep saying that but if you want your kids to understand these things…they've got to go back and say, well what does it mean to be a sixth?" (2-26, p. 4). The purpose that can be deduced from such statements is that the ultimate purpose of these explanations is to be able to clearly explain a concept to children later on, and that although children are not the immediate audience of these explanations, the students need to keep children in mind as they write.

The conflict here lies not so much in the students' sense of two different audiences. Ultimately, the two audiences should coincide in that the type of explanation that this particular teacher as an evaluator of student knowledge expected was also the type of explanation that would be most clear and helpful to an audience outside of students' classroom experience. Instead, the conflict is that the immediate audience is one that is familiar with the concepts that students have been learning and the idiosyncrasies of the classroom language, whereas the intended, never present, and far more abstract audience is an outsider who has not had the same experiences and conversations that these students have had. It is possible that writing to this vague and perhaps very unfamiliar audience might make it difficult for the writer to judge how to best communicate with the unseen reader.

The purpose of the writing is also in conflict. As seen above, students were told that they should write in such a way that their explanations made sense. But at the same time the evaluative function of their writing meant that the teacher was the ultimate authority who could judge whether or not the explanation made sense. This meant that the students' purpose of explaining in order to make sense of the mathematics and communicate their understanding in such a way that it would make sense to a reader might have conflicted with the students' reliance on the authority of the teacher to determine what types of explanations "make sense" rather than their own understanding.

Still, although the emergence of an audience and purpose for writing may have presented certain conflicts that in turn may have led to student difficulties in creating written explanations, the sense of purpose and audience that emerged through the instruction were nevertheless essential to the students' mathematical learning. On a large scale, students' awareness that the ultimate audience for their explanations would be the students whom they would teach as elementary school teachers shaped how they approached learning and how they understood the concepts. The teacher frequently couched mathematical explanations in terms of a child's understanding. For instance, at one point early in the semester the teacher equated one and one half with three halves, and when students accepted this equality without question because of their familiarity with fractions, he suggested that a child probably would not see three-halves in a picture of one and one-half (1-10, p. 3).

The students themselves frequently showed an awareness of children as the eventual audience of their explanations by asking how the concepts made sense from a child's perspective and referring to how they themselves might teach the concepts so that their students would understand. One student explained the reason that $3 \times 4 = 4 \times 3$ by referring to a way of understanding that made sense to her as a young child (2-12, p. 23). Another student recalled learning that multiplying the numerator and denominator of a fraction by the same number in order to find a common denominator was the same as multiplying the whole fraction by one, and struggled to make sense of how this fit in with the concepts they were learning by asking how and when such an idea might be taught to children (2-5, p. 2). These and similar classroom events pushed students to be aware of reasons behind mathematical connections that they were able to make with little thought, and thus shaped how they thought about the concepts and about what it meant for them to understand the concepts. Had students' ultimate audience and purpose been different, both the nature of their understanding and their approach to learning in this class would likely have been very different. Thus audience and purpose not only emerged in the course of the students' conceptual understanding, but crucially affected the way that students came to understand mathematics in this class.

Conceptual Concerns as a Driving Force for Writing Instruction

In the previous section I discussed how particular components of written explanations played a role in classroom instruction and student understanding, even when explanations were not the explicit focus of instruction. I will now discuss those episodes of instruction in which writing played a prominent role. I should note that the instructional episodes that I have chosen to include in this section do not constitute a comprehensive list of such episodes, but are representative of the types of instructional events in the data I analyzed that make use of student writing. My analysis of these episodes serves two purposes. First, the analysis helps to show more of the ways that students in this class were given access to the components of writing that I identified above. And second, the analysis of these particular episodes strengthens the argument that the ways in which students learned to write explanations were inseparably connected to the ways in which students learned the mathematical concepts. The relationship of writing to concepts in apparently writing-oriented instructional episodes is similar to the relationship of writing to concepts in the apparently concept-oriented instructional episodes that I discussed above. That is, just as above, components of writing both contributed to conceptual understanding and were themselves motivated by issues of conceptual understanding. The only difference is that in these episodes, student writing was a noticeable and explicit motivation for the instruction.

In this section I will describe three instructional activities in which writing was explicitly addressed. In the first activity, the teacher used examples of student writing to spur discussion of writing and concepts. In the second, he explicitly addressed student writing errors in class discussion. And in the third, he gave students explicit guidance on writing explanations before engaging students in a peer review of written work. I will describe each of these activities, and analyze each episode for both its relevance to students' understanding of how to write mathematical explanations, and the interrelationship between the writing instruction and students' conceptual understanding.

Using examples of student writing

In instructional activities in which the teacher used examples of student writing, he chose examples of student writing from their assignments, then had students read through the different explanations and used this as a springboard for discussion about the explanations and related concepts. One such episode of instruction occurred near the beginning of class when the instructional activities were still focused on helping students to develop an explicit understanding of the meaning of fractions and useful ways of talking about fractions. The teacher's intention at this point in the semester (as mentioned above) was to help students to develop two complementary definitions for fractions that would help them to understand fractions as quantities. "Iterating" defined $1/n$ as the quantity such that n copies of that quantity together made one, and "partitioning" defined $1/n$ as the quantity created by breaking one into n qualitatively equal pieces. Students had spent time working with manipulatives and pictures representing fractions, and had talked about fractions in groups, as a class, and on their written homework assignments, but the difference between iterating and partitioning conceptions of fractions had not yet been explicitly discussed in class.

In this context, the teacher selected six written responses [Figure 4.2] to a question about the meaning of 1/5 from the students' homework assignment and copied them onto a transparency which he then projected for the class to see. He told students to discuss each response in groups and to talk about which ones made sense, and whether there were ways to make the explanations stronger (1-17, p. 4). The students discussed the explanations as small groups and then the teacher brought them back together for a whole class discussion. The teacher read through the written responses one by one and asked for student feedback on each response. The whole-class discussion itself was relatively brief, and really touched on only two particular issues—making sure the equality of the five pieces came through in the explanation (1-17, p. 5), and attaching meaning to possibly ambiguous terms (in this case, the term “unit”) (1-17, p. 6).

1. It takes five whole blocks to make one whole. So you take five little blocks and you get one fifth.

2. One-fifth means it will take five little bars of the same size to make one whole bigger bar.

3. It takes five parts to make a whole.

4. One fifth means that one moment there are five equal parts and one fifth is one of those five equal units that makes up the whole. Five is for one.

5. One fifth means that there is a whole that can be divided into five pieces. One fifth would be the value of each piece.

6. One fifth means that five units are equal to one.

Figure 4.2
Student Responses

Instruction on writing explanations was explicit here because students knew that they were examining written work and evaluating the effectiveness of the writing in communicating the concepts with which they were becoming familiar. Components of explanations were addressed as students used their understanding of the concepts as a foundation for evaluating whether the language used in the explanations was sufficient to communicate that understanding. That is, in order to do this activity, students needed to think explicitly not just about what they understood about the meaning of the fraction 1/5, but also about how they might communicate their understanding and how such communication might appear to a reader who did not have as complete an understanding of the concept. When the class talked about the effect of including or failing to include the word "equal" in reference to the five pieces of the whole, the question was not whether or not the writer had actually understood that the pieces must be equal (it can probably be safely assumed that the students were all at least implicitly aware of the necessity of equality of pieces) but whether their explanation had clearly communicated all the necessary components of the writer's understanding. And when the class talked briefly about the use of the word "unit" the implication was not that the word itself had no meaning to the user, but that if it was used in communication, such meaning had to be made clear to the reader.

But even though this episode of instruction quite clearly addresses students' written explanations, this episode was also quite clearly an episode of conceptual instruction, and may not have even been viewed by the teacher or students as anything other than conceptual instruction. Later in the class period the teacher told the students that the reason he had put the student explanations up on the overhead was because he "wanted to bring out these two different images [iterating and partitioning] and talk about them" (1-17, p. 14). So although characteristics of effective written explanations had been addressed quite clearly in this instructional episode, the teacher's statement to the class suggests that one of the purposes of the activity was to get students to think about the difference between the two conceptions of fractions, which is itself a conceptual goal.

The activity was further conceptually focused in that the discussion of how concepts were communicated through writing served to draw students' attention to particular aspects of conceptual understanding that may have been implicit to them. When a student pointed out that they needed to explicitly refer to the equality of the five pieces, the students were made aware of the fact that equality was an important part of the concept itself. And when they recognized that the term "unit" was ambiguous in the context of the explanation, students were given the opportunity to think about what was really meant by the term "unit," which itself brought up new concepts for students to think about and understand. This particular example, then, despite the fact that it outwardly appears to be primarily an example of instruction on students' explanations, illustrates the interconnectedness of students' conceptual understanding and their knowledge of how to write explanations.

Explicitly addressing students' writing errors

Instructional episodes that explicitly addressed students' writing errors occurred when the teacher noticed a particular way in which student explanations were problematic that students might not be aware of, or talked about potential difficulties with written communication before such difficulties actually surfaced. One of these episodes occurred several weeks after the first episode, but addressed the same basic mathematical ideas. At this point in the semester, language for discussing fractions had been addressed in nearly every lesson and was central to the instruction, given that the first half of the semester was to be spent learning about fractions, fraction relationships, and fraction operations. Students had become aware of the complementary definitions of fractions in terms of processes of iterating and partitioning language in all explanations involving fractions, and had been encouraged to carefully use iterating and partitioning. By now, students had described fractions from both perspectives in class and on paper many times, and the teacher frequently modeled the language used to describe fractions, particularly from an iterating perspective. Perhaps more than any other concept, students had access to the language and structure of explanations for the definition of fractions.

After the first exam had been graded and returned, the instructor allowed students to ask questions about the exam. In a lull, as students looked over their exams, the instructor made a comment about the way students were writing their explanations of fractions.

> Some people said one third, is…something like 1/3 is repeated three times to get one. Okay? And I see this a lot. 1/3 has become instead of a quantity, or an amount, it has become an operator. In a lot of your explanations that’s [how] you’re talking about 1/3. You’re talking about 1/3 as being iterated something three times. That’s not what 1/3 is. Okay. 1/3 is not the process of iteration. That’s an operation. 1/3 is an amount. What amount is 1/3? It’s the amount that if you made 3 copies of it, and put them together, you get one. (2-7, p. 6)

This brief discussion on the meaning of a unit fraction is clearly centered around students’ written explanations. The teacher identifies students’ written responses as the motivation for the discussion, and the discussion itself is about the way that students are writing about fractions more than it is about students’ understanding of fractions. In fact, the teacher himself suggested that it was likely the phrasing of the explanation rather than the students' understanding that was problematic when he said, "Now I think that most of us when we talk about this, you probably have this idea [that 1/3 is the amount such that three copies of it makes one] in mind" (2-7, p. 6). In other words, students were able to conceive of 1/3 in terms of iteration, but were struggling to put that conception into language that could be clearly understood by a reader.

In this way, the episode of instruction here served to provide students with an understanding of how to write explanations. The language students had developed to communicate this mathematical idea obscured their understanding and could be easily misinterpreted by a reader, so when this was brought to the attention of the students, they were able to become aware of how their language reflected particular conceptions of mathematical concepts, and of how their written explanations might be interpreted by a reader. These are issues of language, purpose and audience.

At the same time, this particular diversion from the lesson also fulfilled a conceptual goal. Students' understanding of 1/3, even based in images, was necessarily grounded in language. It could be argued that if students could not explain 1/3 as a quantity resulting from the process of iteration, then they did not understand it as such, and that their understanding was incomplete until they *could* explain 1/3 appropriately. Learning to communicate their understanding effectively again called particular aspects of their understanding to the students' attention and thereby strengthened their awareness of the meaning of fractions. Furthermore, the teacher himself could not possibly be aware of whether students had an understanding of the meaning of 1/3 if the way that they communicated their understanding in writing reflected an incorrect conception of fractions. The only way to address a possible misconception was by addressing the language through which the possible misconception had been communicated.

Peer review with explicit guidance

Peer review with explicit guidance, in which the teacher pointed out general characteristics of writing and allowed students to read and discuss their own explanations in small groups according to these general characteristics, was rarer than the first two types of writing instruction. In fact, in the data I analyzed there was only one such episode of significance. This episode also occurred shortly after the first exam. After answering students' questions about problems from the exam, the teacher commented that, although many students were becoming quite good at creating explanations, he wanted to help those students who were still struggling by talking about what constitutes a good explanation (2-7, p. 7). The teacher then presented the characteristics in Table 4.3 to the students on a series of overheads (2-7, pp. 7-8).

1. **Characteristics of good explanations**
    a. The explanation is based on images and models (pictures, manipulatives, etc.) rather than just symbols.
    b. Every number is carefully linked to some quantity in the model.
    c. There are no "mysterious operations"—operations are couched in terms of actions upon quantities in the model.
2. **Things to ask oneself when writing an explanation**
    a. Can I draw a picture from this?
    b. Numbers are numbers of what? (For instance, 1/4 is 1/4 of what?)
    c. Why does it make sense to perform a particular operation?
3. **Signs of problematic explanations**
    a. The explanation focuses mainly on manipulating symbols.
    b. Numbers are not explained; they are just symbols to be manipulated.
    c. Operations are sets of steps.
    d. The writer assumes that what is written and drawn is transparent to the reader.

Table 4.3
Characteristics of Good Written Explanations

After this exposition, the teacher had the students choose a problem from their homework on addition and subtraction of fractions from the night before. In groups of two or three, the students read their written explanations and then took turns giving each other both positive feedback and constructive suggestions based on the characteristics of good and problematic explanations that the teacher had already talked about with the class. They then had the opportunity to redo their explanations as part of the following night's homework.

Of all the instructional episodes that I analyzed, this particular episode is not only the most clearly centered around student writing, but is also the most clearly focused on helping students to become better writers of conceptual explanations. All of the guidelines together help students specifically with the necessary components of written explanations. In particular, rather than specifically informing students, in some way, of the type of language they needed to use or what they need to include in a particular explanation, these guidelines referred more broadly to how students could judge for themselves the appropriate language and components of explanations. Language, students were told, must specifically and clearly refer to images or models, to actions on quantities, to the meanings of numbers and operations. There were certain details that students were told they could not leave implicit, such as the meanings of quantities, reasons for performing certain operations, and the conceptual purpose of actions and pictures. Even purpose and audience was addressed at one point, when the teacher explained, "these characteristics of good explanations are characteristics I would place on explanations given to kids, or explanations given to this class. Not every math class would appreciate explanations that are like this" (2-7, p. 7), thus placing explanations that met the characteristics in a very specific context.

Nevertheless, this episode, in which students' ability to write conceptual explanations was the clear focus of instruction, was also very conceptually motivated, and helped to provide students with a foundation for their conceptual understanding of the mathematics that they were learning. On the one hand, by talking about their own mathematical writing, students were also engaged in discussing the mathematical content of that writing. Therefore, through discussing the writing students were undoubtedly strengthening their understanding of the content of that writing. On the other hand, the characteristics of good explanations that were stated and implied in the instruction here could also be seen as instruction that would help the students to better understand mathematical concepts in general, not just to better put their understanding in writing. These characteristics outlined not just what counted as good writing, but also what counted as real understanding. A student's understanding was equated with their ability to connect all symbols and manipulations of symbols to models and actions, to be able to define numbers as specific quantities and operations as actions on quantities, and to be able to explain why he or she performed certain operations in the context of the problem. By understanding what was necessary and acceptable in an explanation of a mathematical concept, students were also learning how to make sense of the concepts themselves.

Summary

In this chapter I have addressed my first research question. I discussed both how aspects of writing were incorporated into the instruction of the classroom, and how these aspects of writing were a crucial component of students' conceptual learning. Three components of writing in particular emerged throughout conceptual instruction: language, what to include in an explanation, and purpose and audience. Not only were these components addressed in conceptual instruction, but they themselves were crucial elements of students' conceptual understanding.

Although these components of writing emerged primarily through conceptual instruction that was not directly focused on student explanations, there were several classroom episodes that centered explicitly on student writing. These episodes took various forms, and in each form of writing instruction students were given access to knowledge about how to write conceptually oriented explanations. The integral relationship between writing and mathematical learning could be seen in these episodes, as well, as each episode was motivated by conceptual concerns and was intended to contribute to the development of students' conceptual understanding. Thus, not only were writing and conceptual instruction related, but they were enmeshed in such a way that each was essential to the success of the other.

Chapter 5 — Results: The Development of Written Explanations

In this second chapter of results I will focus even more closely on the second of my two research questions: How is a classroom in which the teacher values students' written mathematical explanations structured to support the students' ability to write good mathematical explanations? Although showing, as I did in the first chapter, that components of written explanations were addressed both implicitly and explicitly throughout the two months of classroom instruction contributes to an understanding of how students were given access to knowledge about writing explanations that were appropriate to the classroom setting, it does not explain how these details fit together to help students create complete explanations. Nor does it show how an understanding of how to write explanations of particular concepts developed over the course of the semester. In this section I will discuss the dynamic process by which particular aspects of writing emerged in classroom instruction, and how the class was structured to support the development of students' ability to write mathematical explanations. I will first discuss the role of modeling in the development of explanations. Then, in the last section of this analysis, I will explain how the ways that students were given access to knowledge about the creation of written explanations progressed within individual mathematical concepts and over the course of the semester.

The Role of Modeling

Explanations of concepts by both teacher and students occurred constantly in the course of classroom instruction and discussion as the class sought to make meaning of the mathematics they were learning and to refine their understanding of that mathematics.

Although language, expectations for what needed to be included in an explanation, and purpose and audience developed in the course of these explanations, as well as other events that took place in class, particular types of in-class explanations seemed to serve as models for students' written explanations, and were thus a relatively clear source for students' understanding of what an appropriate explanation for a given concept might look like. In general, an explanation either served to explain an unfamiliar concept, or served to model an explanation for a concept. This does not represent a strict dichotomy of explanations, since explanations given for the purpose of explaining could model particular language or other explanatory characteristics to students, and explanations that served as a model had explanatory purposes as well. Nevertheless, certain characteristics set model explanations apart from other explanations, and I define the term *model* for the purposes of this analysis according to these characteristics. An explanation is here defined as a model if it meets the following criteria:

1. *The explanation is given in response to a question or problem that has previously been posed and fully explained in class*. Initial questions and explanations about new or unfamiliar concepts, whether spoken by the teacher or by students, were motivated by the need for students to make sense of the concepts. There came a time, though, when the concepts had been developed to the point that they could be taken-as-shared within the classroom discourse. But even at this point, explanations sometimes still continued, suggesting that the focus of the explanation had shifted from understanding to communication.

2. *The explanation is complete, and satisfactorily answers the question that was posed and explains each aspect of the solution process*. In general, models consisted of a complete explanation to a given problem. In some cases, however, an explanation of a concept was given for purposes of building understanding or answering a student's questions rather than modeling the communication of that concept, and yet still contained an interjection that served as a model of some related sub-concept. In such a case, the explanation of the main concept would not serve as a model because the modeling portion of the explanation did not explain the entire concept. The explanation of the sub-concept, however, could be considered a model because the sub-concept itself was fully explained by modeling.
3. *Each part of the explanation refers directly to the question or problem that was posed*. Any significant additional details in a model are restatements for the purpose of clarification, rather than analogies, anecdotes, examples, or questions that have not been explicitly addressed in previous class sessions. An explanation interspersed with too much additional commentary on the content could not stand as a clear model, and frequently such commentary served explanatory purposes, diverting attention from the modeling function of an explanation.

Once again, a strict dichotomy of model and non-model is not sufficient for describing all explanations that took place in class. For some students, explanations that were given for the purpose of modeling communication of concepts may have served the purpose of supporting their understanding. And even students for whom an explanation does serve as a model are likely to gain additional conceptual understanding through participating in the modeling process as a speaker or listener. Nevertheless, the distinction between models and other types of explanations does serve to highlight important aspects of the data and offer insight into how the instruction was structured to support the creation of students' written mathematical explanations. Below I will discuss three specific forms that modeling took—teacher models, student models, and actively constructed models—and describe how each of these forms of modeling played a role in classroom instruction and gave students access to understanding about how to create a written explanation.

Teacher Models

Teacher models consist of those models in which the teacher himself modeled particular concepts without direct input from the students. This did not occur with all concepts and was, in fact, somewhat rare because of the teacher's reliance on student feedback as students came to understand each concept. The teacher's models tended to serve more as a reminder of how to explain a concept that students had already frequently explained or used, and only when these concepts were a small part for the explanation of another concept. They were models, then, not of an explanation of the main question being addressed, but of explanations for smaller concepts that were relevant to the main question. In the units on multiplication and division of fractions, for instance, the teacher occasionally reiterated the explanation of fractions themselves that had been developed nearly a month before: "How do I know [1/2] is an eighth of 4? Because if I take it and make 7 more copies, what do I get? Four." And later, "How do I know [1/20] is a fifth of 1/4? Because it takes 5 of those to make a 1/4"(2-26, p. 8).

Note that such examples, although brief and of minor importance to the concepts of multiplication and division of fractions, still served as models of language for explaining sub-concepts that were related to these larger concepts. Students were already assumed to have an understanding of the meaning of 1/8 and 1/5, because an understanding of the meaning of fractions underlay every previous lesson on fractions and fraction operations. The teacher's explanation in each case was complete and concise. Although for some students the explanation may have served to help them to understand, such understanding at this point was taken-as-shared within the classroom. These explanations, then, were not for the purpose of clarifying a confusing or unclear idea and therefore served as direct models of the language and form of an explanation for that particular concept. Again, though, models from the teacher were rare. Most teacher explanations served the function of explanation (either to introduce a concept or to try to clarify a concept that students had already learned but perhaps struggled with) rather than the function of model.

Student Models with Feedback

Just as with the teacher's explanations, student explanations also filled the functions of explanation and model. Students' explanations filled the role of model more frequently than did the teacher's explanations, however. The teacher's explanations were directed towards the students, generally for the purpose of helping students to better understand the mathematics given that this was the teacher's primary goal. In contrast, students' explanations were usually given less to help other students understand than to demonstrate their own understanding of the mathematics, as well as the norms for communicating that mathematics, and therefore naturally fell into place as models.

Students certainly had occasion to explain their thoughts for the purpose of helping someone else to understand their thinking, but the general nature of their explanations was such that modeling was more frequent among student explanations than among teacher explanations.

Student explanations were always accompanied by some sort of feedback, occasionally from other students who sought to clarify their own understanding of the speakers' explanation, but most often from the teacher. Such feedback served to implicitly alert students to strengths and weaknesses of their model explanations and therefore to characteristics that made one explanation more acceptable or clear than another explanation, or to how they could improve their own explanations of mathematical concepts.

One example of student modeling comes from the follow-up to a lesson on the relationship between ratios and fractions. An unexpected difficulty that had surfaced as the class talked about the previous day's topic of the relationship between ratios and fractions. Conflicting student responses to a question about how many times larger one piece was than another revealed that some students were unclear about the difference between the phrases "how many times larger than" and "how many times as large as." This led to a discussion of the language and the related quantitative relationships. The homework problem that motivated this discussion involved two lengths: length A, which was two units in length, and length B, which was three units in length. The teacher had already explained the difference between "as large as" and "larger than," and had even modeled explanations for both concepts with another pair of lengths. He then turned the explanation over to the students, asking, "Are we happy to say B is three halves times as large as [A]? Can we really see that?...How do you see it?" This gave one student the opportunity to model an explanation. The concept had already been explained thoroughly, and the explanation itself had been modeled. The students' response was not to explain how she was thinking about a new question, or to explain a concept she already understood to classmates who did not understand the concept, but rather to demonstrate her own understanding of the concepts and the norms for communicating those concepts (and possibly her classmates' understanding by proxy) through modeling the language needed to accurately explain this particular concept. She replied:

> Well, um, they're all equal pieces, so, but there's three pieces of B and there's only two pieces of A so, um. I forgot where I was going with this. Um, if you take half of A you get three of it. (1-24, p. 7)

An underlying understanding of this concept can be seen in this explanation, but the student's attempt to put her understanding into words is somewhat unclear. The teacher then built upon the students' words in order to clarify her model:

> If I take half of A I have three of those in B, right? So I have three-halves A's in B…so we would say that when we're comparing these signs that B is three-halves times as large as A. (1-24, p. 8)

Notice how the instructor draws upon the student's words, essentially transforming her somewhat hesitant explanation into a more refined and complete explanation. The student's statement, "if you take half of A you get three of it," is modified to, "If I take half of A I have three of those [halves] in B," which is then carried forward to its logical conclusion: "So I have three-halves A's in B…so we would say that…B is three-halves times as large as A." The instructor emphasized the language that the student had used correctly, clarified that language for the purpose of addressing an audience, and added the necessary pieces of the explanation that had been missing from the student's attempt.

Teacher feedback not only helped to point to what was missing in a student model or to revise a model that could be improved, but also sometimes gave approval to a particular model of an explanation. An example of a successful student model is found at the conclusion of the unit on ratios. Although the explanation given was in response to another student's question about a problem on the test that she had not understood, the example below serves as a model because the concept had already been covered thoroughly and most students in the class were expected to understand and be able to explain the concept. The explanation is in response to a question from a test given shortly after the lesson on ratios. In the question, a piece of licorice had been shared among two girls in such a way that the ratio between the two girls' pieces was seven to four. The students were asked to explain how they could see the fraction 7/4 in the ratio. The teacher directed this question to the students, and one student responded:

Student: Because you have four on the one side and then you used it to represent a whole, they're, you can divide it into four and then you have 1/4 in each piece. So if you count up all the other side's [pieces] cause they're equal to those side's [pieces], you have seven 1/4 pieces."

Teacher: Say that again. Say that again.

Student: Ok. On this side [referring to a picture on the chalkboard] if you just pretend that this one side was a whole, Teacher: This right here?

Student: Yeah. You've got four pieces. And if it takes four pieces to make up a whole you know that they're 1/4. Each piece is 1/4. So if you—and all the other pieces are equal, so the same size as the 1/4—so if you count 'em up you have seven 1/4 pieces.

Teacher: Seven 1/4 pieces of what?

Student: The whole, of Jessica's whole.

Teacher: Of Jessica's whole. Yeah.

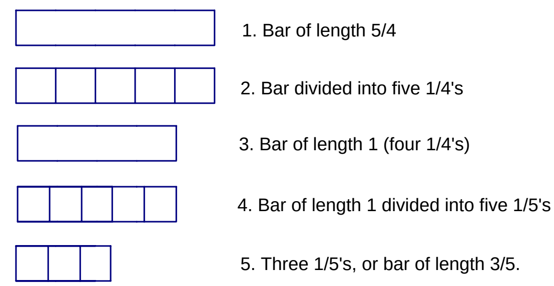
(2-7, p. 5)

While the instructor gave some clarifying input, first by having the student repeat her explanation, and then by making sure she clarified what her 1/4 pieces were 1/4 *of*, the instructor gave implicit approval of the student's explanation by his lack of clarifying input. This explanation then stands as a model explanation for this concept, and while the reliance on context (in particular the picture on the chalkboard that the student was able to refer to in her verbal explanation) would make the explanation as it stands in its oral form somewhat lacking if written word for word, the basic elements of an appropriate explanation are there, in the concepts, language, and structure.

Actively Constructed Models

Not all modeling occurred in an initially complete form. In some cases, a model explanation was constructed by the teacher and students. In the following classroom episode, students had completed a classroom activity and homework assignments that involved using Cuisennaire rods or pictures of bars that were said to represent a given length in order to find another rod or draw a picture of another bar representing a different fractional length. One student explained her solution to a problem in which students had been asked to draw a picture of a bar of length 3/5 given another picture of a bar of length 5/4 [Figure 5.1], and to explain their reasoning.

Figure 5.1
Finding 3/5 Given Length 5/4



The student explained her reasoning to the class, with her language relying heavily on her concurrent construction of the picture on the chalkboard, and then the teacher engaged the students in a dialogue to help them expand and complete the explanation:

Student 1: So that's 5 and just do that, and just do 5 divided by something out of 5…So that would be one. Equals 5/4. And then I compared…So then I divided the whole, um, divided this into 5 parts. And that's five of five. And this is 3/5.

Teacher: All right. Good. Any questions? Great. Okay. How did she know to take this 5/4 and divide it up into five parts at the beginning? Why do you do that?

Student 2: Because that's the given, it said the box equals 5/4.

Teacher: So whenever I say the box equals 5/4, I automatically know I'm supposed to take it and divide it into five parts. Why do I do that? What am I trying to find when I divide it into five parts?

Student 1: Trying to find a whole.

Teacher: No.

Student 3: Trying to find 5/4.

Teacher: (Laughing with the class) No, I know 5/4. 5/4 is right here.

Student 4: You’re trying to find 1/4.

Teacher: Okay. I’m finding the length of 1/4. How do I know if I divide it into five parts it will be 1/4 in each part?

Student 5: Because there’s five 1/4’s.

Teacher: Because there’s five parts and we know it’s five fourths…so they’re equal. They must be a fourth. Does everyone agree with that? Okay. Good. Then I go down to here. Now what am I thinking here? Why do I go from here to here? Now I know what one is, right? So if this is 5/4 I take away 1/4 and that gives me 4/4, which is one. Okay. Then we do something really interesting here. What’s going on here?

Student: One into five parts.

Teacher: Good. She’s thinking, this is one. Now she knows what the one is, she’s forgetting about how many partitions are in [the 5/4]. Right. What we’re really interested in is in terms of what this is in fifths. So I repartition it up into fifths, and I want 3/5, so I take three of those 1/5’s. Okay? Good.

(1-10, pp. 6-7)

This particular interchange has much in common with other episodes of classroom instruction in which new concepts are developed by the students and the teacher. However, this and a handful of other similar exchanges stand out from the rest as models because of the nature and context of the exchange. The purpose of the discussion is not so much to expand or change students' understanding (although students are expected to come to a better understanding by means of the discussion) but rather to push students to think about what needs to be explained and how a concept that already makes sense to the students can be effectively and explicitly communicated. Students have already solved the problem successfully and have explained it to each other in a way that they deem satisfactory. No new or unfamiliar concepts are introduced here. In fact, the teacher's first question to the class ("Any questions?") and the lack of student response seems to demonstrate that students were comfortable with the explanations, and so the primarily purpose of the subsequent dialogue is not explanatory. If anything, students are being forced to address aspects of their understanding that, up to this point, they had been comfortable leaving implicit. And so instead of building conceptual understanding, this exchange pushes students to think about how they might explain the understanding that they already have, at the same time exposing students to expectations for the nature of their explanations, what is and is not sufficient, and how to phrase ideas that remained implicit in their verbal communication. Students are involved in the creation of an explanation for the sake of learning to create an explanation.

The significance of actively constructed modeling is that, while it focused on the concepts as much as teacher or student modeling, it focused less directly on the language used to describe those concepts and more directly on the process involved in creating an explanation, and on the structure and necessary components of the explanation. Students were able to see the process of explaining as a dynamic process, and their attention was drawn to what needed to be explained at the same time that they were exposed to *how* to explain these things. This type of instruction gave students access to the process involved in writing an explanation and the types of thoughts and questions that should guide them as they sought to explain their reasoning and solutions to these mathematical problems.

The Progression

In this final section I will discuss the progression by which explanations were developed for each concept in the unit on fractions. The teacher of this class had well-defined goals for the direction of the instruction, but the classroom structure was flexible. Depending on the progress of the students and the questions that arose in class, different problems and activities were introduced; instruction lengthened, shortened, or modified; topics revisited; and assignments or class requirements revised. Nevertheless, within this flexible structure there was a general pattern by which each individual mathematical concept was developed. Parallel to this development of conceptual understanding was another pattern by which students developed the ability to write explanations of this instruction. Students explored the concept using their own language, solidified their understanding with the use of modeling of explanations, and then refined the understanding and ability to write explanations with more explicit instruction on specific aspects of written explanations as they emerged in class. This pattern was not completely rigid, but was fairly consistent from one concept to the next. I will illustrate this progression with examples from instruction on the definition of fractions, the first concept that students were exposed to, and from instruction on the multiplication of fractions, one of the concepts near the end of the data that I analyzed. In so doing I will not only illustrate the progression, but also the ways in which the progression changed over the course of the semester.

Exploration and Student Language

New concepts were first introduced by providing students with a problem or question that was within students' capability but beyond what they had previously done in class. Students were given time to work on the problem in groups and to explore the question using their own language and understanding. Conceptually, this initial exploration allowed students to make connections between ideas that they were already familiar with, and these connections formed the basis for their understanding of new ideas. Such exploration was also relevant to students' written explanations. Just as the explorations allowed them to make connections between familiar ideas, so too did the explorations give them the opportunity to use familiar language in new ways and thus develop a foundation for the explanations that they would eventually write.

The very beginning of the course was devoted to helping students develop solid and meaningful definitions of fractions that would underlie the students' later understanding of fraction relationships and fraction operations. The teacher had two specific conceptions of fractions that he wanted the students to become aware of, the iterating and partitioning conceptions that have been discussed elsewhere in this analysis. He did not, however, push them to become explicitly aware of these target conceptions of fractions until there had been ample time for iterating and partitioning ideas to emerge in the students' own language through the initial activities and discussions. In the exploration stage of this concept, students worked to solve problems involving comparisons of fractions using manipulatives and pictures rather than symbols and algorithms. They worked on these problems in groups in class, presented and discussed some of their solution processes as a whole class, and wrote up explanations for their solution processes as part of their homework (1-10).

Multiplication of fractions was introduced in a similar way, following a discussion on multiplication of whole numbers to help students begin to think about the meaning of multiplication. After creating and discussing story problems and solutions for the more familiar whole number problems 3×4 and 4×3, students were given the problems $4\times1/2$ and $1/2\times4$ and asked to again create story problems, to solve them using pictures, and to be able to explain their solutions. Students worked in groups, discussing the meaning of the problems and the solutions to the problems, before they came together as a class (2-12).

Two things contributed to the creation of a setting in which students were able to begin to understand new concepts through use of their own language and explanations. First, the teacher provided students with open problems that involved questions that students had not yet had the opportunity to think about in class, and that required them not only to find a solution, but to think carefully about how they would solve the problem and why their solution method made sense. Because the problems had unfamiliar components, students had not yet been exposed to a common language with which to discuss the new concepts. This left them free to develop their own explanations as they developed their understanding. Second, the students began working on these questions without the presence of the teacher. They did not have an experienced authority to make judgments about what they were explaining or how they were explaining it, and could therefore focus on what type of explanations were effective for them in that particular situation.

The first whole-class discussions following students' group explorations were generally focused more on exploring the concepts than on refining explanations. Explanations that were given in class at this point were usually for the purpose of explaining rather than modeling. Such explanations, whether given by the teacher or by students, were motivated by questions that the students had not yet discussed as a whole class, and teacher explanations were frequently interspersed with analogies and examples to help students make useful connections between elements of their prior understanding.

When students first began discussing the meaning of fractions, the teacher did not initially push them to think in terms of iterating and partitioning but rather let students say what they were thinking for the rest of the class to hear and then let the class discuss what they had heard in terms of mathematical correctness and the communication of mathematical ideas. They only gradually began to develop a common language and only gradually were made aware of how some explanations were more effective than other explanations. The foundations of students' ability to explain the definition of fractions were being developed at this point, but little actual modeling occurred.

This phase of instruction, then, can be described not only as exploration of concepts, but as simultaneous exploration of ways of explaining those concepts. In fact, this exploration of ways of explaining, particularly exploration of ways of using language, played a crucial role in students' understanding. For example, when students first discussed the meaning of $1/2 \times 4$, students' experimentation led to successful and unsuccessful ways of conceptualizing multiplication problems, and to class discussions of particular relevant concepts. When one student suggested that she had found it useful to think about 1/2 × 4 as “1/2 of four groups” (2-12, p. 24) the class was required to think about how this use of language differed from the use of language they had been using thus far in their discussion (“1/2 of a group of four” or “1/2 of a copy of four”). The language that students used to talk about the concept both reflected and affected their understanding.

Modeling and Feedback

The second stage of instruction, after students’ initial introduction to and exploration of the concept, involved a development of their understanding and ability to explain their understanding of the concept marked by the transitioned into modeling of explanations. Students were given problems that were similar to the problems they had worked through in the exploration stage, or built upon those problems. Students presented explanations and received feedback on their explanations from the teacher and sometimes from other students, and the teacher helped to create complete explanations of the concepts by eliciting feedback from students. The whole-class discussions served to solidify students’ initial understanding. Furthermore, because these dialogues frequently resulted in the development of model explanations, students were given precedents for their own written explanations.

Modeling of explanations for the meaning of fractions first occurred on the second day of instruction. Students at this point had explored situations involving comparisons of fractions and had been given the opportunity to explain their understanding of the meaning of a function. Effective and ineffective definitions of fractions had been discussed at the beginning of the class period. The instruction that followed marked a transition point at which definitions of fractions from an iterating perspective gradually became knowledge that could be taken-as-shared and explanations began to fill the function of model.

The transition into modeling began when students were asked to apply the ideas from the previous section to fractions that they created with a set of twelve beans. The class discussed the different fractions that could be made from a set of twelve objects, and then the teacher referred to the fraction 3/4 and asked the students to explain the meaning of 3/4 in that situation "in terms of iterating" (1-10, p. 17). The subsequent discussion could be considered an active construction of a model explanation.

Teacher: Three sets of three beans makes three-fourths, but why?

Student: Because four sets of three beans makes one whole.

Teacher: Okay. If I take a set of three beans and I make three more copies of that so I have four copies of that set of three beans, what do I have?

Students: A whole.

Teacher: A whole. I've explained why it's fourths. Now I need to explain the three part of it, right? So I take three of those fourths and that gives me three fourths.

(1-10, p. 17)

Shortly after this explanation, students had another opportunity to practice the language of iterating with an explanation of the fraction 5/3 in terms of the set of twelve objects:

Student 1: [Referring to a picture of 5/3 of a group of twelve on the chalkboard] …if you cover up two of the groups right there you've got three-thirds.

Teacher: How do you know it's three thirds?

Student 1: Because you have three one-thirds.

Teacher: How do you know they're one-thirds? You said it so nicely last time.

Student 1: Three-thirds of a whole…Because it takes three one-thirds to make a whole.

Teacher: So if I take this one-third—if I take this group and I repeat it three times, I get a whole, so it's a third.

Student 2: So if you count it five times you get five thirds.

Students had already discussed and used iterating definitions, and so although this segment of instruction may have served the purpose of strengthening students' conceptual understanding, the instruction was primarily focused on helping students effectively put this strategy into words.

The explanation for multiplication of fractions using pictures was more complex because it required the ability to explain several individual concepts and connect these explanations in a coherent whole. Students had to be able to explain the meaning of fractions, of multiplication, and of how these two fit together to define fraction multiplication. They had to be able to explain a solution process and their reason for choosing to perform certain actions on the pictures they had drawn. Possibly because of this complexity, a complete model of this explanation, including all necessary details, was rarely elicited from a student without extensive feedback, and was never given by the teacher in full. The teacher modeled small parts of the explanation, students explained a problem in front of the class to be discussed, and models were actively constructed in the course of the instructional event.

A student explanation of the solution to the problem 3/4 × 6, given after students had already seen and tried several multiplication problems themselves, serves as a good example of a model because similar problems had already been solved and explained in class. The student explained her solution for the class (relying more on the picture than on her own words) as she drew six objects and found 3/4 of each object, then consolidated each of the six 3/4's in order to find 3/4 of six. The teacher subsequently filled in that explanation, helping to point to what was sufficient and what was insufficient in the explanation:

Student 1: All right. This is 6. 1, 2, 3, 4, 5, 6. And there's just these, just can I show you what a whole is, um, but there are 3/4 of each one, as 3/4 of 6…

Teacher: …Can you say it again? Just a little bit slower. Okay?

Student 1: Yeah. There's six. 1, 2, 3, 4, 5, 6. And there are three…let me move these. There are 3/4 of each translates to 3/4 of six.

Student 2: So then to get the answer you'd put 'em all together.

Student 1: You'd just be like, okay, na na na na. So you'd have 1, 2, 3, 4 and a half.

Teacher: Okay? Good. Thanks. All right. So what we can do here is we can start with six things here. One way to do it is you start with six things like this. All right? I have six things. And then one of the strategies to get 3/4 copies of six is I take 3/4 copies of each of the

> 1’s that make up the six. Right? If I do take 3/4 of each one of those then that’s going to leave me with 3/4 of the six.

(2-12, p. 27)

Again, this stage of instruction involved the transition from explanations for the purpose of aiding students’ conceptual understanding to modeling explanations for the purpose of establishing norms of communication. In this case the student’s explanation and the teacher’s clarification may have served to help students understand multiplication of fractions in terms of actions on pictorial representations of fractions. But because the problem and solution were similar to problems that had already been discussed, the explanation that was constructed in the course of the interchange between student and teacher served modeling purposes as well. In the process of clarifying a student explanation, the teacher implicitly told students what needed to be explained by the parts of the student explanation that he kept in his own explanation and the parts he chose to expand. The student who explained the solution appeared to understand the basic concepts, and therefore the teacher’s support addressed the student’s ability to communicate those concepts.

Once again, the purpose of this stage of student learning was ostensibly to help students understand the concepts. But as students became more familiar with the concepts, the form of the instruction changed in such a way that ways of explaining began to emerge for students to rely upon when they wrote up their own explanations of the concepts.

Explicit Instruction

In the final phase of learning for a particular concept, difficulties that surfaced after students had had the opportunity to learn and explain the concept were addressed through explicit and detailed instruction. This level of instruction was based on difficulties, gaps in understanding, or misconceptions that had arisen as students spent time working through the concept, and therefore this was the stage in which more explicit instruction on particular aspects of students' explanations took place. Such instruction addressed particular issues that the instructor had seen in student explanations, or issues that he anticipated, and ranged from very detailed instruction to small diversions in classroom discussion.

Two of the examples of explicit writing instruction that were discussed earlier in this analysis took place at this stage in students' learning about the meaning of fractions. It was after students had experience exploring and refining their conceptual understanding that the teacher showed students examples of written explanations and had them think about what made those explanations effective and how they might be made more effective. And even later in the semester, after students' experience with writing about the meaning of fractions was even more solid, the teacher identified a problematic way of explaining fractions from an iterating perspective that had appeared on students' tests and drew students' attention to the conception of fractions that their way of explaining seemed to suggest. Because these two examples have been discussed previously, I will not go into further detail on how they influenced students learning and ability to write conceptual explanations, other than to mention their position in this phase of instruction.

One way that literacy issues pertaining to explanations arose in the unit on fraction multiplication was when the teacher addressed the difference between 5/8 × 4 and 4 × 5/8. In the conversation, it was clear that students were aware that the problems were solved by different processes, but that the similarity between two pictures that accompanied the processes necessitated a reiteration of the fact that pictures and descriptions of the pictures were insufficient as explanations, and that students needed to explain not just what was happening in the picture, but how they were seeing the problem so that it was clear whether they were solving the problem 5/8 × 4 or 4 × 5/8 (2-14, p. 15).

However, in the lesson on multiplication, discussions of writing issues were more brief and less central to the daily instruction than in the lesson on the definition of fractions. This may have been because the explanations of multiplication of fractions involved an integration of explanations of other concepts that had already been frequently addressed throughout the semester up to that point and therefore the particular writing issues involved in this particular explanation were less apparent. It may also have been that at this point in the class the concepts were becoming more complex and therefore the extent of the classroom focus on concepts left less room for classroom focus on issues of literacy or how to explain those concepts. Also, less time was spent on multiplication of fractions than on the development of a basic, workable definition of fractions. The definition of fractions was fundamental to everything the students would do later in the unit, and furthermore took place at the beginning of the class when norms for learning about and talking about mathematical concepts were still being established. These things may have influenced the amount of time that was spent focusing specifically on issues of writing.

If the first phase of the progression can be considered exploration of concepts and explanations, and the second phase can be considered the development of concepts and explanations, then this third phase could be considered the refinement of concepts and explanations. Students had been concurrently introduced to the concepts and to ways of explaining the concepts, and had had the opportunity to develop an understanding of the concepts. At this point, misconceptions or gaps in understanding that the students may or may not be aware of were addressed, and at the same time specific aspects of their own explanations that proved to be problematic were brought to their attention and contributed to the way that they understood the concept.

Summary

The second of my two research questions focused on the ways in which the instruction in a mathematics class that made use of writing was structured to support the development of students' written explanations. Explanations of mathematical concepts similar to the types of explanations students would be required to write on assignments and assessments occurred frequently during classroom discussion and instruction. Although many of these in-class explanations were given for the purpose of increasing understanding, explanations continued even when such understanding had become established enough to be taken-as-shared within the class. Such explanations seemed to serve as models of how written explanations could be constructed.

Throughout the semester as different concepts were addressed through instruction, the use of in-class explanations, including modeling, seemed to develop in a reasonably consistent progression that supported the gradual development of the understanding necessary to write explanations. Students explored concepts and the language used to explain those concepts, developed their understanding and ability to explain that understanding through the gradual introduction of model explanations, and then refined the ability to explain through explicit instruction based on their written explanations. Each of these phases could be found in the development of most concepts throughout the course of the unit on fractions.

Chapter Six — Conclusion

This study was motivated by perceived shortcomings in the literature on writing to learn mathematics. The majority of empirical studies of the use of writing in mathematics classrooms involve the introduction of some form of writing into a mathematics classroom, followed by an analysis of the effects of the writing on student understanding as measured by tests of mathematical knowledge, or by an analysis of the content of student writing. Such studies, by divorcing writing from other factors of students' classroom experience, are unable to answer crucial questions about the place of writing in learning and instruction. In particular, current research fails to adequately address two issues involved in the use of writing in mathematics classrooms. First, while some studies show that writing appears to affect learning in some way, these studies are unable to describe or explain the actual relationship between writing and learning. Second, the majority of writing studies fail to take into account the actual process of writing and the requisite knowledge about writing mathematics students must possess in order to appropriately express their mathematical thinking and understanding. These shortcomings have both theoretical and practical implications. It is difficult to study writing if we do not fully understand its role in the learning process, and it is also difficult to successfully implement writing as a tool for learning and participating in mathematical activity if we don't understand the factors that influence students' ability to create written mathematical explanations, or how we might be able to support this ability.

Contributions

Unlike previous studies that implicitly view writing as somehow separable from other aspects of the classroom learning environment, my study situates the use of writing in the context of classroom instruction and other everyday classroom activities. I examined the instruction and whole-class discussion that took place in a classroom in which writing was an important part of students' learning experience and studied the role that writing played in this setting. Doing this allowed me to address both of the shortcomings with prior studies. By looking at the role that writing played in students' wider classroom experience, I was able to identify relationships between the learning atmosphere of the classroom and the importance of writing in that classroom. This allowed me to study the actual relationship between writing and learning. Also, looking at how writing appeared in classroom instruction allowed me to address the process by which writing was developed, which in turn provides insight into how students learn to create mathematical writing and how the use of writing can be supported in a mathematics classroom.

The Relationship between Writing and Learning

Previous studies of writing in mathematics have attempted to show that writing supports students' learning of mathematics, but have been unable to explain empirically why this might be. Rather than looking at the effect of writing on learning as an end product, I examined the classroom environment in which learning occurred and how writing was situated in this environment. I found that instruction which addressed aspects of students' writing also addressed students' understanding of mathematical concepts, and that instruction in mathematical concepts similarly addressed particular aspects of students' written explanations. In fact, in much of classroom instruction, mathematical concepts and aspects of writing were inseparable.

This suggests a much stronger relationship between writing and learning than the assumed relationship that has formed the impetus for previous research on writing. Not only can the use of writing support learning, but the type of writing students are expected to produce and the type of writing instruction they receive will influence the type of mathematical understanding that students develop, and the mathematics content instruction students receive will influence the type of mathematical writing that students produce. On the one hand, I found that the way in which learning occurred in the classroom shaped how students were given access to knowledge about how to convey their understanding in writing. Specifically, the conceptual instruction in the classroom contributed to a class understanding of how to use language to convey particular concepts, how to determine what needed to be included in a written explanation, the purpose for which explanations should be written, and the audience to whom they should address. This in turn led to the predominance of certain types of explanations over others. But on the other hand, as each of these critical aspects of written explanations emerged in the class, the ways that they were addressed and supported also shaped the way that mathematical concepts were understood in the classroom. That is, the type of writing students were encouraged to produce led directly to particular mathematical conceptions and ways of understanding, while also leading away from other conceptions.

This finding contributes to the research on writing to learn by helping to describe a very specific relationship between writing and learning. Previous studies have argued that there is a relationship between writing and learning because the introduction of writing in the classroom accompanied some measurable change in students' performance on some test or task. My study, however, helps to explain why and how such a relationship exists under conditions in which writing is an integral part of instruction. Part of the reason this relationship can be described here where it could not be described in previous studies is that I looked specifically at how writing played a role in the *practices* of the classroom rather than looking only at concrete results of those practices (such as student writing or tests of learning). This has implications for future research on writing. Researchers cannot gain an understanding of how writing may play a role in students' mathematical learning without situating writing in the context of the classroom practices, because such practices influence how students learn, how students write, and the nature of the relationship between their writing and conceptual learning.

This finding has implications for the use of writing in a mathematics classroom as well. The fact that aspects of knowledge related to the creation of written explanations emerged through classroom instruction, sometimes perhaps unintentionally, implies that a teacher who uses mathematical writing for a classroom must be aware that everyday classroom events have a direct influence on the writing that students produce. This awareness can help a teacher to better understand the sources of student writing errors and be able to address student understanding as communicated through writing. Even more importantly, if a teacher is aware of the way that particular aspects of writing and conceptual understanding affect each other in the course of classroom instruction, that teacher is better able to shape the way these aspects of writing and understanding emerge in the classroom in order to help students both to better understand mathematics and to better communicate that understanding in writing.

Supporting Writing through Classroom Instruction

One of the difficulties with the use and study of writing in mathematics classrooms is that there is an abundance of evidence to suggest that writing does not lead automatically to the type of learning the teacher may have envisioned. However, little research addresses the question of how writing can be supported in such a way that students *can* use mathematical writing effectively in such a way that the writing supports the teacher's instructional goals. Because my research was situated in classroom instruction, where expectations about writing are conveyed to the students, I was able to describe some of the ways that writing was supported by specific practices in order to achieve the teacher's content goals.

Modeling appeared to play an important role in giving students access to knowledge about how to write mathematically and in developing expectations and understanding about what it meant to write mathematically. Although components of writing appeared throughout instruction, modeling took place once students had already had significant exposure to the mathematical concept being explained. This allowed a shift in the focus of in-class explanations from conceptual understanding to the language used to convey that conceptual understanding. Without this shift into model explanations, much of students' access to knowledge about how to create written mathematical explanations would have taken place when the students' primary focus was on the foundational conceptual understanding or other details not directly related to the communication of that understanding. Also, the fact that different types of models occurred throughout classroom instruction suggests that students need not only access to models (a role fulfilled by teacher-generated models), but also support in generating their own models (what I referred to as student models with feedback) and the opportunity to see the dynamic process of creating a written explanation (actively constructed models).

In addition, I found that in this classroom, modeling and other forms of student access to knowledge about written explanations occurred in a progression that paralleled the development of mathematics concepts. The parallel nature of this progression can be seen most clearly by describing phases by which written explanations were addressed side by side with phases of conceptual instruction, as in Table 6.1.

Conceptual Instruction	**Written Explanations**
1. Exploration of new concepts; making connections between familiar ideas in order to understand new ideas.	1. Exploration of new concepts by using familiar language in new ways; students try out explanations of new concepts for effectiveness.
2. Students given problems similar to problems from the first stage in order to develop and solidify understanding.	2. Explanations presented in class as understanding comes to be taken as shared; model explanations developed gradually.
2. Difficulties in student understanding that surfaced in the course of instruction addressed explicitly.	3. Problematic aspects of student explanations addressed through explicit writing instruction.

Table 6.1
Parallel Development of Concepts and Written Explanations

On one hand this parallel development of concepts and written explanations is another manifestation of the integral nature of writing and conceptual understanding. Furthermore, this particular finding suggests that support for written explanations occurs as a natural and integral part of mathematical instruction. One common reason that teachers of mathematics give for not teaching writing in their classroom is time used for teaching students to write is time spent away from the ultimate goal of teaching students mathematics. The fact that the development of aspects of written explanations was so closely related to the development of concepts themselves in this classroom suggests that writing instruction does not have to be considered separate from conceptual instruction. Rather, it already plays an integral role in that instruction, and therefore deliberate instruction intended to support good written explanations could occur just as naturally. This assertion is even more strongly supported by the relationship between writing and learning addressed above. Because the way that concepts are taught supports students' writing, and the way that writing is addressed supports students learning, then not only can writing be developed naturally in the course of developing understanding, but addressing writing can be an important part of students' mathematical learning.

Limitations and Future Directions

One obvious limitation to this study was that the data was drawn only from classroom events and not from student writing itself. Although I was able to draw several important conclusions by analyzing classroom data, there are also important questions that this analysis cannot answer. While I can point to ways that writing was addressed in classroom instruction, ways that explanations were supported by the structure of the classroom, and the apparent relationship between the role of writing in the classroom and the nature of conceptual instruction, I cannot say whether this relationship was evident in the written work that students produced. Nor can I say whether or how student writing was influenced by the classroom instruction. These are both important aspects of writing in mathematics classrooms, and while they cannot be answered by the nature of this particular study, they are relevant to the original purposes of my study. That is, these questions address both how writing can be supported in a classroom and how writing affects students' mathematical learning. However, even though my study cannot address these questions, it does help to give direction to the questions by suggesting avenues by which the questions can be answered. This is something that previous research has not done.

Specifically, future research can approach the analysis of student writing for its relationship to student learning by looking at how students develop in the three components of writing that I identified. Research can investigate how students make use of language in their writing, what details they choose to explain, and the apparent purpose and audience of their explanations, as well as how these components of writing change over time as students become more familiar with the concepts and with the norms of writing and communication within the classroom. In addition, such an analysis of student writing should be carefully tied to the classroom practices that form the context in which this writing takes place. This sort of analysis can provide useful insight into the overall development of students' understanding, not just the development of their understanding through the use of writing.

Such an analysis can also provide insight into the ways in which classroom practices actually affect the type of writing that students produce. As language is addressed in class, for instance, how does that language carry over into student writing? Do the details that students choose to include in their explanations follow the patterns that are set in class through modeling and other implicit or explicit writing instruction? What other factors influence student writing, and how much does the classroom instruction affect what students write about mathematics? These questions and others can be at least partially addressed by studying writing in close conjunction with the events that occur in everyday classroom instruction.

Future research can also directly address how writing is supported in the classroom. Because students' writing is more than simply a reflection of their mathematical conceptualizations, and because support for students' writing can also directly support students' mathematical learning, then such support could be an important area of investigation. Research could address not just how writing is supported naturally in an environment where writing is an important part of instruction, but could also address the effects of deliberate writing instruction both on the nature of student writing and their ability to use writing in order to learn mathematics, and on the students' mathematical understanding. If a mathematics teacher not only valued writing, but also viewed every episode of content instruction as an episode of writing instruction as well, and subsequently implemented more overt and direct ways of addressing writing, would such instruction lead to changes in students' writing and learning? Such a study may involve purposefully organizing direct writing instruction in a classroom according to the parallel structure of instruction outlined above, as well as making conscious use of modeling as a necessary component of learning and of learning to write.

In an educational environment in which mathematical writing plays an increasingly important role as a legitimate mathematical practice, such questions are both important and worthwhile to address. These questions are relevant not just to those who are interested in the specific relationship between writing and learning, but to all mathematics education researchers and practitioners because the issues addressed by these questions go to the very root of what it means to learn and do mathematics, how such learning takes place, and ways that educators can help students in the process of learning. As such, the potential directions for future research posed at the conclusion of this study, as well as other unique research questions related to the conclusions of this study, can and should be addressed by other interested researchers in the field of mathematics education.

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