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Abstract: The paper presents modelling complex socio-ecological problems, where relations among individual quantities vary in time. This includes dynamics of processes in the model, which enables to understand time as a continuous quantity and to describe dynamic processes by a system of differential equations with delay. Implementation of these models in Maple system enables both analytic and numerical solutions and their visualizations. The Maple solution of the specific example of the model with delayed argument is shown, where all input parameters can be interactively changed during the solution process, i.e. delay, interval of the solution, or other parameters which have an immediate impact on the given model. The results obtained, provided that the mathematical model is correct, enable us to model the impact of history and its influence as well as the impact of all the mentioned characteristics, and to deepen the knowledge of how the model functions.

***Keywords**:* modelling; socio-ecological problems; differential equations with delay; numerical solution; Maple; environmental damage

1 INTRODUCTION

Current complex environmental problems require new methods of research linking knowledge of both natural sciences and social sciences (including various methods within the social sciences), whereby decision-making processes will be supported at all levels (international, national, regional, and local) of protection and formation of the environment. Problems to be resolved by Preston et al. (2015) in connection with an effective environmental protection system setting and long-term sustainability of social systems very often follow from the overall man-environment interaction.

One of the typical problems connected with environmental protection was studied by models dealing with the response of ecosystems to change of exogenous factors and try to grasp human´s behaviour in everyday life. For example climate change studies also include indicators describing the abilities of institutions and individuals to control risks. Fraser et al. (2013) and Rouge et al. (2015) say that these abilities are geographical, biophysical, social and institutional condition dependent.

Studies dealing with the sensitivity and resistance to interventions damaging the environment increasingly focus on behavioural analysis of socio-ecological systems. It was shown Schlueter et al. (2012), Cote and Nightingale (2012), Klapwijk et al., (2014) and Fischer et al. (2015).

The analysis by Binder et al. (2013) contributes to deeper understanding of the dynamics of human behaviour and evolution. However, it is not easy to model a socio-ecological system involving environmental impact. Most of the environmental dynamic models discussed in literature have their solutions based for example on nonlinear optimisation and game theory. Another publication using nonlinear optimisation is for example Coutts et al. (2013). These authors analysed a model of weed spread control, considering nonlinear behaviour (weed spreading) in an ecological-economic system, caused by weed properties and reactions of the growers. Growers doing nothing to kill weeds are assumed to get under social pressure from their surroundings. The study therefore concludes that social pressure significantly affects the behaviour of the grower.

Challies et al. (2014) said that a deeper understanding of these relations might considerably contribute to improved environmental protection and support sustainable development entailing an improved quality of human life. That is why it is necessary to develop mathematical and computation models able to cover not only ecological but also social processes affecting the target system.

A typical drawback of socio-economic system by Challinor et al. (2010) modelling following from simplified human behaviour is the neglected ability of individuals to adapt to changes in their surroundings. In this context the strong adaptation ability was noted in agriculture and significant elements determining this ability were identified (Panda et al. 2013). Preston et al. (2015) provided quantitative data to understand the extent and scale of the effect of socio-economic conditions and trends on the human adaptability to changing environmental conditions. One of the approaches to the development of a prognostic tool of this kind with the help of landscape typology for the classification of different geographical areas and their type definition based on socio-economic criteria is described by Metzger et al. (2013).
This paper presents a new approach to solutions of non-linear mathematical models described by the systems of standard differential equations with delayed argument, demonstrated on a model of environmental damage based on partisan control and public opinion. In these differential equations, when compared to the traditional model where the relations in question use solely the data for a particular time *t*, the effect of the data from the previous period is used – i.e. the so-called delay. The theory is briefly explained in the introductory chapter and serves as a background for formulation of quantity relations studied by the paper. The results are demonstrated on a specific example and model behaviour is presented by means of a computer simulation with the system of computer algebra Maple and the graphic representation of the results uses this system. The solution is based on the theory of functional differential equations, specifically its part dedicated to the solution of differential equations with delay.

2 MODEL OF DAMAGE TO THE ENVIRONMENT

A gradual increase in the scope and intensity of human activities has brought about a negative impact on the environment. Originally, the adverse impact was of local character. However, the impact led to the damage to or demise of a range of ecosystems, deteriorating living conditions of inhabitants in many areas, as well as significant economic damage in a range of territories.
Therefore, it is crucial to understand how the environment may be impacted by political changes, changes in the public opinion, and economic impact of damage to the environment.
As political and public service models are currently far from being perfect, a new model may be regarded as an opportunity for the existing methodological tools. Obviously, despite the dominance of models arising from thoughts considering linear relationships and stable dynamics, this reality tends to result from rather insufficiently elaborated methods which might take into consideration non-linearity, instability and uncertainty associated with the real life.
The model of damage to the environment, impacted by political changes in the country's leadership, and clean-up spending is described by Brown (1994, 2007). It consists of three equations of state quantities depending on time *t*, $t \geq 0$:

$$\frac{dX}{dt}t = rX(1-X) - pXY - kX$$
$$\frac{dY}{dt} = X_{old}\,(1-Y) - Z \qquad (1)$$
$$\frac{dZ}{dt} = XY(1-Z) - Z$$

where:
X - environmental damage;
Y - public concern over the environment;
Z – spending for environmental clean-up.
and parameters
r – the pollution growth rate parameter;
p – the effectiveness of governmental policies to reduce current levels of environmental damage;
k – natural pollution decay rate that reduces environmental damage based on current values of that damage and
X_{old} - a lagged value of environmental damage concern since public opinion usually reacts to rather than predicts environmental damage.
The parameter k is set equal to the expression $\frac{-ln(.5)}{half-life}$, where half-life is the number of years taken for the damage to fall to falf its original value. More details can be found in Brown (1994).
The first equation in (1) represents changes brought about by the damage to the environment.

The second equation in (1) describes a change in the public interest in the environment.
The third equation in (1) demonstrates the dynamism of governmental spending on the environment.

If parameter p is replaced with the function $P \equiv P(t)$, the model (1) can be supplemented and improved by a fourth equation which demonstrates how political goals of Democratic and Republican party may change the environmental policy:

$$\frac{dP}{dt} = eP\left(g_{dem} + g_{rep} - P\right), \tag{2}$$

where:
g_{dem}- ideal Democratic policy response goal;
g_{rep} – ideal Republican policy response goal;
e – parameter determining speed of government policy changes towards partisan goals.

Let us have Δ resp. $\Delta(t)$ constant or variable time delay between a change in the environment and the public interest on a modelled process, the model (1), (2) after modification shall be the following

$$\begin{aligned} \frac{dX(t)}{dt} &= (r-k)X(t) - pX(t)Y(t) - rX^2(t) \\ \frac{dY(t)}{dt} &= X(t-\Delta(t))(1-Y(t)) - Z(t) \\ \frac{dZ(t)}{dt} &= X(t)Y(t)(1-Z(t)) - Z(t) \\ \frac{dP(t)}{dt} &= eP(t)\left(g_{dem} + g_{rep} - P(t)\right). \end{aligned} \tag{3}$$

From the mathematical point of view, system (3) is a non-linear system of four ordinary first-order differential equations and delay $\Delta(t)$.
Functions X, Y, Z and P are generally unknown functions, $r, k, e,$ g_{dem} and g_{rep} given invariables and Δ is either a constant or variable delay of impact X on the solution of system (3).
Thus, system (3) is a so-called system of ordinary differential equations with delays, or, more generally – system (3) counts among ordinary differential equations with delayed arguments, possibly among so-called systems of functional differential equations. Even though specific cases of such systems or individual equations have been known for centuries (Bernouli, Euler, etc.), systematic theory of these equations and their systems did not come into existence until the mid-20th century. One of the first publications were Azbelev (1991) or Kolmanovski and Myshkis(1992).

2.1 Methodology of numerical solutions

The system of ordinary differential equations with delays is possible to solve only numerically (Brown, 2007). The methodology of numerical solutions of such equations has not yet been sufficiently established following to the global nature of solutions of differential equations and systems with delays and only a recently discovered possibility to use as the basis the general theory of such equations. Only modifications of local methods Runge-Kutta can be applied, and a specific method of steps is used for equations with delays – it is particularly suitable for equations with constant delays.
On the other hand, methodical procedures employed by I. Kiguradze and his colleagues are based on methods of a priori estimates of solutions of such equations and systems and they form a natural basis for the method of gradual approximations on an appropriately affiliated operator equation with a contractive operator.
This method was illustratively by Kiguradze (1988) in order to construct the solution of the initial and periodic problem of a non-linear system of ordinary differential equations multi-point and periodic problem for a system of linear differential equations (textbook). In cooperation with Gelshvilli, he published a detailed analysis of a numerical solution of non-linear differential equations in Gelshvilli and Kiguradze (1995).
The above mentioned procedures have been modified and applied in solutions of a range of other specific problem, modelling, among others, even economic processes with delays.
Analogically with procedures applied in the quoted works with economic models and provided that we know function $P(t)$, we look for the solution of our system (2) as an (even) limit of progression of problems' solutions ($n \in N$):

$$\frac{dX_n(t)}{dt} = (r-k)X_{n-1}(t) - pX_{n-1}(t)Y_{n-1}(t) - rX_{n-1}{}^2(t)$$
$$\frac{dY_n(t)}{dt} = X_{n-1}(t-\Delta(t))(1-Y_{n-1}(t)) - Z_{n-1}(t)$$
$$\frac{dZ_n(t)}{dt} = X_{n-1}(t)Y_{n-1}(t)(1-Z_{n-1}(t)) - Z_{n-1}(t) \tag{4}$$
$$X(0) = X_0(\Delta(0)), Y(t) = Y_0(t)\ \ Z(0) = Z_0(0)$$

where $X_0(t), Y_0(t)$ and $Z_0(t)$ are conveniently selected continuous (in reality, given the modelled situation) „initial“ functions.
In doing so, the continuity of the right sides of the system (3) and of the context of "historic part" function X ($t \geq 0$) follows the unique solvability reduced role containing the first three equations of the system (3) and corresponding initial conditions. From the general theory of numerical solution of these tasks implies unequivocal solvability of the problem (4) and convergence solutions designed sequence of tasks (4) to deal with reduced system (3). Fundamentals of our approach to the construction of solutions are given in Kiguradze and Půža (2003) and the illustration uses can be found for example in Bobalová and Maňásek (2006).

3 MAPLE IMPLEMENTATION

The Maple system has been developed by the company Maplesoft (Maplesoft, 2016) since 1990. Maplesoft's core technologies include the advanced symbolic computation engine in Maple environment and physical modelling techniques in MapleSim environment. Combined together, these technologies enable the creation of cutting-edge tools for design, modelling, and high-performance simulation. There is implements a real-valued delay differential initial value problem finds a numerical solution for the delay initial value problems in Maple. It uses numeric methods: the Fehlberg fourth-fifth order Runge-Kutta (Fehlberg, 1970); a variable order Runge-Kutta Method for initial value problems with rapidly varying right-hand sides (Cash, Karp, 1990) or the implicit Rosenbrock third-fourth order Runge-Kutta (Shampine, Corless, 2000) to obtain the numerical solution.
The Maple system is able to detect the presence of delays automatically, but it requires additional information for the case of variable delays in the procedure *dsolve/numeric/delay* (Online Help, 2016), where *delaymax* parameter option can identify real or positive integer delays.
The delay computation implemented for the numeric solvers utilizes the natural interpolant built into the solvers, storing solution values back to memory as the computation proceeds. On initialization, the dependent variable values are assumed to be constants set to the initial values for all times $t < t_0$, and once the delay time for a delay term exceeds $t - t_0$, the interpolant comes into play. Derivatives of the dependent variable values are assumed to be zero for all times $t < t_0$.
In some cases, such as delays for derivatives, use of the initial value or zero is not desirable, and in these cases a constant value for $t < t_0$ can be specified as the second argument to the delay function. For example, $y(t - 1)$ specifies that for $t < 1$ the initial value of y is used, while $y(t - 1, 0)$ specifies that for $t < 1$ the value 0 be used. Similarly, $\frac{d}{dt}(y)(t - 1,0)$ specifies that for $t < 1$ the value 0 be used, while $\frac{d}{dt}(y)(t - 1,1)$ specifies that for t < 1 the value 1 be used.
In the event that a requested delay is so small as to be past the end of the last computed step, extrapolation using the interpolant of the last available step is used.
Note that delays can be used with initial value problems of differential algebraic equations (DEA) as well as with events, though events do not support setting of a past value on the trigger of an event.

4 CASE STUDY: ENVIRONMETAL DAMAGE. NUMERIC SIMULATION IN MAPLE

The system of differential equations with delay (4) was implemented in Maple in the procedure dsolve/numeric/delay (Online Help, 2016) and we tested above mentioned numerical methods for solving systems of differential equations with delay. The best response gave the Fehlberg fourth-fifth order Runge-Kutta method.
We considered 16-years period of simulation environmental damage to illustrate the solution of the system (4) with the delay. Firstly, the delay of the public to respond to potential changes in the environment, has been set at two years.

The input values on the model (4) have been specified by the team experts as

$$X(0) = 0.3,\ Y(0) = 0.5,\ Z(0) = 0.19,\ p = 0.9,\ k = 0.023,\ r = 1. \qquad (5)$$

With a view to demonstrating possibilities of the new approach to the solution of the original problem, we assume „historic development“ before time t = 0, which is simulated by the function

$$X(t) = 0{,}003 sin(t) + 0{,}3. \qquad (6)$$

4.1 Results of simulation

Figure 1 represents a phase portrait showing the solution trajectory for the above model (4) with input value (5) and delay function (6). The three axes of Figure 1 represent three state variables of the system: *X- environmental damage*, *Y- public concern for the environment*, and *Z- environmental clean-up costs*.

Figure 1 shows the situation, where political parties act in agreement, their behaviour does not differ significantly and they act with respect to the environment. The response *Y* of the public to changes in the environment is approximately six years in this simulation. The graph at Figure 1 shows that the trajectory of the spiral heads towards a stable, balanced state. Furthermore, this situation involves a relatively low level of damage to the environment. Therefore, we may assume that the ruling parties' policy supports the protection of the environment. The initial fluctuation and damage *X* to the environment are minimized in the course of time and the system stabilizes.

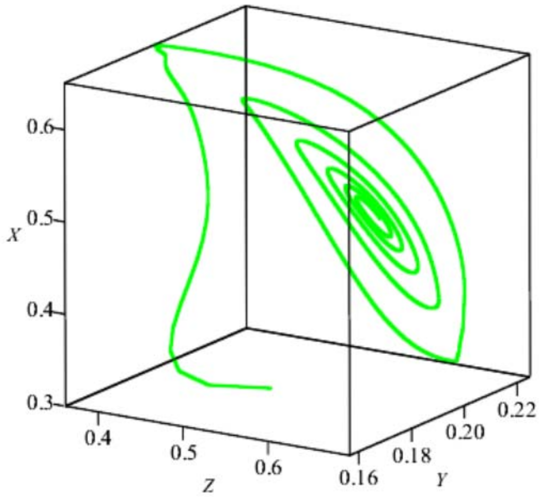

Figure 1. The solution trajectory of the above model (4) - (6).
Source: Authors

Figure 2 shows the situation if the public react with a delay of four years. This situation is different, it stabilizes gradually, but significant damage to the environment *X* occurs repeatedly and very slowly. The graph at Figure 2 clearly shows that the system responds sensitively to changes in time representing the public interest *Y*.

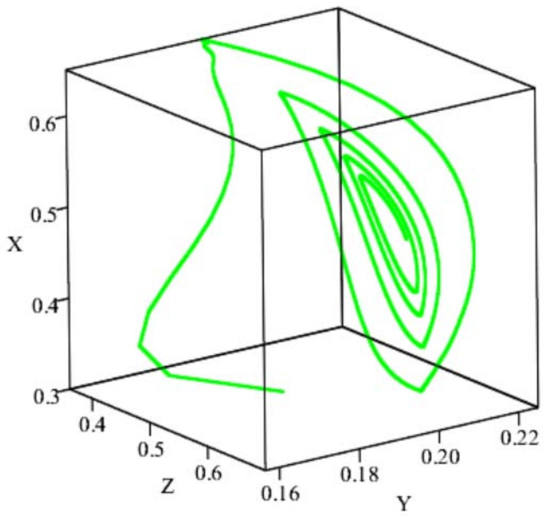

Figure 2. The general public reacts with a delay of four years ;
Source: Authors

In the third case was examined a situation in which the public does not respond to changes in the environment with a constant delay, but their interest fluctuates slightly for the period of about two years. Figure 3 shows that despite the unstable interest of the public the situation stabilizes and gradually achieves a balanced state.

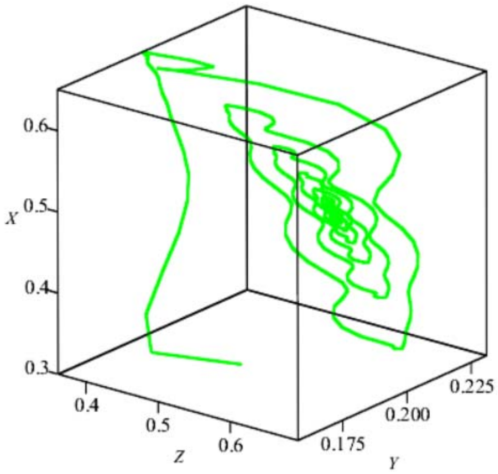

Figure 3. The unstable interest of the public; Source: Authors

These three examples prove quite positively that the public interest in the environment *Y* may significantly influence politicians' behaviour and contribute the stabilization of the whole system.

5 CONCLUSIONS

When modelling complex socio-ecological problems, we face the fact that relations among individual quantities vary in time. One way to include dynamics of processes in the model is to understand time as a continuous quantity and to describe dynamic processes by differential equations and implement these models in Maple system, which enables both analytic and numerical solutions and their visualizations. A range of applications and findings of theoretical mathematics have recently shown that models using non-linear differential equations (with delays) may describe complex behaviour of socio-ecological system more precisely.

As the specific example of the model with delayed argument shows, all input parameters can be changed during the interactive solution process, i.e. delay, interval of the solution, and other parameters which have an immediate impact on the given model. The model used in the paper is stable within the explored solutions. There are other methods used for the analysis of unstable solution behaviour in unstable models that are based on other procedures. The results obtained, provided that the mathematical model is correct, enable us to model the impact of history and its influence as well as the impact of all the mentioned characteristics, and to deepen the knowledge of how the model functions.

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