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Desired Precision in Multi-Objective Optimization: Epsilon Archiving or Rounding Objectives?

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Abstract: Multi-objective optimization (MO) aids in supporting the decision making process in environmental engineering and design problems. One of the main goals of solving a MO problem is to archive a set of solutions that are well-distributed across a wide range of all the design objectives. To this end, some of the state-of-the-art MO algorithms use the epsilon dominance concept to define a mesh-grid with pre-defined grid-cell size (often called epsilon) in the objective space and archive at most one solution in each grid-cell. Moreover, epsilon archiving helps the MO algorithm control the number of archived solutions. This is particularly important when solving problems with a large number of objectives, because as the number of objectives increases, the non-dominated portion of the objective space increases exponentially and therefore the chance of finding any dominating (new better) solution decreases.

The epsilon archiving process is a computationally demanding process. This study introduces a similar but computationally more efficient solution archiving approach where each objective function is rounded to the desired precision level before being compared to the set of archived solutions that already have rounded objective function values. The epsilon archiving and the proposed archiving approaches are compared in terms of the quality of final archived solutions for solving a five-objective benchmark mathematical test problem and a six-objective hydrologic model calibration problem. Results show promises in the proposed solution archiving approach in comparison with the epsilon archiving of ε-NSGA-II.

***Keywords**:* Multi-objective Optimization, Solution Archiving, Desired Precision, Epsilon Dominance, Rounding Objectives

1. INTRODUCTION

Multi-objective evolutionary algorithms (MOEA) search across the decision space to identify and *archive* high-quality solutions whose map in the objective space is the Pareto front. The proximity, measured by the dominance rank, and the diversity of solutions are the main factors that define the quality of a Pareto front. Therefore, the solution archiving operator of MOEAs is mainly based on the proximity and the diversity of generated solutions.

In light of the fact that non-dominated space grows exponentially with increasing objective counts, Laumanns et al. (2002) listed three reasons for binding the archive. First, computation time to check the dominance rank grows as the number of archived solutions grows. Second, the decision making process will not necessarily benefit from a large number of solutions (policies or designs). Third, limiting the archive size allows the MO algorithm to focus on regions of attraction rather than the whole set of non-dominated solutions. So, MOEAs with bounded archive size have to select and discard some extra solutions when the number of first rank solutions (i.e. non-dominated solutions) is more than the size of bounded archive. This process is often based on a measure of solution diversity such as niching in NSGA (Srinivas and Deb 1994), crowding distance in NSGAII (Deb et al. 2002), strength in SPEA2 (Zitzler et al. 2001) and hypervolume contribution in SMS-EMOA (Emmerich et al. 2005).

Hanne (1999) explained why binding the archive can cause deterioration: discarding some non-dominated solutions in one generation and archiving some worse (dominated) solutions in future generations. Hanne (1999) solved the deterioration issue by archiving only dominating solutions. Laumanns et al. (2002) proposed a more advance archiving approach based on the epsilon dominance concept that guarantees both the proximity and diversity of solutions for MOEAs with bounded archive size. This approach discretizes the objective space into grid cells and archives at most one solution (the dominating one) in each cell. This archiving strategy is utilized in some of the state-of-the-art MO algorithms including ε-MOEA (Deb et al. 2003), ε-NSGAII (Kollat and Reed 2005), and Borg (Hadka and Reed 2013).

Epsilon can be set to the desired precision level of each objective function to make sure that the difference between each pair of archived solutions is meaningful. Kollat et al. (2012) used ε-NSGAII and calibrated two different hydrologic models (two different levels of model complexity) for 392 catchments across the United States in a four-objective automatic calibration framework. They showed that when epsilon (grid size) is set to the proper numerical precision level of each objective function, a large number of MO model calibration problems in their study collapse to 10 or fewer solutions on the final Pareto front meaning that, with appropriate numerical precision levels, the conflict between all the four objectives almost disappeared.

2. METHODS

2.1. Epsilon Archiving versus Rounding Objectives

In this study, the idea to set the grid size of the epsilon archiving to the desired precision of objectives is used to mimic the epsilon archiving but completely get around its computational burden. In the proposed solution archiving approach, the objective function values for each recently generated solution are rounded to the desired precision of objectives before the dominance check takes place. Moreover, all currently archived solutions have rounded objectives. Therefore, any difference between two solutions beyond the desired precision of objectives is ignored in the dominance check and a solution is considered to be dominating only if all its rounded objective function values are better than or at least equal to those of the dominated solution. The desired precision level in the proposed solution archiving approach resembles the grid cell size (or epsilon) in epsilon archiving.

2.2. Multi Objective Optimization Algorithm

The proposed solution archiving is compared to the epsilon archiving of ε-NSGAII which is a variant of NSGAII equipped by the epsilon archiving. The epsilon archiving discretizes the objective space into grid cells with epsilon as the grid cell size and archives at most one solution in each grid cell. The solution is the one that is closest to the ideal corner of a cell, e.g. the bottom left corner of each cell in a minimization problem. The following two different settings of ε-NSGAII represent the epsilon archiving and the proposed solution archiving approaches, respectively:

a) Epsilon Approach: epsilon parameters of ε-NSGAII are set to the desired precision of objectives, but objective functions are calculated to their full precision
b) Rounding Approach: epsilon parameters of ε-NSGAII are set to very small numbers (i.e. 10^{-6} here), but the objective functions are rounded to their desired precision

2.3. Study Cases

The aforementioned two variants of ε-NSGAII are applied to a benchmark mathematical test problem and a hydrologic model calibration problem. The mathematical test problem DTLZ2 (originally introduced in Deb et al. 2001) is scalable both in the decision and objective spaces. A modified and more complex version of this problem called R2_DTLZ2_M5 was introduced for the MO algorithm performance competition of the Congress on Evolutionary Computation (CEC09) and is used in this study. R2_DTLZ2_M5 has 5 objective functions and 30 decision variables, and its decision variable space is rotated and extended to add more complexities to the problem. The 5000 equally spaced points on the known Pareto front of R2_DTLZ2_M5 generated as a reference set for CEC09 is used for algorithm comparison purposes in this study.

The second case study is a modified version of the hydrologic model calibration problem introduced in Tolson and Shoemaker (2007) for the 32 km^2 Town Brook sub-watershed of the Cannonsville watershed. The original problem was set up to calibrate 26 parameters of SWAT2000 and optimize a weighted-sum objective function that represented the model performance in simulating daily streamflow (cms), total suspended sediment transport (kg), and total phosphorus delivery (kg). Interested readers are referred to Tolson and Shoemaker (2007) for more details about this case study. Here, the weighted-sum objective function of the original model calibration problem is disaggregated into six objective functions to simultaneously maximize NSE (Equation 1) and minimize the absolute bias ABS (Equation 2) calculated for all the aforementioned three measured datasets. In these equations, m_t and s_t denote measured and simulated data, respectively, for daily streamflow, total suspended sediment and total phosphorus over the calibration period (1096 days).

$$NSE = 1 - \frac{\sum_{t=1}^{1096}(m_t - s_t)^2}{\sum_{t=1}^{1096}(m_t - \bar{m})^2} \tag{1}$$

$$ABS = \frac{\left|\sum_{t=1}^{1096} m_t - \sum_{t=1}^{1096} s_t\right|}{\sum_{t=1}^{1096} m_t} \tag{2}$$

2.4. Numerical Experiment and Results Assessment

A numerical experiment is conducted to assess the proposed solution archiving approach that rounds the objective function values to the desired precision level in comparison with the epsilon archiving of ε-NSGAII. The two variants of ε-NSGAII explained above are compared at three different precision levels for five objectives of R2_DTLZ2_M5: 0.001, 0.01, and 0.1, and a desired precision level for the six objectives of the SWAT2000 model calibration problem: 0.01.

For the mathematical test problem, results are compared at two levels of computational budget: 1. a relatively limited budget of 10,000 solution evaluations, and 2. a relatively large budget of 100,000 solution evaluations. However, results for the model calibration problem are only compared at the computational budget of 10,000 solution evaluations due to the limited computation budget for this study.

Beside epsilon and the computational budget (stopping criterion), ε-NSGAII has several other parameters that are selected based on the recommended values in the literature, e.g. see Tang et al. (2006) and are reported in Table 1.

Table 1. Parameter values of ε-NSGAII

	Computational Budget	
Algorithm Parameter	**10,000**	**100,000**
Populations size	100	400
Generation	100	250
Crossover probability	1.0	
Mutation probability	1.0 / decision variable count	
Crossover distribution index	15	
Mutation distribution index	20	

The final solution of a MO problem is an approximation of the Pareto optimal set in the decision space and its map to the objective space is an approximation of the Pareto optimal front. Unary MO performance metrics assess the quality of each approximate set by a single number. Results of this study are assessed by the unary performance metric additive epsilon indicator introduced by Zitzler et al. (2003). It measures the smallest distance by which a Pareto approximate front must be shifted in the objective space to weakly dominate a reference set of points. For R2_DTLZ2_M5, the 5000 points provided by CEC09 is used as the reference set. For the model calibration problem though, the Pareto optimal front is unknown. In this case the aggregate Pareto front which is the best set of points obtained by all optimization trials is used as the reference set. Results are compared based on the full precision of the objective function values to make sure that the proposed archiving approach does not get awarded or penalized due to the rounded value of objective functions.

ε-NSGAII is a stochastic search algorithm and therefore one should expect some variations in its performance as the initial random seed changes. Typically, this variation diminishes as the computational budget increases. To consider this variation in the analysis of results, 10 independent trials of ε-NSGAII are run for each set of algorithm parameter settings, and the distribution of the algorithm performance metric are compared.

3. FINDINGS AND ARGUMENT

Figure 1 shows the distribution of the additive epsilon indicator (MO algorithm performance metric) based on 10 independent trials of the two variants of ε-NSGAII, with the epsilon or rounding archiving approaches, applied to the mathematical test problem R2_DTLZ2_M5. This figure suggests that as the desired precision level changes from three decimal places to one decimal place, rounding the objective function values become more preferred compared to the epsilon archiving of ε-NSGAII. This preference is most evident when the objective functions are rounded to one decimal place. In this case, results after 10,000 solution evaluations are even comparable to the results after 100,000 solution evaluations with higher precision of the objective functions.

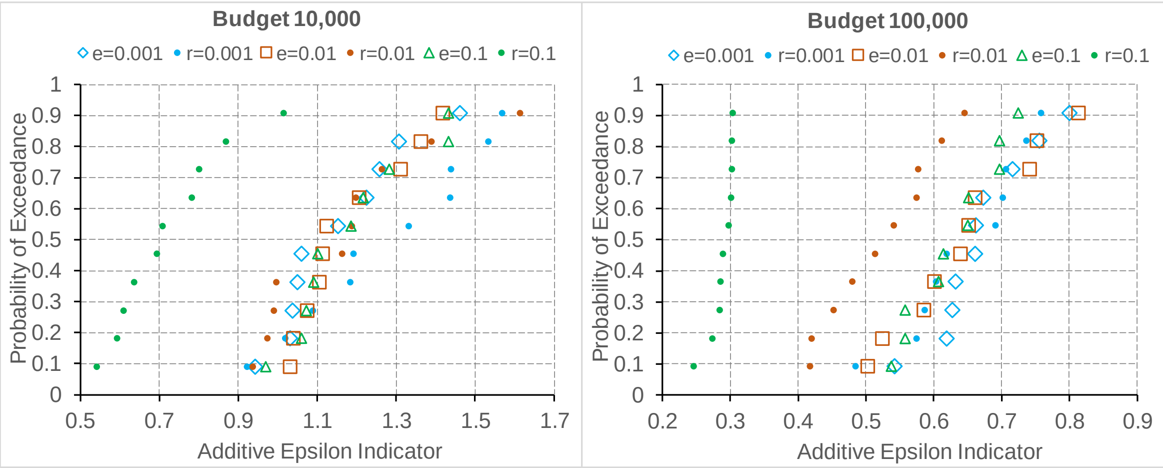

Figure 1. Empirical cumulative distribution of additive epsilon indicator based on final results of 10 independent trials of ε-NSGAII with epsilon archiving (e) or rounding archiving (r) applied to R2_DTLZ2_M5

Figure 2 shows that in case of calibrating SWAT2000 to maximize NSE and minimize absolute bias for daily streamflow, total suspended sediments, and total phosphorus with the precision of two decimal places, the proposed archiving approach is clearly preferred to the epsilon archiving of ε-NSGAII.

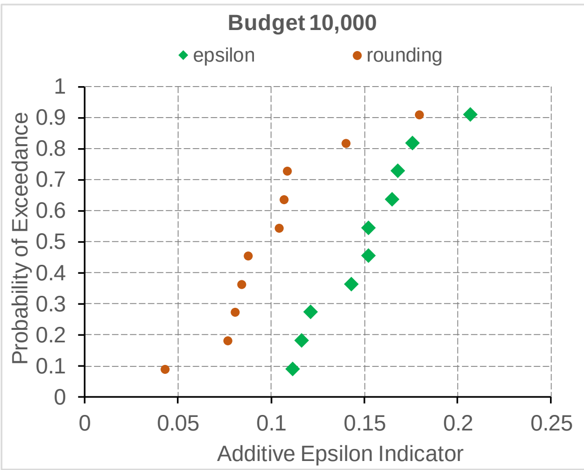

Figure 2. Empirical cumulative distribution of additive epsilon indicator based on final results of 10 independent trials of ε-NSGAII with epsilon or rounding archiving approaches applied to SWAT2000

Table 2 shows that on average, the number of archived solutions identified by both archiving approaches are very similar except for the R2_DTLZ2_M5 case with the objective functions rounded to one decimal place and solved with the budget of 10,000 solution evaluations. Therefore, it is concluded that the preference of the proposed approach based on the additive epsilon indicator is not only due to the better distribution of the archived solutions but also to the better proximity of results. Moreover, the similar number of archived solutions show that the proposed solution archiving approach is as effective as the epsilon archiving in controlling the number of archived solutions.

Table 2. Comparison between the two solution archiving approaches (ε: epsilon archiving r: rounding to the desired precision level) based on the average value of additive epsilon indicator and the number of archived solutions in 10 independent trials.

		10,000		**100,000**	
Case Study	**Precision**	**Add. epsilon Ind.**	**Sol. Count**	**Add. epsilon Ind.**	**Sol. Count**
R2_DTLZ2_M5	$\varepsilon = 10^{-3}$	1.153	614	0.669	4908
	$r = 10^{-3}$	1.272	673	0.646	5107
	$\varepsilon = 10^{-2}$	1.179	614	0.647	4201
	$r = 10^{-2}$	1.171	643	0.523	4919
	$\varepsilon = 10^{-1}$	1.184	368	0.629	1413
	$r = 10^{-1}$	0.725	593	0.288	1457
SWAT2000	$\varepsilon = 10^{-2}$	0.151	362	-	-
	$r = 10^{-2}$	0.101	399	-	-

4. Conclusions and Recommendations

This study represents the first step in introducing a new solution archiving approach that mimics the epsilon archiving approach implemented in some of the state-of-the-art MO algorithms including ε-NSGAII and Borg but gets around the corresponding computational burden. The proposed approach is successfully implemented for solving a benchmark mathematical test problem and a hydrologic model calibration problem.

One of the possible drawbacks of the proposed solution archiving is that it can increase the probability of multi-modality in special cases. Future work should focus on advancing the proposed solution archiving to resolve this potential issue. Moreover, future work should test the effectiveness of the proposed approach for solving more diverse environmental and water resources engineering MO problems.

Finally, future work can implement the proposed solution archiving approach for MO algorithms that have unbounded archive. The unbounded archive is another solution for the deterioration issue explained in the introduction section and has been implemented in several other MO algorithms including the ones introduced in Asadzadeh et al. (2013), Kaylani et al. (2010), Smith et al. (2008), and Fieldsend et al. (2003).

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