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Long Memory Inflationary Dynamics: The Case of Brazil

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Long Memory Inflationary Dynamics: The Case of Brazil

Valderio A. Reisen, Francisco Cribari-Neto, and Mark J. Jensen

Abstract

It has been argued by several authors that the inflationary dynamics in Brazil follow a unit root process, thus displaying some inertia. Indeed, Cati, et al. (Journal of Applied Econometrics, 1999) have found that the inflationary dynamics in Brazil are nearly fully inertial. We estimate the fractional differencing parameter using an ARFIMA specification for the inflation rate in that country and our results suggest that the inflationary dynamics are better modeled by a long memory process than by a unit root mechanism, thus implying that there is no inertia in inflation, contrary to what has been found by other researchers. We also found that the estimates of the fractional parameter are invariant to first differencing.

1. Introduction

It is common to find time series that exhibit long memory dynamics in a wide number of fields, such as astronomy, hydrology, computer science, economics, among others. The ARFIMA (autoregressive–fractionally integrated–moving average) process has been widely used to model time series with such a property; see Beran (1994) for a review. When d , the integration parameter in the ARFIMA specification, belongs to the interval $(-0.5, 0.5)$, the process is stationary, invertible, and may display either the persistence or the antipersistence property. The most interesting case corresponds to $d > 0$ (long memory or persistence). Here, the autocorrelations are not summable and the spectral density tends to infinity as frequencies approach zero.

Many estimators of the parameter d based on parametric and semiparametric estimation have been proposed in the literature. Among the semiparametric estimation approaches, there are those of Geweke and Porter-Hudak (1983), Reisen (1994), and Robinson (1995). They all make use of the regression equation of the log of the spectral density of the process. Robinson (1994) and Lobato and Robinson (1996) develop an alternative semiparametric approach based on averages of the periodogram over a band of frequencies around the origin. The parametric estimators of the degree of fractional differencing are usually obtained using maximum likelihood and pseudo-maximum likelihood methods by assuming that the form of the spectral density is known (see Dahlhaus, 1989; Fox and Taqqu, 1986; Sowell, 1992; and Taqqu, et al., 1995). Recent simulation studies comparing different estimation procedures for long memory processes are Bisaglia and Guegan (1998), and Hurvich and Deo (1999). They focus on the estimation of the parameter d . In the ARFIMA(p, d, q) model, however, all parameters, including the autoregressive and moving average ones, have to be estimated. These parameters can be simultaneously estimated using the *parametric* approach. In the case of *semiparametric* methods, the parameters are estimated in two steps: at the outset one estimates d ; the autoregressive and moving average parameters are estimated in a second step. Smith, et al. (1997) and Reisen, et al. (2001) investigated the bias in both the estimate of the fractional integration parameter d and in the the short-run autoregressive (AR) and moving average (MA) parameter estimates in ARFIMA models using parametric and semiparametric estimation methods. Recent interest has been devoted to the situation where $d > 0.5$. Here, the process is still long memory but becomes non-stationary. Special interest lies with the case where $0.5 < d < 1.0$ (level reversion); see Hurvich and Ray (1995) and Lopes, et al. (2003).

The performance of traditional unit root tests under long memory has also been investigated. As shown in Diebold and Rudebush (1991) and Hassler and Wolters (1994), Dickey–Fuller type tests have low power against fractional alternatives. Consequently, such tests may often suggest the need to take (integer) differences of the data when this operation is not appropriate. A discussion of the misspecification of stationary ARFIMA processes as nonstationary ARIMA models can be found in Crato and Taylor (1996). The consistency of the KPSS stationarity test against fractionally integrated alternatives, i.e., the ARFIMA process, is shown by Lee and Amsler (1997). The authors show how the KPSS statistic can be used to distinguish between the following: (a) short memory, i.e., $d = 0.0$; (b) stationary long memory, i.e., $0.0 < d < 0.5$; (c) nonstationary long memory, i.e., $0.5 \leq d < 1$ and (d) unit root ($d = 1$). The results suggest that one can consistently distinguish (a) and (b) from (c) and (d), but cannot distinguish between (c) and (d).

The goal of this paper is to investigate whether the inflationary dynamics in Brazil display long memory properties. Brazil has often been cited as a country where inflation displays ‘inertia’. That means that in the absence of further shocks, inflation tends to reproduce itself from one period to the next (see, e.g., Cati, et al., 1999; Durevall, 1998; and Novaes, 1993). Full inertia corresponds to a random walk in inflation, where innovations are fully persistent in the sense that a one-percent shock to inflation today changes one’s long-run inflation forecast by exactly one percent. When inflation follows an ARIMA($p,1,q$) process, it displays *some* inertia, which can be small (close to zero) or large (greater than one). When $d < 1$, however, inflation displays long memory (i.e., it takes a long time for shocks to die out) but no inflation inertia (i.e., shocks do die out).¹ A number of unorthodox stabilization plans have been put in effect between the mid 1980s and the early 1990s based on the diagnosis that the Brazilian inflationary dynamics were mainly inertial. Our results suggest that such a diagnosis was incorrect.

2. The ARFIMA(p,d,q) model

The discrete time ARFIMA(p, d, q) model is given by

$$\Phi(B)(1 - B)^d X_t = \Theta(B)\epsilon_t, \quad d \in \mathbb{R}, \epsilon_t \sim WN(0, \sigma_\epsilon^2), \quad (2.1)$$

¹See Baillie, et al. (1996), Hassler and Wolter (1995), and Baum et al. (1999) for empirical evidence of long memory behavior in the inflation data of G7, developing, and high inflation rate countries.

where $t = 1, 2, \dots, n$, B is the lag operator, $\Phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p$ and $\Theta(B) = 1 - \theta_1 B - \dots - \theta_q B^q$ are polynomials of orders p and q , respectively, with all roots outside of the unit circle, and ‘WN’ stands for white noise. When $d \in (-0.5, 0.5)$ the ARFIMA process is stationary and invertible, and its spectral density, $f(w)$, is given by

$$f(w) = f_u(w) \{2 \sin(w/2)\}^{-2d}, \quad w \in [-\pi, \pi], \quad (2.2)$$

where $f_u(\cdot)$ is the spectral density of an ARMA(p, q) process. As shown by Jensen (1999), the variance of the ARFIMA process wavelet coefficients equals

$$\sigma_m^2 = 2^{m+1} \int_{2^{-m}\pi}^{2^{-m+1}\pi} f(w) dw, \quad (2.3)$$

where m is the wavelet coefficients scaling parameter. See Beran (1994), Hosking (1981) and Reisen (1994) for detailed accounts of the ARFIMA process.

We consider five alternative estimators for the parameter d . Three of them are obtained using semiparametric procedures based on regression equations constructed from the logarithm of expression (2.2); the fourth estimator is an approximate, parametric, maximum likelihood estimator (MLE) in the frequency domain proposed by Fox and Taqqu (1986); and lastly the fifth is an approximate, parametric, maximum likelihood estimator in the wavelet domain proposed by Jensen (1999). These estimators are briefly described below.

The first estimator, hereafter denoted by $\hat{d}_p$, was proposed by Geweke and Porter-Hudak (1983) who used the periodogram function $I(w)$ as an estimate of the spectral density function in expression (2.2). The number of observations in the regression equation (bandwidth) is a function of the sample size n , that is, $g(n) = n^\alpha$, with $0 < \alpha < 1$.

The second estimator, hereafter denoted by $\hat{d}_{sp}$, was introduced by Reisen (1994). This regression estimator is obtained by replacing the spectral density function in expression (2.2) by the smoothed periodogram function based on the Parzen lag window. In this estimation method, $g(n)$ is chosen as above and the truncation point in the Parzen lag window is $m = n^\beta$, $0 < \beta < 1$. The appropriate choices for α and β were investigated by Geweke and Porter-Hudak (1983) and Reisen (1994), respectively.

The third estimator we consider is the one proposed by Robinson (1995). It is hereafter denoted by $\hat{d}_{rb}$. This estimator is a modified form of the log-periodogram one which regresses $\{\ln I(w_i)\}$ on $\ln \{2 \sin(w_i/2)\}^2$, for $i = \ell, \ell + 1, \dots, g(n)$, where ℓ is the lower truncation point, with ℓ diverging to infinity more slowly than $g(n)$.

The fourth estimator is a parametric estimator suggested by Fox and Taqqu (1986), and will be hereafter denoted by $\hat{d}_W$. The estimator $\hat{d}_W$ is based on the periodogram and involves the function

$$Q(\zeta) = \int_{-\pi}^{\pi} \frac{I(w)}{f_X(w, \zeta)} dw, \quad (2.4)$$

where $f_X(w, \zeta)$ is the known spectral density function at frequency w and ζ denotes the vector of unknown parameters. The Whittle estimator is the value of ζ which minimizes the function $Q(\cdot)$. For the ARFIMA(p, d, q) process, the vector ζ contains the parameter d and also all of the unknown autoregressive and moving average parameters. For more details, see Beran (1994), Dahlhaus (1989), and Fox and Taqqu (1986). In practice, the estimator $\hat{d}_W$ is obtained by using the discrete form of $Q(\cdot)$, as in Dahlhaus (1989, p. 1753), that is,

$$\mathcal{L}_n(\zeta) = \frac{1}{2n} \sum_{j=1}^{n-1} \left\{ \ln f_X(w_j, \zeta) + \frac{I(w_j)}{f_X(w_j, \zeta)} \right\}. \quad (2.5)$$

The fifth estimator, constructed by Jensen (1999), is an alternative, approximate maximum likelihood estimator to Fox and Taqqu (1986). Based on the wavelet representation of the ARFIMA process likelihood function, the wavelet MLE of d , which we will denote as $\hat{d}_a$, is found by maximizing the ARFIMA process wavelet coefficients likelihood function

$$\mathcal{L}_w(\zeta) = -\frac{1}{2} \sum_{m=1}^{\lfloor \log_2 n \rfloor} \left\{ \frac{n}{2^m} \log \sigma_m(\zeta) + \frac{\mathbf{w}_m' \mathbf{w}_m}{\sigma_m^2(\zeta)} \right\}, \quad (2.6)$$

where $\mathbf{w}_m' = (w_{m,1}, \dots, w_{m,n/2^m})$, is the vector of wavelet coefficients, $w_{m,j}$ corresponding to scale m , and translation j , and $\lfloor \cdot \rfloor$ is the integer operator. If the number of observations n is not a power two the data is zero padded for estimation of the wavelet MLE.

The asymptotic properties of some of these estimators under nonstationarity have recently been derived; see, e.g., Velasco (1999a,b). Monte Carlo results on the finite-sample performance of these (and other) estimators under nonstationarity are presented in Lopes, et al. (2003). Their results suggest that the estimators perform well even when $0.5 \leq d < 1$.

3. Data and results

The data consist of the inflation index known as IGP-DI (‘Índice Geral de Preços, Disponibilidade Interna’) computed by the Getúlio Vargas Founda-

Figure 1: Sample autocorrelation function for Brazil's inflation rate where the major ticks on the x-axis represent the lag argument in years.

tion, and ranges from February 1944 through February 2000 (673 observations). Figure 1 displays the sample autocorrelation of the series function up to lag 100. A visual examination of the correlogram suggests that the data are nonstationary and possibly characterized by long memory behavior since the sample autocorrelations decay very slowly.

The semiparametric estimation results of d for different bandwidths, $g(n)$, are presented in Table 1. The truncation point β in the smoothed periodogram was set equal to 0.9 (see Reisen, 1994, for further details), and the lower truncation ℓ in the Robinson method was taken to be 2. The results in Table 1 show that, regardless of the value of $g(n)$, the estimates strongly indicate nonstationary long memory behavior with $0.5 < d < 1.0$ (level reversion). Also, when all frequencies are used ($g(n) = 336$), the estimated values of d are very similar to the one obtained with the Whittle method (see the case ARFIMA(0, $\hat{d}_w$, 0) in Table 2). The standard errors (s) are also close to their asymptotic values ($\sigma_{\hat{d}}$).

The best fitted ARFIMA(p, d, q) models by the parametric and nonparametric estimators are reported in Table 2, along with the parametric Whittle and wavelet estimates of the ARFIMA(0, d , 0) model. Other than the

bandwidth	smoothed periodogram	periodogram	Robinson
$g(n) = 25$	$\hat{d}_{sp} = 0.705$ $\sigma_{\hat{d}_{sp}} = 0.065$ $s_{\hat{d}_{sp}} = 0.096$	$\hat{d}_p = 0.753$ $\sigma_{\hat{d}_p} = 0.157$ $s_{\hat{d}_p} = 0.143$	$\hat{d}_{rb} = 0.769$ $\sigma_{\hat{d}_{rb}} = 0.102$ $s_{\hat{d}_{rb}} = 0.179$
$g(n) = 95$	$\hat{d}_{sp} = 0.615$ $\sigma_{\hat{d}_{sp}} = 0.029$ $s_{\hat{d}_{sp}} = 0.039$	$\hat{d}_p = 0.595$ $\sigma_{\hat{d}_p} = 0.072$ $s_{\hat{d}_p} = 0.053$	$\hat{d}_{rb} = 0.585$ $\sigma_{\hat{d}_{rb}} = 0.052$ $s_{\hat{d}_{rb}} = 0.058$
$g(n) = 336 \star$	$\hat{d}_{sp} = 0.832$ $\sigma_{\hat{d}_{sp}} = 0.017$ $s_{\hat{d}_{sp}} = 0.022$	$\hat{d}_p = 0.841$ $\sigma_{\hat{d}_p} = 0.040$ $s_{\hat{d}_p} = 0.033$	$\hat{d}_{rb} = 0.856$ $\sigma_{\hat{d}_{rb}} = 0.027$ $s_{\hat{d}_{rb}} = 0.039$

Table 1: Semiparametric estimation results where $s_{\hat{d}}$ is the estimated standard error and $\sigma_{\hat{d}}$ is the asymptotic standard error. The $\star$ denotes the result obtained using all possible frequencies, i.e., $g(n) = [\frac{n}{2}]$, where $[\cdot]$ denotes the greatest integer function.

ARFIMA results where $p = q = 0$, the results reported for the parametric estimators are of the ARFIMA(p, d, q) model whose Akaike Information Criterion (AIC) was the smallest from the array of models where $p, q = 0, 1, 2, 3$.

For the semiparametric estimators, when a model with autoregressive and/or moving average components is estimated, we first filter the inflation series with $\hat{d}$ ($g(n) = n^{0.5} = 25$, see Reisen, 1994) and use the filtered series, $(1 - L)^{\hat{d}} X_t$, in the estimation of the ARFIMA($p, \hat{d}, q$) model's short-memory parameters. Lastly, the best semiparametrically estimated model is chosen as the model with the smallest AIC from among the ARFIMA models with $p, q = 0, 1, 2, 3$. The semiparametric estimation of d and the parametric estimation of d jointly with AR and/or MA parameters were performed using FORTRAN code linked to IMSL libraries. The estimation of AR and/or MA parameters for the filtered series obtained from semiparametric estimates of d was carried out using S-PLUS.

method	model	AR and/or MA estimates	AIC/ p -values *
periodogram	ARFIMA(3, d ,0) $\hat{d}_p = 0.753$	$\hat{\phi}_1 = 0.2021$ $\hat{\phi}_2 = 0.0517$ $\hat{\phi}_3 = -0.1627$	1790.2; 0.51, 0.14
smoothed periodogram	ARFIMA(3, d ,0) $\hat{d}_{sp} = 0.705$	$\hat{\phi}_1 = 0.2447$ $\hat{\phi}_2 = 0.0619$ $\hat{\phi}_3 = -0.1553$	1788.0; 0.65, 0.19
Robinson	ARFIMA(3, d ,0) $\hat{d}_{rb} = 0.769$	$\hat{\phi}_1 = 0.1882$ $\hat{\phi}_2 = 0.0481$ $\hat{\phi}_3 = -0.165$	1791.0; 0.45, 0.12
Whittle	ARFIMA(0, d ,0) $\hat{d}_w = 0.867$ $\sigma_{\hat{d}_w} = 0.049$	—	1827.2; 0.0, 0.0
Whittle	ARFIMA(1, d ,0) $\hat{d}_w = 0.4874$ $\sigma_{\hat{d}_w} = 0.049$	$\hat{\phi} = 0.4658$	1796.3; 0.005, 0.04
wavelet MLE	ARFIMA(0, d ,0) $\hat{d}_a = 0.7942$	—	3723.7; 0.0, 0.0
wavelet MLE	ARFIMA(2, d ,1) $\hat{d}_a = 0.9333$	$\hat{\phi}_1 = 0.8256$ $\hat{\phi}_2 = -0.4665$ $\hat{\theta} = -0.6818$	3412.6; 0.0, 0.0
unit root	ARFIMA(1,1,0)	$\hat{\phi} = -0.0012$	1834.0; 0.0, 0.0
unit root	ARFIMA(3,1,2)	$\hat{\phi}_1 = 0.2673$ $\hat{\phi}_2 = 0.5156$ $\hat{\phi}_3 = -0.1689$ $\hat{\theta}_1 = 0.3194$ $\hat{\theta}_2 = 0.5631$	1789.0; 0.37, 0.14

Table 2: Estimation and identification results. The p -values $\star$ are for the modified Box–Pierce χ^2 statistic with 4 and 32 degrees of freedom respectively.

The reported estimates in Table 2 reveal that the value of d range from 0.49 through 0.94. The smallest estimate (0.49) is obtained from the ARFIMA(1, d ,0) model using Whittle's method, and the largest one is obtained with the wavelet estimate of the ARFIMA(2, d ,1) model. The smallest AIC value corresponds to the ARFIMA(3, d ,0) where estimation was carried out by smoothed periodogram. In all cases, the estimates are below one, i.e., below the unit root case.

In short, the results of Table 2 suggest that the series is nonstationary and displays long memory. However, the order of integration is less than one, thus implying that there is no inflation inertia in the Brazilian inflationary process. These findings are similar to what Baillie et al. (1996) found for Brazilian inflationary process using an ARFIMA model with conditionally heteroskedastic innovations.

A remark on the Whittle estimated ARFIMA(0, d ,0) model versus the ARFIMA(1, d ,0) model is in order. As is well known, in the Whittle estimation the short and long memory parameters are estimated simultaneously in the frequency domain with the parameters entering the objective function through the power spectrum found in (2.2). In comparison with the Whittle estimate of the integration parameters when no short-memory parameters are present ($\hat{d}_w = 0.8967$), the long memory behavior becomes diluted into a smaller valued integration parameter, $\hat{d}_w = 0.4874$, and into the autoregressive parameter, $\hat{\phi} = 0.4658$, when short memory parameters are included. Our estimates suggest that the Whittle estimator of d may be sensitive to the presence of short memory parameters.

Our findings with the wavelet estimator reveal that including short memory parameters in a long memory model does not present a problem for the wavelet estimator. In both the ARFIMA(0, d ,0) and ARFIMA(2, d ,1) model, the wavelet estimator of the integration parameter are slightly less than one, with $\hat{d}_a = 0.7942$ when no short memory parameters are found in the model, and $\hat{d}_a = 0.9333$ when the model is a ARFIMA(2, d ,1).

Since in both models the wavelet estimates of d are near a unit root, we estimate the ARIMA(2,1,1) model:

$$(1 - 0.891B + 0.092B^2)(1 - B)X_t = (1 - 0.946B)\epsilon_t.$$

Other than the autoregressive parameter estimate for the second lag, the short memory parameter estimates are similar to the those estimated with the wavelet estimator; with the first-order autoregressive parameter estimates being most alike, and the signs of the short memory parameters estimates being the same.

bandwidth	smoothed periodogram	periodogram	Robinson
$g(n) = 25$	$\hat{d}_{sp} = -0.290$	$\hat{d}_p = -0.250$	$\hat{d}_{rb} = -0.248$

Table 3: Estimation results based on first differences. This invariance to first differencing was also found to hold for the semiparametric estimates of d when $g(n) = 95$ and $g(n) = \lfloor \frac{n}{2} \rfloor$.

Hurvich and Ray (1995) investigated the invariance property of the Geweke and Porter-Hudak (1983) estimator. They showed that this estimator, in general, is not invariant to first differencing. However, our estimates of the first order differenced series indicate that such invariance does hold for Brazilian inflation. Overall, the semiparametric estimates have proved invariant to first differencing, i.e., the estimates of d based on the original data are quite close to one plus the estimates of d based on the differenced data (see Table 3). This was also observed for the Whittle and wavelet estimates in the case where there are no AR and/or MA parameters. The Whittle and wavelet estimates were, respectively, $\hat{d}_W = -0.1269$ and $\hat{d}_a = -0.1238$. The invariance to first differencing is taken as further evidence that $d < 1$.

The next step is to investigate the forecasting performance of the different estimated models, in order to identify which one has the best forecasting ability. The forecasting experiment consists of estimating the model's parameters with the information found in the first 620 observation (February, 1944 to September, 1995) and predicting out-of-sample 1, 5, 10, and 20 months into the future. We choose not to iteratively update the parameter estimates with an ever increasing information set because when we re-estimated the models with each additional monthly observation added to the original 620 observations we find that the parameter estimates are only different at the fourth order of precision from the estimates found with the first 620 observations.

The h -step ahead forecasts of inflation are obtained using a truncated version of the infinite AR representation of the ARFIMA process as in Brockwell and Davis (1991, p. 533) and Reisen and Lopes (1999). The latter analyzes forecasting issues in long memory processes, including the effect of the truncation point in the forecast expression on the generated forecasts. Based on their results, we chose the truncation point where the estimate of the last AR coefficient in the infinite AR representation of the ARFIMA process is smaller than 0.0001. [See also the discussion in Brodsky and

Hurvich (1998).]

The forecasts are evaluated on three measures; the mean squared forecast error (MSE):

$$\text{MSE} = \frac{1}{n-h-m+1} \sum_{k=m}^{n-h} e_m(k+h)^2,$$

the mean absolute percentage error (MAPE):

$$\text{MAPE} = \frac{100}{n-h-m+1} \left(\sum_{k=m}^{n-h} \left| \frac{e_m(k+h)}{X_{k+h}} \right| \right),$$

and the mean forecasting error (MFE):

$$\text{MFE} = \frac{1}{n-h-m+1} \sum_{k=m}^{n-h} e_m(k+h),$$

where $e_m(k+h) = X_{k+h} - \hat{X}_m(k+h)$ is the out-of-sample forecast error of the $k+h$ observation given information up to time period $m = 620 \leq k$.

Table 4 summarizes each estimator and model's forecast performance.² The best performing models are the ARFIMA(0, d ,0) model estimated with the Whittle and wavelet methods. In the immediate short run ($h = 1, 5$) the wavelet estimated ARFIMA(0,0.794,0) model produces the best forecasts as gauged by all reported measures. For the intermediate ($h = 10$) and long-run ($h = 20$) it is the forecasting measures of the Whittle estimated ARFIMA(0,0.867,0) model that are smallest. These two models delivered the best out-of-sample forecasts, although they did not display the smallest AIC values (see Table 2).

It is noteworthy that the three measures of accuracy of forecasts considered (MSE, MAPE and MFE) point to the same conclusion, namely: for moderately short forecasting horizons ($h = 1, 5$), the best forecasts are obtained from the ARFIMA(0, d ,0) model, where the fractional integration parameter is estimated by wavelet methods; on the other hand, for moderately long forecasting horizons ($h = 10, 20$), it is best to employ forecasts from the same class of models, but with the Whittle estimate of d . The results show, indeed, that the choice of estimation method for the fractional integration parameter has a sizeable effect on the quality of the forecasts of the ARFIMA(0, d ,0) model. For instance, for $h = 10$ (forecasts ten months

²We do not report the MSE, MAPE, nor the MFE for the Robinson estimator because this estimator's forecast measures are nearly indistinguishable from those compute from the periodogram estimator.

		<i>h</i>			
method/model		1	5	10	20
MSE	periodogram; ARFIMA(3,0.753,0)	0.070	2.587	7.036	17.840
MSE	smoothed periodogram;				
	ARFIMA(3,0.705,0)	0.035	7.720	18.594	37.840
MSE	Whittle; ARFIMA(0,0.867,0)	0.379	0.687	0.517	1.994
MSE	Whittle; ARFIMA(1,0.487,0)	0.194	10.272	26.426	51.139
MSE	wavelet; ARFIMA(0,0.794,0)	0.007	0.645	2.468	8.613
MSE	wavelet; ARFIMA(2,0.933,1)	34.910	55.650	44.148	35.503
MSE	unit root; ARFIMA(3,1,2)	3.690	50.310	106.800	162.410
MSE	unit root; random walk	1.688	4.283	4.430	3.713
MAPE	periodogram; ARFIMA(3,0.753,0)	4.780	26.610	48.673	75.955
MAPE	smoothed periodogram				
	ARFIMA(3,0.705,0)	3.380	47.290	80.743	113.400
MAPE	Whittle; ARFIMA(0,0.867,0)	11.080	14.377	12.799	23.635
MAPE	Whittle; ARFIMA(1,0.487,0)	7.930	57.154	97.410	132.969
MAPE	wavelet; ARFIMA(0,0.794,0)	1.480	11.857	27.204	50.356
MAPE	wavelet; ARFIMA(2,0.933,1)	106.400	154.687	138.178	115.806
MAPE	unit root; ARFIMA(3,1,2)	34.58	134.474	202.500	244.370
MAPE	unit root; random walk	23.39	42.920	44.043	38.656
MFE	periodogram; ARFIMA(3,0.753,0)	0.265	−1.077	−2.221	−3.781
MFE	smoothed periodogram				
	ARFIMA(3,0.705,0)	−0.188	−2.253	−3.833	−5.687
MFE	Whittle; ARFIMA(0,0.867,0)	0.616	0.596	0.075	−0.868
MFE	Whittle; ARFIMA(1,0.487,0)	−0.441	−2.717	−4.614	−6.656
MFE	wavelet; ARFIMA(0,0.794,0)	0.082	−0.374	−1.206	−2.509
MFE	wavelet; ARFIMA(2,0.933,1)	5.909	7.328	6.502	5.631
MFE	unit root; ARFIMA(3,1,2)	−1.920	−6.358	−9.562	−12.161
MFE	unit root; random walk	1.299	1.979	2.029	1.844

Table 4: Measures of *h*-step ahead out of sample forecasting errors.

ahead) and ARFIMA(0, d ,0), the MSE and MAPE measures when d is estimated using the Whittle approach are -79% and -53% smaller than the same accuracy measures when d is estimated using the wavelets approach.

It is also interesting to compare the long memory model's forecast to the unit root model (last two rows in each grouping of forecasting measures of Table 4; the final row corresponds to the fully inertial case). Both long memory models are superior to the unit root models at a forecast horizon of five months. When $h = 10, 20$ the strength of the long-memory model forecast is only found in the Whittle estimated model, with the margin of improvement decreasing with the length of the time horizon. We conclude from the results in Table 4 that the forecasts of the inflationary dynamics can be greatly improved by taking into account inflation's long memory behavior.

It could be argued that the long memory, no unit root behavior described above was the result of bias due to the presence of potential 'inliers' in the data. These correspond to observations for which the inflation level was artificially low due to the implementation of 'shock plans' that were destined to fail a few months later, when inflation would resume its natural path; see Cati, et al. (1999) for details. In order to check whether that is the case, we have estimated the fractional integration parameter d for a truncated series that ranges from February 1944 through December 1985 (503 observations). This period is known to not contain inliers since the first shock plan (the so-called 'Cruzado Plan') took place in February 1986. The estimates of d corresponding to the estimation methods/models described in Table 2 were now $\hat{d}_p = 0.934$, $\hat{d}_{sp} = 0.860$, $\hat{d}_w = 0.576, 0.586$ ($p = 0$ and $p = 1$, respectively) and $\hat{d}_{rb} = 0.583$. Also, $\hat{d}_a = 0.543, 0.658$ ($p = 0$ and $p = 1$, respectively). These values also suggest long memory behavior, as did the estimates in Table 2. It is noteworthy that the difference between the Whittle estimates for $p = 0$ and $p = 1$ is now quite small since they are both approximately equal to 0.6. The Robinson estimate of d is also around 0.6, thus being smaller than the figure obtained for the complete series (see Table 2). If the argument of bias induced from the presence of possible inliers were valid, one would expect to obtain estimates of d much closer to one for the truncated series and long memory behavior for the complete series. This is not what we observe. Overall, the results seem to be insensitive to the inclusion in the sample of the period characterized by shock plans and large fluctuations.³

³Using G7 inflation rate data, Bos et al. (1999) also find their long memory estimates to be robust to level shifts. Modeling these level shifts with a ARFIMA model with mean

4. Concluding remarks

Brazil has often been cited as a country where the inflationary process is mainly driven by inflation inertia, and as a consequence traditional monetary and fiscal policies would fail to curb inflation. Several papers have identified some inertia in the inflationary dynamics in that country. In particular, Cati, et al. (1999) find that inflation in Brazil is almost entirely driven by inertia. A number of stabilization plans have been put into effect by Brazilian policymakers between the mid 1980s and the early 1990s, based on the diagnosis that inflation followed an inertial dynamics. Our results suggest, however, that there is no inflation inertia in this country. Instead, the inflationary dynamics display long memory. In short, it takes a long time for shocks to inflation to die out, but they eventually do die out. That is, shocks to inflation are not (totally or partially) persistent.

Colophon

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shifts that are fixed at exogenous points in time, they find that the differencing parameter decreases as the number of level shifts are added, but the presence of long memory is resistant to the level shifts.

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