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Duverger's Law and Polarization in a Ranked Choice Citizen-Candidate Model

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DUVERGER'S LAW AND POLARIZATION IN A RANKED CHOICE CITIZEN-CANDIDATE MODEL

by

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ABSTRACT

DUVERGER'S LAW AND POLARIZATION IN A RANKED CHOICE CITIZEN-CANDIDATE MODEL

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This paper expands on a citizen-candidate model of electoral competition under both plurality rule and ranked choice voting. The paper finds that ranked choice voting nominally avoids Duverger's Law by accumulating many identical candidates but yields fewer viable equilibrium policy positions than plurality rule. Additionally, ranked choice voting favors moderate candidates and policies, increasing the probability of their implementation compared to plurality rule. This moderate bias leads to lower polarization in equilibrium than is possible under plurality rule.

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I. Introduction

The electoral system in the United States has come under increased scrutiny in recent years, whether such scrutiny is justified or not. Some voters complain about the lack of more than two "serious" options. Advocacy groups promote switching the electoral system from the traditional plurality rule, where the candidate with the largest vote share wins, to alternative voting rules, such as ranked choice voting (RCV). Under RCV, voters rank all the candidates from most favored to least favored, after which the votes are tallied. Then, using only voters' top preference, the candidate with the smallest vote share is eliminated, and voters who ranked that candidate first have their votes transferred to their second choice. This process continues until there is only one candidate left, who is the winner. Groups such as FairVote claim that RCV will, among other things, allow "more voter choice," "allow diverse groups of voters … to elect their candidates of choice," and "minimize strategic voting" so that voters don't "feel the need to vote for the 'lesser of two evils'" (FairVote 2023).

Most complaints about plurality rule stem from a finding called Duverger's Law, named after the French sociologist who published a book in which he observes that plurality rule systems tend to collapse to two options, whether two candidates, two parties, or two coalitions (Duverger 1954). Duverger's Law can be conceived along two main lines of thought: the voter perspective and the candidate perspective. From a voter's perspective, Duverger's Law can hold because voters strategically ignore all candidates but the top two, the candidates with the best chance of winning (from this comes the complaint of "needing" to vote for the lesser of two evils). From a candidate's perspective, Duverger's Law can hold because only the top two candidates should bother running, since they are the only ones who are likely to win. In this paper, we will assume that voters will vote sincerely (even if they vote for a candidate who is unlikely to win), and we will focus on the candidates' entry and exit decisions. Along with Duverger's Law itself, voters may also feel that the two options they have are too polarized; they would rather have a policy in between the two options presented to them. RCV, according to its supporters, is supposed to solve or at least mitigate both of these issues. However, existing economic literature has not fully explored this point.

In order to answer whether RCV can actually accomplish this, we utilize a variation of Osborne and Slivinski's (1996) citizen-candidate model, which is itself a variation of the Downsian spatial competition model of elections (Downs 1957). The citizen-candidate model utilizes a system where, instead of a candidate choosing a policy and entering the election with that policy as a platform, each citizen has her own preferred policy and may choose whether or not to enter the election with that policy platform. Thus, the set of candidates, their positions, and their entry/exit decisions are endogenous to the model. One of the main appeals of this type of model as opposed to a more standard Downsian model is the presence of pure-strategy Nash equilibria. The main differences in our version of the model are that the policy space is bounded, citizens are uniformly distributed, and citizens are purely office motivated in their role as candidates. Of course, the most crucial difference is that we also derive the equilibria of the model under RCV along with plurality. Our main results are as follows:

- RCV appears to avoid Duverger's Law in name only; candidates accumulate, but they are nearly always identical to a candidate that has already entered.

- RCV restricts the number of policies that are viable in equilibrium to be closer to the median compared to plurality rule, generally reducing platform polarization.
- RCV does not significantly increase the range of policy options available to voters, compared to what is available to voters under plurality rule.
- RCV has an intrinsic bias towards moderate policies in situations where candidates take three different positions. The median policy is in some cases more than twice as likely to succeed in RCV as the most moderate policy in plurality rule.

Other literature in this area of inquiry is scant, but not nonexistent. Peres (2008) considers RCV in an analysis of whether various voting rules always select the Condorcet winner, finding that RCV is not guaranteed to do so. Miller (2017) considers what is called a “monotonicity failure” under RCV, where the candidate that gets the most votes in the first round fails to win in the second round. This finding is related to our findings that moderate candidates are favored to win, even without getting the most votes initially. Dellis (2013) considers the two-party system and Duverger’s Law from the perspective of strategic voting under a variety of voting rules, not including RCV. Yonk et al. (2011) find that RCV and other voting rules can significantly change the outcomes of multi-candidate elections.

Of particular interest is Dellis et al. (2017). They consider RCV in a citizen-candidate model, though with pure policy motivation in place of hybrid office-policy motivation (as in Osborne and Slivinski (1996)). They find, similar to this paper, that RCV results in less polarization than plurality rule. However, RCV sustains only one- or two-position equilibria in their model because of policy motivation. There are no equilibria with candidates at identical positions for the same reason. It is important to further consider pure office motivation because it seems likely that real-world candidates place at least some value on winning office, other than the ability to implement their preferred policies. Pure office motivation is a relatively standard assumption in the literature and is a good first step prior to attempting a hybrid office-policy motivation scenario. Additionally, the equilibrium logic with office motivation will be different than in Dellis et al. (2017), because candidates will not care about the outcome of the election and are therefore willing to risk a worse policy for a chance of winning the election.

II. Model

"Citizens" have preferred policies that are uniformly distributed across a policy space $[-1,1]$. Each citizen can choose to enter the election at her preferred policy or stay out of the race. Citizens who choose to enter the election are called "candidates." All citizens and candidates have perfect information, and entry decisions are made simultaneously, after which the votes are cast and tallied. "Clone" candidates are permitted; there could be any number of candidates who enter with the same preferred policy. There is a positive cost of entry $k > 0$, a positive benefit of winning $\beta > 0$, and the benefit is greater than the cost $\beta > k$. A candidate is purely "office" motivated before and after entry: her utility is affected only by winning or losing the election, not by the policy that is enacted after the election. Candidate A's expected utility is defined as $EU_A = \mathrm{P}(win)\,\beta - k$.

Citizens have single peaked preferences over the policy space, which ensures transitivity in each citizen's ranking of candidates. Each citizen votes sincerely: she votes for the candidate whose chosen policy is closest to her own preferred policy. Citizens are indifferent both between two candidates running with the same policy platform and between multiple candidates running with different platforms that are equidistant from the citizen's own preferred policy. If multiple candidates with the same preferred policy enter, all the citizens that would have voted for a single candidate with that platform randomize with equal weight among each candidate that entered with that platform, because they are indifferent as to which candidate they support. Citizens do the same in the case of multiple candidates with platforms equidistant from the citizen's preferred policy. In case of a tie (that is, equal vote shares) between n candidates, each candidate is assigned $\frac{1}{n}$ probability to win the tie. A candidate's platform is denoted with a lowercase letter, while the candidate herself is denoted with a capital letter (e.g., candidate A's platform is a). The leftmost candidate will be denoted as A, with the rest of the candidates named in alphabetical order from left to right.

The election rule is either plurality rule or ranked choice voting. Under plurality rule, citizens vote as described above, and then the candidate with the largest vote share wins (with ties settled as described above). Under RCV, each citizen reports her entire ranking of all candidates that have chosen to enter the election. The votes are tallied using the citizens' top preferences and the candidate with the smallest vote share is eliminated. The votes of the citizens that ranked the eliminated candidate first are then transferred to the second ranked candidate of each citizen. The votes are tallied again, and the candidate with the smallest vote share is again eliminated. The process repeats until there are only two candidates left. At that point, the candidate with the larger vote share wins the election.

III. Plurality Rule Equilibrium Analysis

Before considering the equilibria that result under RCV, it is useful to consider the equilibria of the model under plurality rule, taking insight from Osborne and Slivinski (1996). Their conclusions, due to the differences between their model and ours (office motivation and uniformly distributed citizens), do not apply exactly here. In this version of the model, there are infinitely many possible equilibria, most of which follow a similar pattern, which is that candidates are located in evenly spaced "pockets" along the policy continuum. Since $\beta > k$, there cannot be a no-candidate equilibrium. If no other candidates enter, any citizen gains $\beta - k > 0$ by entering and being elected.

One-candidate equilibrium

In the one-candidate equilibrium, the median citizen enters and gains all votes (see Figure 1, Position Set 1). To maintain this equilibrium, the condition $k > \frac{1}{2}\beta$ must hold. This condition ensures that a clone candidate does not enter. An equilibrium where two or more clone candidates at the median enter and split the votes evenly is impossible,

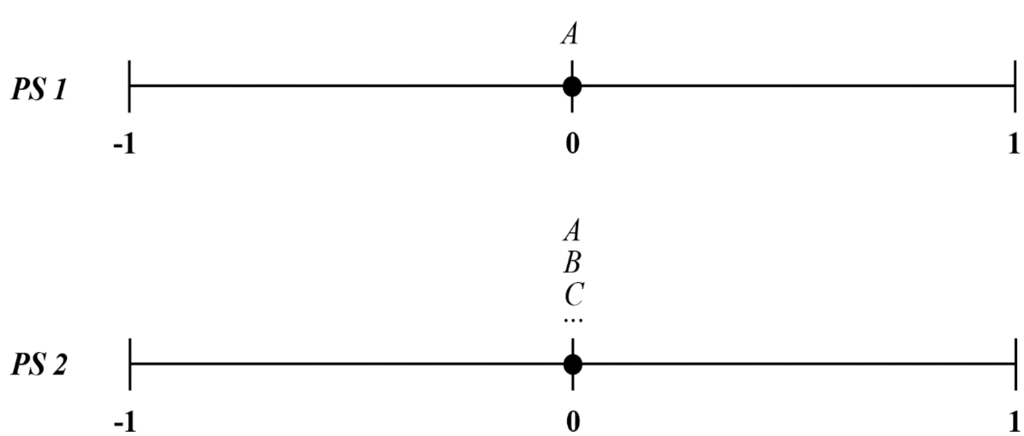

Figure 1: *One-Candidate Plurality and One-Position RCV Equilibria*

because an additional candidate located just to the left or right of the median could enter and gain a plurality.

Two-candidate equilibria

Next, there is an infinite set of non-clone two-candidate equilibria where one citizen enters at a position b, where $b \in \left(0, \frac{2}{3}\right)$, and another citizen enters at $a = -b$ (see Figure 2a, Position Set 1). They split the vote evenly. To maintain this equilibrium, the condition $\frac{1}{2}\beta > k$ must hold, so that each candidate finds it worthwhile to enter. Note that a condition ensuring that no other candidates enter, similar to the condition for the one-candidate equilibrium, is unnecessary for this category of equilibria because no other citizen, including a clone of either candidate, could enter and win, provided the candidate's positions are as described. All other citizens will therefore choose to stay out.

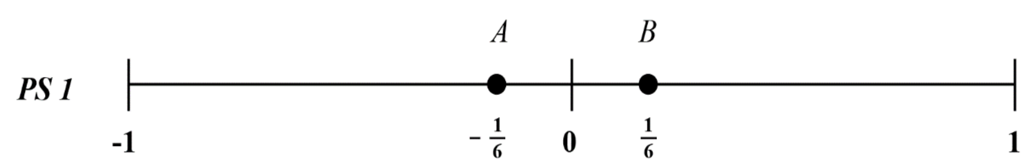

Figure 2a: *Two-Candidate Plurality Equilibrium*

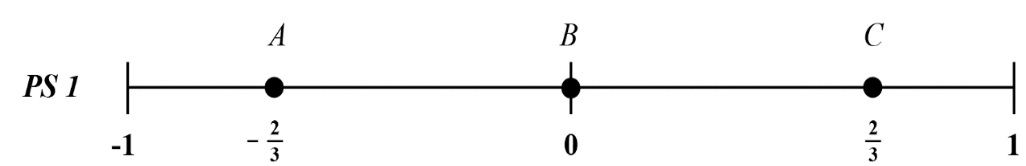

Figure 2b: *Two Extreme Candidates With Median Candidate Entry*

If the candidates are too extreme, a third candidate at or near the median could enter and potentially win the election (see Figure 2b, Position Set 1).

N-candidate equilibria

The equilibria for cases with more than two candidates become more complex. Since an equilibrium requires firstly that every candidate be satisfied with entering, we can lay down a general condition: in equilibrium, the n candidates must be distributed across the policy space such that they are in an n-way tie (that is, each candidate's vote share is equal). Given this condition, the benefit-cost condition required for each equilibrium with n candidates is $\frac{1}{n}\beta > k$. This means that given determinate values of β and k, we can determine the maximum number of candidates that can enter in any equilibrium. Specifically, if $\frac{1}{n}\beta < k$, there can be at most $n-1$ candidates in equilibrium.

We can further deduce from the equal-vote-share condition that whenever a clone candidate is included among the set of candidates, there is an opportunity for an additional candidate just to the side of the clones to enter and gain a vote share greater than or equal to the other candidates. We can be certain of this because the presence of clones in a tied election entails that the clones are positioned further away from the other candidates in the election. If the clones were closer or the same distance, the clones would have smaller vote shares than the other candidates. If an additional candidate can enter and win the election, she will do so, which means the initial distribution of candidates was not an equilibrium. If the best that the new candidate can do is tie with the other candidates, she wins with probability $p > \frac{1}{n}$, since the clones have strictly smaller vote shares after the new candidate enters and are thus excluded from the tie. In order to deter this new candidate from entering, we would need $p\beta < k$ to hold. Given that $p > \frac{1}{n}$,

$p\beta < k$ and $\frac{1}{n}\beta > k$ cannot both be true, which means either that the new candidate is not deterred, or at least one of the candidates gains by staying out. Therefore, we can rule out all cases with clones.

The equal-vote-share condition entails an additional property of each equilibrium: each candidate can only be located within a certain "pocket" in the policy space. Consider the three-candidate case. Each candidate's vote share must be $\frac{1}{3}$. Therefore, the policy space must be divided into thirds among the candidates. This entails that the midpoint between a and b is $-\frac{1}{3}$ and that the midpoint between b and c is $\frac{1}{3}$. If the midpoints were any other number, the candidate's vote shares would not be $\frac{1}{3}$ each.

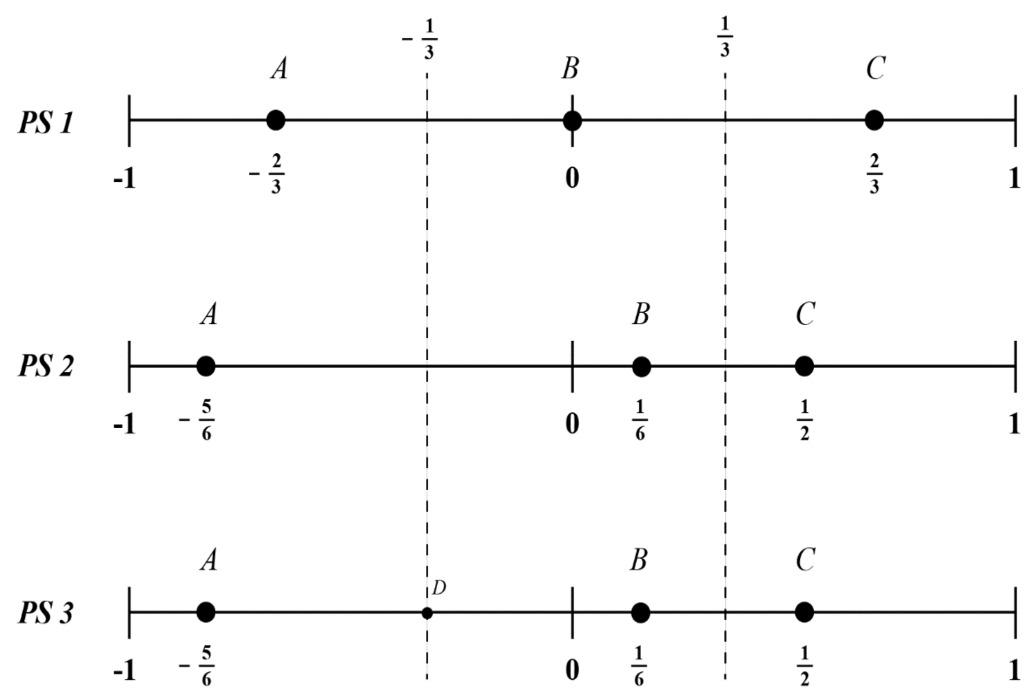

Figure 3: *Three-Candidate Plurality Equilibria*

Therefore, A's pocket is the interval $a \in \left(-1, -\frac{1}{3}\right)$, B's pocket is the interval $b \in \left(-\frac{1}{3}, \frac{1}{3}\right)$, and C's pocket is the interval $c \in \left(\frac{1}{3}, 1\right)$. These intervals are non-inclusive because a candidate locating at either extreme or any of the midpoints would require clones at other positions in order to have equal vote shares, and we have already ruled out all clone cases.

Once one candidate within any pocket is selected to enter, the rest of the candidates that will enter in the other pockets in equilibrium can be found algorithmically. For example, if $a = -\frac{2}{3}$, then in equilibrium $b = 0$, and $c = \frac{2}{3}$ (see Figure 3, Position Set 1). However, if $a = -\frac{5}{6}$, then $b = \frac{1}{6}$, and $c = \frac{1}{2}$ (see Figure 3, Position Set 2). Both of these results are three-candidate equilibria given the condition $\frac{1}{3}\beta > k$. Any clone who enters splits a vote share with one of the other candidates, so her vote share is strictly lower than $\frac{1}{3}$ and she loses. Any other citizen located between the candidates who have already entered captures a vote share strictly lower than $\frac{1}{3}$ if she enters. In the second case, for example, the maximum vote share a fourth candidate could obtain is $\frac{1}{4}$, if the citizen located at $-\frac{1}{3}$ entered. This would decrease the vote shares of A and B, but C's vote share would still be $\frac{1}{3}$, causing the new candidate to lose (see Figure 3, Position Set 3). The small dot for D in Figure 3, Position Set 3 means that D could not win the election if they entered. A similar algorithm can be applied in cases with more candidates, after identifying the candidate pockets.

Plurality rule equilibria summary

Under plurality rule there are no zero-candidate equilibria and there is a unique one-candidate equilibrium given that $\frac{1}{2}\beta < k$ holds. There is a continuum of symmetric two-candidate equilibria where citizens whose preferred policies have an absolute value less than $\frac{2}{3}$ can enter, given that $\frac{1}{2}\beta > k$ holds. There are infinite possible n-candidate equilibrium types, each of which is a continuum of equilibria similar to the two-candidate continuum. The equilibria for each type (that is, the number of candidates) are fairly similar to each other, with candidates always located within their respective pockets. In these equilibria, the maximum possible value of n will depend on the cost-benefit ratio. Generally, if the cost-benefit ratio is such that $\frac{1}{n}\beta > k > \frac{1}{n+1}\beta$, then there are an infinite number of n-candidate equilibria, where the n candidates are located within the n evenly spaced pockets, but there cannot be an equilibrium with $n+1$ candidates. For that same cost-benefit ratio, however, there are still an infinite number of $(n-1)$-, $(n-2)$-… candidate equilibria, where the candidate positions are as described above.

IV. Ranked Choice Voting Equilibrium Analysis

One-position equilibria

Under RCV, there cannot be a zero-candidate equilibrium by the same reasoning as plurality rule. A one-candidate equilibrium, with the median citizen entering, can also be sustained with the same condition, $\frac{1}{2}\beta < k$. However, unlike plurality rule, if this condition does not hold and a clone enters, a two-candidate equilibrium where both candidates are median-clones can also be maintained. If a citizen slightly to the right or left of the median entered, she would "survive" (avoid elimination in) the first round, and one of the clones would be eliminated. However, the new candidate would then lose the election to the remaining candidate located at the median. If a third clone entered, she would survive the first round with probability $\frac{2}{3}$. She would then tie and win the election with probability $\frac{1}{2}$. Her conditional probability of winning is therefore $\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$. Thus, in order to maintain the two-candidate median-clone equilibrium, the condition $\frac{1}{2}\beta > k > \frac{1}{3}\beta$ must hold. The left side of the inequality ensures that both clones find it worthwhile to enter, while the right side of the inequality deters the third clone from entering. Generally, an n-candidate one-position equilibrium can be maintained provided $\frac{1}{n}\beta > k > \frac{1}{n+1}\beta$ (see Figure 1, Position Set 2). Since the upper bound of the next equilibrium is the lower bound of the previous equilibrium, given determinate values of the parameters β and k, only one median-clone equilibrium will be possible, with the number of candidates depending on the ratio between the parameters.

Two-position equilibria

Next, we consider whether there can be a two candidate non-median-clone equilibrium. Similar to plurality rule, infinitely many two-candidate two-position equilibria can be maintained. However, the interval of possible policies is smaller. As before, candidate positions must be symmetric (ensuring that vote shares are equal and neither candidate regrets entering) and $\frac{1}{2}\beta > k$ must hold. One candidate enters at a position b, where $b \in \left(0, \frac{1}{2}\right]$ and another citizen enters at $a = -b$. If candidates locate at positions with an absolute value greater than $\frac{1}{2}$, then a candidate located either at the median or just to the right or left of the median can enter, survive the first round, and potentially win the election (See Figure 4a, Position Set 1). If candidates locate at positions with an absolute value of $\frac{1}{2}$ or less, then a candidate located between them cannot enter and win the election (see Figure 4b, Position Set 1). As before, the small dot in Figure 4b, Position Set 1, indicates that B cannot win that election. However, regardless of the location of the candidates, a clone can enter and tie with the original candidate in the first round. The clone then survives the first round with probability $\frac{1}{2}$, after which she ties the final round and wins the election with probability $\frac{1}{2}$. The conditional probability of the clone winning is therefore $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. Therefore, in order to deter the clone from entering, the condition $k > \frac{1}{4}\beta$ must hold. This condition is compatible with the previous condition, yielding the inequality $\frac{1}{2}\beta > k > \frac{1}{4}\beta$ as the complete condition for this set of two-candidate two-position equilibria.

Suppose that a clone enters at either a or b. In order for each candidate to be satisfied with entry, $\frac{1}{4}\beta > k$ must hold, since $\frac{1}{4}\beta > k$ entails $\frac{1}{2}\beta > k$ (the cloned candidates win with probability $\frac{1}{4}$ and the single candidate wins with probability $\frac{1}{2}$). However, an additional clone could enter at the other position and win with probability $\frac{1}{4}$. In order to deter entry from this clone, $\frac{1}{4}\beta < k$ would also have to hold. $\frac{1}{4}\beta > k$ and $\frac{1}{4}\beta < k$ cannot hold at the same time, so there cannot be an equilibrium with two clones on one side and one candidate on the other. This holds generally, so that there cannot be a two-position equilibrium with n clones at one position and m clones at the other position when $m < n$.

Suppose further that an additional clone enters at the other position. The vote shares are equal at $\frac{1}{4}$ per candidate. Each candidate has three possible outcomes in the first round: with $\frac{1}{4}$ probability she is eliminated, with $\frac{1}{4}$ probability her clone is eliminated, and with $\frac{1}{2}$ probability one of the other candidates is eliminated. If her clone is eliminated, then her vote share becomes $\frac{1}{2}$, the outcome of the second round is irrelevant to her, and she ties the final round, winning the election with probability $\frac{1}{2}$. If one of the other candidates is eliminated, then she ties the second round with her clone, winning with probability $\frac{1}{2}$. If she survives the second round, she ties the final round, winning the election with probability $\frac{1}{2}$. Her overall probability of winning is therefore $\left(\frac{1}{4} \cdot \frac{1}{2}\right) + \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) = \frac{1}{4}$. So, in the case where there are two clones located at each of two symmetric policies, each candidate wins with probability $\frac{1}{4}$. A fifth clone could also enter, bringing the total number of candidates at one position to three. This new clone survives the first round with probability $\frac{2}{3}$. After the first round, the game collapses into the previous game where there were only two candidates located at each policy. Therefore, the fifth clone has a conditional probability $\frac{2}{3} \cdot \frac{1}{4} = \frac{1}{6}$ of winning the entire election. This means that if $k > \frac{1}{6}\beta$ holds, this additional clone is deterred from entering. The set of equilibria with two clones at each symmetric position in the interval $\left(0, \frac{1}{2}\right]$ can therefore be maintained if $\frac{1}{4}\beta > k > \frac{1}{6}\beta$ holds, ensuring that each of the four clones finds it worthwhile to run, but the fifth clone is deterred from entry. The pattern continues with parallel reasoning: an n-candidate symmetric two-position equilibrium (with $\frac{1}{2}n$ candidates at each position) can be maintained if $\frac{1}{n}\beta > k > \frac{1}{n+2}\beta$ holds (see Figure 4b, Position Set 2). With determinate values for the parameters, only one set of these equilibria will be possible, dependent on the ratio of the parameters. Any pair of symmetric policies with an absolute value less than or equal to $\frac{1}{2}$ is a possible n-candidate two-position equilibrium for any values of β and k (given $\frac{1}{n}\beta > k > \frac{1}{n+2}\beta$).

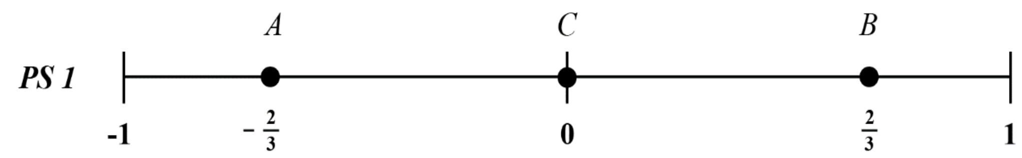

Figure 4a: *Two-Position RCV Non-Equilibrium*

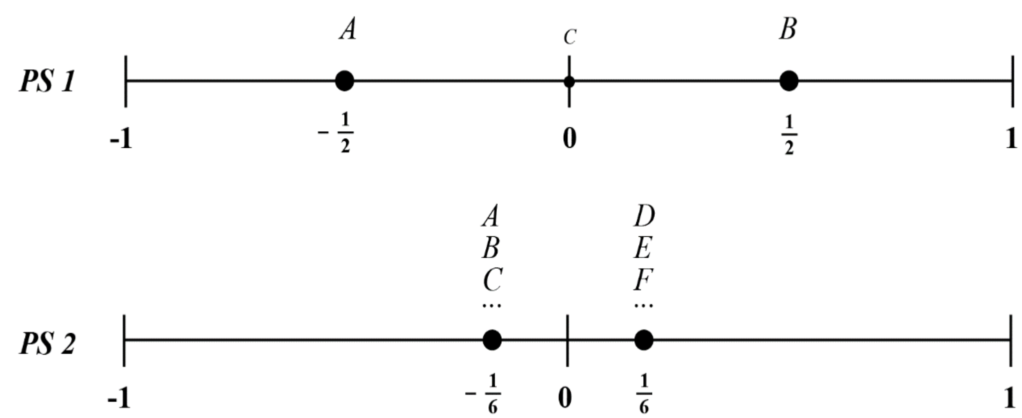

Figure 4b: *Two-Position RCV Equilibria*

Three-position equilibria

Consider now a case with three candidates located at different positions. Unlike plurality rule, vote shares and win probabilities need not be equal. An easily seen condition is that each candidate must at least have a positive probability to survive the first round (otherwise, she would always be initially eliminated). For example, if $a = -\frac{2}{3}, b = 0$, and $c = \frac{2}{3}$, each candidate survives the first round with $\frac{2}{3}$ probability. If A or C is eliminated, B always wins. If B is eliminated, A and C tie the final round and each wins the election with probability $\frac{1}{2}$. Overall, B wins with $\frac{2}{3}$ probability and A and C win with $\frac{1}{6}$ probability each. If a were instead $-\frac{2}{3} - \varepsilon$, where $\varepsilon > 0$, A would always be eliminated in the first round and would therefore not enter. However, simply guaranteeing that each candidate can survive the first round is not sufficient. Consider also the same alternate policy set as before, $a = -\frac{5}{6}$, $b = \frac{1}{6}$, and $c = \frac{1}{2}$. In this case, each candidate still survives the first round with probability $\frac{2}{3}$. As before, if A or C is eliminated, B always wins. However, if B is eliminated, C now always wins. A has no chance of winning overall, despite potentially surviving the first round, and would therefore choose to stay out (see Figure 5a, Position Set 1). The small dot in the figure again indicates that A cannot win. Therefore, in order to have an equilibrium with three or more positions, candidates located in each position must have a positive probability to win overall.

Returning to the $a = -\frac{2}{3}, b = 0$, and $c = \frac{2}{3}$ case, we see that A and C have a lower probability of winning overall $\left(\frac{1}{6}\right)$ than B $\left(\frac{2}{3}\right)$. To ensure that each candidate is willing to enter, $\frac{1}{6}\beta > k$ must hold. Then, to determine whether this position set is an equilibrium, we must also determine whether a fourth candidate will enter. If a candidate entered at any other position, she may be able to survive the first round, but would always be eliminated in a later round or lose in the final round. If a clone located at a or c entered, she would survive the first round with probability $\frac{1}{2}$, after which she would eventually win with probability $\frac{1}{6}$. Therefore, her conditional probability of winning would be $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$. If a clone located at b entered, she would survive the first round with probability $\frac{1}{2}$, and then win with probability $\frac{2}{3}$. Her conditional probability of winning would be $\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$, which is higher than the clone at the outer position. Therefore, in order to deter clone entry, $\frac{1}{3}\beta < k$ must hold. $\frac{1}{6}\beta > k$ and $\frac{1}{3}\beta < k$ are incompatible given $\beta > 0$, so this set of candidates cannot be an equilibrium. The pattern continues, with clones located at the median entering until the probability of one of the median clones winning is the same as the probability of one of the outer candidates winning. This occurs when there are four clones at the median. At this point, a median-clone's probability of surviving the first round is $\frac{3}{4}$, after which she survives the second round with probability $\frac{2}{3}$, and then survives the third round (becoming the only median candidate) with probability $\frac{1}{2}$. Her probability of winning is then $\frac{2}{3}$, which means that her conditional probability of winning overall is $\frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{6}$. Therefore, every candidate in the election wins with probability $\frac{1}{6}$. At this point, $\frac{1}{6}\beta > k$ is still necessary to ensure that every candidate is satisfied with entry. If a clone located at one of the sides enters, her conditional probability of winning is still $\frac{1}{12}$. If a fifth clone located at the median enters, her conditional probability of winning is $\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}=\frac{2}{15}$, which is greater than $\frac{1}{12}$ but less than $\frac{1}{6}$. Therefore, clone entry at either position is deterred when $\frac{2}{15}\beta < k$. This is compatible with $\frac{1}{6}\beta > k$. Therefore, a six-candidate three-position equilibrium can be sustained with $a = -\frac{2}{3},\ b, c, d, e = 0$, and $f = \frac{2}{3}$, given $\frac{1}{6}\beta > k > \frac{2}{15}\beta$ (see Figure 5b, Position Set 1).

If a sixth clone located at the median enters, her conditional probability of winning is $\frac{5}{6}\cdot\frac{2}{15}=\frac{1}{9}$. $\frac{1}{9}$ is less than $\frac{1}{6}$, but greater than $\frac{1}{12}$, so median clones are still better off entering than outer clones, but median clones who have already entered now have a lower probability of winning than any of the outer candidates. Therefore, $\frac{2}{15}\beta > k > \frac{1}{9}\beta$ must hold to both deter additional entry and make entry worthwhile for the seven candidates that have entered. Therefore, given this condition, a seven-candidate three-position equilibrium can be sustained. By parallel reasoning with the condition $\frac{1}{9}\beta > k > \frac{2}{21}\beta$, an eight-candidate three-position equilibrium can be sustained, as can a nine-candidate three-position equilibrium with the condition $\frac{2}{21}\beta > k > \frac{1}{12}\beta$. At this point, however, clones entering at the median have the same probability of winning as do clones entering at the outer positions, $\frac{1}{12}$. Therefore, in order to have a ten-candidate three-position equilibrium, we would need $\frac{1}{12}\beta > k$ in order for the tenth candidate to stay in but also $\frac{1}{12}\beta < k$ to deter an additional clone from entering. There is therefore no ten-candidate three-position equilibrium, nor is there an eleven-candidate three-position equilibrium. However, if the set of candidates is $a, b = -\frac{2}{3},\ c, d, e, f, g, h, i, j = 0$, and $l, m = \frac{2}{3}$, then a twelve-candidate three-position equilibrium can be sustained, as follows.

Each candidate wins the election with probability $\frac{1}{12}$. A ninth median-clone's probability of winning is $\frac{8}{9} \cdot \frac{1}{12} = \frac{2}{27}$, while a third outer-clone's probability of winning is $\frac{2}{3} \cdot \frac{1}{12} = \frac{1}{18}$, which is lower than the median-clone's probability. Therefore, to both keep all twelve candidates in and deter additional entry, the condition $\frac{1}{12}\beta > k > \frac{2}{27}\beta$ must hold.

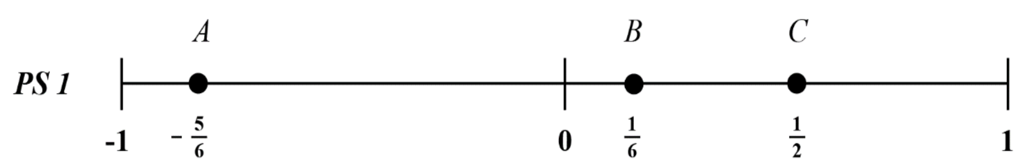

Figure 5a: *Three-Position RCV Non-Equilibrium*

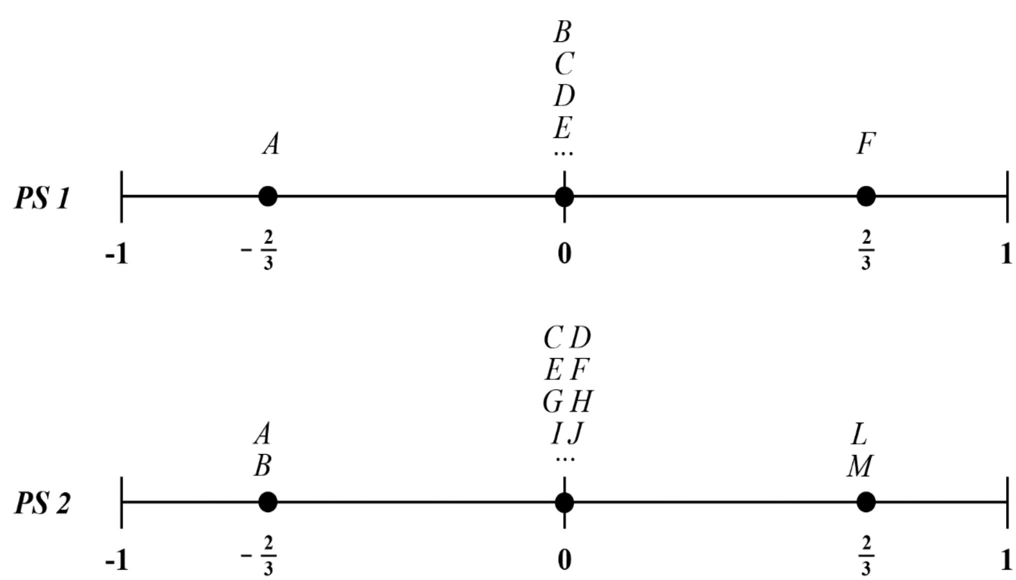

Figure 5b: *Three-Position RCV Equilibria*

The pattern continues in that manner, until the probability of winning for a new outer-clone is equal to a new median-clone, at which point a new outer-clone on both sides and several new median-clones must enter before a new equilibrium is reached. There are infinitely many possible equilibria with this identical three-position position set (any other position set leads to one candidate certainly losing), with the general feature that if there are n outer-clones there must be at least $4n$ median-clones (see Figure 5b, Position Set 2). If γ is defined as the number of median-clones, the probability that a median-clone wins the election is $\frac{2}{3\gamma}$. Then, the general equilibrium condition is $\frac{2}{3\gamma}\beta > k > \frac{2}{3(\gamma+1)}\beta$. Note that a condition concerning an outer-clone's probability of winning is unnecessary, since the only points at which that probability would matter are when it is equal to the median-clone's probability of winning. Since the lower bound of each equilibrium condition is equal to the upper bound of the next condition, given a determinate value of the parameters β and k only one of these many n-candidate three-position equilibria will be possible, depending on the ratio of the parameters. The number of candidates will be the number of median-clones plus the number of outer-clones, so $n = \gamma + 2 \cdot trunc\left(\frac{\gamma}{4}\right)$, where the $trunc$ function returns the value of its argument (in this case, the quotient of γ divided by 4) truncated to a whole number. $trunc\left(\frac{\gamma}{4}\right)$ is the number of outer-clones at each outer position.

Equilibria with more than three positions

No other multi-candidate or multi-position equilibrium exists. To see this, consider four candidates positioned along the policy space. Taking the evenly spaced case, $a = -\frac{3}{4}, b = -\frac{1}{4}, c = \frac{1}{4}$, and $d = \frac{3}{4}$, we see that it begins with a four-way tie in the first round. If A or D is eliminated, all her votes go to B or C, respectively. Then, B and C tie to win the election. If B is eliminated, half her votes go to A, and the other half go to C. D is then eliminated, at which point C always wins. Parallel reasoning applies for $C's$ elimination, with B always winning the election. Therefore, this position set (and any other set that begins with a four-way tie) cannot be an equilibrium, because A and D cannot win in any scenario (see Figure 6, Position Set 1). The small dots in this figure indicate that A and D cannot win.

Consider further a position set that begins with only a two-way tie, such as $a = -\frac{1}{2}, b = -\frac{1}{4}, c = \frac{1}{4}$, and $d = \frac{1}{2}$. With this set, only the inner two candidates, B and C, tie

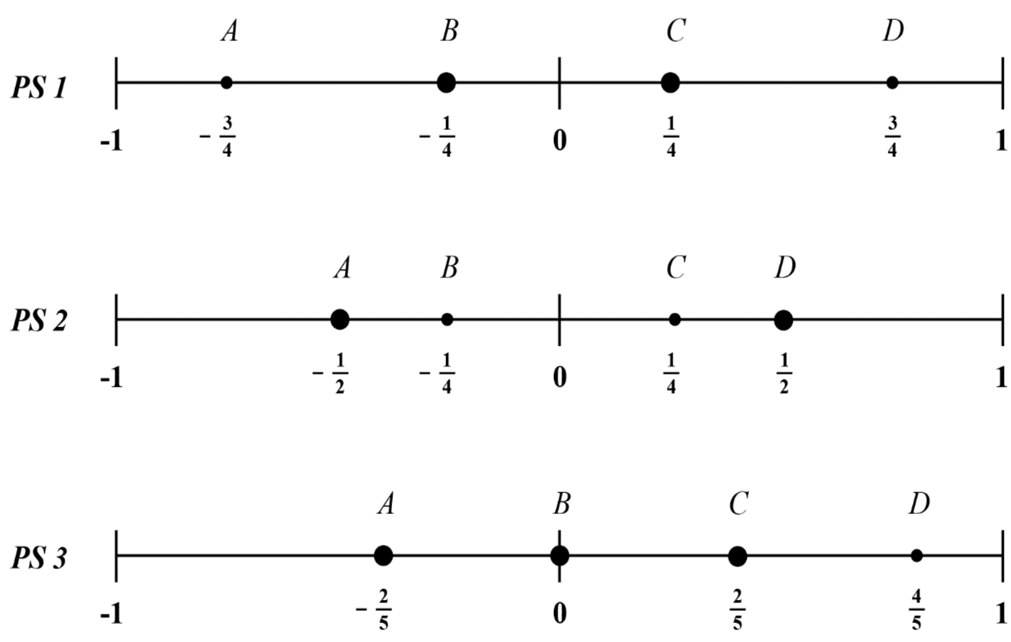

Figure 6: *Four-Position RCV Non-Equilibria*

in the first round, but the inner candidate that survives cannot gain enough votes to survive the second round. Thus, neither B nor C wins in any scenario (see Figure 6, Position Set 2). The small dots in this figure indicate that B and C cannot win. A different four-position set such that there is a two-way tie among outer candidates in the first round proceeds in an analogous manner, because if A is eliminated in the first round, all her votes go to B and D is eliminated in the second round. Parallel reasoning applies if D is initially eliminated. In this case, A and D cannot win in any scenario.

Finally, consider a four-position set such that there is a three-way tie in the first round, such as $a = -\frac{2}{5}, b = 0, c = \frac{2}{5}$, and $d = \frac{4}{5}$. If B is eliminated in the first round, D will then be eliminated, and then A and C will tie to win the election. If C is eliminated in the first, B and D tie in the second round. If B is then eliminated in the second round, A wins, and if D is eliminated in the second round, B wins. If D is eliminated in the first round, B will be eliminated in the second round, and then A and C will again tie to win the election. Thus, there is no scenario where D can win the election (see Figure 6, Position Set 3). The small dot in this figure indicates that D cannot win. As before, none of these cases can be equilibria, because there are candidates that always lose and would therefore gain by staying out. Any n-position set where no candidates are immediately eliminated will either be or eventually collapse to one of these cases (a four-, three-, or two-way tie in the third-to-last round), meaning that at least one candidate will certainly lose in any of these position sets. Therefore, there cannot be an equilibrium with four or more positions under RCV.

RCV equilibria summary

Under RCV there is no zero-candidate equilibrium, and for any cost-benefit ratio, there is one n-candidate one-position equilibrium where all candidates are clones at the median. n will depend on the cost benefit ratio, such that n clones enter if the ratio is such that $\frac{1}{n}\beta > k > \frac{1}{n+1}\beta$. There are infinite symmetric n-candidate two-position equilibria where $n \geq 2$. In these equilibria $\frac{1}{2}n$ clones enter at a position with an absolute value less than or equal to $\frac{1}{2}$, $\frac{1}{2}n$ candidates enter at the symmetric position, and n again depends on the cost-benefit ratio such that n candidates enter if the ratio is such that $\frac{1}{n}\beta > k > \frac{1}{n+2}\beta$. Thus, for a given cost-benefit ratio only one continuum of equilibria will be possible. For a given cost-benefit ratio, there is one n-candidate three-position equilibrium where $n \geq 6$. γ clones enter at the median, and $trunc\left(\frac{\gamma}{4}\right)$ clones enter at each outer position, so that $n = \gamma + 2 \cdot trunc\left(\frac{\gamma}{4}\right)$. γ depends on the cost-benefit ratio such that γ median-clones enter if the ratio is such that $\frac{2}{3\gamma}\beta > k > \frac{2}{3(\gamma+1)}\beta$. There are no equilibria with more than three positions.

V. Conclusion

Comparing the results under plurality rule and RCV is interesting in the context of Duverger's Law. Under plurality rule we see that, depending on the relative values of β and k, the benefit and cost of running respectively, there can be many equilibria with more than two candidates, none of which can be clones. This implies a wider variety of options for voters to choose from. Under RCV, candidates (in the form of clones) start to multiply, but policy positions do not. For example, comparing the one-candidate plurality equilibrium with the n-candidate one-position RCV equilibria yields the insight that RCV does not change the outcome for voters in this case. Either way, the implemented policy will be 0. The same idea holds for the two-candidate plurality equilibria and the n-candidate two-position RCV equilibria. RCV might allow for equilibria with more than two candidates, but they still only have two positions, which might end up being the same positions that obtain under plurality. In fact, the range of potential symmetric positions is smaller under RCV than under plurality rule. The most interesting case, however, is the comparison between the three-candidate plurality equilibria and the n-candidate three-position equilibria. Here RCV's tendency to multiply clones can be clearly seen, as clones, especially median-clones, start to accumulate in the various equilibria. Under plurality rule, if there are three positions, there are three candidates, but under RCV, if there are three positions, there are at minimum six candidates. However, voters are still choosing between the same three policies. Further, RCV cannot support higher numbers of positions, while plurality rule supports as many different positions as the cost-benefit ratio allows. Thus, RCV may nominally avoid Duverger's Law through the accumulation of clone candidates, but in principle the range of options for voters to choose from is the same, or even restricted when plurality equilibria with a greater number of positions are considered.

RCV also changes the lotteries that voters face in three-position equilibria. Under plurality rule, each three-candidate equilibrium is an equally weighted lottery of three fairly evenly spread policies. Under RCV, every three-position equilibrium skews heavily towards the middle policy, which is the median voter's position. This result comes about because of the structure of the voting rule. No matter what, every election will end with exactly one candidate competing for votes against exactly one other candidate. Therefore, the candidate closer to the median voter is victorious. This confers an advantage on moderate candidates. Under RCV, the middle policy is more than three times as likely to be the outcome as one of the side policies in the three-position equilibria. Further, RCV cannot sustain an equilibrium with more than four policies on the table. The reason for this is again because of the intrinsic moderate bias within RCV that this analysis suggests is present. Since candidates closer to the median are favored in the final round, any candidate who must enter at a more extreme side position in order to survive the first few rounds sacrifices victory in the final round, which is the one that matters most. Meanwhile, plurality rule in this model is able to sustain equilibria with a comparatively large number of candidate positions, each of which has an equal chance of being implemented. RCV, in contrast to plurality rule, reduces the number of viable positions.

There is also something interesting to be said about citizen preferences over different voting rules. Intuitively, it seems likely that citizens with more extreme preferred policies might prefer plurality rule over RCV, because plurality rule allows for more extreme policy positions in equilibrium. Citizens with moderate preferred policies might prefer RCV for a similar reason. However, depending on the loss function of the citizens, their risk tolerances will change. For example, in a three-candidate or -position scenario, if the loss function were linear (that is, the citizen is risk neutral), an extreme citizen would be indifferent between the equally-weighted lottery between their preferred policy, the median policy, and the other extreme policy that would result under plurality rule, and the median-weighted lottery that results under RCV. Alternatively, however, an extreme citizen with a concave loss function (that is, the citizen is risk averse) will prefer

RCV (and thereby a higher chance of getting the median policy) over plurality rule, to reduce the likelihood of the other extreme policy being implemented. Finally, if the citizen's loss function were convex (that is, the citizen is "risk loving"), they will prefer plurality rule, because they are closer to being indifferent between the median policy and the other extreme policy, and therefore want to maximize the chance that their preferred policy is implemented. If citizens are risk neutral or averse, a social planner would choose to implement RCV. In the risk neutral case, RCV makes moderate citizens better off without making extreme citizens worse off. In the risk averse case, everyone is better off under RCV. The risk loving case is more ambiguous, however, because social welfare would depend on the exact spacing of the candidates and the exact form of the loss function.

Considering our results in conjunction with those of Dellis et al. (2017) yields a few interesting comparisons. Candidate behavior in equilibrium varies widely between the two models, with office-motivated candidates willing to take larger risks than their policy-motivated counterparts with respect to the policy outcome of the election. This is reflected in the fact that Dellis et al. find that candidates at the more extreme positions will drop out to allow the moderate candidate to win if they are policy motivated, but we find here that these candidates are willing to stay in the race under office motivation. This explains why under office motivation RCV can support three-position equilibria, while RCV can support only one- or two-position equilibria under policy motivation. However, whether candidates are policy- or office-motivated, we find with Dellis et al. that RCV reduces polarization and restricts the number of viable policy positions in equilibrium compared to plurality rule.

There are a few limitations to these results. First, in this paper we have concentrated on strategic candidates, but Duverger's Law can also be considered from the perspective of strategic voting. Future research could examine what occurs under RCV with strategic voters. Next, this paper utilized a spatial competition model, but there are other ways of modeling elections. Future research could examine these alternate models under RCV. Finally, we have assumed pure office motivation and Dellis et al. (2017) have already explored pure policy motivation, so future research could concentrate on a hybrid policy/office motivation scenario (candidates care about the policy that is implemented, but there are also "perks" or benefits to winning, which is how Osborne and Slivinski (1996) originally specified the citizen-candidate model). It seems likely that such results would depend on the relative size of the benefit of winning compared to the cost of running. Where a candidate might exit in order to get her second-choice policy for certain under pure policy motivation, perks of office may induce her to enter (despite the risk of a worse policy), if the office perks were sufficiently large.

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