Theory and Application of Motion Compensation for LFM-CW SAR

David G. Long
david_long@byu.edu

Evan C. Zaugg

Follow this and additional works at: https://scholarsarchive.byu.edu/facpub

Part of the Electrical and Computer Engineering Commons

Original Publication Citation

BYU ScholarsArchive Citation
Long, David G. and Zaugg, Evan C., "Theory and Application of Motion Compensation for LFM-CW SAR" (2008). Faculty Publications. 164. https://scholarsarchive.byu.edu/facpub/164

This Peer-Reviewed Article is brought to you for free and open access by BYU ScholarsArchive. It has been accepted for inclusion in Faculty Publications by an authorized administrator of BYU ScholarsArchive. For more information, please contact ellen_amatangelo@byu.edu.
Theory and Application of Motion Compensation for LFM-CW SAR

Evan C. Zaugg, Student Member, IEEE, and David G. Long, Fellow, IEEE

Abstract—Small low-cost high-resolution synthetic aperture radar (SAR) systems are made possible by using a linear frequency-modulated continuous-wave (LFM-CW) signal. SAR processing assumes that the sensor is moving in a straight line at a constant speed, but in actuality, an unmanned aerial vehicle (UAV) or airplane will often significantly deviate from this ideal. This nonideal motion can seriously degrade the SAR image quality. In a continuous-wave system, this motion happens during the radar pulse, which means that existing motion compensation techniques that approximate the position as constant over a pulse are limited for LFM-CW SAR. Small aircraft and UAVs are particularly susceptible to atmospheric turbulence, making the need for motion compensation even greater for SARs operating on these platforms. In this paper, the LFM-CW SAR signal model is presented, and processing methods are described. The effects of nonideal motion on the SAR signal are derived, and new methods for motion correction are developed, which correct for motion during the pulse. These new motion correction algorithms are verified with simulated data and with actual data collected using the Brigham Young University μSAR system.

Index Terms—Motion compensation, synthetic aperture radar (SAR).

I. INTRODUCTION

EVEry small low-cost synthetic aperture radar (SAR) systems have recently been demonstrated as an alternative to the expensive and complex traditional systems [1]–[7]. The use of a frequency-modulated continuous-wave (FMCW) signal facilitates system miniaturization and low-power operation, which make it possible to fly these systems on small unmanned aerial vehicles (UAVs). The ease of operation and low operating costs make it possible to conduct extensive SAR studies without a large investment.

Recently, new processing methods have been developed to address issues specific to FMCW SAR [8]. The range-Doppler algorithm (RDA) [9] and the frequency (or chirp) scaling algorithm (FSA or CSA) [10] can be modified to compensate for the constant forward motion during the FMCW chirp. Nonlinearities in the chirp can also be corrected [11], and squint-mode data can be processed [12]. In this paper, the problems caused by nonideal motion of a linear frequency-modulated continuous-wave (LFM-CW) SAR system are addressed, and new compensation algorithms are developed.

II. BACKGROUND

Stripmap SAR processing assumes that the platform is moving in a straight line, at a constant speed, and with a consistent geometry with the target area. During data collection, whether in a manned aircraft or a UAV, there are deviations from this ideal as the platform changes its attitude and speed or is subjected to turbulence in the atmosphere. These displacements introduce variations in the phase history, the signal’s time of flight to a target, and the sample spacing, all of which degrade the image quality. If the motion of the platform is known, then corrections can be made to the SAR data for more ideal image processing. Small aircraft and UAVs are more susceptible to atmospheric turbulence; thus, the need for motion compensation on these platforms is greater.

Motion compensation algorithms for traditional pulsed SAR have extensively been studied [13], [14], but the underlying differences with an LFM-CW signal make it a challenge to extend existing motion compensation methods to LFM-CW sensors. In pulsed SAR, the platform is assumed to be stationary during each pulse, and the motion takes place between pulses. With LFM-CW SAR, the signal is constantly being transmitted and received; thus, the motion takes place during the chirp.

Section V presents simulation results in which a SAR system images a few point targets with nonideal motion. The known deviations are used to compensate for the effects of nonideal motion in the simulated data. A quantitative analysis of the simulation results is performed, which compares the proposed motion compensation algorithm to the traditional method. The new motion compensation scheme is applicable to a number of FMCW SAR systems, which are summarized in Section VI. The developed algorithm is applied to actual data from the Brigham Young University (BYU) μSAR data, and the results are presented. Motion data are provided by an inertial navigation system (INS) and GPS. The flight path data are interpolated between samples to provide position data for each sample of SAR data. The motion data are used to determine the necessary corrections that are introduced into the SAR data, effectively straightening the flight path.
Fig. 1. (Top) Frequency change of a symmetric LFM-CW signal over time, together with the signal returns from two separate targets. (Bottom) Frequencies of the dechirped signal, with the times of flight $t_1$ and $t_2$, due to the range determining the dechirped frequency. The relative sizes of $t_1$, $t_2$, and $T_p$ are exaggerated for illustrative purposes.

II. LFM-CW SAR Signal Processing

In a symmetric LFM-CW chirp, the frequency of the signal increases from a starting frequency $\omega_0$ and spans the bandwidth $BW$, at the chirp rate $k_v = BW \cdot 2 \cdot$ PRF. The frequency then ramps back down, as shown in Fig. 1. This up-down cycle is repeated at the pulse repetition frequency (PRF), giving a pulse repetition interval of $T_p$. The transmitted up-chirp signal can be expressed in the time domain as

$$s_t(t, \eta) = e^{j\left(\phi + \omega_0 t + \pi k_v t^2\right)}$$  \hspace{1cm} (1)

where $t$ is the fast time, $\eta$ is the slow time, and $\phi$ is the initial phase. The down-chirp signal can be expressed similarly to the up-chirp signal, but with $\omega_0 + BW$ as the starting frequency and $-k_v$ as the chirp rate.

The received signal from a target at range $R(t, \eta) = \sqrt{R_0^2 + v^2(t + \eta)^2}$, with time delay $\tau = 2R(t, \eta)/c$, is

$$s_r(t, \eta) = e^{j\left(\phi + \omega_0 (t-\tau) + \pi k_v (t-\tau)^2\right)}$$  \hspace{1cm} (2)

where $R_0$ is the range of closest approach of the target.

The transmit signal is mixed with the received signal and low-pass filtered in hardware, which is mathematically equivalent to multiplying (1) by the complex conjugate of (2). This results in the dechirped signal

$$s_{dc}(t, \eta) = e^{j\left(\omega_0 \tau + 2\pi k_v \tau^2 + \pi k_v \tau^2\right)}.$$  \hspace{1cm} (3)

These raw data are then processed to create a SAR image.

Options for data processing include the RDA and the FSA (or CSA), as shown in Fig. 2. For RDA processing, this signal is range compressed with a fast Fourier transform (FFT) in the range direction and then taken to the range-Doppler domain with an FFT in the azimuth direction. Using standard interpolation methods, range cell migration (RCM) can be compensated, but this is computationally costly. The Doppler shift introduced by the continuous forward motion of the platform can also be removed [9]. Azimuth compression is performed by multiplying by the azimuth-matched filter, i.e.,

$$H_2(f_r, f_0) = e^{j\frac{4\pi R_0}{\lambda} D(f_r, v)}$$  \hspace{1cm} (4)

where $D(f_r, v) = \sqrt{1 - \lambda^2 f_r^2/4v^2}$ is the range migration factor, $v$ is the platform along-track velocity, and $\lambda$ is the wavelength of the center transmit frequency.

Alternatively, the FSA (or CSA) [15] can also be modified to work with the dechirped data [16]. With the FSA, RCM can be compensated without interpolation. This advantage makes the FSA the preferred method for LFM-CW SAR processing. Fig. 3 compares the RDA (without RCM correction) and the FSA in processing data from the BYU $\mu$SAR.

FSA processing involves a series of Fourier transforms and phase multiplies. If a known nonlinearity exists in the FMCW chirp, it can be compensated by modifying the functions as in [10]. To start the FSA processing, an FFT is performed in the azimuth direction on the signal from (3). The resulting signal in the dechirped Doppler domain is

$$S(t, f_\eta) = e^{-j\frac{4\pi R_0 D(f_\eta, v)}{\lambda} - j\frac{4\pi k_v R_0 f}{\lambda}} e^{j2\pi f_\eta t} e^{-j\pi k_v t^2}.$$  \hspace{1cm} (5)

The frequency scaling function is applied with an additional term that removes the Doppler shift [10], i.e.,

$$H_3(f_r, f_\eta) = e^{j\left(2\pi f_\eta t + \pi k_v t^2 \left(1 - D(f_\eta, v)\right)\right)}.$$  \hspace{1cm} (6)

A range FFT is performed, and the second function is applied, which corrects the residual video phase. Thus

$$H_4(f_r, f_\eta) = e^{j\left(-\pi f_r^2 \right)/\left(k_v D(f_r, v)\right)}.$$  \hspace{1cm} (7)

An inverse FFT in the range direction is performed, followed by a function that performs inverse frequency scaling, i.e.,

$$H_5(f_r, f_\eta) = e^{-j\pi k_v f_r^2 \left[D(f_r, v)^2 - D(f_\eta, v)^2\right]}.$$  \hspace{1cm} (8)

Here, the secondary range correction and bulk range shift phase function can be used, as in [16].

We again take the range FFT and apply the final filter that performs azimuth compression (4). An azimuth inverse FFT results in the final focused image.

III. Nonideal Motion Errors

The SAR processing algorithms described in Section II assume that the platform moves at a constant speed in a straight line. This is not the case in any actual data collection, as the platform experiences a variety of deviations from the ideal path. These deviations introduce errors in the collected data, which degrade the SAR image.

Translational motion causes platform displacement from the nominal ideal path. This results in the target scene changing in range during data collection. This range shift also causes inconsistencies in the target phase history. A target at range $R$
Fig. 2. Block diagram showing LFM-CW SAR processing using (left) the FSA and (right) RDA, including the proposed two-step motion compensation.

Fig. 3. Taken from a larger image, i.e., a 286 m × 360 m area of North Logan, UT. Imaged with the BYU μSAR operating at 5.62 GHz with a bandwidth of 80 MHz. The data were processed with the RDA and FSA. The horizontal axis is a slant range with the aircraft moving upward at image left. (a) Data processed with the RDA, without RCM correction. The entire collection was processed in 30.4 s. (b) Data processed with the FSA. The entire collection was processed in 84.3 s. (c) Aerial photograph for comparison purposes. It is clear that the focusing is better with the FSA and is worth the extraprocessing load.

is measured at range \( R' = R + \Delta R \), resulting in a frequency shift in the dechirped data. The dechirped signal in (3) then becomes

\[
\Delta \text{dc}(t, \eta) = e^{i\left(\omega_0(t+\Delta t) + 2\pi k_r(t+\Delta t) - \pi k_r(\tau+\Delta \tau)^2\right)}
\]

where \( \Delta \tau = 2\Delta R/c \). Targets that lie within the beamwidth that have a nonzero Doppler frequency experience a different change in range that is dependent on the azimuth position. This is illustrated in Fig. 4, where the range to target \( A \) differently changes with motion than the range to target \( B \).

Variations in along-track ground speed result in nonuniform spacing of the radar pulses on the ground. This nonuniform sampling of the Doppler spectrum results in erroneous calculations of the Doppler phase history.

Changes in pitch, roll, and yaw introduce errors of different kinds. The pitch displaces the antenna footprint on the ground,
the roll changes the antenna gain pattern over the target area, and the yaw introduces a squint. Pitch and yaw shift the Doppler centroid, with the shift being range dependent in the yaw case. If the Doppler spectrum is shifted so that a portion lies outside the Doppler bandwidth, then aliasing occurs. Azimuth compression produces ghost images at the azimuth locations where the Doppler frequency is aliased to zero.

the roll changes the antenna gain pattern over the target area, and the yaw introduces a squint. Pitch and yaw shift the Doppler centroid, with the shift being range dependent in the yaw case. If the Doppler spectrum is shifted so that a portion lies outside the Doppler bandwidth, then aliasing occurs. Azimuth compression produces ghost images at the azimuth locations where the Doppler frequency is aliased to zero.

IV. MOTION COMPENSATION

Previously developed methods of motion compensation are limited for correcting the nonideal motion of an LFM-CW SAR system. Methods like those of [17] apply bulk motion compensation to the raw data and a secondary correction to the range-compressed data. This works as an approximation for motion correction but relies on assuming that the platform is stationary during a pulse. In range compressing the data, we lose the ability to differentiate the motion over the chirp, which is problematic for LFM-CW SAR.

For an LFM-CW SAR signal, motion corrections can directly be applied to the raw dechirped data (9) or Doppler-dependent corrections are applied to the range FFT of the raw data, in the dechirped Doppler domain. Because each data point contains information from every range, and the corrections are range and azimuth dependent, any corrections applied in the dechirped Doppler domain are only valid for a single range and a single azimuth value. However, with approximations, these restrictions can be relaxed.

A. Theoretical Treatment

In general, motion data are collected at a much slower rate than SAR data. For LFM-CW SAR, motion data must be interpolated so that every sample of SAR data has corresponding position information (as opposed to only each pulse having position data). Each data point also needs to have a corresponding location on the ideal path to which the error is corrected. For a target at range \( R \), \( \Delta R \) is calculated from the difference between the distance to the ideal track and the actual track. The flat-terrain geometry of Fig. 4 is assumed. If more precise knowledge of the terrain is available, then the model can be adjusted [18], [19]. Knowing the coordinates of target \( A \), the actual flight path (point \( P_A \)), and the nominal flight path (point \( P_N \)), the distances \( R \) and \( R + \Delta R \) can be calculated from the geometry. Again, \( \Delta \tau = 2\Delta R/c \), but \( \Delta \tau \) is updated for every data sample. The motion errors are corrected using our correction filter

\[
H_{MC}(\tau, \Delta \tau) = e^{-j\left(\omega_0 \Delta \tau + 2\pi k \tau \Delta \tau - \pi k \left(2\tau \Delta \tau - \Delta R\right)^2\right)}.
\]  

When applied to the data (in the raw dechirped time domain or the dechirped Doppler domain), this shifts the range of the target and adjusts the phase.

There are targets in the beamwidth at the same range but different azimuth positions that experience a different range shift due to translational motion. For a given azimuth position, as shown in Fig. 4, target \( B \) is at a position where it has the Doppler frequency \( f_0 \). The angle to target \( B \) is

\[
\theta(f_0) = \sin^{-1}\left(\frac{f_0 \lambda}{2v}\right).
\]

Working through some particularly unpleasant geometry (see Appendix B), the angle on the ground (as defined in Fig. 4), i.e., \( \vartheta(\theta(f_0), R_B, G, H_A) \), is found. From \( \vartheta \), we find the ground range, i.e.,

\[
B_g(f_0) = -\cos(\vartheta)G \pm \sqrt{\cos^2(\vartheta)G^2 + R_g^2 - G^2}
\]

and the actual range to target \( B \), i.e.,

\[
R_B(f_0) = \sqrt{H_A^2 + B_g^2}.
\]

We then find \( \Delta R = R - R_B(f_0) \) and \( \Delta \tau = 2\Delta R(f_0)/c \) and apply (10) in the dechirped Doppler domain. This correction is valid for a single range and a single azimuth position.
Exactly correcting the motion errors in this way is computationally taxing. For every pixel of the final image, the correction is applied in the dechirped Doppler domain for the given range and azimuth position. The data are processed through range compression, and a single data point is kept. A composite range-compressed image is created from these individual points, and the final image is formed through azimuth compression. Fortunately, there are approximations that can be made to reduce the computational load while still maintaining the advantages of this method.

B. Simplifying Approximations

If the beamwidth is narrow, then the errors due to motion can be assumed to be constant for a given range across the Doppler bandwidth. This is called center-beam approximation [20] and is used in many motion compensation algorithms. The errors induced by this approximation are detailed in [21]. A number of methods have been proposed as alternatives to this approximation, as discussed in [22]. When using this approximation, we apply the correction filter (10) to the raw data. The correction is valid for only a single range; thus, a composite range-compressed image is created and then azimuth compressed to form the final image.

Further simplification is possible by splitting the correction into two steps. The first step applies the correction for a reference range and azimuth position. For the proposed correction, this approximation is discussed in [22]. When using this approximation, we apply the correction filter (10) to the raw data. The correction is valid for only a single range; thus, a composite range-compressed image is created and then azimuth compressed to form the final image.

Further simplification is possible by splitting the correction into two steps. The first step applies the correction for a reference range and azimuth position. The data are processed through range compression, and a single data point is kept. A composite range-compressed image is created from these individual points, and the final image is formed through azimuth compression. Fortunately, there are approximations that can be made to reduce the computational load while still maintaining the advantages of this method.

Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chirp Bandwidth (MHz)</td>
<td>250</td>
</tr>
<tr>
<td>Center Frequency (GHz)</td>
<td>5.62</td>
</tr>
<tr>
<td>Azimuth Beamwidth (°)</td>
<td>12</td>
</tr>
<tr>
<td>Pulse Repetition Frequency (Hz)</td>
<td>320</td>
</tr>
<tr>
<td>Sample Rate (Hz)</td>
<td>327680</td>
</tr>
</tbody>
</table>

Table II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range Resolution (meters)</td>
<td>0.5996</td>
</tr>
<tr>
<td>Azimuth Resolution (meters)</td>
<td>0.1274</td>
</tr>
</tbody>
</table>

VI. FMCW SAR SYSTEMS

The proposed motion compensation scheme is suitable for a number of recently developed FMCW SAR systems. The
Fig. 5. Simulated LFM-CW SAR data of an array of point targets and a single point target. The first column shows the motion errors, whereas the second column shows an ideal collection without any motion errors. The third column shows motion correction using the traditional compensation method, whereas the fourth column shows the proposed motion compensation method. The power is normalized to the peak of the ideal collection. In this example, the nonideal motion is sinusoidal. The principal component of the translational motion while the target is in the main beamwidth is moving toward the target. This results in the corrected targets appearing "squinted."

BYU μSARs are small, student-built, low-power, LFM-CW SAR systems [1]. They weigh less than 2 kg, including antennas and cabling, and consume 18 W of power. The μSAR systems operate at C- or L-band with bandwidths of 80–160 MHz. Units have successfully been flown in manned aircraft and on UAVs. Imaged areas include the arctic and areas in Utah and Idaho.

The BYU μSAR is flown with an MMQ-G INS/GPS unit from Systron Donner Inertial, which measures the motion of the aircraft. SAR and motion data are collected together and stored onboard or downlinked to a ground station. As discussed, the motion data are interpolated and matched with the actual SAR data. The results are shown in Fig. 6, where the data collected...
from a UAV show that the motion corrections improve the focusing of an array of corner reflector targets. A more detailed description of the μSAR system is found in Appendix A. A number of other small FMCW SARs are described in the following.

The MISAR [2] developed by EADS was designed to fly on small UAVs having a weight of less than 4 kg and power consumption under 100 W. The system operates at Ka-band (35 GHz) and produces images with 0.5 m × 0.5 m resolution. SAR and motion data (from an onboard INS) are transmitted to the ground via a 5 MHz analog video link. The nonideal motion is compensated using the INS data and autofocus algorithms.

The Delft University of Technology has developed a demonstrator system [3] with a center frequency of 10 GHz and a variable bandwidth of 130–520 MHz. Algorithms have been developed that compensate for nonlinearity in the chirp [11] and the continuous forward motion during the chirp [23]. An INS unit is used to measure the aircraft motion.

The MINISARA [4] from the Universidad Politécnica de Madrid is a portable SAR system with a center frequency of 34 GHz and a bandwidth of 2 GHz. The system is small (24 × 16 × 9 cm) and lightweight (2.5 kg). The motion compensation relies on autofocus algorithms. The ImSAR NanoSAR [5] is a 1-lb SAR that is specifically built for use on small UAVs.

The DRIVE from ONERA [6] has a central frequency of 35 GHz and a bandwidth of about 800 MHz. The goal is to develop a 3-D SAR imaging system, and studies include compensating for the motion of the wings that carry the antennas. The FGAN ARTINO [7] is a 3-D imaging radar similar to the ONERA project that operates at Ka-band. A custom INS was developed for the project and is used for motion compensation and UAV control.

VII. Conclusion

In this paper, the effects of nonideal motion on an LFM-CW SAR signal have been explored, and corrective algorithms have been developed. Motion compensation has successfully been applied to simulated and real SAR data by taking into account the motion during the pulse and correcting the motion-induced range shift without interpolation. The small size of LFM-CW SAR, like the BYU μSAR, makes it possible for LFM-CW SAR to operate from a small aircraft or UAV, which is particularly susceptible to the effects of atmospheric turbulence. With motion measurements, the negative effects of nonideal motion can be corrected, extending the utility of small LFM-CW SAR.

APPENDIX A

BYU μSAR System Description

The BYU μSAR meets the low power and cost requirements for flight on a UAV by employing an LFM-CW signal that maximizes the pulse length, allowing LFM-CW SAR to maintain a high signal-to-noise ratio while transmitting with less peak power than pulsed SAR [24]. While continuously transmitting, the frequency of the transmit signal repeatedly increases and decreases at the PRF.

The μSAR dechirps the received signal by mixing it with the transmitted signal. This simplifies the sampling hardware by lowering the required sampling frequency, although a higher dynamic range is needed.

A simplified block diagram of the μSAR design is shown in Fig. 7. The BYU μSAR system is designed to minimize size and weight using a stack of custom microstrip circuit boards without any enclosure. The system measures 3 in × 3.4 in × 4 in and weighs less than 2 kg, including antennas and cabling. Component costs of a few thousand dollars are kept low by using off-the-shelf components as much as possible (Fig. 8).

The UAV supplies the μSAR with either +18 V dc or +12 V dc. The power subsystem uses standard dc/dc converters to supply the various voltages needed in the system. Power consumption during operation is nominally 18 W, with slightly more required for initial start-up. The μSAR is designed for “turn-on and forget” operation. Once powered up, the system continuously collects data for up to an hour. The data are stored onboard for postflight analysis.

The core of the system is a 100-MHz STALO. From this single source, the frequencies for operating the system are derived, including the sample clock and the radar chirp. The LFM-CW transmit chirp is digitally created using a direct digital synthesizer (DDS), which is controlled by a programmable IC microcontroller. Switches control the PRF settings, allowing it to be varied (128–2886 Hz) for flying at different heights and speeds. The programmable DDS also generates the sample clock coherent with the LFM signal.

The μSAR signal is transmitted with a power of 28 dBmW at a center frequency of 5.56 or 1.72 GHz and a bandwidth of 80–160 MHz. The range resolution of an LFM chirp is inversely proportional to the bandwidth of the chirp; thus, the μSAR has a range resolution of 1.875 m at 80 MHz and 0.9375 m at 160 MHz.

At C-band, two identical custom microstrip antennas, each consisting of a 2 × 8 patch array, are used in a bistatic configuration. The antennas are constructed from coplanar printed circuit boards sandwiched together, a symmetric feed structure on the back of one board and the patch array on the front of another, with pins feeding the signal through the boards. The antennas are approximately 4 in × 12 in and have an azimuth 3-dB beamwidth of 8.8° and an elevation 3-dB beamwidth of

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>MEASURED SIMULATION VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>Ideal Collection</td>
</tr>
<tr>
<td>Resolution (m)</td>
<td>0.647</td>
</tr>
<tr>
<td>PSLR (dB)</td>
<td>13.37</td>
</tr>
<tr>
<td>ISLR (dB)</td>
<td>10.69</td>
</tr>
<tr>
<td>Azimuth</td>
<td>Resolution (m)</td>
</tr>
<tr>
<td>PSLR (dB)</td>
<td>14.49</td>
</tr>
<tr>
<td>ISLR (dB)</td>
<td>10.62</td>
</tr>
<tr>
<td>Processing Time (s)</td>
<td>7.525</td>
</tr>
</tbody>
</table>
50°. The azimuth resolution is approximately equal to half the antenna length (0.15 m) in azimuth. Multilook averaging is used for creating images with the azimuth resolution equal to the range resolution. The L-band system uses antennas consisting of a 2 × 4 array of fat dipoles.

The return signal is amplified and mixed with the transmit signal. This dechirped signal is filtered and then sampled with a 16-bit analog-to-digital converter at 328.947 kHz. A custom field-programmable gate array board was designed to sample the signal and store the data on a pair of 1-GB Compact Flash disks. The data are continuously collected at a rate of 0.63 MB/s and either stored onboard or transmitted to a ground station.

Data processing for the BYU μSAR [25] follows the procedures outlined in the main body of this paper.

The Systron Donner Inertial MMQ-G INS/GPS unit is used with the BYU μSAR because of its small size (9.4 in³) and weight (< 0.5 lb). It provides position and attitude solutions at a rate of 10 Hz. The solution messages are transmitted to the μSAR using RS-232 and stored with the SAR data.

APPENDIX B
NONZERO DOPPLER GEOMETRY CALCULATIONS

As defined in Fig. 4, the range \( R + \Delta R \) and the angle \( \theta \) are derived from the SAR data, and the height \( H_A \) and the range \( R \) are found from the motion data. We define a right triangle on the ground with hypotenuse \( B_g \) and sides \( B_g \cdot \cos(\theta) \) and

\[ H_A = \frac{B_g}{\sin(\theta)} \]

\[ R = \frac{B_g}{\tan(\theta)} \]


\[ B \cdot \sin(\varphi) \] We take the angle \( \varphi \) to be between the lines \( R_g \) and \( G \). From this, we get the following equalities:

\[
\sin(\varphi) = \frac{B \cdot \sin(\varphi)}{R_g} \quad (19)
\]

\[
\cos(\varphi) = \frac{G + B \cdot \cos(\varphi)}{R_g} \quad (20)
\]

\[
\arcsin \left( \frac{B \cdot \sin(\varphi)}{R_g} \right) = \arccos \left( \frac{G + B \cdot \cos(\varphi)}{R_g} \right). \quad (21)
\]

This gives us two unknowns, namely, \( B \) and \( \varphi \). Then, we introduce another equation, i.e.,

\[
\tan(\theta) = \frac{B \cdot \sin(\varphi)}{\sqrt{(B \cdot \cos(\varphi))^2 + H_A^2}}. \quad (22)
\]

Equations (21) and (22) are simultaneously solved for \( \varphi \) and \( B \). The closed-form solution can easily be found using a symbolic solver. Unfortunately, the resulting equation is too large to be printed here. However, tedious the solution may be, given the input values, a computer has no difficulty in producing the correct result.

REFERENCES


