2023-11-13

Inconsistent Correlation and Momenta: A New Approach to Portfolio Allocation

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A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Science

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ABSTRACT

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Correlated stocks should, in equilibrium, have correlated momenta, but in practice momenta do not always correlate. We use short-term inconsistencies between correlations and momenta to predict price corrections, produce more meaningful investment indicators, and improve upon accepted investing strategies. In particular, our approaches integrate inconsistencies within an entire security class rather than relying only on individual or pairwise security data. We use this theory to improve upon not only the standard momentum portfolio but also Pair Trading and Momentum Reversion methods. This results in three strategies for portfolio allocation that outperforms overlying indices and market benchmarks by $5\% - 10\%$ in annual gain with an increase of CAPM α over the standard momentum portfolio from $-.1$ to 5.4. We expand on these strategies by showing applications generalized to comparable investing indicators including volatility.

Keywords: Network Theory, Graph Theory, Portfolio, Momentum, Pair Trade, Relative Momentum, Finance, Correlation, Correlation Matrix

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Chapter 1. Introduction

Correlation between financial securities is vital to Modern Portfolio Theory (MPT) and financial risk management. By investing in securities with minimal correlation, we build a more stable, more diverse, and less volatile portfolio. More recently, the correlation matrix has been used in network-based approaches to study the structure of global and local markets [1]. Each individual stock is represented by a vertex within the network. An edge represents significant correlation between the price fluctuation of two stocks where a significant correlation is determined by a chosen threshold $\theta \in [-1, 1]$.

We propose a modification to the Correlation Matrix, the Momentum Network, derived from positive Spearman rank correlation [2] integrated with security indicators to construct a weighted, signed network. In order to study the Momentum Network, we use four distinct centrality measures that each possess intuitive advantages and application. We use the Momentum Network to improve upon indicators comparable between securities, producing informative and less reactive investing indicators. Moreover, we apply the Momentum Network to improve upon three common methods of portfolio allocation, each of which demonstrate quantified historical edges over market expectation.

1. Link Momentum: Momentum Trading
2. Index Link Momentum: Momentum Reversion
3. Correlative Trading: Pair Trading

While these methods are commonly used in investing, they each have functional weaknesses. Most notably, they lack both important market wide context and are fundamentally based on lagging indicators.

We apply Degree Centrality, Closeness Centrality, Multi-Step Centrality, and Leverage Centrality to the Momentum Network formed on six distinct stock groupings by sector within the S&P 500: Energy, Finance, Health, Industry, Real Estate, and Technology. We show that Link Momentum, Index Link Momentum, and Correlative Trading each have a quantified edge over the ten year period from 2011-2021, measured by annual gain and alpha. Furthermore, we find that each portfolio allocation strategy out-gains a standard momentum while producing greater alpha measured against common benchmarks. Link Momentum outperforms a standard momentum portfolio with $\alpha = 5.4$ vs $\alpha = -.1$ and out gains the market by 5-10% annually. We show Index Link Momentum picks high performing securities within indices while predicting indices that will under-perform, producing a low volatility portfolio and compounding annual gain exceeding 22%, with the capabilities of producing a market-neutral portfolio. Additionally, Correlative Trading outperforms a standard momentum portfolio with α between 1.6 and 3. We test our methods relative to signal strength, number of connections included, and scale of momentum to produce more confidence as more context is given.

We expand on Link Momentum by exploring the generalization of the network structure and measures to other financial indicators including volatility, other momentum measures, and oscillation measures. We find that similar trends appear and that our strategy maintains an edge in portfolio construction over a variety of indicators. We also note several applications of future work, most notably in modern portfolio theory, option trading, and volatility arbitrage. We show that the use of correlation integrated with security indicator data and graph theory may be applied to a variety of portfolio allocation strategies and arbitrage methods, as well as a generalized improvement to security indicators.

A standard momentum portfolio allocates capital towards securities that have recently performed well. The assumption is that securities that outperform the market are more likely to continue to do so. Momentum Portfolios rank performance based on a look-back period, typically over 3-24 months, and are re-balanced on a monthly basis. Momentum portfolios have been observed to outperform market expectation by 7.96% [3] over the 20-year period $1990 - 2009$. Furthermore, we find that a standard 30-day momentum portfolio over the 10-year period 2011-2020 gains 15.75% yearly. The proposed Link Momentum improves upon the standard momentum portfolio over this 10-year period by over 6% yearly, gaining on average 21.89 % annually.

Momentum reversion seeks to identify securities that have been moving in a particular direction for a prolonged period of time and then to trade against those securities in the expectation that they will eventually revert to their mean. Intuitively, prices tend to revert to the mean over time, thus by trading against securities that have been moving in a particular direction for a prolonged period of time, a trader should profit from price reversals. Momentum reversion has been observed to gain 20.7% yearly [4] for long securities over the 20-year period $1980 - 1999$, while maintaining a 13.4% spread between long and short securities. The proposed Index Link Momentum returns on average 22.1% annually over the 10-year period $2011 - 2021$, out-gaining market expectation by up to 8.47% yearly. While Index Link Momentum gains more on average than a portfolio formed by a Momentum Reversion strategy, it maintains a 12.9% spread between long and short securities.

A pair trading strategy seeks to identify two securities with a high historical correlation whose prices have diverged in the short term. When a divergence has been found, a trader will short the security trading high while longing the security that is trading low. Intuitively, the price of two highly correlated securities with diverging prices should eventually revert to mean. Thus, by trading between the two securities, we may profit from the temporary divergence. Correlation is typically measured over a 1-2 year period. A mean price ratio is calculated over a given short term period to detect divergence from historical pricing. When a threshold ratio is met, the trade is triggered. In practice, a pair trading portfolio will be re-balanced daily or weekly. Pair trading has been reported to have a quantitative edge. Over a 40-year period this method was observed to outperform market expectation by close to 1% yearly [5] while maintaining low exposure. The proposed Correlative Pair Trading exceeds this performance with an average 2% annual gain over market expectation while maintaining low volatility.

In practice, the correlation matrix is typically formed by using either Pearson or Spearman Rank Correlation. The correlation matrix was first implemented in financial markets as a tool for portfolio optimization, replacing the covariance matrix as an improved method for measuring risk. Correlation is a standardized measure of covariance, accounting for differences in the scale of price. It was later used to create network structure for financial markets.

Within this structural context securities were grouped by clustering algorithms and other network science tools. This network structure is a critical point of contemporary research, yielding important context into both emerging [6] and global financial markets [7]. Alternative methods such as random matrix theory [8] and minimum spanning trees [9] have been used to improve upon the financial correlation matrix and produce more accurate relationships. More recently, centrality measures have been employed to find influential securities within financial markets. In 2010, it was found that the variation of securities within an entire financial market are influenced by a small number of securities [10]. Furthermore, network methods and structure have been used to develop portfolio allocation strategies using centrality measures [11].

3.1 Correlation Matrix

The data consisted of close prices for stocks within the S&P 500 and examined by sector. The price change was normalized in order to form more consistent price correlations. Let $G_i(t)$ denote the difference between the log-normalized prices at open on day t and the next day $t + \delta t$ (where $X_i(t)$ denotes the price of stock i and t is time in days):

$$G_i(t) = \ln\, X_i(t + \delta t) - \ln\, X_i(t). \tag{3.1}$$

The correlation matrix C is defined with each entry C_{ij} being the correlation between normalized security prices G_i and G_j. In testing, longer periods of correlation resulted in stronger performance, however for ease of comparison we will use a correlation period of 240 days, approximately a one year period. The Spearman rank correlation [12] is calculated by

$$C_{ij} = \frac{\mathrm{Cov}(R(G_i), R(G_j))}{\sigma_{R(G_i)}\sigma_{R(G_j)}} \in [-1, 1] \tag{3.2}$$

with rank standard deviation $\sigma_{R(G_l)}$ and price rank $R(G_i)$. For a correlation period of 240 days, C_{ij} will rarely be negative. This is likely a result of the historical upward trend of the examined security classes. For simplicity, we use a correlation threshold $\theta = 0$ to eliminate anomalies in data. Thus C is non-negative and $\forall i, j$

$$C_{ij} \in [0, 1].$$

3.2 Momentum

Momentum indicates the direction and the speed at which the price is increasing or decreasing. While there are varying momentum methods, we use a standard Momentum Percentage defined by

$$\mathrm{M}(X_i) = \frac{X_i^t - X_i^{t-\delta t}}{X_i^{t-\delta t}} \tag{3.3}$$

where δt is the time frame in which momentum is calculated. In practice, $\mathrm{M}(X_i) \in (-1, 1)$, as our security class consists of only large cap securities. However our methods remain well-defined for momentum outside these bounds. We will use a standard momentum period of 30 days. It is important to note that our methods may be applied to any indicator so long as it is taken as a percentage value allowing comparison between securities. For a security class of size N we will denote $m \in R^N$ as the momentum vector with $m_i \in (-1, 1)$, the momentum for each security i.

3.3 Security Groupings and Procedure

We will use six sector grouping consisting of sector indices. These sectors are: Energy, Finance, Health, Industry, Materials, Technology. We will test our strategies on the sectors individually, as well as a portfolio with positions split equally between all sectors. We will rank securities by percentile according to centrality. Rank one indicates the 10% highest valued securities and rank ten indicates the 10% lowest valued securities. We will examine both the average performance across all sectors and the performance of a portfolio split between all sectors uniformly. In general, our procedure consists of the following:

1. On the first business day of each month, calculate the correlation matrix from the previous year.
2. Calculate the Momentum from the previous 30 business days.
3. Value securities as defined in strategy.
4. Long rank one valued securities, short rank ten valued securities.

4.1 The Momentum Network

We first introduce the Momentum Network, a means by which we produce more informative indicator data and give structure to indicator data. We define the Momentum Network by,

$$A = \begin{pmatrix} 0 & C_{12}m_2 & \dots & C_{1n}m_n \\ C_{21}m_1 & 0 & \dots & C_{2n}m_n \\ \vdots & \vdots & & \vdots \\ C_{n1}m_1 & C_{n2}m_2 & \dots & 0 \end{pmatrix}. \tag{4.1}$$

As correlation is non-negative, the sign of each entry is dictated by the momentum of neighbor j for security i. A positive entry dictates correlation to a security trending positively and is an indication of price strength. Similarly, a negative entry indicates correlation to a negatively trending security and shows weakness in price strength.

4.2 Strategy Performance

We gauge the performance of each strategy relative to three measures:

1. Volatility relative to the benchmark

$$\beta = \frac{1}{N}\sum_{s\in P}\beta_s. \tag{4.2}$$

   Calculated with respect to N equally weighted securities within the portfolio P and volatility β_s for each security $s \in P$. The volatility, β_s for each individual security s is calculated by

$$\beta_s = \frac{\text{Cov}(X_s, X_M)}{\text{Var}(X_M)} \tag{4.3}$$

   where X_S is the price vector for security s and X_M is the market price vector (measured by the S&P 500) taken at monthly intervals.

2. CAPM-α [13] defined by

$$\alpha = R_P - R_f - \beta(R_M - R_f). \tag{4.4}$$

   Calculated with respect to portfolio return R_P, market return R_M, portfolio volatility β, and risk free return $R_f \equiv 1.5\%$. CAPM indicates excess return over the market, adjusted for relative increased or decreased risk.

3. CAG: The average compounding annual gain of portfolio. Calculated by

$$\text{CAG} = (1 + R_P)^{1/y} - 1 \ , \tag{4.5}$$

where R_P is the percentage gain of portfolio and $y = 10$ is the number of years to calculate over. $R_P = 1$ indicates no gain y, $R_P < 1$ indicates loss, and $R_P > 1$ indicates gain.

We will rank securities by centrality, grouping securities into ten percentile groupings or ranks. We will then measure performance of rank one valued securities versus the performance of rank ten valued securities annually.

5.1 Strategy

It is suggested that securities historically outperforming market expectation are more likely to continue to do so [3]. A standard momentum portfolio is produced by ranking securities by a momentum indicator over a look-back period P and then buying securities with the highest momentum. We propose a modification to broaden the scope of the momentum indicator. Rather than examining the momentum of a security i itself, we measure the momentum of stocks that significantly correlate historically. In theory, the security will follow a similar trend in momentum as its highly correlative neighbors. Using information for the entire asset class for the valuation of each individual security allows us to make more informed trades.

An additional consideration is that momentum is a lagging indicator. High momentum indicates the security has already increased in value. While momentum trading is a lagging indicator, presuming securities with strong performance are more likely to perform strongly in the future, Link Momentum is not necessarily a lagging indicator. In theory, a security with low momentum and correlating neighbors with high momentum should increase in momentum. While a security with high momentum, and correlating neighbors with high momentum is more likely to maintain high momentum.

Our first approach is to rank a security by the momentum of each security with significant correlation. Since momentum is taken as a standardized percentage, we may value each security by adding the momentum of highly correlative securities. We further develop this by scaling momentum of a neighbor security by correlation, adding weight to higher correlative neighbors:

$$\begin{aligned} a_i &= \sum_{j \in N(i)} C_{ij} m_j \\ \implies a_i &= \sum_j A_{ij}. \end{aligned} \tag{5.1}$$

This is precisely in-degree centrality for the Momenta Network previously defined. To further this approach, we consider a multi-step centrality measure. We first make the observation, that

$$\begin{aligned} a = \sum_j A_{ij} &= \sum_{j \in N(i)} C_{ij} m_j = C_i m \\ \implies a &= Cm. \end{aligned} \tag{5.2}$$

As previously defined, we use a correlation threshold $\theta = 0$ which implies that the matrix C is non-negative.

We next note that index i, j of nth power of the non-negative matrix C yields the combined weight of all walks of length n between securities i and j. That is for k steps considered, we define centrality vector a by

$$a^{(k)} = \sum_{n=1}^{k} (rC)^n m. \tag{5.3}$$

In order for the summation to converge we choose a sufficiently small r term. We will define r by

$$r = \frac{0.85}{\|C\|_\infty} = \frac{0.85}{\max_i \sum_j C_{ij}} = \frac{0.85}{\max_i d_i}$$
$$\implies \|(rC)\|_\infty \leq 0.85$$

where 0.85 accounts for the diminishing weight each step contributes. The constant 0.85 is a typical factor used in centralities such as PageRank and Katz [14]. We find that as $k \to \infty$, the Multi-Step series converges. For $a^{(k)} = \sum_{n=1}^{k} (rC)^n m$,

$$\lim_{k\to\infty} \left\| a^{(k)} \right\|_\infty = \left\| \sum_{n=1}^{\infty} (rC)^n m \right\|_\infty$$
$$\leq \sum_{n=1}^{\infty} \|(rC)^n m\|_\infty$$
$$\leq \sum_{n=1}^{\infty} \|rC\|_\infty^n \|m\|_\infty$$
$$\leq \sum_{n=1}^{\infty} \|rC\|_\infty^n$$
$$\leq \sum_{n=1}^{\infty} (0.85)^n.$$

This is a convergent series by the geometric series test.

5.2 Results

Rank one securities 2011-2021 - α, β						
Sector	Momentum		Degree		Multi-Step	
	α	β	α	β	α	β
Energy	-1.8	0.93	5.4	1.09	**6.8**	1.2
Finance	**5.1**	0.81	4.9	.94	4.9	92
Health	-1.6	1.16	**4.8**	.98	2.8	.96
Industry	**2.3**	1.00	-2.2	.88	-.1	.9
Materials	-6.5	0.86	-4.1	.95	**1.8**	.97
Tech	**11.8**	1.21	-6.7	.99	2.8	.98
Average	1.2	0.995	1.4	.97	**3.2**	.99
Combined	-.1	1.15	3.4	1.063	**5.3**	1.098

Table 5.1: Compares CAPM α and β of rank one valued securities for each sector valued by a standard momentum portfolio, Degree Centrality on the Momentum Matrix, and 5 term Multi-Step Centrality (at convergence) on the Momentum Matrix. We measure both the average across all sectors and the combined performance of portfolio with the highest valued security from each sector. Both Degree and Multi-Step Centrality have a quantifiable edge over a standard momentum portfolio.

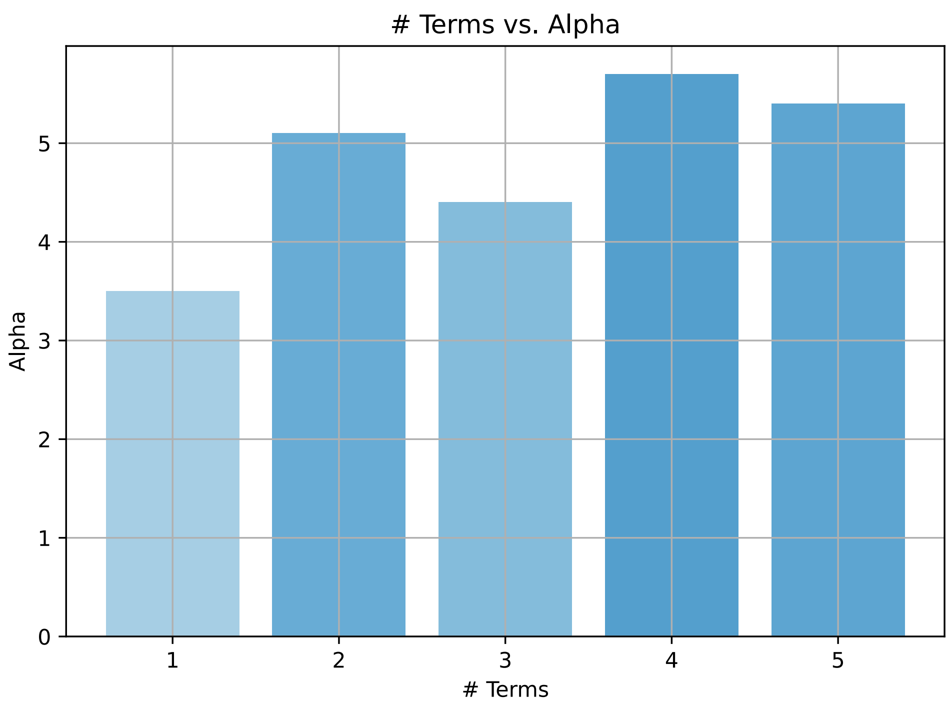

Figure 5.1: Strategy Alpha relative to number of terms. As the number of terms increase, both compounding annual gain and alpha increase.

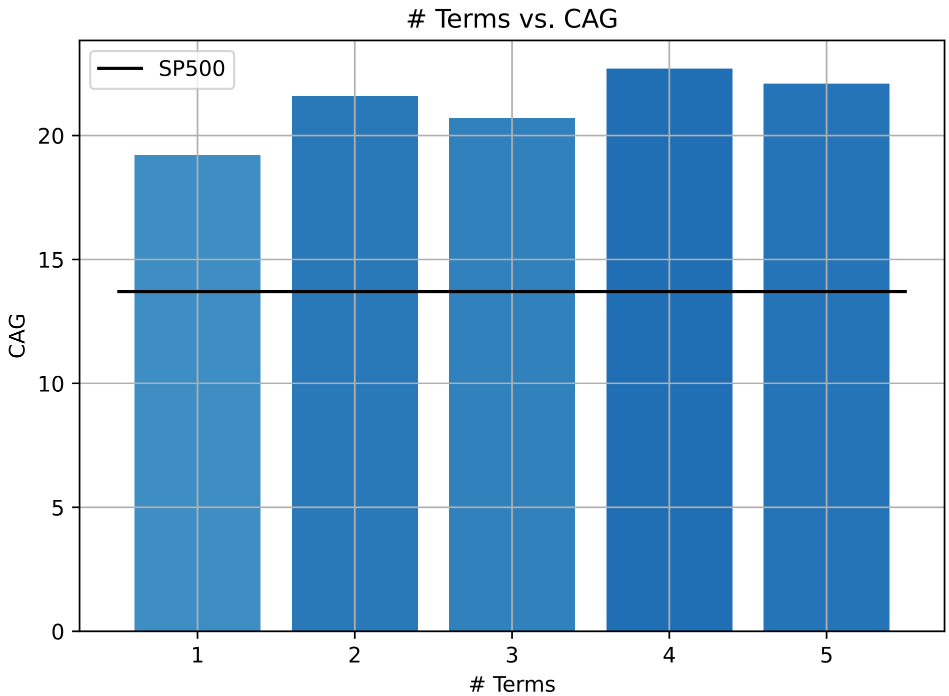

Figure 5.2: Strategy compounding annual gain relative to number of terms.

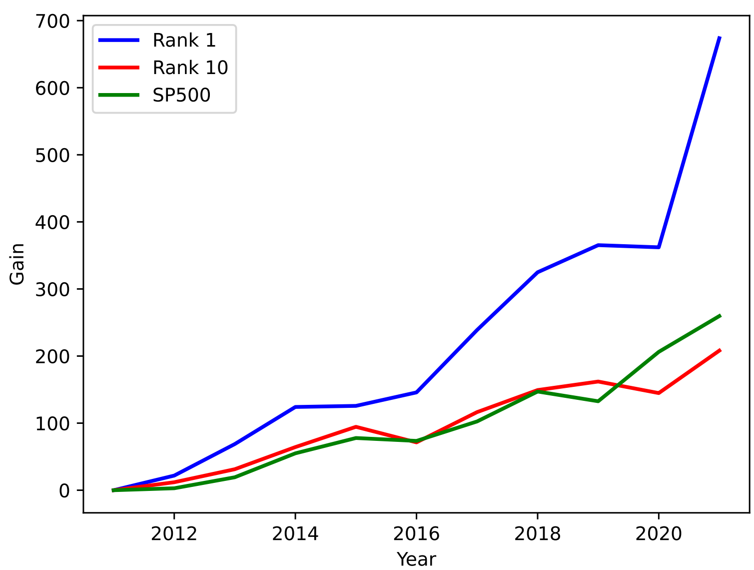

Figure 5.3: Plots the strategy total percentage gain by year. At the start of each month securities were ranked by centrality. The performance of both rank one and rank ten securities were tracked. Rank one securities greatly outperform, showing a quantified edge over the benchmark. However, while we would expect rank ten securities to under-perform, they perform close to benchmark expectation.

5.3 Discussion

A portfolio formed by Link Momentum outperforms a standard momentum portfolio. This is supported by performance on both the individual sector groupings as well as the combined grouping of all sectors. Multi-step centrality outperforms the momentum portfolio with a 3.2/5.4 alpha vs. a $1.2/-.1$ alpha on the individual and combined groupings. Moreover, we find a positive trend relative to the number of terms computed in centrality. Degree centrality, or single term, yields a combined alpha of 3.4 with noticeable jumps in performance when one more term is included. Top performance is achieved at four terms with convergence, relative to signals given, achieved at five terms.

Multi-step rank one performance outperforms the benchmark every year except for the intervals 2014-2015 and 2019-2020. At convergence, it gains 8.19% more than the benchmark over the time frame. Interestingly, the rank one valued securities showed it's strongest gain on the year interval 2020-2021. A similar effect takes place in the market recovery from 2012-2014. While rank one performance is promising, the rank ten grouping performs only marginally worse than the benchmark. We would expect rank ten performance to perform substantially worse than the benchmark. In fact, in nearly half of the yearly intervals it outperforms the benchmark. We conclude, that high centrality securities are likely to outperform benchmark while low centrality provides no meaningful information.

Further research should be done investigating momentum periods, correlation periods, diminishing factor r, and the impact of high correlation neighbors with high momentum vs the impact of high correlation neighbors with low momentum security. Specifically, we would examine the predictive nature of Link Momentum for low momentum securities with high centrality and similarly the maintenance of strong performance. Lastly, further research should be dedicated to performance relative to market conditions. As noted, performance was strongest in periods of market recovery.

6.1 Strategy

In theory, we expect an overlying index to follow the same trends as the sum of it's composite securities. However, in practice an overlying index will not always represent it's underlying indices completely. These inconsistencies may be found in volatility, trend, and other financial indicators. We will focus our efforts first on momentum but later note the applications with volatility. A traditional approach is to compare the momentum of underlying securities within an index to the momentum of the overlying index. We expect the index to accurately depict the price and trend of securities within. Therefore, an inconsistency is found when the weighed average momentum of securities within the index is significantly lower than the index itself. In this case, we would expect a correction between the underlying and overlying so that price and trend are represented accurately. When the weighed average momentum is lower than the overlying index momentum, the momentum of underlying securities should increase and the index momentum should decrease. To exploit these inconsistencies, we buy securities with the lowest momentum and sell the index itself, profiting when the composite securities align with the index. We propose an alternate method, specifically, first using centrality to find the securities most influential within the index. We then form a long position on the securities of rank one centrality with momentum lower than the index and a short position on the index. As the index should represent it's underlying securities, the most influential securities within should have the strongest connection to the index.

Using Leverage Centrality, we project a ranking of the most influential securities relative to momentum within an index. The most influential securities represent an expectation for the securities within the index, and as such should follow the same trend. We then compare the momentum of these securities to the overlying index, where we long the security if momentum is less than index momentum and short if momentum is greater than index momentum.

Leverage Centrality [15] is defined on a weighted, non-negative, connected network A by

$$b_i = \frac{1}{a_i} \sum_{k \neq i} \frac{a_i - a_k}{a_i + a_k} \tag{6.1}$$

with degree for node i, $a_i = \sum_j A_{ij}$. For positive a_k, $a_i - a_k$ will be positive when $a_i > a_k$ and negative when $a_i < a_k$. $a_i + a_k > a_i - a_k$ will be strictly positive. Thus $\frac{a_i - a_k}{a_i + a_k}$ represents a percentage of influence node i has over node k with positive, negative values representing influence or lack of influence. As we sum over all k, Leverage Centrality measures the influence each node has on all of it's neighbor nodes. However, Leverage Centrality remains well-defined on a modified Momentum Matrix A despite being a signed network. First, note that for a similarly defined degree centrality,

$$a_i = \sum_j A_{ij} = \sum_j C_{ij} m_j = C_i \cdot m.$$

So then when we consider Leverage Centrality,

$$\begin{aligned} b_i = \frac{1}{a_i} \sum_{j \neq i} \frac{a_i - a_j}{a_i + a_j} &= \frac{1}{C_i \cdot m} \sum_{j \neq i} \frac{\sum_k C_{ik} m_k - \sum_k C_{jk} m_k}{\sum_k C_{ik} m_k + \sum_k C_{jk} m_k} \\ &= \frac{1}{C_i \cdot m} \sum_{j \neq i} \frac{\sum_k (C_{ik} - C_{jk}) m_k}{\sum_k (C_{ik} + C_{jk}) m_k} \\ &= \frac{1}{C_i \cdot m} \sum_{j \neq i} \frac{(C_i - C_j) \cdot m}{(C_i + C_j) \cdot m}, \end{aligned} \tag{6.2}$$

where C_i, C_j are non-negative vectors for all i, j. In order to ensure b is well-defined we will use a modified momentum, setting $m'_i = 1 + m_i \in (0, 2)$ with $m' = m + e$ and the one vector e. Since C_i is non-negative for all i, $C_i m' > 0$. Substituting $m + e$ for m, we have the final equation

$$b_i = \frac{1}{C_i \cdot (m + e)} \sum_{j \neq i} \frac{(C_i - C_j) \cdot (m + e)}{(C_i + C_j) \cdot (m + e)} \tag{6.3}$$

which is well-defined for all i.

Intuitively, the vector $C_i - C_j$ describes the difference in correlation between the neighbors of securities i and j. When $(C_i - C_j) \cdot (m+e) > 0$ security i has positive influence over security j while $(C_i - C_j) \cdot (m+e) < 0$ indicates negative influence. In the denominator, $(C_i + C_j) \cdot (m + e)$ measures the total momentum weighted correlation both securities have to all other securities in the asset class. Together,

$$\frac{(C_i - C_j) \cdot (m + e)}{(C_i + C_j) \cdot (m + e)}$$

measures a positive/negative influence as a percentage of combined degree. As we normalize by the total momentum weighted correlation of i, we gain a complete measure of the influence security i has in the asset class.

Once we compute the influence each security i has within the index, we then measure the momentum of the rank one valued securities vs the overlying index itself. A buy signal for security x in the rank one grouping $R_1(c)$ of centrality vector c, index y and momentum values m_x, m_y is given by

$$B(x) = \begin{cases} 1 & m_x > m_y \\ 0 & m_x \leq m_y. \end{cases} \tag{6.4}$$

Similarly, we determine short signals for each index by the average momentum of the rank one underlying securities. Let N be the number of securities in $R_1(c)$ of centrality vector c, then

$$B(y) = \begin{cases} 1 & \frac{1}{N} \sum_{x \in R_1(c)} m_x > m_y \\ 0 & \frac{1}{N} \sum_{x \in R_1(c)} m_x \leq m_y. \end{cases} \tag{6.5}$$

6.2 Signal Strength

We introduce the confidence scalar ω allowing for a measurement of signal strength. Larger gaps between the momentum of stock x and the momentum of overlying index y, in theory, should be larger market inconsistencies. The minimum signal is given with a threshold denoted by ω. We define this as the confidence level or confidence scalar. We may now define a buy signal with confidence level ω formally as,

$$B(\omega, x) = \begin{cases} 1 & \omega m_x > m_y \\ 0 & \omega m_x \leq m_y. \end{cases} \tag{6.6}$$

As defined, ω is simply a desired ratio of the index momentum to security momentum,

$$\omega \leq m_y / m_x.$$

Similarly, we define a short signal for indices with an equal confidence level ω as

$$B(\omega, x) = \begin{cases} 1 & \frac{\omega}{N} \sum_{x \in R_1(c)} m_x > m_y \\ 0 & \frac{\omega}{N} \sum_{x \in R_1(c)} m_x \leq m_y. \end{cases} \tag{6.7}$$

6.3 Results

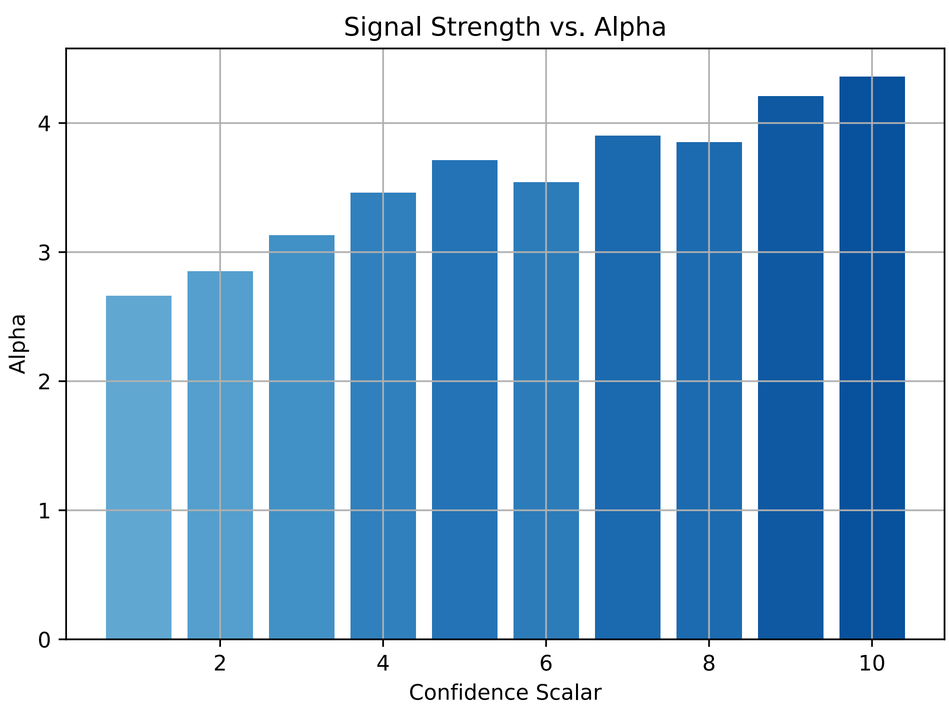

Figure 6.1: Strategy alpha and compounding annual gain relative to signal strength. As confidence levels increase, both compounding annual gain and alpha increase.

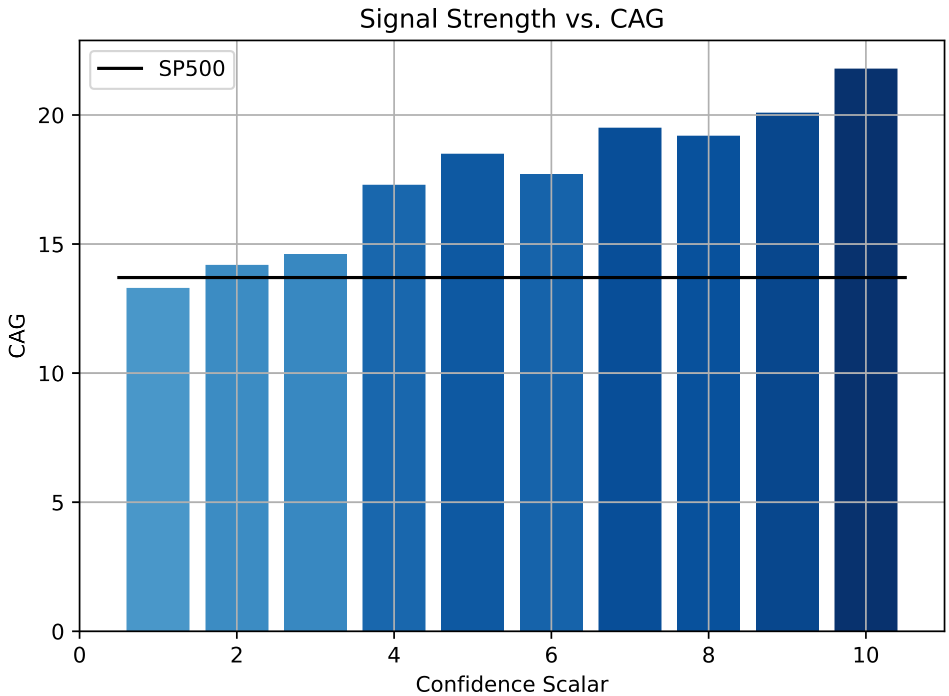

Figure 6.2: Strategy compounding annual gain relative to signal strength.

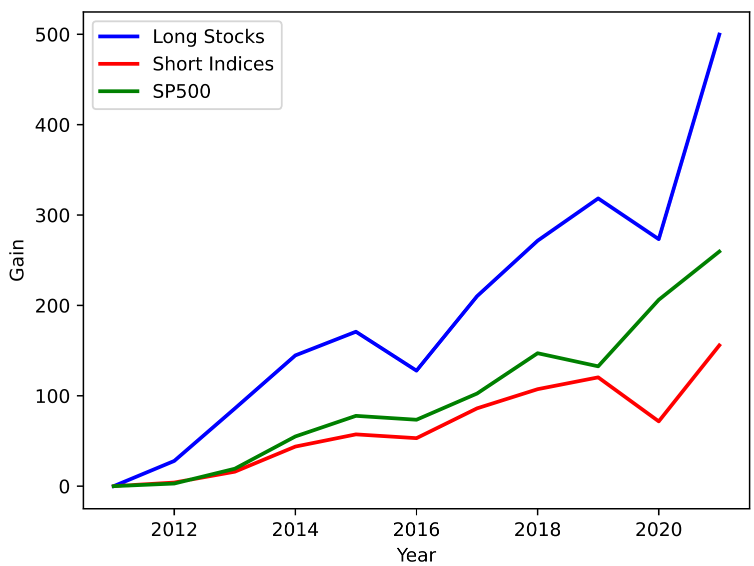

Figure 6.3: Performance comparison of securities with long signals, indices with short signal vs. S&P 500. Securities with long signals outperform the benchmark while indices with short signals consistently under-perform annually. Uses the confidence scalar 8.

6.4 Discussion

Index Link Momentum outperforms the benchmark over seven of the ten intervals. The outliers were in years 2014-2016, in which it lost significantly more than benchmark, and 2017-2018, where it gained slightly less than benchmark. The indices with short signals outperformed the index on yearly intervals 2011-2012, 2016-2017, and 2020-2021, while they under-performed on all other increments. Notably, the shorted indices under-performed by 4.5% annually compared to benchmark.

The strongest evidence supporting Index Link Momentum is the increase in performance as the confidence threshold increases. Unthresholded Index Link Momentum has an average annual gain lower than benchmark but at maximum confidence levels out-gains the benchmark by 8.47%. Performance increases at nearly every successive increase in confidence level. Unthresholded Index Link Momentum maintains positive alpha and as confidence levels are increased a positive trend is again expressed. We find that an increased confidence scalar minimized poor trades while maintaining trades with the highest gains. When both the long and short strategies are implemented there is a 12.9% difference in compounding annual gain.

In brief testing, we find that shorter trading increments produced stronger results. We believe that further research into daily and weekly trading will further enhance performance. Additional research is needed to develop theory with correction time periods, additional indicators such as volatility, and parameter optimization.

7.1 Strategy

First, we examine only two securities, i and j, that historically correlate closely. In the future we would expect these stocks to continue to follow similar price trends. Thus if stock i was trending in a positive direction while stock j was trending in a negative direction, we would expect these trends to revert. Similarly, if two securities correlate inversely but are trending in the same direction, we would expect these trends to diverge. This is the essential idea behind Pair Trading strategies.

In the case of only two correlating securities, a short signal is given for security i and a long signal for security j, when $m_i >> m_j$. Equivalently, this can be described by $m_j - m_i << 0$. We further this approach by not only examining two securities with high correlation, but an entire asset class of correlating securities. We now consider n securities within an asset class. Short and long signals are given for security i when

$$m_i >> \frac{1}{n-1}\sum_{j\neq i}^{n} m_j \implies \frac{1}{n-1}\sum_{j\neq i}^{n} m_j - m_i << 0 \tag{7.1}$$

$$m_i << \frac{1}{n-1}\sum_{j\neq i}^{n} m_j \implies \frac{1}{n-1}\sum_{j\neq i}^{n} m_j - m_i >> 0 \tag{7.2}$$

respectively. We will denote this momentum difference for security i by c_i. As securities within the asset class will vary in levels of correlation with i, we scale the momentum difference for each i, j by the correlation of securities i and j. This provides the equation

$$c_i = \frac{1}{n-1}\sum_{j=1}^{n-1} C_{ij}(m_j - m_i) \tag{7.3}$$

which is precisely degree centrality for a square, n dimensional, matrix B defined by

$$B_{ij} = (m_j - m_i)C_{ij} \in [-2, 2] \tag{7.4}$$

$$B_{ji} = (m_i - m_j)C_{ij} \in [-2, 2]. \tag{7.5}$$

We will first improve upon this method with an analogous Multi-Step Centrality measure as defined in Link Momentum. In this case, we again compute the centrality of the correlation matrix separately. However the centrality vector c now simply equates to the differences in Link Momentum a_i, a_j, converging by the same analysis.

$$c_i^{(k)} = \sum_j (\sum_{k=1}^{n} C^k)(m_j - m_i)$$

$$= \sum_j \sum_{k=1}^{n} C^k m_j - \sum_j \sum_{k=1} C^k m_i$$

$$= a_j^{(k)} - a_i^{(k)}. \tag{7.6}$$

After computation of centrality we long rank one securities while shorting rank ten securities. As we long/short the securities with maximal/minimal correlative distance to neighbor securities.

Secondly, we will use the average shortest distance as a centrality measure on the Momentum Matrix A defined by

$$c_i = \sum_{j=1}^{n} \frac{d(i,j)}{n-1}$$

where $d(i,j) > 0$ is the shortest distance between securities i and j in the network A. This simplifies with respect to the fully connected network A as we rank securities by centrality, becoming

$$\text{argmax}_i c_i = \text{argmax}_i \sum_{j=1}^{n} d(i,j). \tag{7.7}$$

Average shortest distance is an ideal choice for centrality, as we wish to find securities that, on average, are trading the highest and lowest relative to their neighbor nodes, signaling an inconsistency. The shortest distance from security i to each j indicates how far out of equilibrium security i is with j. As we sum the shortest distance for all neighbors of i, we determine how inconsistent security i is within it's asset class. Of the rank one securities, we long securities that have the lowest momentum difference and short securities that have the highest momentum difference. This yields the familiar formula

$$c_i = \sum_{j=1}^{n} d(i,j) C_{ij} (m_j - m_i). \tag{7.8}$$

Once we have computed our centrality measures and ranked the security grouping accordingly, we short rank one securities and long rank ten securities.

7.2 Results

Rank one securities 2011-2021 - α, β						
Sector	Momentum		Multi-Step		Average Shortest	
	α	β	α	β	α	β
Energy	-1.8	0.93	**7.2**	1.12	6.8	.875
Finance	**5.1**	0.81	3.4	0.99	1.8	.905
Health	-1.6	1.16	**4.0**	1.06	-1.2	1.008
Industry	**2.3**	1.00	.5	1.11	1.4	1.058
Materials	-6.5	8.6	**-.7**	1.09	7.5	.928
Tech	**11.8**	1.21	3.9	1.08	.8	1.066
Average	1.2	0.995	3.1	1.07	**3.75**	.973
Combined	-.1	1.15	1.6	1.18	**3.0**	1.096

Table 7.1: Compares α and β of rank one valued trades for each sector valued by a standard momentum portfolio, Multi-Step on the Momentum Matrix, and Average Shortest Distance Centrality on the Momentum Matrix. Both Multi-Step and Average Shortest Distance outperform a standard momentum portfolio. Multi-Step produces the strongest results.

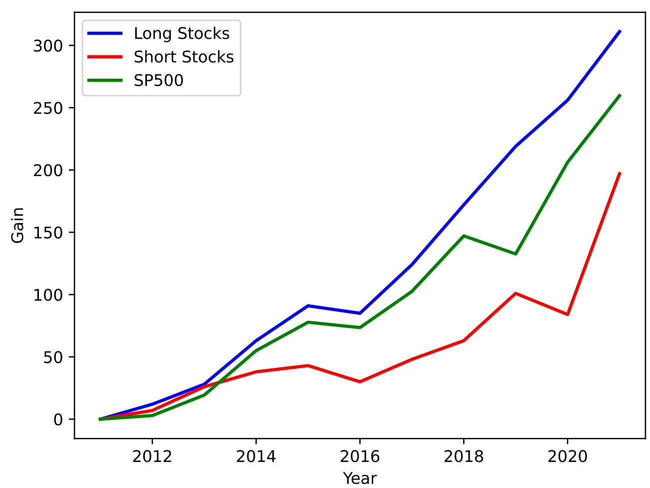

Figure 7.1: Performance comparison of securities with long signals, and short signals vs. S&P 500.

7.3 Discussion

Both the Multi-Step and Average Shortest Distance centrality measures outperform the standard momentum portfolio. Multi-Step provides the most consistent performance over the individual sectors. However, Average Shortest Distance has the strongest average performance over the sectors. Average Shortest Distance benefits from strong performance of the Materials sector, which both the standard momentum portfolio and Multi-Step centrality under-performed on. Additionally, Average Shortest Distance centrality has the strongest performance over the complete security pool of all securities. It is likely that, again, this performance is a result of performance on the Materials sector. While gain was only slightly higher than the benchmark, the volatility of the portfolio formed by Average Shortest Distance centrality was nearly 10% lower than multi-step and 6% lower than the momentum portfolio.

The rank ten securities, those that were shorted, under-performed vs the benchmark every year after 2013 except for 2020-2021. Additionally, rank ten securities performed worse than the long stocks on every yearly interval except for 2020-2021.

While results were promising, there are many improvements to be implemented. As with all portfolio allocation strategies, trades were made on monthly intervals. We believe that shorter trading intervals will increase performance further as it allows for trades to be closed when discrepancies are corrected. Furthermore, a confidence threshold analogous to that defined in Index Link Momentum may produce similarly enhanced performance.

8.1 Volatility

The Momentum Network may be applied to any indicator that is comparable between individual securities. Most significantly, we find that the network structure can be applied to volatility, predicting securities that will have a quantifiable increase in volatility. All three strategies presented are applicable to Volatility.

1. Link Momentum: Calculate a more informed prediction of future volatility.
2. Index Link Momentum: Compare Volatility of Index relative to volatility of most influential composite securities.
3. Correlative Trading: Can exploit by selling/buying volatility rather than shorting/longing the security.

Additionally, option contracts are priced primarily by implied volatility. A volatility premium denotes a contract bought at a lower volatility than actual volatility over the life time of contract. Therefore if we can predict an increase in volatility we may buy valuable contracts with a premium, and if we predict a decrease in volatility we may sell overpriced contracts. In combination with a momentum trading strategy where we may predict the trend of a security, we produce a complete option trading strategy.

	Volatility Increase	Volatility Decrease
Momentum Increase	Buy Call	Sell Put
Momentum Decrease	Sell Call	Buy Put

Lastly, it is worth noting possible applications in risk management. We expect correlative volatility to be more stable and predicative than historical volatility. Rather than using historical or implied volatility to assess and manage risk, we may use correlated volatility to asses risk and construct portfolio.

8.2 Additional Indicators

We use a sample of indicators, and compare their alpha, beta with respect to Absolute Leverage and Absolute Index Leverage. This is neither an exhaustive listing nor were the parameters optimized for performance.

Rank one securities 2011-2021 - α, β						
Sector	Standard		Link Momentum		Index Link	
	α	β	α	β	α	β
SMA 20	-4.7	1.06	3.4	1.25	**3.5**	1.25
EMA 20	-4.6	1.06	3.7	1.26	**3.8**	1.26
OBV	-.1.9	1.15	**-0.4**	1.12	**-0.4**	1.12
STO	-2.5	1.02	-1.3	.97	**-1.1**	.965
ULTSOC	-1.9	1.07	**1.1**	1.023	.3	1.026
RSI	-3.5	1.18	-.6	.974	**-.4**	.974
Average	-2.6	1.10	.95	1.08	**.97**	1.08

Table 8.1: Compares α and β for portfolio's of rank one securities using a standard rank one portfolio, Absolute Leverage and Absolute Index Leverage on a variety of indicators. Both Link Momentum and Index Link Momentum maintain an advantage over a standard formed portfolio when applied to new financial indicators.

	Link	Index	Correlative
Formula	$\sum_{n=1}^{k} C^n m$	$\frac{1}{C_i(m+e)} \sum_{j \neq i} \frac{(C_i - C_j)(m+e)}{(C_i + C_j)(m+e)}$	$\sum_j d(i,j) C_{ij} (m_j - m_i)$
Concept	Uses momentum of high correlation neighbors. In theory, more predictive.	Exploits differences between highly central securities and overlying index.	Pair trading using Link Momentum. Adds information to pair trading strategy.
α	5.4	4.6	3.0

Table 9.1: Compares the formulas, concepts and alpha of the three proposed portfolio allocation strategies: Link Momentum, Index Link Momentum, and Correlative Trading.

All three portfolio allocation strategies show promise. Most notably, Link Momentum showed the strongest compounding annual gain, 24%, and an alpha of 5.4 for long securities, outer-performing the standard momentum portfolio over the same time period with $\alpha = -.1$ while trading with 5% volatility lower. Link Momentum shows the strongest performance in periods of market recovery, however there is no evidence that rank ten valued securities under-perform relative to benchmark. As such, Link Momentum can only be applied as a long strategy and thus far can not be applied to a market neutral strategy.

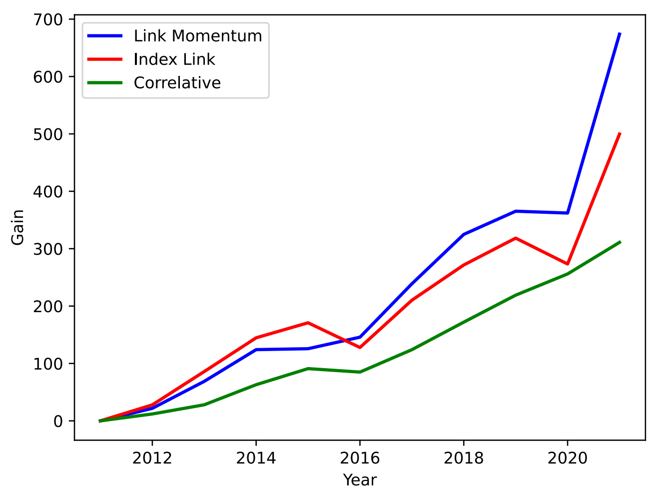

Figure 9.1: Performance comparison of securities with long signals valued by Link Momentum, Index Link Momentum (with confidence scalar 8), and Correlative Trading. Link Momentum achieves the strongest performance.

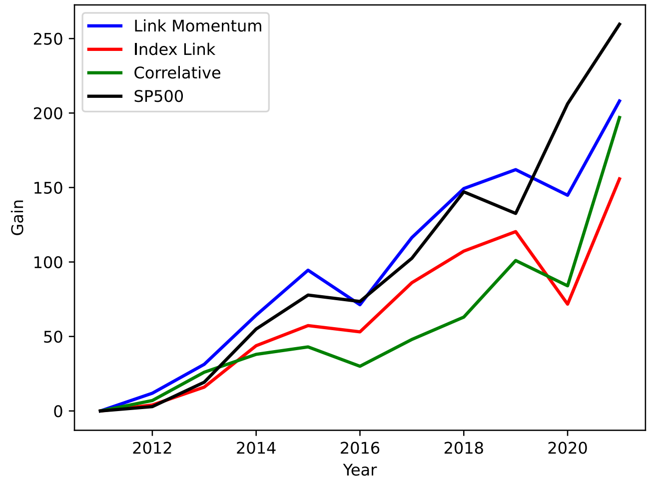

Figure 9.2: Performance comparison of securities with short signals valued by Link Momentum, Index Link Momentum (with confidence scalar 8), and Correlative Trading. Correlative Trading more consistently under-performs while Index Link Momentum yields the worst performance.

Index Link Momentum out-gained the benchmark by $> 9\%$ while providing the lowest volatility and the ability to predict under-performing indices. Additionally, it also has the strongest evidence of performance, as increasing confidence levels relate to improved performance. Index Link Momentum, also produced the strongest performance for short signals. Indices with short signals under-performed on average by more than 4.5% annually. However, it also appears to have the weakest performance during stagnant market conditions. Additionally, at it's strongest performance with high confidence levels there are periods where large portions of capital are not invested.

Correlative Trading, while showing promise for both long and short signals, produced the weakest results with alpha of 3.0 while producing 15%-19% compounding annual gain, as valued by Average Shortest Distance Centrality and Multi-Step Centrality. With Average Shortest Distance Centrality only slightly exceeding market expectation. However Correlative Trading has the most room for growth and can be improved by confidence ratings analogous to Index Link Momentum, as well as through the implementation of shorter time increments.

Each strategy relies on discrepancies between historical correlation and recent trends. However, many of these discrepancies may be a result of information outside of the scope of the allocation strategies. Major changes in firm earnings, fundamentals, sentiment, or other pertinent variables may lead to false signals. Strong trade signals produced by divergences between long term and short term correlation are vulnerable to information outside of the network's context. This may explain some of the periods of increased volatility. Using the strategies in unison with other important data may greatly improve performance. Additionally, the use of analyst ratings, price targets, and longer term trends should in theory reduce false signals. Lastly, in order to avoid false signals, we may avoid securities with upcoming earnings reports.

In future work, we believe it is most important to include firm fundamentals and risk factors in order to eliminate false-positive signals as noted previously. Additionally, it was a conscious decision to neglect parameter tuning specific to momentum and correlation time frames. In future work, these parameters may be considered further. Specifically, future work should be done on time increments, correlation periods, indicator look-back periods, diminishing factor r, and correlation threshold. Lastly, the application of the Momentum Network to option trading is readily apparent. Link Momentum may be applied to security volatility and momentum to value option contracts. Furthermore, Index Link Momentum and Correlative Trading yield long and short positions that both have quantifiable edges over the benchmark. These signals may easily be applied to option contracts. For either of these allocation strategies, a major obstacle is the time delta in which projected discrepancies will correct. Lastly, in future work we recommend the use of the Momentum Network and our proposed allocation strategies to additional security indicators. Most importantly, we note the future applications of our work relevant to the volatility indicator.

10.1 Link Momentum

```
class LinkMomentumDegree(QCAlgorithm):

    def Initialize(self):
        self.SetStartDate(2011, 1, 1) # Start of Backtesting WIndow
        self.SetEndDate(2021, 1, 1) # End of Backtesting WIndow
        self.SetCash(100000) # Initial Cash to Use
        self.AddEquity("SPY", resolution=Resolution.Daily) # Used for Benchmark
        self.SetBenchmark("SPY") # Benchmark (Measure Performance Against)
        self.corr_period = 240 # window to determine correlations
        self.maketrade = False # Make trades when True.
        self.threshold = 0 # Correlation Threshold

    # Build Correlation Matrix
    def build_matrix(self):
        # Calculate Correlation Matrix for each Sector
        for sector in self.sector_codes:
            symbols = self.symbols[sector]
            history = self.History(symbols, self.corr_period, Resolution.Daily)
            history['log_ret'] = np.log(1+history.opena.pct_change())
            a = spearmanr(history['log_ret'].unstack(level=0).dropna())[0]
            if type(a) is  float or len(a[0]) != len(a.T[0]):
                self.Log("FOUND")
                return
            num_stocks = len(symbols) # Number of Stocks
            # Correlation Matrix
             self.adj[sector] = np.array([[0 if j==i or a[i][j] < self.threshold else a[i][j]
                     for j in range(num_stocks)] for i in range(num_stocks)])
             # Momentum Vector
             self.v[sector] = np.array([self.data[symbols[j]].macd.Current.Value for j in range(num_stocks)])


    # Compute Centrality - returns symbol corresponding to largest valued index
    def centrality(self,sector):
        A = self.adj[sector]
        num_stocks = len(A[0])
        x = [sum([A[i][j] for j in range(num_stocks)]) for i in range(num_stocks)]


        x1 = self.largest_elements(x) # long
        x2 = self.smallest_elements(x) # short
        return [self.symbols[sector][i] for i in x1], [self.symbols[sector][i] for i in x2]
```

```
class LinkMomentumMulti(QCAlgorithm):

    def Initialize(self):
        self.SetStartDate(2011, 1, 1) # Start of Backtesting WIndow
        self.SetEndDate(2021, 1, 1) # End of Backtesting WIndow
        self.SetCash(100000) # Initial Cash to Use
        self.AddEquity("SPY", resolution=Resolution.Daily) # Used for Benchmark
        self.SetBenchmark("SPY") # Benchmark (Measure Performance Against)
        self.corr_period = 240 # window to determine correlations
        self.maketrade = False # Make trades when True.
        self.threshold = 0 # Correlation Threshold

    # Build Correlation Matrix
    def build_matrix(self):
        # Calculate Correlation Matrix for each Sector
        for sector in self.sector_codes:
            symbols = self.symbols[sector]
            history = self.History(symbols, self.corr_period, Resolution.Daily)
            history['log_ret'] = np.log(1+history.opena.pct_change())
            a = spearmanr(history['log_ret'].unstack(level=0).dropna())[0]
            if type(a) is  float or len(a[0]) != len(a.T[0]):
                self.Log("FOUND")
                return
            num_stocks = len(symbols) # Number of Stocks
            # Correlation Matrix
             self.adj[sector] = np.array([[0 if j==i or a[i][j] < self.threshold else a[i][j]
                     for j in range(num_stocks)] for i in range(num_stocks)])
             # Momentum Vector
             self.v[sector] = np.array([self.data[symbols[j]].macd.Current.Value for j in range(num_stocks)])




    # Compute Centrality - returns symbol corresponding to largest valued index
    def centrality(self,sector):

         C = self.adj[sector] # Correlation Matrix for Sector
         m = self.v[sector] # Momentum Vector for Sector
         num_stocks = len(C[0]) # Length of Centrality Vector

         # Store the original Correlation Matrix
         C_0 = C.copy()
         alpha = 0.85/np.linalg.norm(C, ord=np.inf)

         # First Term in multi-step
         # x - Centrality Vector
         x = C@m

         # add the additional terms
         for i in range(self.num_terms-1):
             C = alpha*C@C_0 # New C matrix (the ith power of C)
             x += C@m

        x1 = self.largest_elements(x) # long
        x2 = self.smallest_elements(x) # short
        return [self.symbols[sector][i] for i in x1], [self.symbols[sector][i] for i in x2]
```

10.2 Index Link Momentum

```
class IndexLink(QCAlgorithm):

   def Initialize(self):
       self.SetStartDate(2011, 1, 1) # Start of Backtesting WIndow
       self.SetEndDate(2021, 1, 1) # End of Backtesting WIndow
       self.SetCash(100000) # Initial Cash to Use
       self.AddEquity("SPY", resolution=Resolution.Daily) # Used for Benchmark
       self.SetBenchmark("SPY") # Benchmark (Measure Performance Against)
       self.corr_period = 240 # window to determine correlations
       self.rank=1 # rank to analyze
       self.percentage=10 # percentage of each rank grouping
       self.maketrade = False # Make trades when True.
       self.threshold = 0 # Correlation Threshold

   # Build Correlation Matrix
   def build_matrix(self):
       # Calculate Correlation Matrix for each Sector
       for sector in self.sector_codes:
           symbols = self.symbols[sector]
           history = self.History(symbols, self.corr_period, Resolution.Daily)
           history['log_ret'] = np.log(1+history.opena.pct_change())
           a = spearmanr(history['log_ret'].unstack(level=0).dropna())[0]
           if type(a) is  float or len(a[0]) != len(a.T[0]):
               self.Log("FOUND")
               return
           num_stocks = len(symbols) # Number of Stocks
           # Correlation Matrix
          self.adj[sector] = np.array([[0 if j==i or a[i][j] < self.threshold else
          a[i][j]*(1+self.data[symbols[j]].macd.Current.Value) for j in range(num_stocks)]
          for i in range(num_stocks)])



   # Compute Centrality - returns symbol corresponding to largest valued index
   def centrality(self,sector):
        A = self.adj[sector]
        num_stocks = len(A[0])
        d = [sum([A[i][j] for j in range(num_stocks)]) for i in range(num_stocks)]
        x = [sum([(d[i]-d[j])/(d[i]+d[j]) for j in range(num_stocks)])/d[i] for i in range(num_stocks)]

       x1 = self.largest_elements(x) # long
       x2 = self.smallest_elements(x) # short
       return [self.symbols[sector][i] for i in x1], [self.symbols[sector][i] for i in x2]
```

10.3 Correlative Trading

```
class CorrelationPair(QCAlgorithm):

   def Initialize(self):
       self.SetStartDate(2011, 1, 1) # Start of Backtesting WIndow
       self.SetEndDate(2021, 1, 1) # End of Backtesting WIndow
       self.SetCash(100000) # Initial Cash to Use
       self.AddEquity("SPY", resolution=Resolution.Daily) # Used for Benchmark
       self.SetBenchmark("SPY") # Benchmark (Measure Performance Against)
       self.corr_period = 240 # window to determine correlations
       self.rank=1 # rank to analyze
       self.percentage=10 # percentage of each rank grouping
       self.maketrade = False # Make trades when True.
       self.threshold = 0 # Correlation Threshold

   # Build Correlation Matrix
   def build_matrix(self):
       # Calculate Correlation Matrix for each Sector
       for sector in self.sector_codes:
           symbols = self.symbols[sector]
           history = self.History(symbols, self.corr_period, Resolution.Daily)
           history['log_ret'] = np.log(1+history.open.pct_change())
           a = spearmanr(history['log_ret'].unstack(level=0).dropna())[0]
           if type(a) is  float or len(a[0]) != len(a.T[0]):
               self.Log("FOUND")
               return
           num_stocks = len(symbols) # Number of Stocks
           # Correlation Matrix
           self.adj[sector] = np.array([[0 if j==i or a[i][j] < self.threshold else a[i][j]
                   for j in range(num_stocks)] for i in range(num_stocks)])

   # Compute Centrality - returns symbol corresponding to largest valued index
   def centrality(self,sector):
       adj_matrix = self.adj[sector]
       n = adj_matrix.shape[0]
       dist_matrix = np.where(adj_matrix == 0, np.inf, adj_matrix)
       np.fill_diagonal(dist_matrix, 0) # Set diagonal elements to 0

       for k in range(n):
          for i in range(n):
              for j in range(n):
                  if dist_matrix[i, j] > dist_matrix[i, k] + dist_matrix[k, j]:
                      dist_matrix[i, j] = min(dist_matrix[i, j], dist_matrix[i, k] + dist_matrix[k, j])

        total_distances = np.sum(dist_matrix, axis=1)
        x = (n - 1) / total_distances
        x = [x[i] / self.data[self.symbols[sector][i]].macd.Current.Value for i in range(n)]

       x1 = self.largest_elements(x) # long
       x2 = self.smallest_elements(x) # short
       return [self.symbols[sector][i] for i in x1], [self.symbols[sector][i] for i in x2]
```

10.4 General Methods

```python
def Initialize(self):

        # Set Universe
        self.SetUniverseSelection(QC500UniverseSelectionModel()) # Grouping Similar to SP500
        self.UniverseSettings.Resolution = Resolution.Daily # Change Stock Universe Daily

        # Build Correlation Matrix at Month Start
        self.Schedule.On(self.DateRules.MonthStart("SPY"),
        self.TimeRules.At(0, 0),
        self.build_matrix)

        # Signals time to trade at Month Start.
        self.Schedule.On(self.DateRules.MonthStart("SPY"),
        self.TimeRules.At(0, 0),
        self.timetrade)

        # Store universe information
        self.data = {}
        self.symbols = {}
        self.adj={}
        self.v = {}
        self.sector_symbols = {}

        # Sectors to Use
        self.sector_codes = [MorningstarSectorCode.Energy, MorningstarSectorCode.FinancialServices,
                          MorningstarSectorCode.Healthcare, MorningstarSectorCode.Industrials,
                          MorningstarSectorCode.BasicMaterials, MorningstarSectorCode.Technology]
        # Number of Sectors
        self.num_sectors = len(self.sector_codes)

        # Initialize Dictionaries
        for s in self.sector_codes:
        self.symbols[s] = []
        self.adj[s] = []
        self.v[s] = []

        # Warm Up Algorithm
        self.SetWarmUp(self.corr_period)

# Time to make a trade
def timetrade(self):
        self.maketrade = True

# Makes sure any added assets are within the sector
def OnSecuritiesChanged(self, changes):

        for security in changes.RemovedSecurities:
                if security.Symbol in list(self.data.keys()):
                        self.Liquidate(security.Symbol)
                        for sector in self.sector_codes:
                                if security.Symbol in self.symbols[sector]:
                                        s = self.symbols[sector]
                                        s.remove(security.Symbol)
```

10.5 Making Trade

```
# Method called at start open each Business Day
def OnData(self, data):

        # check if still warming up, trade signal, correlation matrix built
        if self.IsWarmingUp: return
        if self.maketrade==True:
        if len(self.adj) == 0: self.build_matrix()

        # loop through each sector and adds  stocks to buy/sell
        b = [] # Buy Symbols
        s = [] # Sells Symbols


        for sector in self.sector_codes:
                if len(self.adj[sector])==0: continue
                if len(self.adj[sector]) != len(self.symbols[sector]): continue
                b1, s1 = self.pr_centrality(sector)
                b.append(b1)
                s.append(s1)

        self.Liquidate()

        # Long
        num_targets = len(b)
        for i in range(num_targets):
        self.SetHoldings(b[i], .98/num_targets/2)

        # Short
        num_targets=len(s)
        for i in range(num_targets):
        self.SetHoldings(s[i], -.98/num_targets/2)

        # delay making next trade
        self.maketrade=False
```

10.6 STANDARD MOMENTUM PORTFOLIO

```
#region imports
from AlgorithmImports import *
from Selection.QC500UniverseSelectionModel import QC500UniverseSelectionModel
import numpy as np
from scipy.stats import spearmanr
#endregion


# Could try Close prices instead
# Try Adding Matrix ve^T with positive and negative indices
# Parse for only positive signals/momentum values
class CasualYellowGreenHyena(QCAlgorithm):

# Initialize
def Initialize(self):
        self.SetStartDate(2011, 1, 1)
        self.SetEndDate(2021, 1, 1)
        self.SetCash(100000)
        self.AddEquity("XLE", resolution=Resolution.Daily) # Used for Benchmark
        self.AddEquity("SPY", resolution=Resolution.Daily) # Used for Benchmark
        self.SetBenchmark("SPY") # Benchmark (Measure Performance Against)

        # Parameters
        self.k=2 # number of stocks to trade (long and buy is 2k total)
        self.rank=1 # rank to analyze
        self.rank_percentage=10
        # Set Universe
        self.SetUniverseSelection(QC500UniverseSelectionModel())
        self.UniverseSettings.Resolution = Resolution.Daily

        # Store universe information (used to determine assets by sector)
        self.data = {}
        self.momp = {}
        self.lookback = 90 # Momentum indicator lookback period
        self.symbols = []

        self.rebalance = False

        self.sector_codes = [MorningstarSectorCode.BasicMaterials, MorningstarSectorCode.Energy,
        MorningstarSectorCode.FinancialServices, MorningstarSectorCode.Healthcare,
        MorningstarSectorCode.Industrials, MorningstarSectorCode.Technology]

        self.sector_symbols = {}

        # self.sector_codes = [MorningstarSectorCode.Energy]

        self.Schedule.On(self.DateRules.MonthStart("SPY"),
        self.TimeRules.AfterMarketOpen("SPY"),
        self.Rebalance)

        # Warm Up Algorithm
        self.SetWarmUp(250)
        def Rebalance(self):
        self.rebalance=True
```

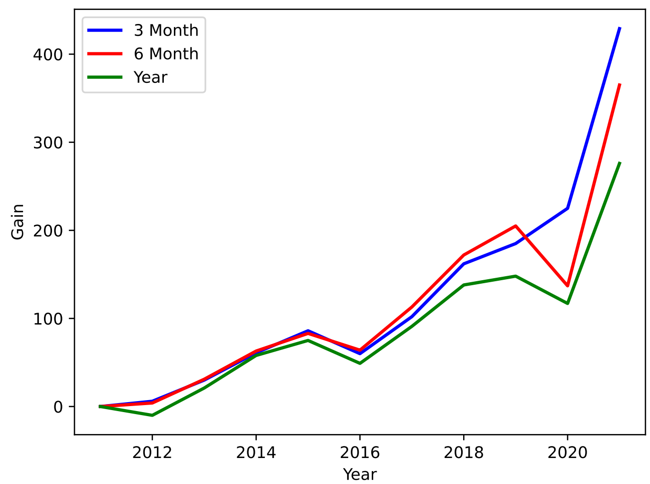

Figure 11.1: Performance for standard momentum portfolio with three month, six month, and one year look-back periods.

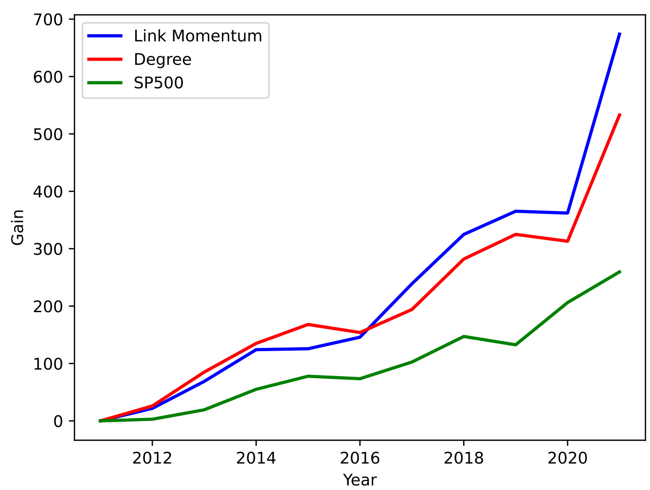

Figure 11.2: Measures performance of rank one Multi-Step Link Momentum vs. Degree Link Momentum.

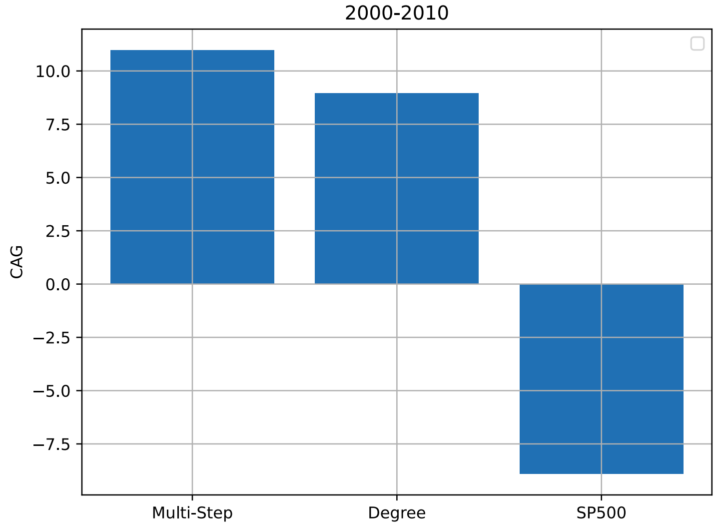

Figure 11.3: Measures performance of rank one Multi-Step Link Momentum and Degree Link Momentum vs. S&P 500 over the time period 2000-2010. Multi-Step outperforms the benchmark by > 15%.

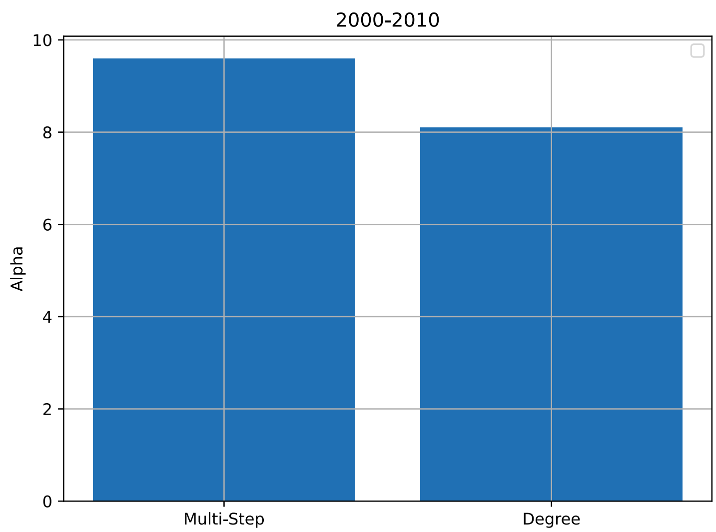

Figure 11.4: Measures performance of rank one Multi-Step Link Momentum and Degree Link Momentum vs. S&P 500 over the time period 2000-2010. Multi-Step outperforms Degree with alpha > 9.

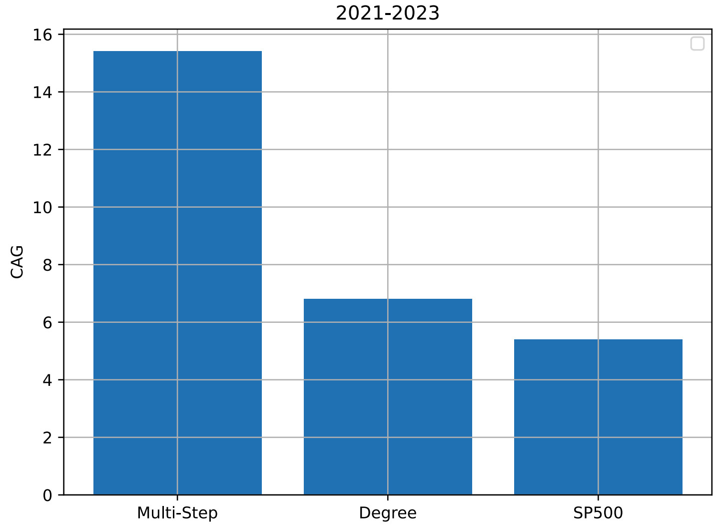

Figure 11.5: Measures performance of rank one Multi-Step Link Momentum and Degree Link Momentum vs. S&P 500 over the time period 2021-2023. Multi-Step outperforms the benchmark by > 10%.

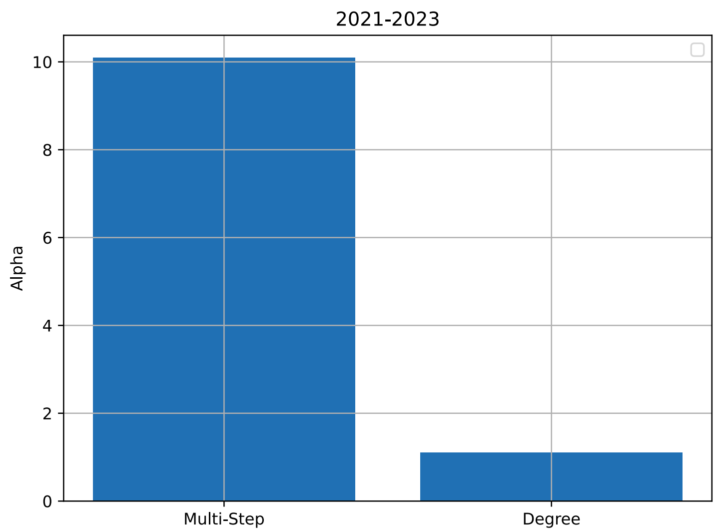

Figure 11.6: Measures performance of rank one Multi-Step Link Momentum and Degree Link Momentum vs. S&P 500 over the time period 2021-2023. Multi-Step outperforms the Degree with alpha > 10.

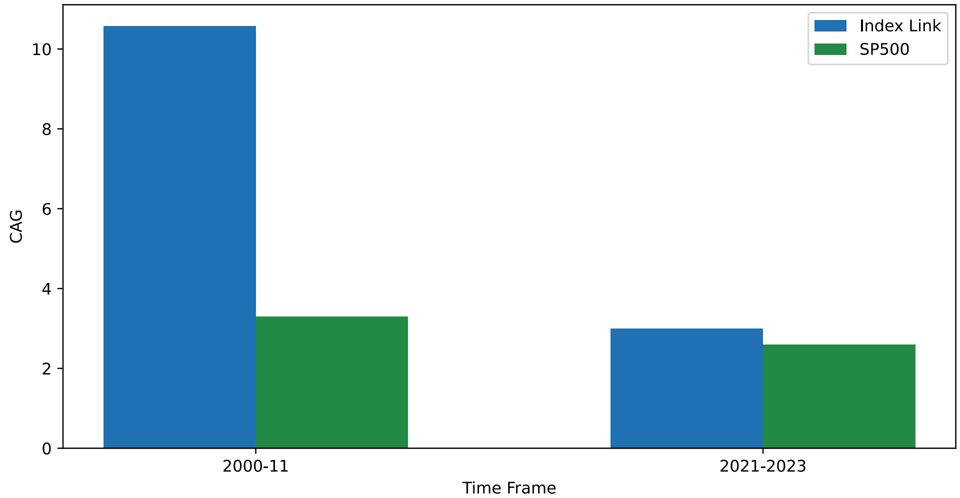

Figure 11.7: Measures Performance of rank one Multi-Step Link Momentum and Degree Link Momentum vs. S& P 500 over the time period 2021-2023. Multi-Step outperforms the Degree with alpha > 10.

Rank one securities 2011-2021 - Annual Gain				
Year	Link	Index	Correlative	S&P 500
2011	**14%**	**6.3%**	**1%**	0%
2012	**45%**	**61%**	**23%**	13.41%
2013	26%	**35%**	29%	29.6%
2014	-3.7%	1.3%	**14%**	11.39%
2015	**14%**	-9%	-14.1%	-.7%
2016	**36%**	**40%**	**40.2%**	9.5%
2017	**44%**	**31%**	**34.2%**	19.4%
2018	**3.4%**	**1.2%**	**2.6%**	-6.2%
2019	**29%**	23%	24.1%	28.9%
2020	**34%**	**27%**	15.3%	16.3%

Table 11.1: Compares compounding annual gain of rank one securities valued by Link Momentum, Index Link Momentum, and Correlative Trading vs. benchmark gain for each year increment. Link Momentum shows the strongest results while all three allocation strategies achieve greater gain than the benchmark.

Rank ten securities 2011-2021 - Annual Gain				
Year	Link	Index	Correlative	S&P 500
2011	3.0%	**-1.6%**	2.3%	0%
2012	20.4%	14.6%	**12.3%**	13.41%
2013	**27.0%**	**23.1%**	**17.2%**	29.6%
2014	13.1%	**9.8%**	**.07%**	11.39%
2015	**-11.6%**	**-6.3%**	**-14.5%**	-.7%
2016	35.2%	27.7%	28%	9.5%
2017	21.5%	24.8%	**16.7%**	19.4%
2018	-4.8%	**-6.9%**	11.6%	-6.2%
2019	**16.9%**	**13.8%**	**16.3%**	28.9%
2020	48.1%	**10.7%**	39%	16.3%

Table 11.2: Compares compounding annual gain of rank ten securities valued by Link Momentum, Index Link Momentum, and Correlative Trading vs. benchmark gain for each year increment. Index Link Momentum consistently chooses indices to short that under-perform relative to benchmark, while Correlative seems to slightly under-perform. However, Link Momentum consistently chooses securities that gain more than benchmark.

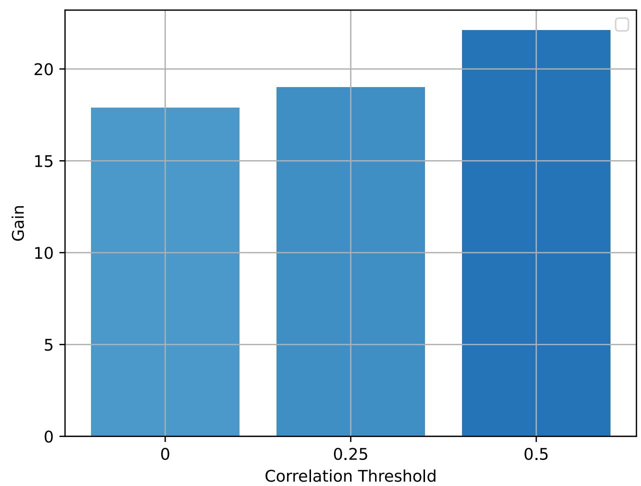

Figure 11.8: Measures performance of rank one Multi-Step Link for correlation thresholds 0, 0.25, and 0.5 over the time period 2021-2023. The correlation threshold 0.5 achieves the largest compounding annual gain at 22.1% yearly.

Link Momentum by rank 2011-2021			
Rank	CAG	α	β
1	16.5%	1.07	1.4
2	14.0 %	1.11	-.5
3	14.9 %	1.07	.5
8	9.7 %	1.08	-3.3
9	14.8 %	1.06	.4
10	14.3 %	1.05	.2
[1]-[10]	2.2	.02	1.2

Table 11.3: Measures the performance of Link Momentum by rank. Rank one outperforms rank ten by 2.2% annually with a 1.2 difference in alpha. In general higher rank groupings outperformed lower rank groupings however it is notable that the rank two grouping produced the second worst performance of the test ranks.

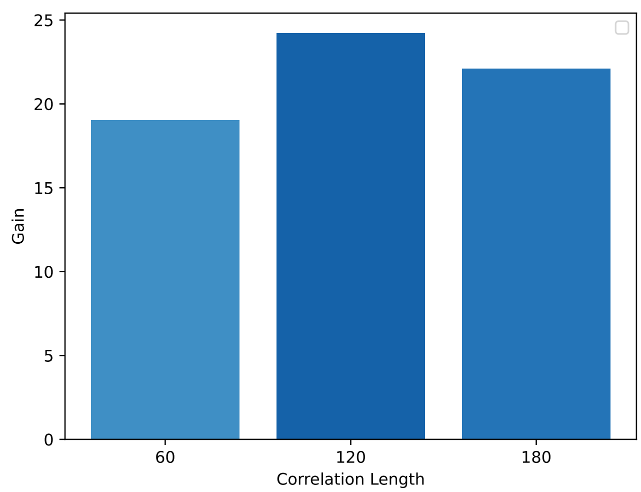

Figure 11.9: Measures performance of rank one Multi-Step Link for correlation periods of 60, 120, and 240 days over the time period 2021-2023. The correlation period of 120 days achieves the largest compounding annual gain at 24.2% yearly.

Chapter 12. References

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