Extending the Bandwidth of Intensity-based Sound Power Estimates

Michael C. Mortenson

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EXTENDING THE BANDWIDTH OF INTENSITY-BASED
SOUND POWER ESTIMATES

by
Michael C. Mortenson

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Mechanical Engineering Department
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Advisor: Dr. Tracianne B. Neilsen
Honors Coordinator: Dr. Brian D. Jensen
ABSTRACT

EXTENDING THE BANDWIDTH OF INTENSITY-BASED SOUND POWER ESTIMATES

Michael C. Mortenson
Mechanical Engineering Department
Bachelor of Science

Sound power is often measured using the intensity-based engineering standard ANSI S12.12-1992:R2017. Traditional methods for intensity-based sound power estimation are limited in bandwidth at low frequencies by phase mismatch between microphones and at high frequencies by microphone spacing—with errors occurring well below the spatial Nyquist frequency. The Phase and Amplitude Gradient Estimation (PAGE) method has been used to extend the bandwidth of intensity calculations [1]. This thesis examines the efficacy of the PAGE method in overcoming bandwidth limitations in estimating sound power. Specifically, the sound fields from three sources—a blender, vacuum cleaner, and reference sound source—were measured according to ANSI S12.12-1992:R2017. The sound power was computed for each source using both the traditional and PAGE methods. The resulting intensity-based sound power estimates are compared against sound power measurements obtained according to the scientific-grade ISO 3741:2010 standard. The PAGE method increases the bandwidth over which reliable estimates are achievable for intensity-based sound power, even exceeding the spatial Nyquist frequency when phase unwrapping is successful. Thus, using existing equipment, industry professionals can extend the bandwidth of sound power estimates with the PAGE method.
Nobody works in a vacuum (even if one-third of your thesis revolves around one), and many people deserve credit and appreciation for their hand in this work. First and foremost, my thanks goes to Dr. Traci Neilsen, whose unparalleled patience, support, and mentoring carried this work from start to finish. Also, to Dr. Kent Gee, for welcoming me as a mechanical engineering student onto the homecourt of physics research. Thanks also goes to Dr. Scott Sommerfeldt for showing me the ropes of the technical—and sometimes mathemagical—theory of acoustics. To my fellow students—Joseph Lawrence, Suzanna Gilbert, Caleb Goates, and Gabe Fronk—who shared their knowledgeable minds and helpful hands, I send a hearty thank you.

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1. INTRODUCTION

1.1 Motivation

Acoustical engineers care about creating products that are both functional and safe. Products that are too loud pose a hearing health risk. According to a 2011–12 study by the Center for Disease Control and Prevention, between 10 and 40 million adults have symptoms of hearing loss in one or both ears from exposure to loud noise [2]. Additionally, the National Institute of Health has estimated that as many as 17% of teens (ages 12 to 19) have features of hearing loss in one or both ears [3]. To combat noise-induced hearing loss, acoustical engineers are tasked with the challenge of finding effective and economical ways to characterize sound.

Noise-induced hearing loss is caused by exposure to sound pressure levels that are too high. Since sound pressure levels decrease the farther away from a source the sound is received, it can be difficult to determine whether a product is dangerously loud. Environmental factors like surface reflection or absorption can influence the spatial distribution of sound, called the sound pressure field. Additionally, differing product sizes and shapes make it difficult to compare products. For example, which is more likely to expose the user to higher sound levels—a blender or a vacuum cleaner? Consumers and companies want to know, but such questions cannot be answered by sound pressure measurements alone.

To solve this problem, acoustical engineers use sound power rather than sound pressure when characterizing products and assessing risk to hearing health. Sound power is a measure of the sound energy radiating from a source per unit time [4]. In fact, the sound energy radiating from a source is what creates a sound pressure field as sound energy interacts with the environment. Because of the law of conservation of energy, the total sound power radiating from a source in steady state through a closed control surface remains constant no matter the size of
the surface [5]. This affords sound power a distinct advantage over sound pressure in characterizing sources and also allows easy comparison of sources of different shapes and sizes.

Methods for determining sound power fall into two main categories: diffuse-field methods and free-field methods [6]. Diffuse-field methods are considered indirect methods, meaning that the sound power of the source in question is found by comparing it to a source with a known sound power. Diffuse-field methods for estimating sound power, such as the scientific-grade method described in ISO 3741:2010 [7], require a certified reverberation chamber and calibrated sound source. While accurate to within 3 dB re 10 pW, most acoustical engineers in industry do not have access to expensive research-grade reverberation chambers or calibrated sound sources [8].

The free-field method takes a different approach, using acoustic intensity to find sound power. Acoustic vector intensity ($\vec{I}$) is the energy flux across a closed control surface containing a sound source. Multiplying acoustic intensity by the surface area of the control surface—usually a hemisphere or rectangular prism—yields sound power ($\Pi$). Traditionally, this method is most popular among acoustical engineers for obtaining sound power because it can be used in a normal room using the floor as a reflective surface. Engineering-grade standard ANSI S12.12-1992:R2017 [9] describes how to take such intensity-based sound power measurements. This thesis aims to improve on the use of intensity-based sound power.

Acoustic intensity is an energetic quantity. Energetic quantities can be calculated from two variables: an effort variable and a flow variable. In the acoustic domain, the effort variable is acoustic pressure ($p$) and the flow variable is acoustic particle velocity ($\vec{u}$). The accuracy of acoustic intensity measurements is determined by the accuracy of these pressure and particle velocity values.
Measuring the acoustic pressure is as simple as setting up microphones, but particle velocity is not so simple to obtain. While there are devices called acoustic vector sensors, designed to measure pressure and particle velocity directly, such devices are expensive to acquire and maintain and inaccurate under extreme temperature and airflow conditions. Therefore, acoustical engineers traditionally opt to estimate particle velocity by finite-differencing the pressure across multiple microphones. This method known as the traditional (TRAD) or p-p method has been used to obtain sound power with the free-field method [5]. Standards such as the engineering-grade standard ANSI S12.12-1992:R2017, utilize the TRAD method to measure acoustic intensity and, thereby, sound power in regular rooms with a single reflecting surface [9].

While the free-field method is preferred by acoustical engineers because it only requires microphones and a regular room, the method has bandwidth limitations. Significant bias errors occur in low frequencies where phase mismatch between microphones is on the same order as the phase of the pressure wave. At high frequencies, bias errors occur due to microphone spacing, when the spacing between microphones approaches half a wavelength. The frequency at which microphone spacing is half a wavelength is called the spatial Nyquist frequency. Because of these frequency limitations in estimating the acoustic particle velocity, acoustic intensity and, by extension, sound power calculations are limited in bandwidth.

For many years, acoustical engineers have dealt with the bandwidth limitations of the traditional method, but an alternative method for estimating particle velocity from multiple microphones has been developed at Brigham Young University. This alternative method is called the Phase and Amplitude Gradient Estimation (PAGE) method [10]. Therefore, the goal of this thesis is to set forth a validation for using the PAGE method for estimating acoustic particle velocity to extend the usable bandwidth for intensity-based sound power estimates.
1.2 Background

1.2.1 Intensity-based Sound Power

Acoustic sound power is a common energy-based quantity. It is used to characterize the energy a source contributes to its environment. Just as the temperature field in a room can be predicted by knowing the power of a heat source, the pressure field in a room can be predicted by understanding the sound power of an acoustic source. Thus, sound power is a useful metric for understanding the potential impact of a product on its environment.

By definition, sound power is the amount of acoustic energy that crosses a surface per unit time—the net energy flux through a control surface enclosing the source. Mathematically, sound power can be written as a surface integral using Gauss’ Theorem

\[ \Pi = \oint \vec{I} \cdot d\vec{S} \]

where \( \Pi \) is sound power, \( \vec{I} \) is the active acoustic intensity, and \( d\vec{S} \) is the unit vector normal to the control surface [11]. In practice, the sound power is approximated with a Riemann sum of the product of the intensity vector perpendicular to the surface and the area at \( n \) discrete points. This approximation is written

\[ \Pi \approx \sum_{i=1}^{n} I_i A_i \]

where \( I_i \) is the \( i \)th acoustic intensity vector perpendicular to the \( i \)th area, \( A_i \). Thus, the sound power of a source can be directly obtained over a control surface once acoustic intensity has been determined.

Acoustic intensity is a vector quantity that describes both the direction and magnitude of acoustic energy. This energy is distributed between potential and kinetic energy. In other words, some acoustic energy is stored in the motion of the particles of a medium and some acoustic energy is transmitted through the medium. The potential energy portion of the intensity is called
reactive intensity while the kinetic energy portion of the intensity is called active intensity. Since sound power is a measure of the energy flux across a control surface, only the active intensity is needed to determine sound power.

Both active and reactive intensity are calculated from pressure and particle velocity values in the frequency domain. Thus, when calculating intensity, the single-sided Fourier transform of the pressure and particle velocity is used to map values from the time domain to the frequency domain. This Fourier transform has two important impacts on the quantities. First, it allows frequency-by-frequency analysis of the pressure and particle velocity signals, and second, it represents the signals as complex values. In this work, complex frequency-domain signals will be indicated in bold.

Active acoustic intensity is formulated from the single-sided complex spectra of the pressure and particle velocity [12]. Symbolically, active acoustic intensity can be expressed

\[
\tilde{I}_a = \frac{1}{2} Re\{P^* \bar{U}\}
\]

where \( \tilde{I} \) is the acoustic active intensity, \( P^* \) is the complex conjugate of the Fourier transformed pressure, and \( \bar{U} \) is the Fourier transformed particle velocity. Thus, intensity-based sound power can be determined from the active acoustic intensity so long as the pressure and particle velocity are readily available.

### 1.2.2 Particle Velocity Estimates

As noted previously, the pressure can be directly obtained from microphones, but particle velocity is often obtained indirectly from the pressure. The expression which relates acoustic particle velocity and pressure is call Euler’s equation. It is written

\[
\bar{U} = \frac{j}{\omega \rho_0} \nabla P
\]
where $\mathbf{\bar{U}}$ is the acoustic particle velocity, $j$ is the imaginary number ($j = \sqrt{-1}$), $\omega$ is the angular frequency of the acoustic wave, $\rho_0$ is the ambient air density, and $P$ is the acoustic pressure. In short, Euler’s equation allows the user to estimate the particle velocity from the gradient of pressures. Thus, if an acoustical engineer is able to determine the gradient of the pressure, it is only a matter of math to calculate the particle velocity, and from there, the acoustic intensity and sound power.

Now this is where an acoustical engineer has a choice: how should the gradient of the pressure be calculated? It is infeasible to obtain a true continuous gradient of the pressure field, so instead the pressure gradient is approximated by sampling the pressure at discrete points. The pressure at these points then becomes the input for a Taylor approximation of the pressure gradient [13]. By using a multi-microphone probe, several points can be sampled, each additional microphone increasing the order of the approximation. For many practical applications, and for the purposes of this thesis, a simple two-microphone probe, as shown in Figure 1, is sufficient.

The one-dimensional probe consists of two microphones a distance $d$ apart. The midpoint between microphones is located a distance $r$ from the sound source. Microphone 1 is nearest the sound source, while microphone 2 is farthest from the sound source. This probe configuration allows for the estimation of the pressure and the particle velocity at the midpoint between the microphones along a line connecting the microphone heads. The microphone spacing determines the spatial Nyquist frequency, the frequency at which $d$ is equal to half a wavelength.
For a two-microphone probe, the traditional way of estimating the gradient of the pressure works on the complex frequency spectra. First, pressure measurements $p_i(t)$ are taken at each microphone location, $i$. Then the time-domain pressure record is recast into the frequency domain with a Fourier transform, $P_i(f)$. The gradient of the pressure is then found by finite differencing the real and imaginary parts of the complex pressure over the microphone spacing $d$, as can be seen in Equation 5:

$$\nabla P = \frac{P_1 - P_2}{d}$$

Alternatively, the Phase and Amplitude Gradient Estimation (PAGE) method [10] approximates the gradient of the pressure differently by using the notation developed by Mann et al. [15] and Mann and Tichy [16–17]. Instead of a complex representation of the pressure, $P_i(f) = \text{Re}(P_i(f)) - j \text{Im}(P_i(f))$, the PAGE method uses a phasor representation,

$$P = Pe^{-j\phi}$$
where $P$ is the amplitude of the pressure, $e$ is Euler’s number, $j$ is the imaginary number, and $\phi$ is the phase [18]. For a two-microphone probe, the pressure at the center of the microphones is estimated by averaging the pressure amplitudes from each microphone such that

$$
(7) \quad P_{\text{PAGE}}^P = P_{\text{ave}} e^{-j\phi_{\text{center}}}
$$

where $P_{\text{ave}} = \frac{P_1 + P_2}{2}$ and $\phi_{\text{center}}$ is the center phase estimate. The center phase estimate is a relative phase and, as such, can be replaced with a zero in intensity calculations [15]. This leads to a new formulation of Euler’s equation

$$
(8) \quad \mathbf{U}_{\text{PAGE}}^P = \left( \frac{e^{-j\phi_{\text{center}}}}{\rho_0 \omega} \right) (P_{\text{ave}} \nabla \phi + j\nabla P)
$$

where

$$
(9) \quad \nabla P = \frac{P_1 - P_2}{d}
$$

and

$$
(10) \quad \nabla \phi = \frac{\phi_2 - \phi_1}{d}.
$$

Thus, the active acoustic intensity using the PAGE method can be written

$$
(11) \quad I_{\text{ave}}^P = \frac{P_{\text{ave}}^2 \nabla \phi}{2 \rho_0 \omega}
$$

This subtle shift in representation—from complex notation to phasor notation—has important implications for bias error in computing the intensity.

### 1.2.3 Bias Error in TRAD and PAGE Processing

One source of bias error in sound intensity measurement is phase mismatch between microphones. Measurement-grade microphones, no matter how well manufactured, have inherent differences in their individual phase responses. Phase mismatch between microphones becomes a significant problem when the mismatch is on the order of the phase of the pressure wave [19]. This occurs at low frequencies and marks the lower frequency limit for microphone probes [11].
One typical solution to this problem is to use phase-matched microphones. Phase-matched microphones have a phase mismatch with a small enough tolerance that the lower frequency limit is sufficient for a given application.

When phase-matching is insufficient or infeasible, increasing the separation distance between microphones can also drop the lower frequency limit. Since the phase of low frequency pressure waves varies slowly spatially, increasing the distance between microphones increases the magnitude of the measured phase above the range where it might be obfuscated by phase mismatch between microphones.

Microphone spacing is another source of bias error. Theoretically, the maximum frequency that can be sampled simultaneously with two microphones is twice as long as the spacing between the microphones. In other words, the upper frequency limit of a microphone probe is set at the frequency where the microphone spacing is equal to half a wavelength [20]. This frequency is called the spatial Nyquist Frequency.

In truth, the traditional (TRAD) method for estimating sound intensity incurs bias errors due to spatial sampling well below the spatial Nyquist frequency. The work of Whiting et al. in [14] shows that bias errors of 5% and more occur at around 18% of the spatial Nyquist frequency for TRAD active intensity estimates. The typical solution to this problem has been to decrease the separation distance between microphones. A spacing of 12 mm, which is fairly typical, can give accurate intensity results up to 5 kHz [5].

Thus, when using the TRAD method for estimating acoustic intensity, acoustical engineers are caught between increasing microphone spacing to accommodate low frequencies and decreasing microphone spacing to accommodate high frequencies. However, the PAGE method presents an alternative way to achieve accurate measurements over a wider frequency
range. The PAGES method offers the capability to obtain accurate acoustic intensity estimates up to the spatial Nyquist limit, and, for broadband sources, extend accurate estimates well beyond the spatial Nyquist limit with phase unwrapping algorithms.

The theoretical foundation for comparing bias errors between TRAD and PAGES for measuring intensity levels with a two-microphone probe can be found in the work of Whiting et al. [14]. The plot in Figure 2 shows the bias error for measuring the sound intensity level of an ideal plane wave as a function of $kd$, where $k$ is the wave number and $d$ is the microphone spacing. For a two-microphone, the spatial Nyquist occurs when $kd = \pi$. In Fig. 2 the spatial Nyquist limit is shown on the plot as a vertical black line where $kd = \pi$.

![Figure 2. Bias error in a.) active intensity levels (upper) and b.) direction of the intensity vector (lower) using a two-microphone probe, comparing TRAD, PAGES, and PAGES with phase unwrap processing.](image)

The errors in active intensity levels using the traditional method processing, shown in black with circle markers, behave according to expected limitations. The error in the active intensity level ($L_{e,1}$) begins to fall off well before the spatial Nyquist (see Fig. 2a.) Figure 2b
shows the bias error in the direction of the intensity vector \((\phi_{e_I})\). At frequencies above the spatial Nyquist, the signal becomes spatially aliased, leading to untrustworthy results for both acoustic intensity level and intensity vector direction.

The PAGE method, marked in Fig. 2 with the red dashed line and triangles, does not suffer from the same bias errors as the TRAD method. Notice that up to the Nyquist limit, the PAGE method has zero bias error in intensity level and the direction of the intensity vector. Because of the way it is formulated, the PAGE method’s transfer function can only accept arguments between \(-\pi\) and \(+\pi\) for the gradient of the phase [18]. As a result, values outside of this range get wrapped. For example, \(1.1\pi\) is treated like \(-0.9\pi\). This phase wrapping behavior occurs when \(kd > \pi\), and interferes with the accuracy of the PAGE method above the Nyquist limit.

For broadband sound, there is a solution to the PAGE method phase wrapping problem, a solution that has already been successfully applied to intensity measurement of static rockets [21–22], plane wave intensity measurements [23], military jet noise [24–25], noise contamination testing [26], narrowband intensity with low-level broadband noise [27], and beamforming arrays [28]. If the source contains broadband sound, the phase can be unwrapped by using an unwrapping algorithm, such as MATLAB’s unwrap function. An unwrap function effectively undoes the phase wrapping by adding \(2\pi\) to the argument of the PAGE transfer function every time it exceeds its \(\pm\Pi\) limits [22]. If used with unwrapping, the PAGE method can keep error to zero beyond the spatial Nyquist limit. This is show in Fig. 2 by the blue line with diamond markers. It is this bias error-averse characteristic of PAGE processing this thesis seeks to validate in application to intensity-based sound power.
1.3 Overview

Intensity-based sound power is an important metric for determining the acoustic characteristics of products. The calculation of intensity-based sound power relies upon two main variables: acoustic pressure and acoustic particle velocity. In practice, acoustic particle velocity is estimated from the acoustic pressure. For the TRAD method, the frequency range over which particle velocity estimates are accurate is limited by microphone phase mismatch for low frequencies and by microphone spacing for high frequencies.

The newer Phase and Amplitude Gradient Estimator (PAGE) method shows promise for widening the useable bandwidth for particle velocity estimates up to the spatial Nyquist frequency. For broadband noise, phase unwrapping allows for accurate estimates above the spatial Nyquist Frequency. By extending the bandwidth of particle velocity estimates, the bandwidth of intensity-based sound power is also extended. The goal of this research is to compare and validate the PAGE method and its bandwidth-widening capabilities against the TRAD method for estimating intensity-based sound power.

To test the two processing methods, TRAD and PAGE, three sources are analyzed: a Nor278 reference sound source, a handheld vacuum cleaner, and a household blender. The intensity-based sound power is determined according to ANSI S12.12-1992:R2017, the engineering method for the determination of sound power levels of noise sources using sound intensity in a regular room. As a baseline, the sound power levels are compared against measurements obtained according to ISO 3741:2010, the scientific standard for determining sound power levels from sound pressure in a reverberation chamber. Analysis of test results shows the capability of the PAGE method to extend the bandwidth of intensity-based sound power estimates.
2. METHOD

Sound power was measured for three devices: a Nor278 Norsonic reference sound source, a handheld vacuum cleaner, and a household blender. These sound sources are shown in Fig. 3. Measurements were taken in accordance with the engineering standard for intensity-based sound power in a regular room, ANSI S12.12-1992:R2017. For comparison between processing methods, the data was processed with both the traditional (TRAD) and PAGE methods.

![Sound sources tested: a.) Nor278 Norsonic reference sound source, b.) handheld vacuum cleaner, and c.) household blender.](image)

Additionally, the two processing methods were compared against a sound power benchmark. For the first device—the reference sound source—this benchmark sound power spectrum was taken from the factory documentation. For the other two devices—the vacuum and the blender—no factory documentation was available, so benchmark sound power measurements were obtained by measuring the sound power according to the scientific standard, ISO 3741:2010, for sound power in a reverberation chamber.

The intensity-based sound power measurements were taken in room U117 of the Eyring Science Center at Brigham Young University. A one-meter cube control surface was established, the footprint of which was marked out in black tape on the floor. The measured sound source
was mounted securely in the center of the cube, 0.5 m from all faces. The experimental setup is shown in Fig. 4a.

A simple, two-microphone probe was used to measure the pressure normal to the control surface. Throughout the measurements, the microphones were arranged such that the line connecting the heads of the two microphones remained perpendicular to the control surface. The microphones used were phase-matched half-inch G.R.A.S. 46AE CCP microphones with spherical black windscreens to mitigate flow noise. The microphone probe is shown in Fig. 4b.

![Image of experimental setup]

Figure 4. Experimental setup. a.) Gantry system positioned to move microphone probe to points on the 1 m cube control surface. b.) Two-microphone probe with windshields protecting the ½-inch GRAS 46AE CCP microphones.

The microphone probe was attached to a digitally controlled gantry system. Signals from the microphones passed through a Texas Instruments PXI data acquisition card and were saved
using a LABview software program called Acoustic Field Recorder (AFR) developed by Dr. Kent Gee (BYU) and collaborators. AFR also controlled the gantry system.

Since the gantry system only moved in two physical dimensions, up-down and left-right, the sound source had to be rotated 90° between measurements to properly scan each of the four faces of cube-shaped control surface. The top face of the control surface was scanned by mounting the microphone probe on a stand with a boom arm and moving it manually. Each face of the cube was decomposed into a 7x7 grid of squares. One measurement was taken at the center of each grid square making 49 measurements per cube face or 245 points in total. Each center point was assumed to approximate the acoustic field for its corresponding grid square. Pressure measurements were taken for 15-second periods at each center point.

The pressure data was processed using MATLAB code, which can be found in the Appendix. Essentially, this code calculated the acoustic intensity at each grid location, multiplying by the area of that grid square, and summing the values across the entire control surface—four sides and the top of the cube. Thus, the sound power was estimated with a summation of the energy crossing the control surface by applying Eq. 2 as follows:

$$
(12) \quad \Pi = \oint \mathbf{I} \cdot \hat{dS} \approx \sum_{i=1}^{245} I_i A_i
$$

Here, $\Pi$ is sound power, $\mathbf{I}$ is the active acoustic intensity, $\hat{dS}$ is the unit vector normal to the surface, $I_i$ is the acoustic intensity at the $i$th position, and $A_i$ is the area of the $i$th grid square.

The intensity at each location was estimated using both TRAD and PAGE processing as described in Sec. 1.2. From this data, plots were generated with MATLAB code to aid in the analysis and evaluation of the methods.

Benchmark sound power measurements were conducted in BYU’s reverberation chamber according to scientific standard ISO 3741.2010. As per the standard, the sound source was
mounted sufficiently far from the walls to ensure a diffuse sound pressure field. Six, half-inch G.R.A.S. 46AE CCP microphones were mounted at irregular heights and locations throughout the room, sufficiently far from reflective surfaces. After turning the source on, the sound was left to reach a steady-state, before diffuse field measurements were taken. The measurements were processed with AFR. Calculations and plots were generated with MATLAB to analyze the benchmark data.

The results of the sound power calculations, processed with both TRAD and PAGE, as well as the benchmark measurements for each source are presented in the Sec. 3.

3. RESULTS

Recalling earlier work done on the bias error of the traditional and PAGE methods, it is expected that the traditional method begins to diverge from the PAGE method well below the spatial Nyquist limit. The gap between the two methods is expected to increase as frequencies increase above the spatial Nyquist, the region where the PAGE method uses phase unwrapping. The experimental results for all three devices—the Norsonic Nor278 reference sound source, the handheld vacuum cleaner, and the household blender—corroborate the bias error theory presented in Sec. 1.2.3.

The narrowband sound power spectra clearly show these differences. In Fig. 5–7, sound power spectra are shown for all three sources when computed with the traditional method (black) and the PAGE method (red). For all three cases, the TRAD estimate and the PAGE estimate match closely until, at about 70% of the spatial Nyquist Frequency, the TRAD estimate begins to drop off. Phase unwrapping is used for the PAGE estimate above the spatial Nyquist frequency. Above the spatial Nyquist frequency, the TRAD line continues to drop more rapidly.
Figure 5. Narrowband sound power spectra for the Nor278 reference sound source. Traditional (TRAD) processing shown in black and PAGE processing shown in red. The spatial Nyquist limit is represented with a vertical dashed green line. Power levels referenced to a sound power of 10 pW.

Figure 6. Narrowband sound power spectra for the household blender. Traditional (TRAD) processing shown in black and PAGE processing shown in red. The spatial Nyquist limit is represented with a vertical dashed green line. Power levels referenced to a sound power of 10 pW.
Figure 7. Narrowband sound power spectra for the handheld vacuum cleaner. Traditional (TRAD) processing shown in black and PAGE processing shown in red. The spatial Nyquist limit is represented with a vertical dashed green line. Power levels referenced to a sound power of 10 pW.

While this comparison highlights the difference between the PAGE and TRAD methods, it does not give information about the accuracy of either method. In order to evaluate the accuracy of the two methods, intensity-based sound power estimates are compared to the benchmark data for each sound source. To improve the visualization of energy distribution in the data, the narrowband data was smoothed by converting to one-third octave bands. One-third octave band sound power estimates are presented in Fig. 8–10 with benchmark sound power spectra in blue, the PAGE processed intensity-based sound power in red with circle markers, the TRAD processed intensity-based sound power in black with stars, and the spatial Nyquist limit marked with a dashed vertical green line. The results for Nor278 reference sound source are shown first.
The PAGE method matches the benchmark Nor278 sound power calibration curve well above the spatial Nyquist frequency. The calibration curve was determined by the Norsonic Calibration Laboratory using ISO 3745:2003. The PAGE method estimate stays within 2–3 dB of the calibration curve up to the spatial Nyquist frequency, while the TRAD method is already 8 dB below the curve by the time it reaches the Nyquist limit. Then, with phase unwrapping, the PAGE method continues to follow the calibration curve closely, within 3 dB, while the TRAD method underestimates the sound power by 20 dB by at 9 kHz.

The handheld vacuum cleaner shows a similar trend with some small differences. In Fig. 8, the benchmark is the one-third octave band sound power found with the scientific standard ISO 3741:2010 for diffuse-field sound power in a reverberation chamber.
As expected, the TRAD method underestimates the benchmark sound power as it approaches the spatial Nyquist limit and worsens as frequencies extend above the spatial Nyquist limit. There are several differences between the reverberation chamber benchmark and the normal room estimates for the vacuum. The difference in the 125 Hz band is due to a hum present in the room during measurements. The vacuum exhibits strong tonal behavior in the 500–1000 Hz range which complicate measurements and causes error in both the TRAD and PAGE estimates. However, the PAGE method estimate matches the reverberation chamber benchmark within 0.5 dB for the 2.5–8 kHz range while the TRAD method estimate underpredicts the level by 10 dB at 2.5 kHz and 14 dB at 8 kHz. The overall effect is that the PAGE method more closely matches the benchmark sound power than the TRAD method.

![OTO Vacuum Sound Power](image.png)

*Figure 9. One-third Octave (OTO) sound power spectra for a handheld vacuum cleaner. The reverberation chamber benchmark is shown in blue. The PAGE method estimate is shown in red with circle markers and the TRAD method estimate is shown in black with star markers. The spatial Nyquist frequency is shown in green with the vertical dashed line.*
Like the results for the vacuum, the results for the blender generally show that the PAGE method performs more accurately than the TRAD method when compared to benchmark sound power measurements taken in the reverberation chamber according to ISO 3741:2010. As with the vacuum, the low frequency difference in the 125 Hz band comes from a hum present in the measurement environment. As with the other measurements, the PAGE method estimate matches the benchmark better than the TRAD method estimate. From 250 Hz, well below the spatial Nyquist frequency, to 8 kHz, well above the spatial Nyquist frequency, the PAGE method matches the benchmark within 1 dB, while the TRAD method estimate begins dropping off in the 630 Hz band. At the spatial Nyquist limit the TRAD method underestimates the benchmark by 7 dB, and by the 8 kHz band it underestimates the benchmark by 14 dB.

Figure 10. One-third Octave (OTO) sound power spectra for a household blender. The reverberation chamber benchmark is shown in blue. The PAGE method estimate is shown in red with circle markers and the TRAD method estimate is shown in black with star markers. The spatial Nyquist limit frequency is shown in green with the vertical dashed line.
In summary, the PAGE method remains accurate within 2–3 dB up to the spatial Nyquist limit when calculating intensity-based sound power. Additionally, in cases where broadband noise is present, the phase can be unwrapped, allowing for accurate sound power estimates well above the Nyquist limit.

4. **CONCLUSION**

The work done here validates that the PAGE method can extend the usable bandwidth for multi-microphone probe, intensity-based sound power estimations. While the traditional method underestimates the sound power level as it approaches the spatial Nyquist limit, the PAGE method is accurate up to the spatial Nyquist limit. Additionally, in cases where the phase can be unwrapped, the PAGE method extends the upper frequency limit well above the spatial Nyquist limit.

This bandwidth extension is great news for acoustical engineers, who can now get better results from the equipment they have by making changes to their signal processing algorithm. For consumers, these results mean more information about the sound characteristics of products. It also means safer, better designed products.

Future work can be done to validate the capacity for the PAGE method to push the lower frequency limit for intensity-based sound power. Others have found that, when using phase-matched microphones, the low frequency limit can be extended by increasing microphone spacing [5, 11, 13]. While the scope of this thesis covers most frequencies of interest as far as audibility is concerned, there is room to extend the PAGE method technique to increase the total bandwidth of intensity-based sound power estimates.
5. REFERENCES


APPENDIX 1: SoundPower3.m

%% Sound Power 3

% DESCRIPTION:
% This code finds the sound power of a source using intensity measurements
% taken at even increments along a cube-shaped measurement surface around
% the source. It creates and saves a figure of the sound power spectrum
% comparing PAGE and TRAD. It also saves the power, intensity, and other
% data in a .mat file that can be used by "SoundPowerPlots4.m" to generate
% more figures and compare with the reverb chamber.

% VERSION:
% SoundPower1.m By Suzanna Gilbert Winter 2018
% SoundPower2.m has modifications by TBeilsen Nov 2018
% SoundPower3.m has modifications by Michael Mortenson, Aug 2019

clear;

%% Function Library

functions = 'C:\Users\Michael Mortenson\Documents\MATLAB\general-signal-processing'; %Path to functions
addpath(functions); %Path to functions
addpath('C:\Users\Michael Mortenson\Documents\MATLAB\PAGE');
addpath('C:\Users\Michael Mortenson\Documents\MATLAB\sound-power');
addpath('C:\Users\Michael Mortenson\Documents\MATLAB\matlab-plotting');
myFigureDefaults

%% Inputs

fileName = 'U117'; %%% Name of Folder in Data Folder
condition = 'U117';
source = 'Norsonic' % Capitalized (Norsonic, VacuumWindscreen, BlenderWindscreen) Used in Saving
measName = '7x7'; %%% Used in Saving / On Plot Title
totScanPts = 245; % # of scan points over entire measurement surface
tw = 15; %time window for measurements
other = 'NewPhaseUnwrap1';

L is distance between scan points in cm
if totScanPts == 54 || totScanPts == 45 || totScanPts == 9
    L = 0.33;
elseif totScanPts == 150 || totScanPts == 125 || totScanPts == 25 % 5x5 case
    L = 0.20;
elseif totScanPts == 294 || totScanPts == 245 || totScanPts == 49 % 7x7 case
    L = 1/7;
end

%% Paths

% path to bin files - pressure data (YOU WILL NEED TO CHANGE THESE BASED ON WHERE YOUR FILES ARE SAVED):
binPath = ['C:\Users\Michael Mortenson\Desktop\Acoustics Master Folder\Sound Power Research\Data\',fileName,filesep,source,'SoundPower',measName,filesep];
% where to save the resulting .mat file with calculated power
savePath = 'C:\Users\Michael Mortenson\Desktop\Acoustics Master Folder\Sound Power Research\Data\Power Calculated\Thesis\';

% where to save figures
figPath = 'C:\Users\Michael Mortenson\Desktop\Acoustics Master Folder\Sound Power Research\Figures\Thesis\';

figName = ['PAGEvsTRADPower',source,measName,condition,other]; % Name of graph to be saved
saveFile = [savePath,source,measName,condition,'PowerData',other]; % Name of .mat file to be saved

%% Mathematical Inputs

d = 4*.0254; % Distance between microphones
fs = 51200; % Sampling frequency
fs = 50000; % Sampling frequency
rho = 1.21; % m/s
rho = 1.21; % kg/m^3
Iref = 1e-12; % Reference intensity
ns = 2^13; % Samples per block
df = fs/ns; % # narrow-band frequency resolution

probe_config = [0,0,d/2;0,0,-d/2];
w = hann(ns); %time window
W = mean(w.*conj(w)); % ^^^

%% Initialize matrices:
IatotPAGE = zeros(1,ns/2); % ns/2 because using single-sided spectrum, "total time averaged intensity calculated with PAGE or TRAD"
IatotTRAD = zeros(1,ns/2);

for a=1:totScanPts % Load bin files and convert to frequency domain
    p1 = binfileload(binPath,'ID',a,0); %signal from top mic, A1
    p2 = binfileload(binPath,'ID',a,1); %signal from bottom mic, A2
    [x1,~,~] = computeBlockFFT(p1(1:tw*fs-1),ns,w,W,fs);
    [x2,numblocks,f] = computeBlockFFT(p2(1:tw*fs-1),ns,w,W,fs);
    Xss(1,:,:)=x1.'; % Put data into matrix for TRAD and PAGE functions
    Xss(2,:,:)=x2.';
    %f = frequency array; x = amplitudes at each f

% Use TRAD and PAGE functions to get active intensity
TRAD = TRAD_func(f,Xss,probe_config,rho,c);
PAGE = PAGE_func(f,Xss,probe_config,rho,c); % I tells it to use Matlab's built-in unwrapping.
IaPAGE(a,:) = PAGE.I_mag; %Active intensity magnitude
IaTRAD(a,:) = TRAD.I_mag; % ^^^
TFphase(a,:) = PAGE.TFphase;
TFphase_wrapped(a,:)= PAGE.TFphase_wrapped;

% Sum intensity
IatotPAGE = IatotPAGE + IaPAGE(a,:); % IatotPAGE = total active intensity magnitude over all sides
IatotTRAD = IatotTRAD + IaTRAD(a,:); % IatotTRAD = total active intensity magnitude over all sides
end

PtotPAGE = IatotPAGE.*L^2; % Multiply by surface area of one measurement location
PtotTRAD = IatotTRAD.*L^2; % ^^^
%Total sound power spectral density
Lp/iPAGE = 10*log10(PtotPAGE./df./Iref); %Divide by df to get power spectral density
Lp/iTRAD = 10*log10(PtotTRAD./df./Iref); %Iref = Piref

%% Total sound power levels over specified frequency band:
freqID1 = find(f>=30,1,'first'); %find the index of the first frequency above 30 Hz
%sum power spectrum to find total sound power level and put into decibels,
%leaving out all the frequencies below 30 Hz

totLp/iPAGE = 10*log10(5*sum(PtotPAGE(freqID1:end))./Iref); %10*log10(sum(PtotPAGE(freqID1:end))./Iref); Added the 5s to account for sides

totLp/iTRAD = 10*log10(5*sum(PtotTRAD(freqID1:end))./Iref); %10*log10(sum(PtotTRAD(freqID1:end))./Iref);

save([saveFile,'.mat'],[c,d,f,fs,IaPAGE,IatotPAGE,IatotTRAD,IaTRAD,IiPAGE,IiTRAD,LpiPAGE,...
' LpiTRAD','ns',PtotPAGE,PtotTRAD,totScanPts,binPath,totLp/iPAGE,totLp/iTRAD]);

%% Plot power spectra comparing PAGE and TRAD

fig1 = figure;
semilogx(f,Lp/iTRAD,f,Lp/iPAGE); %plot power spectrum
grid on; hold on;
NY = c./(2*d); %Spacial nyquist frequency
line([NY NY],[ -40 100],color,[0.2745,0.51,0.70588]); %plot line at Nyquist
xlim([20 9000]) %limit x-values shown on plot
ylim([30 68]) %limit y-values shown on plot
Title = [source,' Sound Power TRAD vs PAGE (' meaName,')'];
title(Title);
xlabel('Frequency (Hz)')
ylabel('Power Level (dB)')
legend('Traditional','PAGE','Nyquist limit','location','best')
hold off;

saveFigure([figPath,figName])
APPENDIX 2: SoundPowerPlots4.m

%% Sound Power Plots 4

% DESCRIPTION:
% Makes One-Third Octave (OTO) plots from the data output of SoundPower3.m
% This code uses .mat files created by "SoundPower3.m" to make
% comparison figures between sets of data. It can plot a single
% narrowband PAGE and TRAD room measurement comparison, and a 1/3-octave
% plot of reverb against normal room measurements.

% VERSION:
% By Suzanna Gilbert Winter 2018
% Modified by TBNeilsen Nov 2018
% Modified by Michael Mortenson, August 2019

clear;

%% Function Library
% Add paths to functions and other files:
functions = 'C:\Users\Michael Mortenson\Documents\MATLAB\general-signal-processing';
addpath(functions);
addpath('C:\Users\Michael Mortenson\Documents\MATLAB\PAGE');
addpath('C:\Users\Michael Mortenson\Documents\MATLAB\sound-power');
addpath('C:\Users\Michael Mortenson\Documents\MATLAB\matlab-plotting');
myFigureDefaults

%% Find Saved Data
% location of the .mat file with calculated power
savePath = 'C:\Users\Michael Mortenson\Desktop\Acoustics Master Folder\Sound Power Research\Data\Power Calculated\Thesis';

% where to save figures
figPath = 'C:\Users\Michael Mortenson\Desktop\Acoustics Master Folder\Sound Power Research\Figures\Thesis';

%Choose source:
source = 'Norsonic';
% 'Vacuum'; % 'Blender';
nsource = 1;
source1 = 'Norsonic';
% Capitalized (Norsonic, BlenderWindscreen, VacuumWindscreen)
name1 = '7x7';
% (same as "measName" from SoundPower4.m code)
condition1 = 'U117';
nside1 = 49;
% Number of measurement locations on one side
Iref = 1e-12;
% Reference Value

%Load in variables from .mat file
load(fullfile(savePath,source1,name1,condition1,'PowerDataNewPhaseUnwrap1.mat'));
IaPAGE1 = IaPAGE;
% rename variables from .mat file so they
IaTRAD1 = IaTRAD;
% won't be overwritten by second file.
LpiPAGE1 = LpiPAGE;
LpiTRAD1 = LpiTRAD;
PtotPAGE1 = PtotPAGE;
$\Pi_{TOT\ TRAD1} = P_{TOT\ TRAD}$;
$\text{totLpiPAGE1} = \text{totLpiPAGE}$;
$\text{totLpiTRAD1} = \text{totLpiTRAD}$;
$dA1 = L^2$;

$\text{df} = f(2) - f(1)$; % define df from frequency array, $f$

%% Plot one Sound Power measurement (Trad v. PAGE)
fig1 = figure;
semilogx($f$, $LpiTRAD1$, 'k-+', $f$, $LpiPAGE1$, 'r--');
grid on; hold on;
$NY = c./(2\times d)$; % spatial Nyquist frequency
line([NY NY], [-40 100], [0,0,5,0], 'LineStyle', '--'); % Plot Nyquist
xlim([20 9000]) % limit x-values shown on plot
ylim([5 80]) % limit y-values shown on plot
Title = [source, name1, 'SoundPower'];
title(Title);
xlabel('Frequency (Hz)')
ylabel('Power Level (dB)')
legend('TRAD', 'PAGE', 'Nyquist limit', 'location', 'best')
hold off;

%% Plot Reverb chamber measurement against intensity-based measurement:

% Let code decide which reverb file to open depending on source:
sourceName = source;
reverbPath = ['C:\Users\Michael Mortenson\Desktop\Acoustics Master Folder\Sound Power Research\Data\Reverb Chamber'];
% ^^^ First cell is for Norsonic, second is for blender, third is for vacuum

% Make file path is correct (and also set y-limits for graph):
if strcmp(sourceName, 'Norsonic')
    reverbFile = filesToLoad{1};
ylims = [45 90];
elseif strcmp(sourceName, 'Blender') || strcmp(sourceName, 'BlenderWindscreen')
    reverbFile = filesToLoad{2};
ylims = [45 90];
elseif strcmp(sourceName, 'Vacuum') || strcmp(sourceName, 'VacuumWindscreen')
    reverbFile = filesToLoad{3};
ylims = [45 90];
end
Re = load([reverbPath, '\\', reverbFile]);

% Plot room measurement against reverb measurement:
ReverbPower = Re.Lw_ave;
fcReverb = Re.fc;
fig3 = figure;
semilogx(fcReverb, ReverbPower, 'b'); % plot reverb data
grid on; hold on;

% Include lines from regular room measurement:
load([savePath,source1,name1,condition1,‘PowerDataNewPhaseUnwrap1.mat’]);
%load([savePath,source1,name1,condition1,‘PowerData.mat’]);
PtotTRAD(1) = 0;

[~,OTOspecPAGE] = FDOTOspec(f,PtotPAGE./df,[20 20000]);
%convert power spectrum from anechoic

[fc,OTOspecTRAD] = FDOTOspec(f,PtotTRAD./df,[20 20000]);
%data into OTO bands to match reverb data

OTOLwPAGE1 = 10*log10(OTOspecPAGE./Iref);
OTOLwTRAD1 = 10*log10(OTOspecTRAD./Iref);

semilogx(fc,OTOLwTRAD1,'k-*',fc,OTOLwPAGE1,'r-o');

%Add Nyquist line, title, legend, and axis limits
NY = c./(2*d);
%Spacial nyquist frequency
plot([NY NY],[-40 105],'color',[0,0.5,0],’LineStyle’,’--’);
%title([sourceName,’ Power (’,name1,’),condition1,’]);

%W%excel
xlim([100 10000])
ylim(ylimits)
title([sourceName,’ Power (’,name1,’),condition1,’]);

xlabel(‘Frequency (Hz)’)
ylabel(‘Power Level (dB)’)
legend(’Reverb ISO3741’,’TRAD’,’PAGE’,’Nyquist limit’)

%% Add Norsonic Calibration Curve

if source == ’Norsonic’;

fNor = [20 25 31.5 40 50 63 80 100 125 160 200 250 315 400 500 630 800 1000 1250 1600 2000 2500
3150 4000 5000 6300 8000 10000 12500 16000 20000];
Lw = [66.6 67.6 70.9 72.0 73.1 75.2 78 79.2 80 80.9 81 81.8 82.4 82.5 83.6 83.8 84.2 84.6 85.8
87.9 88.4 87.8 87.0 86.7 84.9 82.9 82.8 78.7 76.5];
plot(fNor,Lw,’LineStyle’,’-’,’Color’,[0.5, 0, 0.5],’Marker’,’s’);
legend(’Reverb ISO3741’,’TRAD’,’PAGE’,’Nyquist limit’,’Calibration’)
end

hold off;

%% Plot Norsonic without Reverb

if source == ’Norsonic’;

fig4 = figure;

semilogx(fNor,Lw,’b-*’);
hold on; grid on;

semilogx(fc,OTOLwTRAD1,’k-*’,fc,OTOLwPAGE1,’r-o’);

plot([NY NY],[-40 105],’color’,[0,0.5,0],’LineStyle’,’--’);

xlim([100 10000])
ylim(ylimits)
title([sourceName,’ Power (’,name1,’),condition1,’]);

xlabel(‘Frequency (Hz)’)
ylabel(‘Power Level (dB)’)
legend(’Calibration’,’TRAD’,’PAGE’,’Nyquist limit’)