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Summary of Feedback Edge Set Papers

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I. Introduction

The problem of finding a set of edges F from a directed graph $G = (V, E)$ such that the graph $G' = (V, E - F)$ contains no cycles is known as the feedback edge set (FES) problem or the feedback arc set problem. A related problem is the minimum feedback edge set problem, in which the feedback edge set F must be minimal. The feedback edge set problem has application in applying a common reliability technique called triple modular redundancy to FPGA circuit designs where configuration memory scrubbing is employed [1].

II. Summaries

This paper gives a brief summary of five papers that make various contributions in the area of the feedback edge set problem and the minimum feedback edge set problem.

A. Notes on a Minimum Feedback Arc Set

In [2], Kamae presents a collection of definitions, theorems, and lemmas arriving at an upper and lower bound for the order of a minimum feedback arc set. These bounds are given as

$$r \leq |F_m| \leq q(q-1)/2,$$

where r is the order of a set of edge-disjoint dicircuits in the graph, $|F_m|$ is the order of a minimum feedback arc set, and q is the number of vertices in the graph.

B. A Global Feedback Detection Algorithm for VLSI Circuits

In [3], Shih et al. present a heuristic for finding a minimum feedback edge set quickly. The application described in the paper does not require minimal or even near minimal edge sets. It does, however, require a quick computation of the feedback edge sets. The algorithm described in the paper executes in a manner very similar to the *Highest Fanout SCC Decomposition* algorithm presented in [1], except that it executes recursively instead of using a stack.

The paper also describes a partitioning scheme whereby in order to reduce the number of recursive calls required by the algorithm (and hence memory requirements), the circuit is first partitioned into subcircuits. The feedback in each subcircuit is cut independently first. Then the subcircuits are considered as a combined whole and any feedback contained in the interconnections of the subcircuits is cut using the same algorithm.

C. Approximating Minimum Feedback Sets and Multicuts in Directed Graphs

In [4], Even et al. give reductions between multicut, FES, and FVS (feedback vertex set) problems. The paper also gives two polynomial time approximation algorithms for the SUBSET-FVS problem (which by reduction can be used for the FES problem).

D. Computing Minimum Feedback Vertex Sets by Contraction Operation and its Applications on CAD

In [5], Lin and Jou address the minimum feedback vertex set problem, which can be used to solve the minimum feedback edge set problem by reduction. The paper first reviews five graph contraction operations which simplify a graph without changing the results of finding a minimum cutest. It then introduces three new and powerful contraction operations which can greatly reduce the complexity of a graph and hence the time required to find the minimum feedback vertex set. Finally, a branch and bound algorithm incorporating all of the contraction operations is introduced.

E. Minimum Feedback Arc Sets for a Directed Graph

In [6], Younger presents the idea that a feedback arc set is a set of edges (i, j) for which $R(i) \geq R(j)$ for an ordering R of a graph. Then the problem of finding a minimum feedback arc set is reduced to finding an ordering R for which $Q(R)$ (the order of the set of feedback arcs (i, j) for which $R(i) \geq R(j)$) is minimal. Properties of sequential orderings which correspond to minimum feedback arc sets are explored in order to reduce the search space in order to find a minimum or nearly minimum (when computation time is limited) feedback edge set.

References

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