Dynamic Coalescence as a Mathematical Model of Leadership and Empirical Evidence of the Value of Strategic Sacrifice

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DYNAMIC COALESCENCE AS A MATHEMATICAL MODEL OF LEADERSHIP
AND EMPIRICAL EVIDENCE OF THE VALUE OF STRATEGIC SACRIFICE

by
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ABSTRACT

DYNAMIC COALESCENCE AS A MATHEMATICAL MODEL OF LEADERSHIP AND EMPIRICAL EVIDENCE OF THE VALUE OF STRATEGIC SACRIFICE

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The topic of leadership belongs to a rich literature that is deep in historical documentation and broad by the many disciplines that have attempted to understand it. However, the contributions of quantitative theorists are scarce, which has resulted in theories of leadership that are primarily qualitative and often subjective. This paper begins building a bridge between the existing scholarship and a more quantitative approach to the study of leadership by rigorously defining leadership as a strategy of interpersonal investment and presenting a mathematical model of this leadership theory. This model provides a theoretical foundation to explain team dynamics, to study the various components of the team system, and to measure the results of leadership behavior in a team setting. Further, a study is presented which shows empirical evidence that this model proves veritable in practice.
I would like to express deep appreciation to the many people who have mentored me throughout this research. I would especially like to thank Dr. Sean Warnick for paving the way for me to pursue this research in the first place. I would also like to thank my Honors Committee; in particular, I would like to express my gratitude for Dr. Gibb Dyer’s meaningful insights and direction in leadership studies, Dr. Ryan Sharp’s invaluable contributions to the practical connection of servant leadership and strategic sacrifice, and Dr. Griffin’s essential aid helping me see it through.

My deepest gratitude is reserved for my family and their constant support of my academic pursuits. I would like to thank my parents, my first examples of true leadership, for their special love and support. I would like to thank my children for the unequivocal meaning they have added to my life and work, and I would like to dedicate this work to them: Addison, Kohen, and McKay, may you always be the leaders that this world needs. Finally, I would like to thank my wife, Jessica, for her undying love and support. She is my greatest example of leadership. Of all the worthy pursuits available in this world, she has dedicated her life to greatest. As my greatest friend, closest partner, and ultimate confidant, she is the epitome of strategic sacrifice and my daily example of leadership in its truest form.
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I. Introduction

The concept of leadership has been a source of debate “since the first time two people came together for the purpose of completing a task” [1]. For as long as we have recorded history, we have had human organizations. History has shown that from families to cities to entire nations, every human organization of notable success in cooperation and survival has had a leader [2]. One of the major challenges facing contemporary corporations is the lack of competent leadership supply facing them in the labor market today. Many CEOs have cited the lack of qualified leadership talent as their companies’ most significant constraint on growth [8]. In part, this development problem is due to a shrinking supply of potential leaders: the number of 35- to 44-year-olds in the work force, the so-called “key leader age,” has dropped by more than 10% over the last decade according to U.S. Bureau of Labor [9]. Poorly prepared to face the growing scarcity of potential leaders, business organizations are feeling pressured to ensure that the talent they recruit are high potential individuals and the leadership development programs they implement are effective and successful.

Today’s organizations demand a better product from leadership expertise. In order to meet these needs, we must understand leadership as it truly is, and we must define it in a way that allows us to accurately measure and develop its presence in people. There is a rich literature concerning questions of this regard; it is both deep in historical documentation and broad by the many disciplines that have attempted to understand it. From business administration to classroom education, the attempts to define leadership vary widely in both approach and method. Three of the inherent challenges that arise with such a diverse set of thought contributors are, first, how to define leadership [5], then, how to measure leadership [6], and, finally, how to develop a person’s leadership abilities [7] in a meaningful, cross-
disciplinary way. This article seeks to respond to this need by providing a theoretically based mathematical model for leadership.

II. The Evolution of Leadership Theory

In his seminal work, *Leadership for the Twenty-First Century*, Joseph C. Rost identified more than 200 published definitions of leadership between the years 1900 and 1990 [11]. While most people can formulate some basic understanding of what leadership is, Rost’s research showcases the difficulty had by both academic scholars and leadership practitioners. Yet, despite the many different definitions, Peter Northouse [12] recently described how the definitions of leadership follow an evolution of themes through time; that is, while the semantics of any two definitions may only resemble one another, prevailing themes can be found purveying groups of definitions at different time periods of the 20th century. A brief review of this evolution will help situate our proposed theory within the broader leadership literature.

1900-1929

Definitions of leadership in the beginning of the century are characterized by emphasizing control and centralization of power with a common theme of domination. At a conference on leadership in 1927, leadership was defined as “the ability to impress the will of the leader on those led and induce obedience, respect, loyalty, and cooperation” [11].

1930s

The 1930s saw a significant shift in the general perspective of leadership. Rather than the power-centric views of the last three decades, leadership began to be viewed as influence. In particular, leadership came to be identified as the interaction of an individual’s specific personality traits with the traits of those in a group.
**1940s**

The relationship between the leader and the group continued as a focal point of thought on leadership, the definition of which evolved into “the behavior of an individual while involved in directing group activities.” Also during this decade, a distinction was drawn between leadership by persuasion and leadership by coercion.

**1950s**

Northouse points out that three themes dominated leadership definitions during this decade: 1) Group theory maintained its spot in the forefront of leadership thought, framing leadership as what leaders do in groups. 2) The beginnings of a behavioral approach to defining leadership occurred, particularly interpreting leadership as a relationship that develops shared goals based on the behavior of the leader. 3) Effectiveness also entered the definitions, namely that leadership was defined by the ability to influence overall group effectiveness. Group theory, leadership as a relationship that develops shared goals, effectiveness.

**1960s**

For maybe the first time in recent history, a general consensus was reached among leadership scholars concerning the definition of leadership. The behavioral approach became primary approach to defining leadership, characterizing it as “acts by persons which influence other persons in a shared direction.” This definition, though not always accepted outright, would prove fundamental to all definitions of leadership up to the present day.
**1970s**

Rost explains that the prevailing theme in the ’70s would describe leadership as “initiating and maintaining groups or organizations to accomplish group or organizational goals.” However, as Northouse points out, ‘Burns’ 1978 definition was the most important concept of leadership to emerge: ‘Leadership is the reciprocal process of mobilizing, by persons with certain motives and values, various economic, political, and other resources, in a context of competition and conflict, in order to realize goals independently or mutually held by both leaders and followers.’’

**1980s**

This decade marks the apex of the academic and mainstream treatment of leadership as a topic. This is apparent in the wide variety and large number of leadership definitions, ranging from the almost century-old view of leadership as dominion to revisiting the themes of leadership as influence and leadership as a set of personality traits, finally introducing a new movement defining leadership as a transformational process. This idea of transformative leadership was described by Burns [12] as occurring “when one or more persons engage with others in such a way that leaders and followers raise one another to higher levels of motivation and morality.”

**1990-Present**

Rather than developing new definitions of leadership, emerging now in the literature is an emphasis on the process of leadership, “whereby an individual influences a group of individuals to achieve a common goal.” [12] Such processes, Northouse explains, include:
• **Authentic Leadership:** Utilizing the authenticity of the leader and his or her leadership to touch the group of followers.

• **Spiritual Leadership:** Utilizing moral values and the sense of calling and membership to motivate followers.

• **Servant Leadership:** Focusing on the needs of the followers and utilizing “caring principles” to help the followers become more autonomous, knowledgeable, and more capable as leaders and servants themselves.

• **Adaptive Leadership:** Encouraging followers “to adapt by confronting and solving problems, challenges, and changes.”

It is also in this time period that the first debate is drawn as to whether leadership and management are the same things [14].

One major advantage from this relatively high level of convergence of scholarly stances regarding leadership is that it provides both researchers and leadership practitioners with a palpable construct in which we can legitimately analyze the proposed utility of various leadership behaviors and conjecture about the effects of their applications. However, the focus on “styles” of leadership carries the risk of excessive crossover between new styles or approaches to leadership.

For example, if, as Burns said, transformational leadership is raising “one another to higher levels of motivation and morality,” does that mean that ethical leadership, spiritual leadership, authentic leadership, and servant leadership are essentially the same books under different covers? This question was addressed in a recent meta-analysis [20]. This study found that “authentic and ethical leadership display significant construct redundancy as evidenced by their high correlations with transformational leadership and their overall similar
pattern of associations with other measures.” This redundancy is symptomatic of the excessively qualitative approaches to leadership theory that continue to persist in the literature: there are so many ways to talk about how one can influence others that we may often be using different words to describe identical phenomena.

Our goal is to overcome such redundancy and usher in a new generation of leadership studies where scholars quibble less regarding the nuances of the definition and debate more on the efficacies of different processes and behaviors whereby “an individual influences a group of individuals to achieve a common goal” [12].

III. Servant Leadership

Of the many ways to influence others, one of great import and rising scholarly interest is the concept of servant leadership. A Google Scholar search underscored this growing interest as the subject “servant leadership” yielded 2,630 results for 1980 to 2003 versus 16,800 results for the period 2003 to 2016 [20]. The same study also revealed that servant leadership “appears to exhibit a higher degree of conceptual and empirical distinctness from transformational leadership” [20]. The idea of servant leadership began in 1977 when Robert K. Greenleaf proposed that service to followers is the primary responsibility of a leader. Greenleaf stated: “The servant-leader is servant first. It begins with the natural feeling that one wants to serve. Then conscious choice brings one to aspire to lead” [18].

In this construct, a servant leader must empower followers to accomplish an objective rather than dominate them with the asymmetric power inherent to positions of authority. Drawing from Greenleaf’s writings, Larry C. Spears identified 10 characteristics of servant leaders including: listening, empathy, healing, awareness, persuasion, conceptualization,
foresight, stewardship, commitment to the growth of others, and building community. The argument of servant leadership is that by focusing on the development and success of individual followers, the broader organizational objectives will be successfully achieved. In other words, organizational success is, at least in part, the result of reciprocal processes wherein a leader’s focus and concern towards subordinates increases their trust, loyalty, and productivity.

Hoch et al concluded that servant leadership shows promise but is lacking in extant research [20]. While servant leadership continues to emerge as a legitimate academic discipline, there remains a need to provide a quantitative theoretical framework to further substantiate the work being done and to push this leadership approach forward. The purpose of this article is to respond to that gap by providing a quantitative approach to understanding servant leadership, including a fundamentally different approach to explain what it is, why it matters, and how it affects group productivity.

IV. A Mathematical Approach to Leadership

In this approach, we will examine servant leadership as a strategy, or a conscientious selection of behavior in tandem with the behavior of others, shifting the focus from the personal traits displayed by a leader to “a transactional event that occurs between the leader and the followers … [implying] that a leader affects and is affected by followers” [12]. This article proposes a leadership strategy—aligned with servant leadership and grounded in mathematical theory—whereby a leader, upon implementation, facilitates the coalescence of self-interested individuals into a group of persons willing to work together in order to achieve a common goal. The precision of this definition is worth attention, as follows.
Strategy: A strategy ought to be understood as a plan of action designed to achieve some major or overall aim. For quantification purposes, we will take a game theoretic approach and say that a strategy is a set of individual actions which he or she can choose in a setting where the outcome depends not only on his or her own actions but also on the actions of others. A player’s strategy will determine the action he or she will take at any stage of the game.

Coalescence: Coalescence is the formation of a coalition – an alliance for combined action. For what is a leader without a group to lead? Historically, coalitions have been viewed as temporary associations formed with the purpose of acquiring political control. This idea is important because it illustrates the fact that people with many competing allegiances can come together and under a prioritized agenda to accomplish a general objective, at least for the period necessary to accomplish that objective. The complexities of coalescence will be discussed thoroughly throughout this paper, the first of which is self-interest.

Self-Interest: The third principle of economics is that rational people think at the margin [15]. That is, people make decisions to maximize a constrained utility. The underlying assumption is, of course, that consumers behave rationally out of self-interest. We will proceed with the same assumption, namely that people systematically and purposefully do the best they can to achieve their individual objectives. This is the foundation of a competitive setting. Analyzing a business unit, one would quickly admit that each member of the team is working out of self-interest: for wage raises, bonuses, promotions, etc. Despite being housed under one organizational agenda, these individuals often must outperform their teammates in order to get better bonuses, better job-security, etc.
Production: Because of this self-interest, the individuals can be represented as consumers and producers in an economic market, where the market is the group composed of these individuals. The individuals consume incentives and produce work that is meaningful to the team. The aggregate production of these individuals, then, is the market (or team) production, and we assume that this production is the number one goal of any rational organization.

This article argues that team production can be increased when self-interested individuals cooperate to achieve the business-unit goals. We will analyze two facets of this phenomenon: (1) The team’s iterative coalescence, and (2) the strategy which a leader can apply to aid coalescence and improve the equilibrium production of a team. We call them dynamic coalescence and strategic sacrifice, respectively. To quantify this idea, we utilize game theory and robust control theory to postulate and analyze the ideas of coalition formation and strategic interaction.

V. A Background in Economic Game Theory and Robust Control

Game theory is a branch of mathematics that attempts to model conflict and cooperation between intelligent rational decision-makers. To be complete, a game theoretic model must have at least three features: a set of players, a set of actions, and a set of payoffs respective to each player for his or her chosen actions. These features are used in tandem with each other to deduce a set of strategies: the players’ chosen subset of available actions and the associated payoffs. In most games, the players will eventually find an equilibrium -- a set of strategies that will persist until the rules change. The Nash equilibrium is a special equilibrium studied in game theory. This particular equilibrium is provably stable (it will not naturally end) but not optimal because it indicates diminished productivity. The Nash
equilibrium occurs when each player has played a strategy exhibiting a property of mutual best response. That is, in the Nash equilibrium, neither player has an incentive to deviate from his or her current strategy. The Nash equilibrium can be defined formally:

**Definition 1.** Let \( N = \text{Set of players} \ n_i, S = \text{Set of strategies} \ s_i, \) and \( P = \text{Set of payoff functions} \ p_i. \) Then the Nash equilibrium is a list of strategies \( (s_1^*, \ldots, s_n^*) \) \( \in \text{the set of Cartesian products} \ S_i \) for \( 1 \leq i \leq n \) such that \( P_i(s_i^*, s_i^*) \geq P_i(s_i, s_i^*) \forall i \in N, \forall s_i \in S. \)

To illustrate, consider the classic game “The Prisoner’s Dilemma,” (illustrated in Fig. 1). Two players, Bonnie and Clyde, are placed in separate interrogation cells after being arrested for a small-scale crime, the sentence for which is 1 year in prison. The police, however, know that the two criminals have committed a series of bank robberies, though they do not have enough evidence for a conviction on that account. The police offer Bonnie and Clyde each individually the chance to walk free if they will confess, but their partner will be given a 3-year sentence. However, if both criminals confess, they both will receive 2 years in prison.

---

![Figure 1: The Prisoner’s Dilemma](attachment:image.png)

This game is a classic example of a Nash Equilibrium. This figure illustrates the three fundamental parts to the game, (1) the players Bonnie and Clyde in green, (2) the available strategy set in black, and (3) the payoffs of each strategy in blue. The state of Nash equilibrium, with its respective strategies and payoffs, is circled in red.
This particular game exhibits the property of a stable yet suboptimal equilibrium. It is stable because neither Bonnie nor Clyde have an incentive to deviate unilaterally from both confessing, and it is suboptimal because the joint payoff is worse compared to the joint payoff the two would have received if they had cooperated. The cooperative equilibrium is Pareto optimal – that is, the profit of one player cannot be increased without decreasing the profit of another – and it maximizes the overall market profit.

A market is an economic setting where transactions are made. The transactions may be buying and selling goods with money, or it may be transactions of a more abstract nature, like the exchanging of information for a reduction in jail time as shown the prisoner’s dilemma. One market that is familiar to most people is a team. Whether it is a sports team or a team of business professionals in a business unit, any team expects its members to contribute resources – time, capital, skills, engagement, and other means of production – and will reward them with goods like a wage, greater knowledge, career advancement, etc.

Teams are an interesting market to examine because, similar to the prisoner’s dilemma, the performance incentives attributed to individual members seem misaligned with full-team cooperation. Despite the need of cooperative teamwork in order for most organizations to be successful, many organizations incentivize individuals to compete with other team members. To maximize their payoffs (yearly bonuses, managerial recognition, wage raises, etc.), individuals feel the need to outperform their peers. As in the prisoner’s dilemma, such a system will inevitably reach and persist at the Nash equilibrium.

Over the many years through which the corporate nature of the developed world has evolved, this suboptimal equilibrium has become normality, and there is no apparent reason to believe otherwise. Organizations still adopt various methods to increase production,
typically inserting a manager whose responsibility it is to ensure that the team’s production levels meet company requirements. However, these incentives for increasing individual production may be undermining their ability to reach their full production potential. Effective leaders will behave in ways that nullify the unintended consequences of misaligned incentives.

One way that managers can mitigate against this unintended consequence is through the practical application of control theory. Control theory is the foundation of modern systems engineering. It is applied to understand and control continuously operating dynamical systems in engineered processes and machines, like the cruise control of a vehicle. As the system experiences constantly changing inputs, like hills, bumps and curves along a road, a controller is in place to analyze the changing inputs and direct the system’s response accordingly. A controller is engineered with a transfer function, a mathematical model of the relationship between the input and output based on the equations describing the system.

An economic market is an example of a dynamical system where a controller can be engineered to analyze and direct its production. In a situation like the prisoner’s dilemma, one could imagine inserting a controller, or rule, into the game to govern the players’ choices. By so doing, one could, ideally, direct the system to any output we choose, even to the perfectly cooperative equilibrium. In practice, then, a leader could be inserted into a team who, theoretically, could control the outputs of the team by simply adjusting the inputs. If the leader is capable of accurately measuring his or her team’s output as a function of the team’s dynamics – so he must be capable of discerning the dynamics of the team – then he or she could lead the team to its optimal equilibrium. Our proposition is that the most effective
behavior a leader can exhibit to induce cooperation is the servant leadership model: sacrificing his or her own resources to the team in order to calibrate the team’s output.

VI. Dynamic Coalescence and Strategic Sacrifice: A Mathematical Model of Leadership

Ultimately, the beliefs surrounding the efficacy of servant leadership have been guided by a qualitative approach to understanding the phenomenon. However, when we model a team as an economic market, we are able to show that servant leadership is effective not because of a perceived obligation to the sacrificing leader, but because a leader’s servitude reduces the subordinate’s incentive to compete with teammates. As competition decreases, cooperation necessarily increases and the team becomes a positive coalition.

In 2009, Brown, Tran, and Warnick formulated and solved the stability robustness problem for a class of gradient play cooperative differential games with coalition structures allowing for partial participation and symmetric contracts. In 2016, Hurst and Warnick introduced a two-player coalition structure similar to those published in 2009 and presented a particular contract strategy able to produce near-perfect cooperation in the system [10]. They called the firm feedback “side-payments,” because they essentially modeled what would occur if two firms, stuck in a “prisoner’s dilemma”-type situation, began to share payoffs. They were able to show that, by introducing this sacrificial behavior into the market, they could induce a more cooperative price equilibrium (see Figure 2) in simulations.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Payoff 1 ($)</th>
<th>Payoff 2 ($)</th>
<th>Production ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Cooperative</td>
<td>5.98</td>
<td>4.58</td>
<td>10.56</td>
</tr>
<tr>
<td>Perfectly Collusive</td>
<td>8.00</td>
<td>4.00</td>
<td>12.00</td>
</tr>
<tr>
<td>Dynamic Coalescence</td>
<td>6.43</td>
<td>4.90</td>
<td>11.33</td>
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Figure 2: Dynamic Coalescence. The payoffs and total market production for each strategy set.
Not only is this new equilibrium provably stable, it is stable at a point of increased payoffs for each player. Therefore, the team as a whole has an increased level of production as well. In simulations, they have shown that the equilibrium can come within 5% of the perfectly collusive market equilibrium. The mathematics are important to understanding and analyzing the model, but the underlying premise of the model itself is key to understanding what actually happens in a team.

The idea behind the model is portrayed in Figure 3, a basic feedback system where the leader is placed as a controller to calibrate the system input according to the feedback received from the system. He or she performs the calibration by sacrificing the resources needed by the team’s individuals. A leader is anyone who examines the output of a team (i.e. market), understands both the dynamics that produced the output and the change required of the inputs in order to improve the output, and is willing to take the risk of sacrificing his or her own resources to meet the resource distribution required to reach a higher production level.
This is the quantitative foundation of our leadership model, where the mechanism of control could be interpreted as sacrificial activities such as training and mentoring new employees, providing regular direct feedback, researching and providing valuable data, etc. In so doing, leaders can, theoretically, create a cooperative team rather than a fragmented market of individuals competing with each other for resources and rewards.

Borrowing the fundamental idea behind this model, we present its properties as they relate to leadership. In particular, we assert that a leader can understand the dynamics of their organization and then exhibit strategic behavior in a multi-agent non-cooperative setting by offering sufficient side-payments or by facilitating side-payments made by all players throughout an organization. We call these side-payments “Strategic Sacrifices.”

A. The Leader’s Competitive-Cooperative Environment: Human Organizations as Systems of Non-cooperative Coalitions

Defining the market in the leadership model is straightforward as we consider various human organizations (a business, an athletic team, a classroom, etc.). A fundamental
assumption is that rational human beings tend to seek their own self-interests before the interests of those around them [7]. As a result, most organizations, formal or not, almost always have competition.

Consider, for example, Burns’ assertion that “Leadership is the reciprocal process of mobilizing, by persons with certain motives and values, various economic, political, and other resources, in a context of competition and conflict (i.e. a competitive market), in order to realize goals independently or mutually held by both leaders and followers (i.e. induce cooperation).”

Despite organizational reliance on effective teamwork to be successful, there are often competing incentives for individual team members to stray from the goals of the team to seek their own goals. Take, for instance, a marketing team within a business organization: while the success of the team certainly depends—at least in some part—on the synergy of the team, each team member is also incentivized to outperform his or her teammates to earn a promotion. While this behavior might be selfish, it must also help the team reach its goals. In this way, an individual performance bonus can act as a requisite sacrifice in the model.

A business organization or a college basketball team, rather than a market of firms, is still a production market. In fact, the production of these organizations are often very important to a number of different people. From the low-cost emotional stakeholders, like sports fans and brand champions, to the high-risk monetary stakeholders such as private investors and executive leadership teams, and even to the team members themselves, the profitability of an organization is a very real concern for many people. And maybe this is why great leaders are in such high demand.
Opportunities for an immediate increase in personal profitability can change the profitability trajectory of an entire organization in the long run. If these people can be accurately represented as the firms in the preceding model, it would follow that many organizations may already be stuck in their own Nash Equilibrium. A leader then assumes the role of providing or facilitating strategic sacrifices to incentivize the positive collusion of his or her system’s unique agents, bringing the organization to a sustained monopolistic equilibrium.

B. Strategic Sacrifice as an Effective Method of Side-Payments

Sacrificing one’s resources for the sake of altruism alone seems unrealistic and unhelpful. If this is the case, we must ask ourselves, “What makes a sacrifice strategic?” In its simplest form, a strategic sacrifice can be defined as foregoing one’s own consumption of a resource so that it might be consumed by other team members in order to improve overall production of the team. Such sacrifices could be interpreted as a leader teaching those on his or her team how to perform certain tasks correctly or sharing with them information from his or her experience and giving them a direction to follow in solving the problems at hand. This role can be formal or informal, but the actions taken are virtually the same: they can be characterized by a leader sharing a meaningful resource with his or her followers, often at some expense to the leader (like time) in with a hope of some future return on the investment (like sufficiently correct assistance on some technical problem).

In their article examining one’s journey through a professional career, Dalton, Thompson, and Price presented what they deemed the “4 Stages of a Career” [16]. The authors explored the key activities, essential relationships, and important psychological factors of a person through apprenticeship (stage 1), technical expertise (stage 2), mentorship
(stage 3), and company pioneering (stage 4). Interesting to this discussion are the key activities which the authors identified as being performed by those in the third stage of their careers, including:

- Informal mentorship
- Ideation
- Management
- Influencing and directing others
- Taking responsibility for others, especially for their development.

While this list is not exhaustive, it should give the reader an idea of the types of activities that we consider “Strategic Sacrifice.” This raises the question: “What is the actual effect of a leader’s strategic sacrifice on the productivity of his or her team?”

VII. Capturing Strategic Sacrifice in Data

To answer this question, we present an analysis of the Asia Talent dataset, a series of surveys given to different employees within companies of variegated size across Asia. The dataset initially contained 77 variables and 472 total observations. To show practical application of the theory being explored in this article, the independent variable chosen was the productivity of a firm as measured by sales per employee (as reported in the company’s Income Statement). Given the low number of observations within the set, we needed to consolidate and select the variables carefully. The dependent variables were split into three major categories: Information regarding the hiring of employees, the managing of employees, and the firing of employees. Across these three major categories are 14 minor subsets of variables, each of which includes a different number of the variables making up the data set.
Hiring-type categories include:

- Hiring standards
- Hiring and recruiting source
- Screening of potential employees
- Secure the hiring of top candidates

Managing-type categories include:

- Steering employees
- Motivating employees
- Developing employees
- Managing employees
- Communication within the firm
- Identifying leaders among the current staff

Firing-type categories include:

- Retaining top employees
- Removing low performers
- Top 3 reasons for leaving the company

Because this article focuses on the effect of a leader on his or her team within a firm, it will not consider a company’s capacity to hire and fire employees appropriately. This is not to say that these are not important functions of leadership, but this information detracts from the heart of the question at hand, and many of the variables within these categories have a sort of mechanical relationship to productivity; e.g. if a firm fires a low-performing employee, the overall productivity per employee will increase, ceteris paribus.

With only the managing-type categories of variables remaining, we created a new consolidated variable labeled Strategic Sacrifice of particular variables that capture strategic
sacrifice. This new group consists of the following variables, with their original managing-type category indicated in parentheses:

- Provide new employee orientation (steer)
- Provide new employees the tools to do their job (steer)
- Ensure competitive compensation (motivate)
- Listen to our employees (motivate)
- Give employees direct feedback (assess)
- Deliver training programs (develop)
- Give employees development assignments (develop)
- Help employees grow through internal coaching and mentoring (develop)
- Help leaders grow by hiring external coaches (develop)
- Adapt our employee development practices (e.g. training, coaching, etc) to the needs of employees at each stage in their career development (develop)

Still needing to reduce the number of variables overall, we consolidated all the management-type categories of variables to their respective categories by the average of all respective variables. For example, after removing the strategic sacrifice-type variables, we computed the mean of all variables belonging to the “motivate” category by observation and generated a new variable called “motivate” to capture that mean. Note that both variables from the “steer” category are now in the strategic sacrifice group. Thus, the final dataset consists of 6 dependent variables and four control variables. The following dependent variables each capture how well leaders perform in different activities related to leading a team:

- Motivate
• Assess
• Develop
• Manage
• Communication
• Strategic Sacrifice

The control variables are: Number of employees, annual revenue, office’s country, and industry. Note that number of employees and annual revenue are nominalized values. Number of employees has been discretized into 6 ranges of 500 employees each, and annual revenue is in 11 bins of various ranges from 0 to 10 billion dollars, both of which were predetermined by the survey. Figure 4 contains the summary statistics of the final dataset.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>productivity</td>
<td>472</td>
<td>3.6911</td>
<td>.79817</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>motivate</td>
<td>472</td>
<td>3.9105</td>
<td>.75048</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>assess</td>
<td>472</td>
<td>3.7566</td>
<td>.78833</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>develop</td>
<td>472</td>
<td>3.0974</td>
<td>1.0773</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>manage</td>
<td>472</td>
<td>3.8170</td>
<td>1.0650</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>communication</td>
<td>472</td>
<td>3.4166</td>
<td>.92381</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>sacrifice</td>
<td>472</td>
<td>3.6723</td>
<td>.76666</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>n_emp_bin</td>
<td>448</td>
<td>1.7522</td>
<td>1.1810</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>rev_bin</td>
<td>354</td>
<td>2.5423</td>
<td>2.5878</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>country</td>
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<td>2.0593</td>
<td>.83238</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>industry</td>
<td>472</td>
<td>8.4979</td>
<td>3.9573</td>
<td>1</td>
<td>15</td>
</tr>
</tbody>
</table>

Figure 4: Summary statistics
VIII. Models and Analysis

To analyze these data, we utilized three different econometric frameworks: Ordinary Least Squares Regression, Quantile Regression, and Multinomial Logistic Regression.

A. Ordinary Least Squares Regression

We began with an uncontrolled ordinary least squares binary regression examining the causal effect of Strategic Sacrifice on productivity with the following model:

\[ Productivity = \beta_0 + \beta_1 \text{StrategicSacrifice} + e \]

which yielded a very high estimated coefficient \( \beta_1 \) of .4481071(.0433467) and a statistical significance of 1.00. The full results are found in Figure 5.

| Variable         | Coefficient | Std. Error | t     | P > |t|  | 95% C.l.       |
|------------------|-------------|------------|-------|-----|---|----------------|
| StrategicSacrifice| .4481071    | .0433467   | 10.34 | 0.000 | .3629297 | .5332845       |

Figure 5: Binary OLS Regression. Stage3 is the StrategicSacrifice variable.

Next, we added to the model the other categories of activities aimed to boost productivity, namely how well leaders do in motivating, assessing, developing, managing, and inducing communication among employees:

\[ Productivity = \beta_0 + \beta_1 \text{Motivate} + \beta_2 \text{Assess} + \beta_3 \text{Develop} + \beta_4 \text{Manage} + \beta_5 \text{Communicate} + \beta_6 \text{StrategicSacrifice} + e \]

Doing so shows that, absent other controls, strategic sacrifice is one of two statistically significant factors in determining team productivity and is, by far, the most effective method of increasing productivity on the margins with an estimated coefficient \( \beta_6 \) of .2654803(.1059912). The other significant variable was communication, but with a much lower coefficient at .1654636(.055742). The full results are listed in Figure 6.
To better ensure that the results are accurate, we controlled the previous model by the size of the firm in terms of both number of employees and annual revenues, by the country in which the firm resides, and by the industry to which the firm belongs. While adding the controls should add robustness to the estimates and confidence to the inference, it is important to note that missing data causes the number of observations to decrease to \( n = 354 \), a high and unfortunate cost. This yields the final OLS model,

\[
\text{Productivity} = \beta_0 + \beta_1 \text{Motivate} + \beta_2 \text{Assess} + \beta_3 \text{Develop} + \beta_4 \text{Manage} + \beta_5 \text{Communicate} + \beta_6 \text{Strategic Sacrifice} + \beta_7 \text{nemployees in} + \beta_8 \text{Revenue in} + \beta_9 \text{Country} + \beta_{10} \text{Industry} + e
\]

which increased both the significance level and the estimated coefficient \( \beta_6 \) of Strategic Sacrifice to the point that it is clearly the most important and most influential variable in the model. According to this model, a unitary increase in Strategic Sacrifice increased the productivity of a team by more than 33% on average per employee. The full results are captured in Figure 7.
B. Quantile Regression

These results lead to further questioning of the importance of strategic sacrifice for improving team productivity, particularly at what levels of productivity strategic sacrifice is most beneficial. That is, if a team is entirely unproductive, will strategic sacrifice be enough to bring that team to full productivity? Further, if a team is already very productive, say 90% more productive than other, similar teams, will the effects of strategic sacrifice still be significant? To examine these questions, we extended the analysis to include a quantile regression model:

\[
\text{Productivity} = \beta_0(\tau) + \beta_1(\tau)\text{Motivate} + \beta_2(\tau)\text{Assess} + \beta_3(\tau)\text{Develop} + \\
\beta_4(\tau)\text{Manage} + \beta_5(\tau)\text{Communicate} + \beta_6(\tau)\text{Strategic Sacrifice} + \\
\beta_7(\tau)\text{Number of Employees in} + \beta_8(\tau)\text{Revenue in} + \beta_9(\tau)\text{Country} + \\
\beta_{10}(\tau)\text{Industry} + e
\]

for \( \tau = .1,.25,.5,.75,.9 \).

The findings were surprising as we hypothesized that this empirical work support the theory introduced in Section 2B, but there was not an expectation that it would correlate so exactly. Recalling the results of the simulation as recorded in Figure 2, namely that Warnick and Hurst were not able to reach the fully collusive market equilibrium through with the

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t</th>
<th>P &gt;</th>
<th>95% C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motivate</td>
<td>-0.1008353</td>
<td>0.0780251</td>
<td>-1.29</td>
<td>0.197</td>
<td>[-0.2543048, 0.0526341]</td>
</tr>
<tr>
<td>Assess</td>
<td>0.0762411</td>
<td>0.08566</td>
<td>0.89</td>
<td>0.374</td>
<td>[-0.092285, 0.2447672]</td>
</tr>
<tr>
<td>Develop</td>
<td>0.0015116</td>
<td>0.054557</td>
<td>0.03</td>
<td>0.978</td>
<td>[-0.1057979, 0.108821]</td>
</tr>
<tr>
<td>Manage</td>
<td>0.0224384</td>
<td>0.050585</td>
<td>0.44</td>
<td>0.658</td>
<td>[-0.077084, 0.1219353]</td>
</tr>
<tr>
<td>Communication</td>
<td>0.136169</td>
<td>0.0672183</td>
<td>2.03</td>
<td>0.044</td>
<td>[0.0039556, 0.2683823]</td>
</tr>
<tr>
<td>StrategicSacrifice</td>
<td>0.3392569</td>
<td>0.1257003</td>
<td>2.70</td>
<td>0.007</td>
<td>[0.0920139, 0.5864999]</td>
</tr>
<tr>
<td>Number of Employees</td>
<td>0.038898</td>
<td>0.0374483</td>
<td>0.90</td>
<td>0.366</td>
<td>[-0.0397681, 0.1075478]</td>
</tr>
<tr>
<td>Yearly Revenue</td>
<td>0.0186693</td>
<td>0.0169235</td>
<td>1.10</td>
<td>0.271</td>
<td>[-0.014618, 0.0519566]</td>
</tr>
<tr>
<td>Country</td>
<td>0.0944666</td>
<td>0.0474882</td>
<td>1.99</td>
<td>0.047</td>
<td>[0.0010609, 0.1878723]</td>
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<tr>
<td>Industry</td>
<td>-0.0063329</td>
<td>0.0103108</td>
<td>-0.61</td>
<td>0.539</td>
<td>[-0.0265137, 0.0139479]</td>
</tr>
</tbody>
</table>

Figure 7: Fully Controlled OLS Regression
strategic sacrifice strategy, the results of this quantile regression analysis show that strategic sacrifice ascends in effectiveness until peaking around the 75th percentile and the statistical significance of strategic sacrifice follows a similar trajectory (Figure 8). The coefficients and significance levels of both Communication and Strategic Sacrifice can be found in Figure 9.

This seems to imply that strategic sacrifice is an essential factor in reaching an equilibrium that is close to full collusion, but it may not be very helpful in closing the remaining gap. Interestingly, communication trends upward in both significance and marginal effect at the points where sacrifice begins to trend downward. This may indicate a problem of asymmetric information. Once a leader has reached an optimal point of strategic sacrifice, it becomes their task to communicate such sacrifices effectively to the employees and doing so will allow said sacrifices to continue increasing productivity.
C. Multinomial Logistic Regression

As predetermined by the survey, there is a categorical version of the dependent variable Productivity that is nominalized into bins from 1-5 with 5 being the most productive. This is not equivalent to the continuous version of the variable, but it is based on a firm’s real “sales per person.” Thus, the third and final method of analysis was a multinomial logistic regression. By nature, the multinomial regression is more indicative of which stage of productivity a company is in given the observation rather than explaining the causality of the independent variables in the observation, as opposed to quantile regression. That is, multinomial regression is often considered a prediction model rather than a causality model.
Despite a low pseudo $R^2$ value of 0.1466, the results (Figure 9) were not unexpected, and they can be interpreted with similar logic to the quantile regression. While 4 was used as the baseline class for the regression, of the remaining classifications there are only three statistically significant variables: Strategic Sacrifice for class 3 and Communication and Strategic Sacrifice both for class 5.

IX. Limitations

It is important to note that, for all of these analyses, the survey data being used was gathered by a third party. While it is a respected professional organization who conducted the surveys, all of the data was self-reported by employees and customers associated with the organization. This could naturally lead to biased data, as it can be both difficult and highly subjective to rate oneself or the performance of one’s team. Objective data and a larger sample size would add merit to the analysis.

Furthermore, this survey was given only in Asia, and only in 3 countries: China, India, and Singapore. Fortunately, each of these countries have very different cultures and perspectives on leadership; however, a better dataset would include many more countries throughout the world in order to balance any cultural biases in the current responses.

X. Conclusion and Future Direction

Assuming that the models are unbiased and the sample is representative of the population, the analyses presented support the theoretical model advanced in the beginning of the paper. At the very least, it supports the assertion that strategic sacrifice is key to improving team productivity and can potentially provide significant increase in marginal productivity. Additionally, when strategic sacrifice is accompanied by excellent
communication, these factors can increase marginal productivity by more than 50% on average per employee.

Further, the quantile regression has shown that strategic sacrifice may not be sufficient to reach full cooperation in a competitive setting unless it’s accompanied with excellent communication upon nearing the 80th and 90th percentiles. In full, this analysis appears to validate the dynamic coalescence theory of leadership.

There is much work left to be done. First, we need more data. Given the low number of observations and the possible culture bias, sampling from more countries across the globe will both increase accuracy and decrease culture bias. Additionally, more insights could be gained by having “Strategic Sacrifice”-specific questions in the survey rather than having to extract those questions from other categories. Finally, this research will ideally culminate in randomized control experiments. Such experiments would assure the validity of the ordinary least squares and quantile regressions by providing a truly random sample, as well as allow us to objectively observe both the mechanism of strategic sacrifice and a leader’s propensity to participate in those types of activities.

Following these improvements, work can be done to construct an objective leadership score. This score would be given to a leader based on his or her participation and performance in the experiments, and this score could provide a valuable tool to companies as they seek to hire great leaders and improve productivity in the firm. Such a leadership score, however, is not the only valuable outcome of this continued research. An accurate understanding of leadership and the mechanisms that define it will provide a solid foundation for a quantitative reformation of leadership training and identification in many settings across the globe.
REFERENCES

[3] Examine, for example, a sample case of Clinkle and its run-amok CEO Lucas Duplan. There is no formal case study that I am aware of, but the interested reader could begin at https://techcrunch.com/2015/05/15/clunk/.

1 Access to this dataset has been generously granted by Justin Allen and Dave Ulrich, a principals of The RBL Group in Highland, UT to which this dataset belongs