



Valuation of Flexibility for Optimal Reservoir Operation

Erfaneh Sharifi¹, Hamid Bashiri¹, Arturo Leon¹, Yong Chen², Nathan Gibson³

¹Department of Civil and Environmental Engineering, University of Houston, USA

²Department of Applied Economics, Oregon State University, USA

³Department of Mathematics, Oregon State University, USA

ABSTRACT

This research presents a valuation study of flexibility in hydropower generation. Flexibility herein is considered as a stochastic process and is defined as the difference between the maximum hydropower energy generation capacity and the scheduled energy production plan to meet the demand and obligatory sales of electricity. To value the flexibility, this framework combines options theory and the Non-dominated Sorting Genetic Algorithm II (NSGA-II), while this paper focuses only on real option model. Options theory is used in this research to create a framework that can value the flexibility at various levels of risk for a multi-reservoir system. A Black-Scholes-like equation, which is a Partial Differential Equation (PDE), is solved numerically using standard solvers. The value of flexibility is estimated based on the solution of the Black-Scholes-type model. The results could help in assessing the value of flexibility over time and hence, guide allocating hydropower generation capacity from one period to another.

Keywords

Black-Scholes Model, Flexibility, Hydropower, Real Option, Reservoir Operation

1 Introduction

In this paper, we propose to use real option as a tool to evaluate operational flexibility in hydropower generation for Bonneville Power Administration (BPA). As a market clearing agency, BPA is required to ensure that energy supply matches demand at every time point. However, confronted with various sources of uncertainties in the system, BPA has to maintain operational flexibility to buffer unforeseeable energy inefficiencies either due to the uncertainties in the wind/solar energy or unexpected surges in demand. Thus the water reserve in ten dams operated by BPA becomes a major source of operational flexibility. We propose to evaluate the value of this operational flexibility using a real option model.

Real Option (RO) theory is an approach to assess the value of projects under uncertainty. RO is widely used in energy sector investment [Fernandes et al., 2011]. The uncertainty in hydropower projects (one of the most economically attractive types of renewable energy projects) is mainly a function of water inflows. RO can be used to hedge the generation of hydropower plants, or to identify flexibility for designing the plants [Cesena et al., 2013]. A couple of researchers have studied the use of RO for the design of the projects [Bockman et al. 2008, Fleten et al., 2007]. The Black-Scholes, equation which was originally developed in financial engineering, represents the dynamics of asset prices [Sharifi et al., 2014], and it is used widely in real

option analysis. Here, an American put option model adapted from the Black-Scholes model is developed for the real option analysis of the hydropower systems.

The optimal reservoir operation can be derived from optimizing objectives, subject to some feasibility and policy constraints. Depending on the type of the problem, algorithms used in reservoir optimization include Linear Programming (LP), NonLinear Programming (NLP), Dynamic Programming (DP) [Sharifi et al., 2016, Unami et al., 2015], Computational Intelligence (CI) [Ahmad et al., 2014], Harmony Search algorithm [Bashiri-Atrabi et al., 2015], and Genetic Algorithm (GA) [Chen et al., 2016]. Here a GA algorithm is used for the optimization of the reservoir system. The main objective of reservoir operation is to satisfy the demand while maximizing the revenue from electricity generation. Since the focus of this paper is on the real option model, only the results of the optimization will be an input to the real option model. By considering the flexibility to be the available water after meeting demands and obligations, the input to the real option model will be the available flexibility f .

2 Real Option Model

In this paper a real option framework is presented which uses available flexibility from an optimization model [Bashiri-Atrabi et al., 2017; Biswas, 2017] as input in order to value the operational flexibility. Using a real options framework, operational flexibility can be modeled as the American put option. Table (1) summarizes of the key elements used in financial option and how they are used in reservoir operation.

Let $f(t)$ be the flexibility in the system at time t . Shortage occurs if $f(t) < 0$. Let V be the value of h units of flexibility to be allocated. More specifically, if h is to be allocated at current period of time, the increased expenditure in some future time point due to this sale is denoted as $V(t)$. If h is allocated, the effective amount of h (i.e., the amount of h that has some value) at some future period of time denoted as \hat{h} , is given by

$$\hat{h}(t) = \max(-\max(-h, f(t)), 0) = \begin{cases} h & \text{if } f(t) < -h < 0 \\ f(t) & \text{if } -h \leq f(t) < 0 \\ 0 & \text{if } f(t) \geq 0 \end{cases} \quad (1)$$

When $f(t) < 0$, a shortage occurs. If a future shortage amount is greater than the allocated h , ($f(t) < -h < 0$), without allocating h at current time point, the amount of water saved can only be sufficient to buffer shortage up to h . The remaining shortage, $(-f(t) - h)$, has to be purchased from the market anyway. If the future shortage amount is less than the allocated amount h , ($-h \leq f(t) < 0$), without allocating h , we have saved too much water. Only part of h is sufficient to cover the shortage. The amount of purchase avoided is given by $|f(t)|$. If there is no shortage, $f(t) > 0$, there is no need to purchase on the market, and allocating h has no additional cost (i.e., saving h has no value).

The quantity \hat{h} is the amount of energy purchased at time t that could be avoided if h amount of flexibility were not used now. We call it the foregone option of using h at the current time point. This is in a format consistent with typical option theory. The payoff formula in a typical option pricing model $K - S(t)$ is replaced by $-\max(-h, f(t))$ in our model.

Financial Options	Notation in Financial Options	Notation in the RO for Reservoir Operation	Reservoir Operation
Payoff Function	$\max(K - S(t), 0)$	$\max(-\max(-h, f(t)), 0)$	Purchase Cost
Time of Expiry	T	T	Operation Days (Times to Expiry)
Interest rate	r	r	Assumed as Zero
Volatility of Price	σ	σ	Volatility of Flexibility

Table 1: Model parameters in financial option and reservoir operation

In this problem, the option value originates from BPA's (Bonneville Power Administration) obligation to purchase electricity if shortage occurs, which is driven by the quantity ($f(t)$) not the price. For simplicity, we abstract away from price stochasticity. However, because BPA is a federal power marketing agency in the Northwest and the region's major wholesaler of electricity, when BPA tries to sell/buy excess electricity, the price goes down/up. For simplicity, we assume a 30% price margin between the purchase and sales price for BPA. That is, the purchase price is normalized to be $p_{buy} = 1$ and the sales price is assumed to be $p_{sell} = 0.7$.

We assume $f(t)$ follows an Ito process [Karatzas and Shreve, 1991].

$$df = \mu_f dt + \sigma_f dz \quad (2)$$

where z is a standard Wiener process, μ_f is mean and σ_f is volatility. In particular, we assume that $f(t)$ follows geometric Brownian motion that is $\mu_f = \mu f$, and $\sigma_f = \sigma f$. This allows us to focus on the domain where a shortage occurs ($f \leq 0$).

From Ito's Lemma, we have

$$\mu_v = V_t + \mu f V_f + 1/2 \sigma^2 f^2 V_{ff} \quad (3)$$

$$\sigma_v = \sigma f V_f \quad (4)$$

Following Black-Scholes model, let's create a portfolio by buying one unit of the option V and selling θ units of flexibility f . The value of the portfolio is described by

$$dV - \theta df = (\mu_v - \theta \mu_f) dt + (\sigma_v - \theta \sigma_f) dz \quad (5)$$

In order to make it risk free, we set

$$\theta = \frac{\sigma_v}{\sigma_f} \quad (6)$$

Because it is risk free, the portfolio must earn the risk-free rate of return r (interest rate). This is the arbitrage free condition. So the average return on the portfolio should equal the return of putting the money in the bank:

$$(\mu_v - \frac{\sigma_v}{\sigma_f} \mu_f) dt = r(V - \frac{\sigma_v}{\sigma_f} f) dt \quad (7)$$

rearranging the terms gives

$$\frac{\mu_v - rV}{\sigma_v} = \frac{\mu_f - rf}{\sigma_f} \equiv \eta \quad (8)$$

or

$$\mu_v - rV = \eta \sigma_v \quad (9)$$

Substituting the results from the Ito's lemma, we have:

$$rV = V_t + rfV_f + \frac{1}{2} \sigma^2 f^2 V_{ff} \quad (10)$$

From Eq. (8), we have

$$\mu_f - \eta \sigma_f = rf \quad (11)$$

This gives:

$$rV = V_t + rfV_f + \frac{1}{2} \sigma^2 f^2 V_{ff} \quad (12)$$

We impose the following constraint and boundary condition:

$$V(g, t) \leq \max(-\max(-h, f), 0) \quad (13)$$

$$V(\infty, t) = h \quad (14)$$

Because BPA can choose to exercise this option at any sub-period of time, this is a free boundary problem in the sense that at every sub-period of time, there is a critical value of $f^*(t)$ above which, the BPA should exercise the option rather than holding the water for later periods. Below $f^*(t)$, the BPA should hold the water for later.

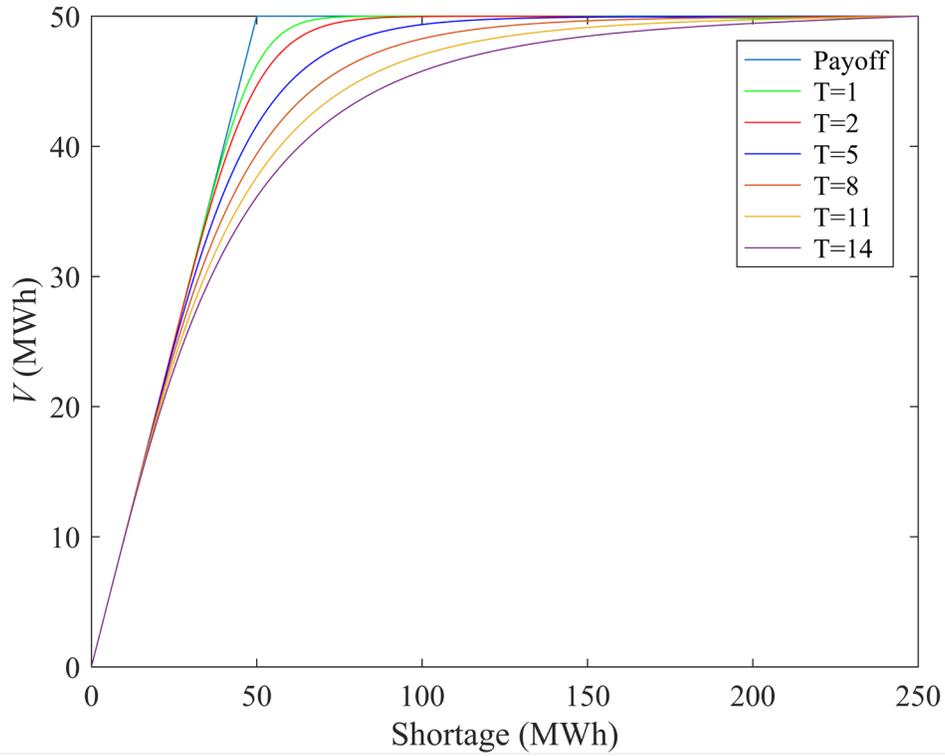


Figure 1 : Numerical solution of Eq. (12) with payoff function (Eq. 13) and boundary condition (Eq. 14) for $T = 14$ days, and $h = 50$ MWh for various values of σ

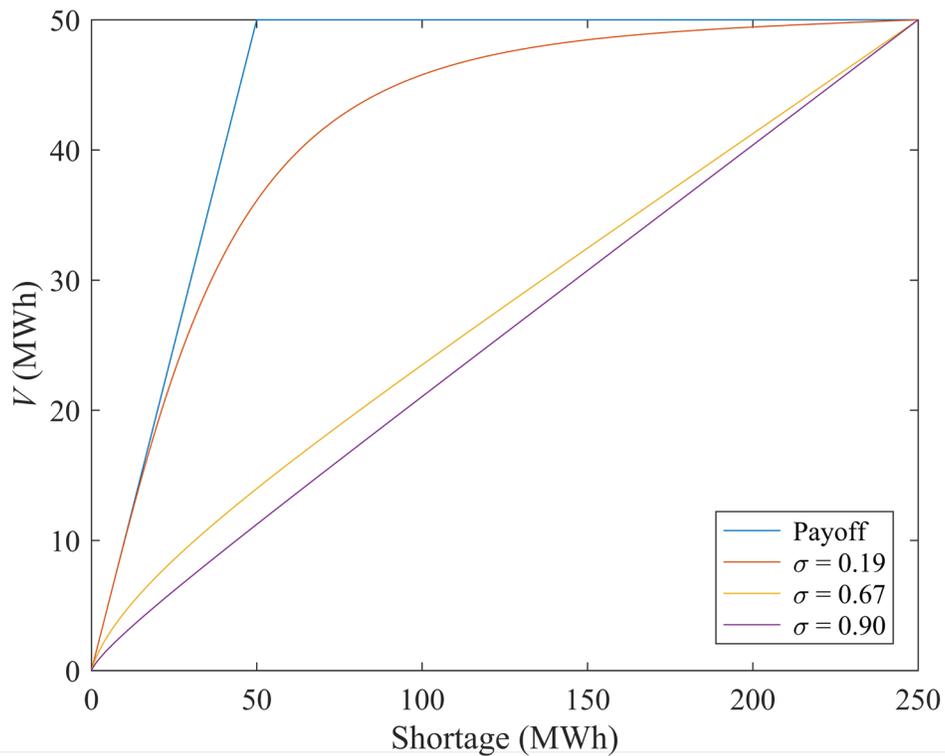


Figure 2: Numerical solution of Eq. (12) with payoff function (Eq. 13) and boundary condition (Eq. 14) with $\sigma = 0.19$, and $h = 50$ MWh for different operation days ($T = 1, 2, 5, 8, 11, \text{ and } 14$)

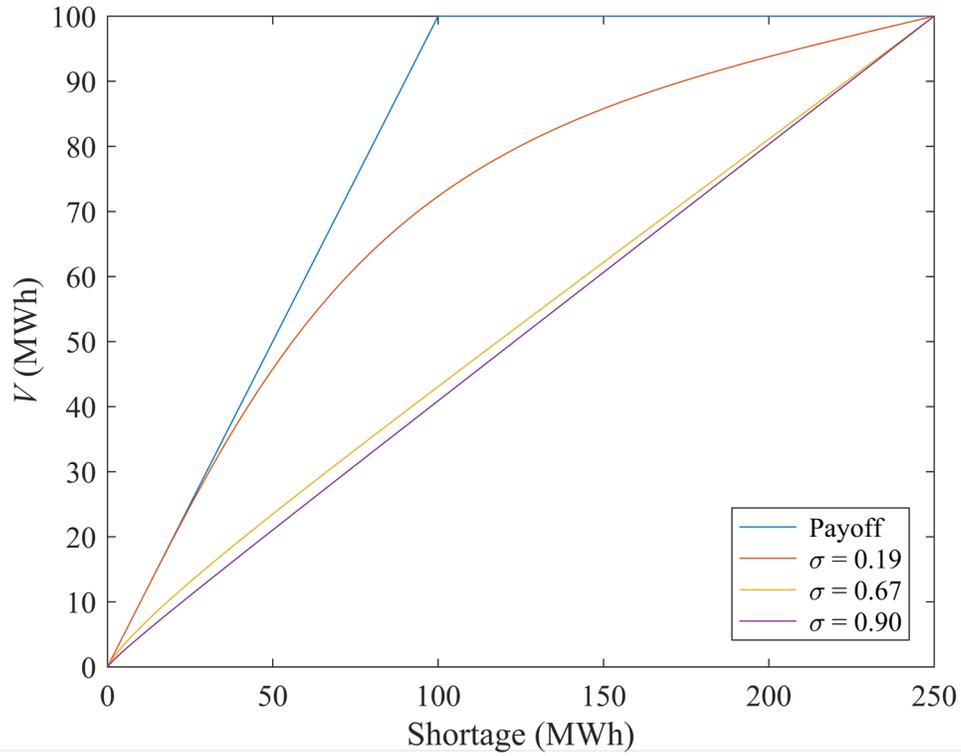


Figure 3: Numerical solution of Eq. (12) with payoff function (Eq. 13) and boundary condition (Eq. 14) with $T = 14$ days, and $h = 100$ MWh for various values of σ

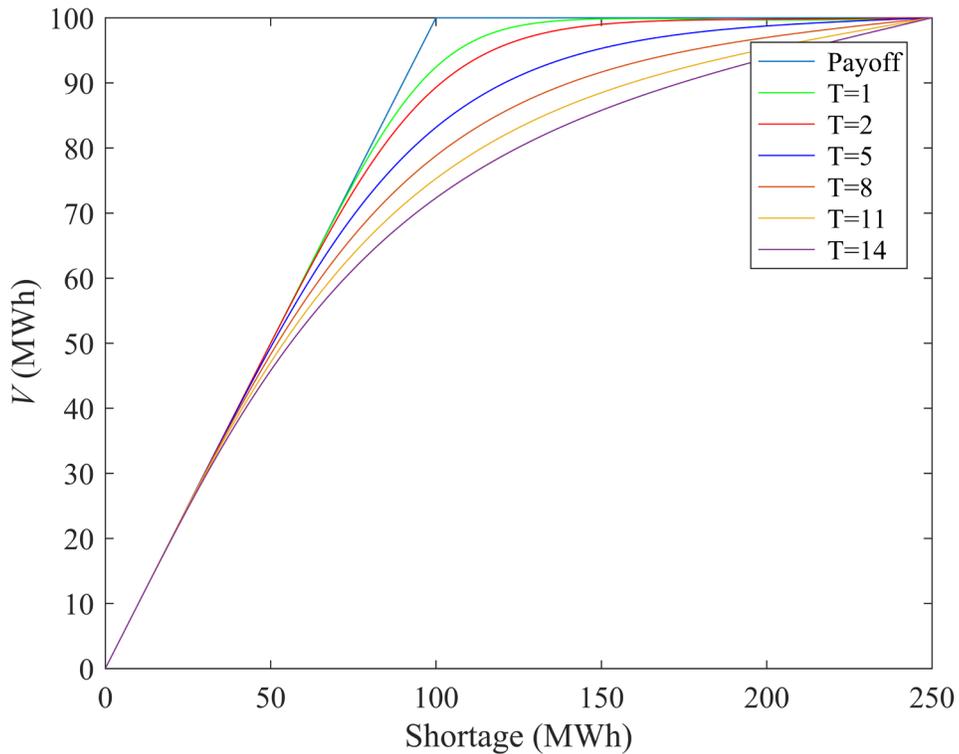


Figure 4: Numerical solution of Eq. (12) with payoff function (Eq. 13) and boundary condition (Eq. 14) with $\sigma = 0.19$, and $h = 100$ MWh for different operation days ($T = 1, 2, 5, 8, 11,$ and 14)

3 Results and Discussion

A Finite difference method with Rannacher smoothed Crank-Nicolson scheme is employed for the numerical solution of the PDE [von Sydow et al., 2015]. The system of linear equations is solved by the UL (Upper Lower) decomposition for the American put option over a finite domain.

We solved Eq. (12) with payoff function (Eq. 13) and boundary condition (Eq. 14) numerically for different parameters. This allows us to compare the results for different levels of uncertainty, and different operation days for the allocated flexibility. We consider $r = 0$ and two values of allocated flexibility ($h = 50$) MWh and $h = 100$ MWh. Then using these parameters the results are compared for different σ and T .

Numerical solution of Eq. (12) for 14 operation days ($T = 14$) and different σ values is shown in Figure (1). Here we allocated 50 MWh of flexibility ($h = 50$) then the option price (value of flexibility) is estimated for all cases. The blue line shows the payoff function based on Eq. (13). By increasing σ the difference between the payoff function and the option price increases. The fact that the option price decreases with σ in this example seems counter intuitive but is actually reasonable in our setting. In this example, small σ corresponds to the case in which future shortage will almost surely occur. That is selling h amount of flexibility in the current period will almost surely require a purchase of h amount of flexibility in the future. So the option value is close to purchase cost (payoff function). As σ increases, the probability of having no shortage increases, that is, the probability of no required future purchase increases. The option value for selling h in the current period therefore decreases.

Figure (2) illustrates the impact of expiration time T on the option value of flexibility. In this figure, the option values of flexibility given different expiration time are plotted with different colors. It is evident that the option value given a larger expiration time T is smaller than the option value for a smaller T . In other words, the difference between the option value and the payoff function, referred to as the time value of an option, increases with the expiration time T . This conforms with common intuition that as time approaches the expiration time (T decreases), the time value of an option decreases, referred to as the time value decay property in standard option theory. By considering different expiration time T in Figure (2) the option price

V is computed at a fixed σ . For example, option value when $T = 14$ is below the option value when $T = 1$.

Figure (3) shows the numerical solution of Eq. (12) when allocating 100 MWh of flexibility ($h = 100$) with different σ for the operation time of 14 days ($T = 14$). Figure (4) illustrates the value of flexibility for different expiration times when allocating 100 MWh of flexibility ($h = 100$). Again, different σ and T resulted in different V for the same available flexibility in current period (day 1). Solution of Eq. (12), payoff function (Eq. 13) and boundary condition (Eq. 14) gives us the solution $V(f_0, 0; h)$. If the revenue from using h in current period (day 1) exceeds $V(f_0, 0; h)$, that is, $p_{sell} V(f_0, 0; h)$ (where $p_{sell} = 0.7$), we should use h in the current period. This comparison will enable BPA to make the decision whether to sell flexibility at the current period of time or hold it for future use.

4 Conclusions

In this research a real option model has been introduced for the valuation of operational flexibility in hydropower generation. The real option model consists of a PDE, which governs the option value to be solved, and uses a payoff function and boundary conditions based on the American put option. A finite difference method with Rannacher smoothed Crank-Nicolson scheme is employed for the numerical solution of the PDE. Application of the model included the valuation of the flexibility, which here is shown by option price and is obtained for different uncertainty and different operation times. Further investigation of the optimal amount of current allocation will be the subject of our future research.

Acknowledgments

This work has been supported by the Bonneville Power Administration (BPA) under project Number TIP342.

References

- Ahmad, A., El-Shafi, A., Razali, S. F. M., and Mohamad, Z. S. (2014). Reservoir optimization in water resources: a review. *Water Resour. Manage.* 28, 3391–3405.
- Bashiri-Atrabi, H., Qaderi, K., Rheinheimer, D. E., and Sharifi, E. (2015). Application of harmony search algorithm to reservoir operation optimization. *Water Resour. Manage.* 29, 5729–5748.

- Bashiri-Atrabi, H., Sharifi, E., Leon, A., Gibson, N. L., and Chen, Y. (2017). Quantification of short-term hydropower generation flexibility. *Open Water Academic Journal*.
- Biswas, A. (2017). Bi-level flexible-robust optimization for energy allocation problems. Master thesis, Oregon State Univ. Oregon, USA.
- Bockman, T., Fleten, S. E., Juliussen, E., Langhammer, H. J., and Revdal, I. (2008). Investment timing and optimal capacity choice for small hydropower projects. *Eur. J. Oper. Res.* 190, 255–267.
- Cesena, E. M., Mutale, J., and Rivas-Davalos, F. (2013). Real options theory applied to electricity generation projects: A review. *Renew. Sust. Energ. Rev.* 19, 573–581.
- Chen, D., Leon, A., Gibson, N. L., and Hosseini, P. (2016). Dimension reduction of decision variables for multireservoir operation: A spectral optimization model. *Water Resour. Res.* 52, 36–51.
- Fernandes, B., Cunha, J., and Ferreira, P. (2011). The use of real options approach in energy sector investments. *Renew. Sust. Energ. Rev.* 15, 4491–4497.
- Fleten, S. E., Maribu, K. M., and Wangensteen, I. (2007). Optimal investment strategies in decentralized renewable power generation under uncertainty. *Energy* 32, 803–815.
- Karatzas, I., and Shreve, S. E. (1991). Brownian motion and stochastic calculus. *Springer, New York*.
- Sharifi, E., Unami, K., and Fujihara, M. (2014). Modeling alternation of dry and wet spells using the Langevin equation. *Vulnerability, uncertainty, and risk: quantification, mitigation, and management*, ASCE, Liverpool, 1994–2001.
- Sharifi, E., Unami, K., Yangyuoru, M., and Fujihara, M. (2016). Verifying optimality of rainfed agriculture using a stochastic model for drought occurrence. *Stoch. Environ. Res. Risk Assess.* 30, 1503–1514.
- Unami, K., Mohawesh, O., Sharifi, E., Takeuchi, J., and Fujihara, M. (2015). Stochastic modelling and control of rainwater harvesting systems for irrigation during dry spells. *J. Clean. Prod.* 88, 185–195.
- von Sydow, L., Joseph Hook, L., Larsson, E., Lindstrom, E., Milovanovic, S., Persson, J., Shcherbakov, V., Shpolyanskiy Y., Siren, S., Toivanen, J., Walden, J., Wiktorsson, M., Levesley, J., Li, J., Oosterlee, C. W., Ruijter, M. J., Toropov, A., and Zhao, Y. (2015). BENCHOP-The BENCHmarking project in option pricing. *Int. J. Comput. Math.* 92, 2361–2379.