Impact of Applying Visual Design Principles to Boardwork in a Mathematics Classroom

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Impact of Applying Visual Design Principles to Boardwork in a Mathematics Classroom

Jennifer Rose Canizales

A thesis submitted to the faculty of Brigham Young University in partial fulfillment of the requirements for the degree of Master of Science

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ABSTRACT

Impact of Applying Visual Design Principles to Boardwork in a Mathematics Classroom

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Though black boards and white boards have been a fundamental tool in the classroom for over a century, little research has been done on how to best design and present information using these boards. My study takes visual design principles and applies them to boardwork in a mathematics classroom to better organize and clarify the content. This research shows that students notice boardwork, have strong opinions on what makes boardwork good, and that the application of design principles on boards has a significant impact on students and the teacher. Students felt their cognitive load was lightened and that they were receiving higher quality instruction and the teacher felt that using the design principles during the planning stages of the lesson reduced their cognitive load while teaching. Findings from this study can inform teachers on best practices for organizing their boardwork, serve as a template for professional development workshops, and inform curriculum for pre-service teacher education programs.

Keywords: boardwork, cognitive load, gestalt theory, making connections, mathematics instruction, mathematics education
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CHAPTER 1: INTRODUCTION

Nearly every person these days has experienced some frustration with the design of software, websites, or printed materials. The online checkout is not where you would expect it to be, the information you are looking for is hidden within tabs and tabs of other topics, or big news is glossed over because it was not given a prominent location within an article. These frustrations can be felt everywhere people consume information, including the classroom. A teacher writes something on the board, but the student does not know how important that idea might be in relation to others. One element of a concept is addressed on the far left of the board, but the other half of the concept is addressed on the far right making it difficult to make connections. Or the chosen mathematical representations are not helping students understand the material properly. Having experienced all these frustrations myself, the question became: Are there principles of website design or printed material layout that teachers could draw on to better organize their boards?

Boardwork can mean a variety of materials in the classroom. For the purpose of this paper, I am taking a broad view of boardwork by defining it as any public record of mathematics and mathematical thinking by either the students or teacher that is accessible to everyone. This definition would include work done on white boards, black boards, and smartboards, and through overhead projectors and document cameras. This definition would not include worksheets handed out to all students in the class, however, because what each student writes individually on the paper is not accessible by everyone.

As a student myself I have struggled to interpret the boardwork of some of my professors, especially in lecture type mathematics classes. Many of my teachers in these classes seem to have put little to no thought into the layout of their board space before class. In fact, the purpose
of the board to them seems to only be for writing up theorems and working through example problems. The most common practice I have witnessed consists of:

- The professor, starting on the far upper left-hand side of the board, writes out the theorem/problem in lines, like when reading a book.
- They go about arms reach before returning to the left-hand side of the board to start writing on the next line down.
- Once they have filled all the space in that column, they draw a vertical line to separate out the next section of the board, repeating this process several times until they reach the far lower right-hand side of the board.
- At this point they begin erasing the columns on the left-hand side and repeat the whole process over again until the end of lecture.

In these types of instances, it feels as though a well typed out Power Point would have sufficed in exchange for the poor and ill organized handwriting on the board. At least then the Power Point could be published for later review by students but since the work earlier in the lesson is almost always erased, it makes it difficult for students to look back on the entire lesson and make connections across the content.

Not only this, but students often rely heavily on the boardwork since purely audio lectures can be hard to remember. Think back to when you were a student and how hard it was to comprehend everything a teacher said at once. Lessons often move very quickly and making sense of what a teacher is saying can be difficult. When a student is having a hard time understanding what they are hearing they likely move to the boardwork to try and make sense of what is going on but if the boardwork is not well organized, or the steps of the example are not laid out clearly, then questions, and repeats of the same question, often get asked. By being more
deliberate in our boardwork we can help students make better sense of the mathematical concepts we are teaching.

There are, of course, some elements of boardwork that many teachers know are helpful for students: writing legibly, underlining important ideas, or color-coding information based on the topic. But there are other areas of visual design that I think can be utilized by teachers as well. In the field of visual design, designers commonly use what are called Principles of Design to guide their design and composition decisions. Principles of Design are tools that designers use when creating a composition such as contrast, balance, and emphasis. These principles help make designs more interesting and engaging for the viewer as well as cleaner, clearer, and aesthetically pleasing.

While little research has been done on the application of design principles with white or black boards, more research has been done on the use and design of technologies such as Interactive Whiteboards (IWB) and Educational Software (Alrwele, 2017; Davison & Pratt, 2003; Miller, 2011; Moller et al., 2012; Yong et al., 2016). The use of these types of technologies has certainly increased over the decades but the use of whiteboards and blackboards are a staple in the classroom. Billman et al. (2018) even highlighted the benefits of physically representing mathematics in front of students as opposed to revealing ready-made slides using presentation software such as Power Point. I believe research on the application of visual design principles to boardwork could be of great pedagogical value.

In this study, I seek to build on existing research on the design of educational technology by exploring how design principles can be implemented with the medium of boardspace. I will also study which specific design principles students notice and appreciate the most and how the implementation of these design principles influences student’s learning experience by measuring:
student recall ability for information from the lesson, influence on what goes into their notes, and influence on students’ perceptions of ease. I hope that by exploring these questions this research can better inform both teacher’s preparatory decisions while lesson planning as well as in-the-moment decisions when using their boardspace.
CHAPTER 2: BACKGROUND

Literature Review

Little research has been done on teachers' use and design of board space in the classroom. Though this area of study is more common in Japan, even a quick google scholar search of “how teachers use blackboard space in Japan” produced few promising results. Only a few books, to my knowledge, and some articles exist outlining the effectiveness of boardwork in the mathematics classroom ((Baldry et al., 2022; Ninomiya & Kamoda, 2010; Nishio & Kubota, 2009; Soma, 1997; Takahashi, 2006; Takahashi, 2021; Yanase, 1990; Yoshida, 2009). In fact, much of this list of references (some of them written in Japanese) did not come from a search on google scholar but came from Professor Hiroyuki Ninomiya, an author of a book on blackboard use (Ninomiya & Kamoda, 2010). The research that has been done shows how important board space planning ought to be when lesson planning.

Japanese Research on Boardwork

Boardwork planning, or keikaku, is a common and important practice in Japan. Many focus on bansho which is the act of writing on the board. Kuehnert et al. (2018, p. 363) described bansho as “the intentional use of board space for facilitating student learning.” Bansho is a highly developed skill, and planning for effective use of the board is an important part of teachers’ professional development. Across Japanese research on boardwork there are several ideas that have been presented as to why boardwork planning is important in the classroom. According to this research, proper boardwork can:

- help students understand the learning process, (e.g., Baldry et al., 2022)
- organize student thinking, (e.g., Yoshida, 2009)
- serve as a model for student notes, (e.g., Ninomiya & Kamoda, 2010)
• serve as a summary of the lesson, (e.g., Takahashi, 2021)
• and help students make connections across content and student work. (e.g., Baldry et al.,)

There are several articles which mention all five of these ideas (Nishio & Kubota, 2009; Soma, 1997; Takahashi, 2006; Yanase, 1990; Yoshida, 2009). Of these ideas the most shared are the latter three – that boardwork can serve as a model for student notes, serve as a summary of the lesson, and should help students make connections across content and student work. I will elaborate on these three ideas in the following paragraphs.

Boardwork planning is important because what goes on the board is often what goes in students’ notes. Takahashi (2021) noted the importance of planning and organizing boardwork because “what is written on the board will also serve as the model which students can record in their notebooks” (p. 26). Because the content of the board is what goes into students' notes, it is important for us, as educators, to be mindful of what we put on the board, how it is organized, and what is emphasized. Yanase (1990) adds that while boardwork should serve as a model for student notes it shouldn’t be so detailed that students just copy from the board. Boardwork needs to give enough room for students to also describe their own ideas and interpret the main concepts from class in their own words.

Boardwork can also serve as a summary of the day’s lesson and allow students to look at the board at the end of the day and see a good synthesis of everything they learned that day. Yoshida (2009) said, “by looking at the blackboard during and after the lesson, students can gain a lot of information, which will go a long way to help them make sense of what they are learning” (p. 97). Without proper planning for the organization of the boardspace, creating a well-formed synthesis of the lesson from the boardwork would be difficult.
Lastly, with proper planning, boardwork can help students make connections across content and student work (Baldry et al., 2022; Greifenhagen, 2014; Kuehnert et al., 2018). When putting content and student work up on the board there are two important factors: how it is represented and how it is organized (Baldry et al., 2022; Kuehnert et al., 2018; Yanase, 1990). In order to make content and student work easy to compare the teacher needs to plan the best way to represent the work. A common practice in Japanese boardwork is to place representations side by side to easily draw connections between them (Takahashi, 2006). Color will often be used as well to highlight similarities and explicit discussions held to find the connections (Baldry et al., 2022). Similarly, if there are multiple examples of student work to be showcased it is also important to consider how to sequence the student work. If the chosen student examples get progressively more abstract one might consider beginning with the most concrete student work and progressing towards the most abstract work (Baldry et al., 2022; Takahashi, 2021).

Helping students make connections is a critical part of mathematical student learning and educators in the United States have recognized this as well (Smith & Stein, 2018; Stigler & Hiebert, 2004). One example is NCTM’s *Principles and Standards for School Mathematics* (2000) which includes in each section a chapter on connections. Smith and Stein’s (2018) 5 *Practices* builds on these NCTM standards, describing connection making in a mathematics classroom in two ways: (1) connecting previous knowledge to new knowledge and (2) drawing connections across multiple representations. They say that making connections may be the most challenging of their 5 Practices. These two ways of making connections align well with the Japanese practice of using boardwork to aid students in making connections.

Ideas like using boardwork to help students make connections, be a model for student notes, and serve as a summary of the lesson are communicating how to use board space most
effectively, how students can benefit from the board’s proper use, and what the board’s purpose should be in the classroom. Where Japanese research falls short, however, is that there is not much discussion about the visual aspect of board space and how to actually implement these practices, like techniques for organizing mathematical information to summarizes the lesson and present information in a clear and easily comprehensible way. The few visual aspects of board space that these papers do mention are using color to highlight similarities and using proximity when planning layout to help students draw connections between representations (Baldry et al., 2021; Kuehnert et al., 2018; Ninomiya & Kamoda, 2010; Soma, 1997).

It is important to note that these Japanese studies are different from traditional studies in mathematics education in the US. In Japan their research on boardwork is not of the form of traditional research articles but more commonly discussed in books. The participants of the studies discussed in these books are often the students of the authors themselves and these Japanese articles on boardwork are more practical than typical US research articles. Their studies are much more practitioner focused. I do not, however, believe that this diminishes their value in the field of research and believe that we, as mathematics education researchers in the US, need to be more accepting of research from other countries even when their techniques and methods are different from our own (Adler et al., 2005).

**Research on the Visual Aspect of Board Space**

I have found only a few studies (DeLeeuw et al, 2021; Friedkin, 2016; McLaughlin, 2016) that have studied the visual design of board space. In a study of boardwork preferences of students enrolled in an English discussion class McLaughlin (2016) examined issues such as maximalist vs minimalist boardwork, if specific locations are noticed more on the board, and other factors like size, symbols, pictures, color use, and compartmentalization of ideas. He found
that students prefer minimalist boardwork, as well as set color schemes and sectioned off areas of the board and that the middle bottom portion of the board draws students’ eyes most.

Friedkin's (2020) article on lesson studies looked at revising student work so that the resulting representations are most true to the student's thinking and appropriate to present to the whole class. These decisions were debated and made while performing a mockup of the lesson, before teaching it to the entire class; This way when the lessons were taught to the students the teacher already had a good idea of what student solution paths might be presented and how to best represent those ideas so that everyone could have equal access to the conceptual ideas therein. I believe mathematical representations, such as diagrams, graphs, models, formulas, equations, expressions, etc., are very important in a lesson and they need to be thoroughly thought about beforehand, so students have the best chance of grasping the concepts at hand.

DeLeeuw et al. (2021) makes an argument that in order to help students see important mathematical structures and instances of repeated reasoning we, as teachers, need to do more than just verbally identify similarities in problems or leave work un-erased on the board. It requires us to plan how we will use the visual record to highlight those important mathematical structures. By doing this kind of planning we can help students better recognize patterns through our mathematical representations on the board. The findings of these three studies have helped inform my decision of which visual design principles to focus on in my study, these will be discussed more in the theoretical framework later.

**Research on Visual Design Within Other Mediums**

Though there is little research on the visual design of white/black board space in the classroom, there is more research in relation to other mediums such as Interactive Whiteboards, Educational Websites, and Infographics. Research on Interactive Whiteboards mainly focuses on
the affordances of the dynamic aspect of this tool, but one study by Davison and Pratt (2003) talks about the visual affordances Interactive Whiteboards offer. They surveyed teachers who use Interactive Whiteboards and found they commonly mentioned that Interactive Whiteboards offer clearer text, better images and representations, and allow you to easily manipulate size and color to add emphasis and organize ideas.

Some research has also been published on the effects of visual design on learning when using Educational Software. Miller (2011) looked at the relationship between visual aesthetic and learning and found that websites with good aesthetic interface design significantly decreased learner cognitive load, and increased learner satisfaction and willingness to continue to use the website. Miller (2011) did not, however, identify which principles of visual design contributed most to these results and in fact called for further research to be done.

Yong et al. (2016) and Moller et al. (2012) took up this challenge of identifying specific design principles that contribute most to learning in education websites. Yong et al. found that the visual principles of balance, harmony, and variety had the strongest correlation with a classical aesthetic, meaning it is “pleasant, clear, clean and symmetric” (p. 320). This finding suggests that by utilizing these three principles in visual design, the results will have a positive impact on learning, though they conclude their paper stating that such a statement requires further study. Moller et al. (2012) found that the principles of similarity and proximity were strongly linked to a viewer's acceptance and recommendation of a website. These studies suggest that there is a relationship between visual design principles and ease of learning, but more research is also needed to show this is true.

Another area of research that links learning to visual design is Infographics. Infographics are visual images used to represent data, information, or knowledge. They utilize a mix of
images, text, and design principles such as color, line, or contrast to present information quickly and clearly. There has been much research already on using infographics in education (Alrwele, 2017; Bicen & Beheshti, 2017; Kaya-Hamza & Beheshti-Fezile, 2017; Naparin & Saad, 2017). The large consensus being that infographics serve as aides in the learning process. Alrwele (2017) used both an achievement test and questionnaire to assess students’ learning experience with infographics and found that the experimental group experienced significant improvement. They state in their study that infographics:

Captured [students’] attention, kept them engaged, helped them to easily understand complex information and determine key words and concepts, helped them to filter out irrelevant data, organized information into logical groups, increased their understanding of hidden relationships, facilitated the connection of new information to old information, improved their critical thinking, [and] facilitated retention. (p. 111)

The creation of infographics commonly utilizes visual design principles to make an aesthetically pleasing image for learning (Kaya-Hamza & Beheshti-Fezile, 2017). This suggests that the use of these design principles on boardwork could have a similar effect and make boardwork a more effective tool to increase interest and learning.

The Teaching Gap

While analyzing the data from my study I found that some of my findings support or can be explained by findings from The Teaching Gap (Stigler & Hiebert, 2009) so I feel it necessary to report on this literature as well. Stigler and Hiebert’s book reports findings from the data gathered in the Third International Mathematics and Science Study (TIMSS). This study gathered video, survey, and interview data from the fourth and eighth grade levels of mathematics in the US, Japan, and Germany. I will focus on findings between the US and Japan.
here. While this book is not focused on boardwork, they make mention of the public records that teachers in the US tend to use in contrast to Japan and report on the teaching culture and general teaching styles between teachers in these two countries.

From their data these authors found that teachers in the US and Japan have different views on what the purpose of public record is and thus tend to use public record differently in their classrooms. For the Japanese a common goal in using public record is to “provide a record of the problems and solution methods and principles that are discussed during the lesson” (Stigler & Hiebert, 1999, p. 74). For this reason, the most popular form of public record in Japan is the use of chalkboards since this form of public record is large enough to accommodate multiple solution methods and, when planned well enough, can capture all the concepts of an entire lesson with minimal erasing. Stigler and Hiebert state that a common US goal in using public record is to “focus students’ attention” (p.74), so the most common forms of public record are chalkboards and overhead projectors because chalkboards and overhead film can be filled then erased or covered to focus student attention on a single concept or activity. The authors noted that a likely corollary to these different goals in using public record explains the differences between the US and Japan in how connected the different segments of their lessons are. Their data analysis showed that within a single lesson only 45% of US lessons had connected segments compared to 92% in Japan, and Japan scored 6 times higher in how interrelated their lessons were compared to the US (p. 63–64).

These findings can be explained by the different teaching cultures and practices in the US and Japan. They analyzed video recordings of lessons in the US and Japan and found evidence for a clear pattern to lesson structures in both countries. Japanese lessons tend to start with a short review, followed with the presentation of a difficult question for the day, then students are
given time to work individually and then in small groups to answer the question, the teacher monitors student work and selects students to present their work, student work is presented, connections made across the various solution methods, and at the end of class the lesson is summarized and students start on their homework (Stigler & Hiebert, 1999).

Lessons in the US also started with a short review but take a much different path. The teacher then demonstrates how to solve a certain kind of problem, students are given a worksheet with many more problems of a similar type and varying difficulty, the teacher walks around and monitors student work, intervening when they begin to struggle, and class ends with time to work on their homework (Stigler & Hiebert, 1999). This lesson structure and boardwork approach in the US dates back to the 1890’s. Black board teaching manuals have been recovered from this period and instruction on teaching arithmetic often used a “demonstrate then imitate” approach where “the end goal of these methods was memorization, achieved through students’ repetition of corrected, acceptable imitations of the teacher’s model” (Wylie, 2012).

Stigler and Hiebert (1999) were somewhat surprised to see that a country as large and diverse as the US still showed a clear pattern to lesson structures. From these findings they draw the conclusion that teaching is a cultural activity with cultural scripts and these scripts “not only guide behavior but they also tell participants what to expect” (p. 86). They share an example where a teacher in the US tried to implement the cultural script for Japanese lessons in his own classroom. He started class with a review and presented a challenging problem to the students. Only the students in his classroom did not respond to the prompt in the same way the Japanese students had. They played their traditional role and waited to be shown how to solve the problem first. This shows that since teaching is a cultural activity it is incredibly difficult to change. They speculate that even “if all teachers in the US started using the chalkboard tomorrow, rather than
the overhead projector, teaching would not change much” (p. 97). Instead, the chalkboard would be used just as the overhead projector currently is – “to catch and hold students’ attention” (p. 97).

Stigler and Hiebert’s (1999) findings from *The Teaching Gap* show that teaching is a cultural activity and because of that it is a complex process with many facets making it often difficult to change. The teaching culture within each country seems to be a large indicator for student success so if we want to improve how our students learn we need to change the way we teach. Their findings also suggest that teachers’ attitudes toward and beliefs about the purpose of boardwork are a part of this culture. These beliefs and attitudes condition students to respond to the board in certain ways. Students in Japan see the board as a public record of what has been discussed in class while students in the US see the board as a place to view and copy down example problems.
Theoretical Framework

Design and Gestalt Theory

In visual design there are a few important terms that are commonly used that I need to define first. I am going to briefly define these ideas here so that the reader can see the overall picture but more detailed definitions of the most important components in relation to board space design will be given later in this section. Elements of design are the tools used for creating art such as color, size, texture, shape, etc. (Taheri, 2020). For example, in Figure 1 some of the elements of the design are the varying shades of blue and the circular shapes. Principles of

Figure 1
Illustration of the Elements and Principles of Design

---

1 According to APA guidelines normally figures in text are placed at either the top or bottom of the page. However, according to my own design principle of proximity, representations and the text describing them should be placed in close proximity, so I am choosing to place my figures closest to where I discuss them in the paragraph.
design are how you use the tools to create art such as contrast, emphasis, hierarchy, etc. (Chapman, 2018a). Figure 1 uses the varying shades of blue to create contrast between the layers of the Venn diagram and uses the size and weight of the text to create hierarchy between the title, sub description, and list of elements/principles in each region. Principles of Gestalt refers to the laws of human perception such as proximity, similarity, common region, etc. according to Gestalt Theory (Chapman, 2018b). Again, using Figure 1, the principle of common region is demonstrated in how the text within each circle is seen as a group by the viewer. Finally, Layout and data visualization are frameworks on which these elements and principles are applied. For example, the nested circles in Figure 1 are both a layout and data visualization choice.

Gestalt Theory consists of principles/laws about human perception to describe how we group similar elements, recognize patterns, and simplify complex images when we perceive objects. Gestalt Theory says that the human brain will attempt to simplify and organize complex images or designs that consist of many elements by “subconsciously arranging the parts into an organized system that creates a whole, rather than a series of disparate elements” (Chapman, 2018b, para. 5). Just as visual design principles have been studied in fields like web design and infographics, Gestalt Theory has also been studied with web design (Graham, 2008; Kapllani & Elmimouni, 2020), infographics (Lu et al., 2020; Steyn et al., 2018), and a third field called instructional design (Moore & Fitz, 1993; Smith-Gratto & Fisher, 1999). Instructional design is the design of instructional material for the acquisition of knowledge or information. The purpose in applying Gestalt Theory principles to these different mediums is to observe what improvements the viewers perceive with their application. Many of these studies found that viewers perceived the revised material to be clearer, easier to comprehend, and easier to use (Kapllani & Elmimouni, 2020; Lu et al., 2020; Moore & Fitz, 1993; Steyn et al., 2018).
Figure 1 shows how all these elements and principles of visual design and Gestalt Theory nest within each other. Though the design world does not have one set list of the elements and principles of design (Adam, 2013) those used in Figure 1 are a good summary of some of the most common elements and principles. At the core of design are its elements, principles of design utilize those elements, principles of Gestalt Theory are how the viewer perceives the art, and these elements and principles are applied to layout and data visualization. I will not use all these principles in my study but have chosen a select few I believe from my research and experience are most applicable to mathematical boardwork.

Why Visual Design

Now that I have reviewed what visual design is I want to briefly address why visual design is useful in general. According to Sandra Rendgen (2012) in her best-selling book *Information Graphics*, graphics are anything visual that communicates information. That information could be data, propaganda, news, a narrative, opinions, emotions, mathematics or much more but at its core visual design is all about communicating information. And information needs organizing because the way in which data is organized is “the critical factor in enabling viewers to understand it quickly” (p. 96). This is the purpose in using visual design principles, to allow information to be understood quickly and easily. When visual design principles are applied to complex systems like data, infographics, or web design the goal is to help viewers navigate the space with ease, provide greater clarity, and make connections easily and quickly (Alrwele, 2017; Berge, 1998; Rendgen, 2012). Jorge Rayna (2013), a leading educational technology consultant also said that “design and aesthetics have a profound impact on how users perceive information and learn, judge credibility and usability, and ultimately assess value to an
experience” (p. 28) So not only does design organize and clarify information but ultimately impacts the viewer’s entire perception of their experience with the content.

**Five Principles of Board Space Design**

The framework for this research heavily centers around my chosen visual design principles that I will be applying to teachers’ boardwork in the classroom. As Miller (2011) points out “aesthetic design, […] is fundamentally established in context” (p. 317). Thus, design elements or principles that work for one medium may not work for another. I minored in Graphic Design as an undergraduate student and worked as a graphic designer for 3 years. From this experience and my literature review I have identified these five principles as the most relevant to boardwork and chosen to study their application: **Visual Hierarchy, Grouping Perceptions, Layout, Representations, and Repetition.** I will define these principles next.

*Visual Hierarchy* is a way of identifying what is most important in a composition and separating out all the different parts. You can accomplish this using contrast, scale, and color. This principle helps guide our eyes when lots of information is presented to us in one image. As seen in Figure 2, the image on the left utilizes design elements such as size and weight to tell us

**Figure 2**

*Example of Hierarchy*
what things in the image are most important and where our eyes should go first, second, and third. The image on the right however does not utilize any of these elements which makes knowing where to look difficult as the message becomes lost in the uniformity. Using contrast, scale, and color to create *Visual Hierarchy* in boardspace will help students identify the most important ideas in the lesson, which things they should be copying down in their notes and which things they should focus on.

Next, we have the *Grouping Perceptions* which are the Gestalt Principles of Similarity, Proximity, and Common Region. These three principles of Gestalt Theory all have to do with how we group objects visually. They tell us that things that look similar, are in proximity, or bound within the same region will be grouped together in our minds and we will automatically associate them together and look for connections between them. These are principles of Gestalt Theory and there are many interesting things your brain notices subconsciously when it looks at an image like this in Figure 3. One, because of proximity it notices there are two main groups, the one on the left and the one on the right. Two, it starts to look for patterns within those groups.

**Figure 3**

*Example of Grouping Perceptions*
like the columns because they are also in proximity and of similar shape. It also groups the items of similar color together (orange vs gray), even though they are not all the same shape. Lastly, your brain also groups the circle and square together at the bottom because it is bounded within the blue rectangular region, even though they are different shapes and in different groups. When discussing two ideas in a lesson that are related the teacher could write them in the same color, place them in proximity or enclose them within the same region with boxes to help students recognize that the two ideas are connected.

Layout is another principle I have chosen for my study. While these previous principles are more “in the moment” applications, layout is something that must be planned beforehand. Layout consists of deciding beforehand which space to dedicate to certain ideas. The example in Figure 4 shows the layout some teachers in Japan use daily when writing on their boards. They have dedicated space for stating the problem, showing multiple examples of student work,

Figure 4

Example of Layout

(Kunimune, personal communication, January 29, 2021)
recording comments from the class discussion, and writing up the final answer and conclusions.

Pre-planning the layout of the board will help ensure that all the important mathematical concepts fit somewhere on the board so that at the end of class students have a good summary of everything that was discussed that day. Furthermore, pre-planning the layout could also help the teacher think about how they would use the other principles like *Visual Hierarchy, Similarity* and *Proximity* within their boardwork.

Next, *Representations* are not so much a principle of design but an entire subfield of graphic design called Data Visualization which utilizes principles like those mentioned above to create visual representations of data that presents the information in a clear and concise format. Figure 5 is a simple statistical example. If you were trying to compare and contrast two data sets, *Figure 5 Example of Representations*

![Example of Representations](image)

would you use pie charts or bar charts? Bar charts is the clear choice. Even though the colors in the pie charts are in the same order, the varying sizes make it difficult to compare one value to the other. The bar charts on the other hand show the differences between these two datasets more clearly and makes it easier to decipher the differing trends. *Mathematical Representations* are often thought of as graphs but could also be diagrams, models, formulas, equations, expressions, etc. *Mathematical Representations* in lessons, similarly, need to be thought out beforehand to best represent the concepts at play so they are clear and comprehensible to the students.
Lastly, the principle of *Repetition* is the repeated use of an Element or Principle of Design. This principle is commonly used when branding an entity like Harvard for example. A well-known University, Harvard has a very strong brand identity; their merchandise, website, and printed materials are easy to identify because of their use of *Repetition*. Figure 6 (next page) shows a portion of Harvard’s graphic identity guide and shows on the left all the ways they restrict the use of elements to create a consistent brand identity: fonts, colors, logos, and format of digital applications, stationery, and merchandise. The right side also shows some of their standard “Do’s and Don’ts” of using their logo. The idea of using *Repetition* to create identity can also be applied to boardspace. Teachers can create *Repetition* by setting a color scheme for their boardwork, always using the same colors to highlight, emphasize, or box off areas of the board, or by using a consistent layout each day as mentioned above with the Japanese lessons so that students know which areas of the board will generally hold certain types of information.

I found support for the selection of these principles through my literature review. Friedken (2020) and Deleeuw et al. (2021) make compelling arguments for the importance of strategically thought-out mathematical *Representations*. Friedken’s research looked at teaching mockup lessons to examine different student thinking that might occur in order to determine the best way to represent this thinking to the class and Deleeuw et al. examines ways to present student thinking so that connections can easily be made across the *Representations*. Mclaughlin (2016) found that students prefer minimalist boardwork, set color schemes, and sectioned off areas of the board. These findings are very similar to the principles of *Layout, Repetition,* and *Grouping Perceptions*, respectively. And lastly Japanese articles such as Takahashi (2021), Yoshida (2005), and Nishio and Kubota (2009) demonstrate the more common practice of *Layout* planning for the board and discuss why planning boardwork is so important.
**Figure 6**

*Example of Repetition*

<table>
<thead>
<tr>
<th>GENERAL GUIDELINES</th>
<th>DO:</th>
<th>DON'T:</th>
</tr>
</thead>
</table>

Using these design principles as my framework has helped me identify specific techniques that teachers can use when both planning their boardwork before a lesson and making in-the-moment decisions while using the board during the lesson. I studied how the use of these principles impacted students’ learning experience by measuring: student recall ability for information from the lesson, influence on what goes into their notes, and influence on students’ cognitive load and perceptions of ease.

**Cognitive Load Theory**

Along with the influence of Gestalt Theory (GT), this research is also influenced by Cognitive Load Theory (CLT). CLT looks at the way in which instruction should be presented to best maximize performance and is based on our knowledge of cognitive architecture, particularly working memory and long-term memory, and their limitations. CLT believes that by decreasing the amount of information students need to process in their limited working memory they can better form schemas to understand and learn new information (Kalyuga et al., 1999; Kirschner, 2002).

Take multiplication for example. For most people multiplying two single digit numbers is simple enough and can likely do this in their head, like 6 times 7 because there is only one thing to keep track of. Some can even handle multiplication of two double digit numbers like 15 times 30 since the numbers are multiples of 5 and there are only about 5 steps involved. But try multiplying 23 times 764 in your head. A problem like this has about 10 steps to keep track of, when performing a traditional algorithm, which makes performing the multiplication difficult without the aid of writing tools. While research on the capacity of working memory has varied over the years, most researchers tend to agree that humans can process around 4 to 7 pieces of information at one time in their working memory (Cowman, 2001; Miller, 1956). Some theories
within CLT believe that limiting the amount of information one needs to process will free up space in working memory to better process and understand new information (Pass et al., 2003).

Within Cognitive Load Theory there are three types of cognitive load recognized: intrinsic cognitive load (ICL), extraneous cognitive load (ECL), and germane cognitive load (GCL). I will briefly define each type of cognitive load below, but this study mainly focuses on extraneous cognitive load moving forward. As defined by Klipesch et al. (2017) ICL is “the load resulting from the inherent complexity of the learning task” (p. 2), ECL is the load “caused by the instructional design of the learning material” (p. 2), and as defined by Sweller (2010) GCL is the “working memory resources that the learner devotes to dealing with the intrinsic cognitive load associated with the information” (p. 126). According to Hogg (2007) good instructional design decreases ECL but increase GCL. I look to do the same, so the nature of my research focuses on ECL and GCL since I am looking to decrease student’s ECL caused by the boardwork design and by default increase student’s GCL.

While no research that I could find has studied the effects of boardwork design on students’ ECL, there is specific research on instructional and mathematical representation design and their impact on students’ cognitive load. Instructional design is the design of instructional materials for the acquisition of knowledge or information. This has been most heavily researched when applied to educational instructions both in print (De Jong, 2010; Sweller, 1994) and on websites (Richardson et al., 2014; Tabbers et al., 2004). These studies look at how the design of educational instructions can decrease the ECL of students and increase the GCL. Other studies, more specific to STEM education, have found that learning with visual representations resulted in higher learning performance and lower cognitive loads (Yum & Paas, 2015) and that
incorporating text into representations and presenting multiple representations side by side lowers ECL and promotes GCL (Cook, 2006; DeLeeuw et al., 2020; Richland et al., 2017).

Richland et al. (2012) said that “any kind of intervention that reduces working-memory demands and helps people focus on goal relevant relations will aid learning of effective problem schemas and thereby improve subsequent transfer to new problems” (p. 194) By applying my list of design principles – Visual Hierarchy, Grouping Perceptions, Layout, Representation, and Repetition – to boardwork I believe working-memory demands will be reduced and students will be able to focus on goal relevant relations, thus lightening their ECL and increasing their GCL allowing them to invest more mental effort into complex learning processes.

**Research Questions**

From this research and my knowledge of these design principles, Gestalt Theory, and Cognitive Load Theory along with my passion for mathematics education, these three research questions were produced.

- Do students notice boardwork and how well do they notice design principles such as Visual Hierarchy, Grouping Perceptions, Layout, Representations, and Repetition?
- How does the application of these 5 design principles to boardwork in the classroom affect students’ learning experience and cognitive load?
- How does the knowledge of these 5 design principles influence the teacher’s in-the-moment decision making?
CHAPTER 3: METHODS

Introduction

This study takes both a quantitative and qualitative approach to its methods and data analysis. To assess the impact of this study on students, surveys included both quantitative questions with 7-point Likert scales and “quiz” type questions that they either got right or wrong where the data was measured for statistical significance. There were also qualitative portions where students justified their responses, and those responses were coded deductively and inductively. The teacher participant in this study was also interviewed at the end of the study and the data from that interview was qualitatively coded both deductively and inductively.

Measuring How Students Notice the Board

To test the extent to which students notice the design principles in classroom boardwork the questions in the survey were crafted to target each one of the five principles of design for mathematical boardwork. Question 1 on the survey asks students how easy it was to determine what information on the board is most important. This is targeting the principle of Hierarchy as this principle is what helps viewers determine which objects they should look at first, second, third, etc. Question 2 asks students how easy it was to recognize and link crucial information. This question is targeting the principle of Grouping Perceptions since the way we group information helps highlight similarities, differences, and connections. Question 3 and 4 both ask students about mathematical representations and how well they aided them in learning new concepts. This question is, rather obviously, targeted at Representations as students are asked about the general use of mathematical representations in class and given a specific example of one from class that day. Question 5 asks students what patterns they notice in their teacher’s use of the board. This question is targeted at Repetition since repetition is most easily expressed in
boardwork through patterns. Layout is the only principle that is not directly target through a single question but in the data analysis section below it will be explained how this principle was still analyzed through student responses.

Measures of Student’s Learning Experience and Cognitive Load

Learning Experience

Students’ learning experience was not measured explicitly with any of the student survey questions but was accounted for by analyzing their notes and interviewing the teacher. Student notes were collected during Part 1 of the student survey and will be discussed in the next paragraph below. When interviewing Brooke, I asked how she felt the design principles influenced her students experience in the classroom and several themes emerged from her interview data which will be given in the discussion.

I studied how good design of boardwork can influence what students write in their notes. Many Japanese researchers have noted that what goes on the board should be a model for what students put in their notes (Ninomiya & Kamoda, 2010; Nishio & Kubota, 2009; Soma, 1997; Takahashi, 2021; Yanase, 1990; Yoshida, 2009). I studied how good boardwork influences the quality of notes students take. I define note taking as anything written down during lecture in a student’s notebook/workbook other than any fill in the blank notes/worksheets already provided. This could include writing down phrases said by a teacher or classmate, copying down something presented on the board, or adding extra emphasis to fill in the blank notes provided by a workbook by highlighting, underlining, etc.

Cognitive Load

Measuring cognitive load is a difficult task but there are three main ways that researchers have attempted this: using surveys to elicit participants self-report measures of cognitive load,
using dual-task measures such as having participants attempt to perform two tasks at once like tapping their foot to a constant beat while solving a problem, or measuring physiological parameters like heart rates and pupil dilation (Paas et al., 2003). The method of measuring cognitive load with self-report measures was first used by Paas (1992) where students self-reported how much mental effort was needed to complete a task using a 9-point Likert scale. Following Paas’ original work, a 9-point scale has been used most frequently (example studies are Kester et al., 2006; Paas, 1992; Paas et al., 2007; Paas & van Merriënboer, 1993; van Gerven et al., 2002) but a 7-point scale has also been used often (see e.g., Kablan & Erden, 2008; Kalyuga et al., 1999; Moreno & Valdez, 2005; Ngu et al., 2009; Pollock et al., 2002). Because my study uses current students in a college trigonometry class, and I have a large data set I collected my data using self-report measures with a 7-point Likert scale. This type of measure is highly subjective to the participants’ own experience and point of view but for that same reason also gives great insight into their self-perception of mental effort and cognitive load.

Along with using the 7-point Likert scale questions to measure students’ cognitive load I also included recall questions on the student survey. I tested both what they remembered about the topics in the lecture that day and what specific aspects of the boardwork and discussion surrounding the boardwork they could recall. There are two types of recall used in this study: free recall and cued recall. Free recall is the ability to recall information given no prompt. For example, “What did you learn from class today?” tests a student's ability to freely recall what they learned. Cued recall is the ability to recall information given some prompt. For example, “What did you learn from our discussion about inverse functions?” tests a student's ability to recall information given the hint that it is related to inverse functions. I used these two types of recall questions in my student surveys.
Participants and Context

The participants of this study consisted of college students ranging from freshman to seniors enrolled in MATH 111, a college trigonometry course, and one MATH 111 teacher who was a graduate student in Mathematics Education. The students participated on a volunteer basis. There were in total about 180 students enrolled in MATH 111 Fall and Winter Semester, of which 60 students elected to participate in the surveys, a rate of 33%. Some students completed all 6 parts of the survey, others only completed 1 part. The 60 students are divided across Fall and Winter semester and two different sections each semester, one section met at 12pm and the second met at 1pm. There were 31 students who participated in Fall semester (19 in the 12pm section and 12 in the 1pm section) and 29 students who participated in Winter semester (17 in the 12pm section and 12 in the 1pm section).

The teacher in this study, Brooke Matthews (pseudonym), was a graduate student in BYU’s Mathematics Education Master’s program. She used a reformed approach to teaching, focusing a lot on group work and discussions and strived to implement the 5 practices (Smith & Stein, 2018) in her teaching. Fall semester was her second time teaching MATH 111 at BYU, but she had no prior teaching experience to this other than being a student teacher at a high school and teacher’s assistant for college algebra. She also studied Mathematics Education for her undergraduate degree at BYU and went straight from her undergraduate program to the graduate program. She was chosen partly for convenience but also because she already had prior experience teaching MATH 111 and would be teaching the same class in two consecutive semesters. This was an important aspect to the study because with prior experience in this curriculum she had a better handle on her scope and sequence for this class which I hoped would result in fewer changes in content from Fall to Winter semester. While this was the intent of
choosing a teacher with prior experience teaching the course, there were some significant changes she made to her scope and sequence between Fall and Winter semester in this study. The extent of these changes will be detailed after the Data Collection section below. She was also chosen because her class did not use a pre-written workbook for student note taking. I believed having a student workbook would influence student note taking too much for that to be an effective part of this study.

The college trigonometry course used Pathways as its curriculum (Pathways, n.d.) which strives to teach concepts over procedures and often presents mathematics through contextual applications. For this study on boardwork I chose three lessons from the curriculum. These lessons were chosen for two reasons: one, they were taught at the beginning of the semester which was necessary for the time frame I had to collect my data, and two, they were each a week apart which allowed enough preparation time between lessons. The topics for the lessons are Day 1: arc length and area of a sector, Day 2: graphing sine and cosine, and Day 3: other trig functions and right triangle trigonometry.

**Data Collection**

At the beginning of Fall semester 2021 I observed Brooke teach three lessons and surveyed her students after each lesson (control group). Then throughout the rest of Fall semester I worked with her to learn and internalize the design principles. More detail on the treatment will be given in the Treatment section below. In Winter semester I observed Brooke give the same three lessons again but now with the design principles applied to the boardwork. I then surveyed her students again after each lesson with the same survey given in Fall semester (treatment group).
Data for this research was collected via survey and personal interview. Each student was surveyed via Google Forms after each of the three lessons during both Fall and Winter semesters. These surveys were split into two parts, one part given the same day as the lesson and the second part given three days after the lesson. The first part of the survey consisted of questions targeting their self-reported cognitive load and is where I collected pictures of their notes from class. To assess perceptions of ease there were four questions for students to rank on a seven-point Likert scale from Very Strongly Agree (1) to Very Strongly Disagree (7). These questions were adapted from a survey given by Klepsch et al. (2017) in assessing cognitive load.

Klepsch et al. (2017) developed and improved on an instrument for measuring ICL, ECL, and GCL via survey questions. They crafted specific learning tasks that were conceptually high or low in each of the cognitive load types. Participants then completed the tasks and rated how much they agreed or disagreed with a given statement with a 7-point Likert scale. For example, one question targeting ECL stated, “It was exhausting to find the important information.” They then compared participants self-reported measures of cognitive load with how high/low they intended the task to be and found a match between their intended amounts of cognitive load and participants self-reported measures of cognitive load indicating that these questions accurately measured the type of cognitive load intended and that participants can naively recognize and distinguish between the three types of cognitive load. Since my study focuses on ECL I adopted my questions from their questions that targeted ECL in the study. A comparison of their questions with my adaptations are presented in Table 1 below.
Table 1

*Adaptation of Extraneous Cognitive Load Questions*

<table>
<thead>
<tr>
<th>Questions from Klepsch et al. (2017)</th>
<th>My Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>It was exhausting to find the important information</td>
<td>It is easy to determine what information on the board is most important</td>
</tr>
<tr>
<td>During this task it was difficult to recognize and link crucial information</td>
<td>The board made it difficult for me to recognize and link crucial information</td>
</tr>
<tr>
<td>The design of this task was very inconvenient for learning</td>
<td>The mathematical representations helped me better understand the topics of the lesson</td>
</tr>
</tbody>
</table>

My questions also each targeted a specific design principle to test students’ perceptions of that design principle in the boardwork. Following each of these Likert questions was a section for the student to explain why they chose the ranking they did. This part of the survey also asked students what patterns they’ve noticed in how their teacher used the boardwork and asked them to select all that applied from several options for how they use the boardwork from class in their learning. More detail on these questions will be given in the results section. A summary of these surveys is found in Appendix A and complete copies of the surveys are provided in Appendix C and D.

The second part of the survey, given three days after the lesson, consisted of questions about the main concepts of the lesson and what they could recall about the boardwork and class discussions. There were two types of recall questions I used, free recall and cued recall. The questions on the student survey varied so that each of these types of recall could be tested. To best assess recall abilities and not risk triggering their cued recall, the questions were given in order of most general to most specific. Given first were the free recall questions which were open ended questions, “What did you learn from mathematics class [insert date]?, “What parts
were easy to understand?,” “What parts were difficult to understand?” Second, to test cued recall, several types of questions ranging from broad to specific were also given. Broad questions like “What can you remember about the conversation surrounding this piece of the board?” As well as some more specific questions like “How did [specific concept] relate to [other specific concept]?” And finally, at the end were two computation questions like those done in class that day. The survey was designed so that each question was given on its own page and students must complete that question before moving on to the next. This way there was no risk of students seeing questions later in the survey that might cue their memory of the lesson that day when answering an earlier question.

In deciding what aspects of the boardwork to include in the survey I asked Brooke what her lesson goals were and picked pieces from the lesson that covered her core goals. This way when changes are made to those parts of the boardwork it would directly affect the goal outcomes of the lesson.

In Winter semester I helped Brooke plan the boardwork for the three treatment lessons. After the teacher taught all the treatment lessons, I interviewed her about how she felt the boardwork influenced her teaching and influenced the students’ learning. The interview was not initially a large part of the study, but findings from it have given me great insight into how these principles benefit the teacher and not just the students. It also helped support the data collected from the student surveys. In preparing the interview questions I reflected on my own use of the design principles and my perceived impact on myself and my students and asked her questions to see if she felt a similar way. These questions were broken into three main categories: her general impressions, influence on her students, and influence
on her teaching. The interview was audio recorded and I took notes throughout. A copy of the interview questions is found in Appendix B.

Changes to Scope and Sequence

Brooke made a few significant changes to the scope and sequence of her course between Fall and Winter semester. These changes impacted day 2 and 3 of my observations. In Fall semester day 2 Brooke covered both the unit circle and the graphs of the sine and cosine functions. But for Winter semester she decided to leave the unit circle as a homework activity and only focus on the graphs of sine and cosine in class. This significantly increased the amount of time students got to spend thinking about these graphs in Winter semester which likely influenced the content specific questions on part 2 of the student survey. In Fall semester day 3 Brooke covered tangent, cotangent, even/odd functions, and right triangle trigonometry. But in Winter semester she cut out even/odd functions and added in secant and cosecant. This affected the questions on part 2 of the student survey this day as any even/odd function content could not be duplicated for the Winter semester survey.

Treatment

In between the three lesson observations at the beginning of Fall semester and the three lesson observations at the beginning of Winter semester I spent several weeks teaching Brooke the design principles and practicing them with her. This treatment consisted of 5 lessons once a week each lasting 1–2 hours and 3 in-class implementations with observation at the end of Fall semester. I believe that group work is key to learning most new material, so I decided to open Brooke’s 5 lessons to other graduate students to participate as well. Every week there were between 1–3 other graduate students participating alongside Brooke. Having multiple students proved beneficial because each participant had unique perspectives on the boardwork and had
additional questions that helped clarify concepts and brought greater understanding for the whole group. Table 2 below is a summary of the events that took place during this treatment period and more detail is given below.

**Table 2**

*Summary of Sequence of Treatment Events*

| Week 1                          | Participants were given a handout with examples of the 5 Design Principles for Mathematical boardwork
|                                | Principles were defined and discussed as a group
| Week 2                          | Lesson focused on the principles of *Hierarchy* and *Grouping Perceptions*
|                                | Participants were given a pre-developed lesson plan and asked to plan the accompanying boardwork focusing on the above principles
| Week 3                          | Lesson focused on the principles of *Layout, Representation*, and *Repetition*
|                                | Participants were given a pre-developed lesson plan and asked to plan the accompanying boardwork focusing on the above principles
| Week 4                          | Participants were given a pre-developed lesson plan and asked to plan the accompanying boardwork focusing on all of the 5 design principles
| Week 5                          | Participants were asked to pick their own topic and plan and present boardwork utilizing all 5 of the design principles.
| Trial Lessons                   | I helped Brooke plan and then observed implementation of boardwork applying all 5 design principles for three lessons at the end of Fall semester before the treatment lessons in Winter semester.

Week 1 was an introduction to all the design principles. Participants were provided with copies of Figures 1–6 above. I defined each design principle, how the image demonstrated that principle, and we discussed the examples. At the end of the lesson the participants were provided with the above example in Figure 7 and asked what principles they could see already being applied and what changes they could make so that more principles were being applied.
7.7 Error Approximations

**Boardwork Example without the Application of Design Principles**

- We are finding the maximum value of the 2nd or 4th derivative of \( f \) on the interval \((a,b)\).
- \( K \) can be any number larger than all the values of the 2nd/4th derivative of \( f \) but smaller values of \( K \) give better error bounds.

\[
|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad |E_S| \leq \frac{K(b-a)^5}{180n^4} \quad |E_M| \leq \frac{K(b-a)^3}{24n^2}
\]

**About Those Pesky \( K \)'s....**
- In some particularly easy cases you can probably find the maximum of \( f''(x) \) by taking its derivative, setting it equal to 0, and finding it's maximum.
- In other cases, you can recognize it's always increasing or decreasing in which case one of the endpoints is the max.
- If they involve cosine or sine, these are always bounded by 1 and -1.
- There's always the graph/make a table of value and graph it.

**Boardwork Example with the Application of Design Principles**

2. Then evaluate \( K = f^n(c) \) (what the max is)

- \( K \) is the output (or \( y \)) value of the input (or \( x \)) value that gives you the max

1. First find \( c \) (where the max is)

\[
K = f^n(c) \quad f^n(x)
\]

**When solving for \( n \) with Simpson’s round \( y \) to nearest even \#**

\[
n \geq 4.00001 \quad \text{let } n = 10
\]

**About Those Pesky \( K \)'s....**
- In some particularly easy cases you can probably find the maximum of \( f''(x) \) by taking its derivative, setting it equal to 0, and finding it’s maximum.
- In other cases, you can recognize it’s always increasing or decreasing in which case one of the endpoints is the max.
- If they involve cosine or sine, these are always bounded by 1 and -1.
- There’s always the graph/make a table of value and graph it.
Then participants were shown Figure 8 and participants discussed the changes they could see and identified which principles are being used now. At the end of our discussion, we summarized techniques that use the design principles and best practices for applying the design principles to our boardwork.

In week 2 and 3 we went into greater depth for the design principles and split them up between the two days. In week 2 we focused on Hierarchy and Grouping Perceptions and in week 3 we focused on Layout, Representation, and Repetition. In these weeks the participants were given pre-developed lesson plan for an algebra topic and asked to look over it and collaborate on some ideas for boardwork that would apply the design principles.

During week 2 the lesson plan was on composition of functions and in week 3 the lesson plan was on concavity. An important observation I made these weeks was that these lessons were a lot more successful when the lesson plan topic was something the participants were already very familiar with. In week 2 both participants had already taught lessons on composition of functions when they were student teachers but in week 3 neither participant had taught concavity before. This led to very rich discussions week 2 about how the design principles would aid the boardwork but in week 3 there were many places I anticipated they would see the need for the design principles, and they did not.

In week 4 we looked at a lesson plan for transformations of functions and planned its accompanying boardwork trying to utilize all five principles. This was a topic all participants were familiar with so the discussion this week was much better than the previous week. This week I also prepared some questions for the participants to consider as they planned the boardwork. These included:
• What parts of transformations are difficult for students?
• How can the principles help them better understand these?
• What representations do we want?
• What are the main topics we could divide this up into?
• How could we better organize the layout, so each topic has its own space?
• What connections do students need to make? What principles can help with that?
• How well does this summarize the lesson?
• If this is a model for their notes, how good are their notes?

As they considered these questions the participants adjusted the boardwork, they had planned to better address the concerns expressed in these questions.

In the final week participants chose their own topic and planned their boardwork accordingly and presented it to the group. Feedback was then given by everyone present on what they thought was good and gave suggestions for additional ways the boardwork could incorporate the principles. Most participants chose a topic for an upcoming lesson of their own. Brooke’s lesson topic was on the law of sine, and she noted later in the week to me that she felt very confident about this lesson now because of all the boardwork planning she had put into it.

After these 5 weeks of learning the design principles, but prior to the three lessons where I gathered treatment data in Winter semester, I worked with Brooke over three lessons to plan her boardwork and incorporate the design principles. I then observed her teach these three lessons Fall semester and after each lesson we had a half hour meeting to discuss how she felt the lesson and boardwork went, concerns she had, and improvements that could be made. These debriefing sessions were not used in my data analysis, they were part of the treatment she received and used as an opportunity for her to self-reflect on her boardwork practices.
In Winter semester I continued to work with Brooke to plan her boardwork for the three treatment lessons. These meetings generally consisted of Brooke planning her lesson beforehand and coming to our meetings with an idea of her layout already planned. We then discussed together how the other principles could best be utilized to help students recognize important information and make the desired connections. We most frequently discussed the use of color and how she wanted to code certain concepts and student approaches. At the end of each planning session we reviewed each of the five principles and discussed how we were using them in the boardwork plan and discussed if there were any other additions or alterations we should make to improve the use of a given principles.

Data Analysis

Student Survey Part 1

The data was analyzed in the order it was collected: survey part 1, survey part 2, student notes, then teacher interview. When initially looking for responses from the student survey part 1 I noted some students may have been confused about the Likert ratings. They gave poor scores, but their justifications were positive. I first sifted through all the student responses to make sure their justification matched their score and changed their score by reversing it if they did not. For example, if a student scored the question a 2 but only had negative things to say in their justification then their score was changed to a 6. I only did this if they gave only positive comments but gave a poor score or vice versa. If the justification had mixed statements and they placed a 3, 4, of 5 for their score then I did not change it. This occurrence was not common but did occur both Fall and Winter semester across all three observation days. Typically, these occurrences happened across all four Likert questions for a single student. Below in Table 3 are the questions asked in part 1 of the student survey.
Student scores on the four Likert scale questions were quantitatively tested for significance using a Fisher Exact Test. The Fisher Exact Test I was able to find online only allowed to six categories, where my Likert scales had 7. For this reason, I decided to combine the counts for students who scored a 6 or a 7 since justifications for a score of 6 or 7 were identical and overwhelmingly negative. Thus, the categories for the Fisher Exact Test were students who scored a (1), (2), (3), (4), (5), and (6, or 7). The reason a Chi Square was not used was because for some categories the combined count not above 5 instances.

Table 3

*Student Survey Part 1 Questions*

| Question 1 | It is easy to determine what information on the board is most important |
| Question 2 | The board made it difficult for me to recognize and link crucial information |
| Question 3 | The mathematical representations helped me better understand the topics of the lesson |
| Question 4 | This mathematical representation helped me better understand [specific concept] |
| Question 5 | Have you noticed any patterns in the way your teacher uses the board from one lesson to the next? Or within one lesson? |
| Question 6 | Which of the following are ways you use the board for learning? Check all that apply |
| | • Model for my notes |
| | • Summarize the lesson |
| | • See the main points of the lesson |
| | • See examples of worked out problems |
| | • Keep track of where we are in the discussion |
| | • To better understand what the teacher said |

Note: For Q1–Q4 students responded using a 7-point Likert scale ranging from 1-Very Strongly Agree to 7- Very Strongly Disagree and were asked to justify their choice.
A deductive coding process was used first for student justifications and responses for Questions 1–5 of part 1 of the student survey by using the 5 design principles from the framework (Table 4). Data was chunked by idea, so grain size varied among student responses. For some student responses this meant their entire response was one chunk because it only contained one idea and thus received one code, other student responses had several ideas, even within one sentence, which created several chunks and thus received several codes.

**Table 4**

*Description of Codes for Student Survey*

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hierarchy</td>
<td>Response referred to the size of objects, how crowded or uncrowded the board felt, or noticed things placed in the same color, same style, underlined, or circled.</td>
</tr>
<tr>
<td>Grouping Perceptions</td>
<td>Response noticed the use of different colors, boxed off areas of the board, or how similar info was kept close together.</td>
</tr>
<tr>
<td>Layout</td>
<td>Response mentioned the amount of material on the board, how close together or spread out the materials was, use of different mediums (i.e. chalk board, projector, document camera), noticed similar info was kept close together, or used language like “clear, concise, organized, disorganized, messy.”</td>
</tr>
<tr>
<td>Representations</td>
<td>Response referenced diagrams, equations or formulas, example problems, labeling, or referenced seeing a “step by step” processes</td>
</tr>
<tr>
<td>Repetition</td>
<td>Response noticed patterns in the teacher’s boardwork by talking about how the teacher repeatedly did something, or used language like “always, often, frequently, never.”</td>
</tr>
<tr>
<td>Other</td>
<td>If no previous code was applicable.</td>
</tr>
</tbody>
</table>

One of the challenges in coding this data was that there was some overlap in the design principles, so some single chunks of data actually received two codes. Particularly when planning *Layout* you tend to use the principles of *Hierarchy* and *Grouping Perceptions*. Some students made mentions to a specific section of the board that contained a summary of “the
important formulas and concepts” from class that day. This justification can be seen as *Layout* because they are referring to a specific section of the board but can also be seen as *Hierarchy* because this section of the board is how they determined what was most important from class that day. Other justifications also mentioned that the board felt “too crowded,” so they had a hard time determining what was important. This could be seen as *Layout* because they are referring to how things were organized and placed on the board, but also *Hierarchy* because a lack of *Hierarchy* principles, like boxing out formulas to make them stand out, has made it difficult for them to determine what was important. These justifications were initially coded with *Layout* but when I later realized how similar to *Hierarchy* these justifications are I decided to double code these single chunks of data with both *Layout* and *Hierarchy* for Question 1. This will be discussed again in the results section.

Similarly, some justifications in Question 2 (Q2: The board made it difficult for me to recognize and link crucial information) commented on the placement of information on the board. If it felt like the information was “scattered across the board” then it was difficult for them to link information together. But if the information was “well organized” and “sectioned off by topic” then they felt it was easier to make these connections. These comments can be viewed as *Layout* because they are referencing the general organization in placement for the board but can also be seen as *Grouping Perceptions* because they are noticing (application or lack of) proximity and common regions. These justifications were initially coded with *Layout* but like above once I noticed how similar they were to *Grouping Perceptions* I decided double code these single chunks of data with both *Layout* and *Grouping Perceptions* for Question 2. This will also be discussed more in the results section.
Once the initial round of deductive coding was finished then a more inductive approach was taken to look at all data that had been given the Other code. These codes were grouped for similarity and according to emerging themes. Some students noted how things moved too quickly so they had a hard time understanding the content, these were given the code Pace. Others talked about how things the teacher said helped clarify concepts, these were given the code Verbal. Some mentioned that spending time working with fellow students was a common classroom practice, these were given the code Group Work. These are just a few examples of emergent codes from the Other data. None of these codes produced significant results about boardwork and will not be reported on further.

The only code different from the design principles that did produce significant results was a subcode of Representations used in Question 4 (Q4: The mathematical representations helped me better understand the topics of the lesson) named Specific. In Question 4 I noted that some justifications only mentioned mathematical representations in a general sense saying things like “graphs always help me better understand.” While others specifically referred to the representation provided in the question saying things like “lines representing sine and cosine should be curved and not straight. The graph helped me to understand that.” I tracked those comments that were specific to the given representation using the code Specific.

After I finished both the deductive and inductive coding, I realized I may have created bias in my data since I knew which files were from Fall semester and Winter semester while coding. I decided to recode all the documents by obscuring the semesters and weeks and randomly shuffling the files so I would not know while coding which file was from which semester. I kept my old codes hidden in blacked out cells and after I finished recoding the data, I compared my new codes to my old codes to see if there was a significant change. There was not.
I only had a dozen changes across all 155 student responses, which are changes I likely would have made if I had instead decided to just double check my original codes. I feel confident my current coding is accurate and impartial.

**Student Survey Part 2**

Table 5 shows the questions from part 2 of the student survey on Day 1. This part of the student survey was more about testing recall than the student observations of the design principles, so the codes used in the student survey part 1 did not fit this data well. Since the content changed with each lesson these codes varied a lot across all three days. Some generalities will be explained here but more detail on the codes used in part 2 of the survey will be given in the results section.

Questions 1–4 focused on what students could recall about the general topics of class that day and what concepts were easy or difficult for them to understand. To code these questions, I first made a list of the topics that were covered each day of class. Most students made specific mention of a topic, like arc length, sine, tangent, etc. so coding these questions was obvious. Anything that didn’t fit into these categories got the *Other* code. Then the *Other* codes were analyzed and grouped by similarities. Most of these codes were simply misnaming a topic that was taught. For example, instead of area of a sector some students wrote area of a circle. But some significant codes that emerged from the *Other* category across all three days were *Formulas, Conceptual, None,* and *All.* The *Formulas* code was used if students commented on the formulas, or equations used in class. This was often with respect to what they felt was most
Table 5

*Student Survey Part 2 Questions Day 1*

| Question 1     | What did you learn from MATH 111 class on September 9th? |
| Question 2     | What was your main takeaway from class on September 9th? What part of the lesson told you this was an important idea? |
| Question 3     | What parts/concepts were easy to understand? Why? |
| Question 4     | What parts/concepts were difficult to understand? Why? |
| Question 5     | What can you remember about the conversation surround this piece of the boardwork? |
| Question 6     | How is finding Arc Length related to finding the Area of a Sector? |
| Question 7     | In your words how would you describe to a classmate when to use one Area of a Sector formula and when to use the other formula? |
| Question 8     | How would you describe the purpose of this section of the board? |
| Question 9     | What is 160 degrees as a measure in radians? |
| Question 10    | Find the Arc Length of S below. |

Note: Because the content for Day 1, 2, and 3 varied the content specific questions (Q5–10) also varied. Here is presented only the questions from Day 1 to give the reader an idea what the questions were like across all days.

Note: Questions 7 and 8 also varied from Fall to Winter semester for Day 1. The questions presented here are from the Winter semester survey.

important from class in Question 2, but also used in Questions 3 and 4 with what they felt was easiest or most difficult. The *Conceptual* code was given if, rather than name a topic, they explained the concepts they learned about that topic. The *None* and *All* codes were used only with Questions 3 and 4 as some students said all/none of what they learned was easy/hard.
Questions 5–8 were more content specific than Questions 1–4 so the codes were never uniform across these questions. The responses to these questions were read first to get a feel for the data and then codes were created. Some of these questions used similar content specific codes as Questions 1–4 but other questions asked students to explain their understanding of a concept. These were just coded with Right if their understanding was correct, Wrong if their understanding was incorrect, and Half if their understanding was partially correct and partially incorrect or not complete.

Questions 9 and 10 quizzed students with problems similar to those performed in class. These questions were always coded as either Right or Wrong. Since there were no justifications given for these questions the use of the Half code from above was not applicable here.

**Student Notes**

During part 1 of the survey students were asked to upload images of their notes from class. The fall semester notes were entirely handwritten notes while in winter semester some notes were pictures of the worksheets Brooke provided in class. Brooke’s use of worksheets began halfway through fall semester and became a central part of her teaching practice that continued into winter semester. Despite this, most student notes in winter semester were still handwritten on their own paper. And those images that were from the worksheet most often had added pieces of information from the boardwork that were not necessarily designed to be written on the worksheet.

Since Brooke make a few significant changes to her scope and sequence, data from days 2 and 3 of my observations were affected. Since the content taught days 2 and 3 varied so much from Fall to Winter semester, only the student notes from day 1 were analyzed. I took an inductive coding approach to the student notes, first taking time to look over all the
notes from both semesters and made initial comparisons. From those initial observations I began to ask myself some questions. How many students recorded both ways of finding arc length in their notes? How many recorded the important underlying concept of percentages? There were many formulas given this day of class, how many students correctly recorded all the formulas in their notes? These are the questions that came to mind as I thought about Brooke’s lesson goals for this day of class and what information I would expect student to record in their notes but could be influenced by how well the boardwork made the information obvious.

Along with looking at the content of student notes I also analyzed them for their organization. I first looked across all the notes within each semester and looked for patterns between the notes from student to student. Then, I reviewed the photos I had taken of the boardwork each semester and compared them to the student notes to look for similarities in organization. Nothing here was tracked or counted, just general observations were made.

**Teacher Interview**

The audio recording from Brooke’s interview was first transcribed using Microsoft Word’s speech to text feature. I then listened to the audio while reading through the transcript to clean up the language, add punctuation where it felt obvious, and separate the two speakers, Brooke and myself. While cleaning up the text I simultaneously started my initial analysis of the data with inductive coding and started to notice some themes and patterns. These themes were her use of the design principles and how she felt the boardwork aided her teaching. The data was chunked by idea. Sometimes an entire sentence or paragraph was encapsulating one idea. Other times a single sentence contained multiple ideas. I first went through and coded every chunk that contained a design principle. Sometimes she named a principle directly and then talked about it,
but other times she only alluded to principles like when she says she can “underline it and use the same color” she is alluding to *Hierarchy* and *Grouping Perceptions* respectively.

**Table 6**

*Description of Codes for Teacher Interview*

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support</td>
<td>Help support her goals for the lesson and general teaching practice.</td>
</tr>
<tr>
<td>Reminder</td>
<td>Serve as a reminder for her lesson plan.</td>
</tr>
<tr>
<td>Discussion</td>
<td>Aid her when orchestrating discussions.</td>
</tr>
<tr>
<td>Connections</td>
<td>Help students make connections across the lesson.</td>
</tr>
<tr>
<td>Learning Goals</td>
<td>Help clarify her learning goals for her students.</td>
</tr>
<tr>
<td>Pacing</td>
<td>Help her decide how much time to dedicate to each section while teaching.</td>
</tr>
<tr>
<td>In-the-moment</td>
<td>Which principles she uses while in the moment of teaching, not planning her lesson.</td>
</tr>
<tr>
<td>Cognitive Load</td>
<td>How a design principle makes something easier for her teaching practice.</td>
</tr>
</tbody>
</table>

After coding for the design principles, I then went back through and coded all other chunks of data where the design principles codes were not applicable (Table 6). This produced a set of codes that all relate to how Brooke felt the design principles aided her with teaching. I read through the transcript several times to double check the codes I had already created and ensure all the codes I created captured what was in the data.

Once the coding was complete, I looked across the codes to analyze if the types of codes and their frequency supported the things Brooke claimed. For instance, Brooke said in her interview that her favorite principles to use were *Layout* and *Grouping Perceptions*, but she
struggled a lot with *Hierarchy*. I looked across the codes to see which of the design principles she most frequently mentioned throughout the interview and which she mentioned the least and found that the frequency of these codes supported her own claims. This was done for several of her claims and more detail will be given in the results and discussion sections.
CHAPTER 4: RESULTS

Introduction

The results below are sorted according to the order in which the data was received and analyzed. I chose this order because it felt most natural for the reader to sift through the data in the same order that I did and make connections in the same order that I did. I will report on the qualitative and quantitative aspects of the data together within each section. First, I will report on part 1 of the student survey which contained 6 questions on how students felt about the boardwork. Next, I will report on part 2 of the student survey which contained 10 questions about students recall abilities. Then, I will report on the students notes which were collected during part 1 of the survey. The data up to this point has both qualitative and quantitative aspects. Lastly, I will report on the data from the teacher interview which is purely qualitative.

Student Survey Part 1

This part of the survey was sent to students the same day as class and collected the next day by midnight. Questions 1 – 4 asked students to report how much they agreed or disagreed with a statement about the boardwork using a Likert scale from 1 to 7, 1 being they agreed completely and 7 being they disagreed completely. After each Question 1 – 4 students also justified their choice of number. Question 5 asked students what patterns they may have noticed in how the teacher used the board. Question 6 asked students how they use the board in class and were given several options to choose from. I combined the data for each of these questions across all three days in part 1 of the survey. This was done because these questions rely on how the boardwork was done in class and not the content that was taught.

I will first make note here of the data that combined all the questions across part 1 of the student survey (the All column in Table 7). This analysis is based on my coding of the student
Table 7

Combined Day 1, 2, and 3 Fall (n=87) and Winter (n=68) codes for Q1 – Q5

<table>
<thead>
<tr>
<th>Code</th>
<th>Q1 F</th>
<th>Q1 W</th>
<th>Q2 F</th>
<th>Q2 W</th>
<th>Q4 F</th>
<th>Q4 W</th>
<th>Q5 F</th>
<th>Q5 W</th>
<th>All F</th>
<th>All W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Principle</td>
<td>34%</td>
<td>54%*</td>
<td>21%</td>
<td>43%**</td>
<td>70%</td>
<td>95%*</td>
<td>39%</td>
<td>65%**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(30)</td>
<td>(37)</td>
<td>(18)</td>
<td>(30)</td>
<td>(21)</td>
<td>(18)</td>
<td>(34)</td>
<td>(44)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hierarchy</td>
<td>13%</td>
<td>43%***</td>
<td>8%</td>
<td>13%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>4%</td>
<td>11%***</td>
</tr>
<tr>
<td>(11)</td>
<td>(30)</td>
<td>(7)</td>
<td>(9)</td>
<td>(1)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(18)</td>
<td>(39)</td>
</tr>
<tr>
<td>Grouping Perceptions</td>
<td>2%</td>
<td>16%**</td>
<td>3%</td>
<td>17%**</td>
<td>3%</td>
<td>21%*</td>
<td>7%</td>
<td>21%*</td>
<td>3%</td>
<td>11%***</td>
</tr>
<tr>
<td>(2)</td>
<td>(11)</td>
<td>(3)</td>
<td>(12)</td>
<td>(1)</td>
<td>(4)</td>
<td>(6)</td>
<td>(14)</td>
<td>(12)</td>
<td>(38)</td>
<td></td>
</tr>
<tr>
<td>Layout</td>
<td>28%</td>
<td>36%</td>
<td>20%</td>
<td>35%</td>
<td>5%</td>
<td>0%</td>
<td>11%</td>
<td>47%***</td>
<td>12%</td>
<td>24%***</td>
</tr>
<tr>
<td>(24)</td>
<td>(25)</td>
<td>(17)</td>
<td>(24)</td>
<td>(1)</td>
<td>(0)</td>
<td>(10)</td>
<td>(32)</td>
<td>(54)</td>
<td>(83)</td>
<td></td>
</tr>
<tr>
<td>Representation</td>
<td>40%</td>
<td>19%**</td>
<td>39%</td>
<td>22%*</td>
<td>23%</td>
<td>5%</td>
<td>60%</td>
<td>26%***</td>
<td>48%</td>
<td>34%***</td>
</tr>
<tr>
<td>(35)</td>
<td>(13)</td>
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<td>(15)</td>
<td>(7)</td>
<td>(1)</td>
<td>(52)</td>
<td>(18)</td>
<td>(210)</td>
<td>(116)</td>
<td></td>
</tr>
<tr>
<td>Repetition</td>
<td>1%</td>
<td>0%</td>
<td>1%</td>
<td>3%</td>
<td>0%</td>
<td>0%</td>
<td>39%</td>
<td>65%**</td>
<td>8%</td>
<td>14%**</td>
</tr>
<tr>
<td>(1)</td>
<td>(0)</td>
<td>(1)</td>
<td>(2)</td>
<td>(0)</td>
<td>(0)</td>
<td>(34)</td>
<td>(44)</td>
<td>(36)</td>
<td>(46)</td>
<td></td>
</tr>
<tr>
<td>Specific</td>
<td>47%</td>
<td>89%**</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(14)</td>
<td>(17)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *p < 0.05; **p < 0.01; ***p < 0.001; ****p < 0.0001

Note: Q4 was a content specific question and could not be combined across all three days. Only Day 1 data is given in this table. More details on this decision will be given below in the section for Question 4.

justifications for their choice of the Likert scale rating. On average student justifications for Questions 1–5 in Fall and Winter semester were 23.5 and 25 words in length respectively. In Table 7 we see that all five of the design principles produced highly significant p values. From Fall to Winter semester there was a significant increase in how many students mentioned Hierarchy, Grouping Perceptions, Layout, and Repetition being used in the boardwork, but a decrease in how many students mentioned Representations.

Along with the significance across all 5 questions, the data was also qualitatively analyzed for how specific each justification was in respect to the boardwork done in class that day. If a response made specific reference to a piece of the boardwork, then it was counted, and totals were compared across Fall and Winter semester for Questions 1–5. For example, some
students made specific reference to where “the teacher circled or drew arrows” for a topic, referenced a specific representation like “the fly on the ceiling fan,” or noted how “all of the important information was boxed off on the right-hand side.” In Fall semester 28% of students had boardwork specific responses in comparison to 44% of students in Winter semester. Using a 2-sample test for proportions, this change produced a significant $p$ value less than 0.00001. More results about the student justifications will be shared in the Discussion section for each of design principles.

In Questions 1–4, students rated how much they agreed or disagreed with a given statement using a 7-point Likert scale (1 being completely agree and 7 being completely disagree) and then justified their choice of number. Their choice of number was quantitatively analyzed, and the results are presented below in Figures 9–12. I used both a Fisher Exact Test and an Unpaired t-test to analyze this data. These measures were used because my Likert scale has 7 points and could been seen as a continuous set of data for an Unpaired t-test or the numbers 1 through 7 on the Likert scale could be seen as separate categories for a Fisher Exact test. Each test has its own benefits and brings a different lens in which to analyze my data. An Unpaired t-test is a test of the means and tells us if there is a significant change in how well or poorly students scored each question. This is important for determining if students’ cognitive load changed but unfortunately does not give much detail into how students scored each question. This is where the Fisher Exact test picks up the slack. It compares each category from Fall to Winter and determines if they are distinct. This allows for a more detailed look into how the data has shifted between semesters. A Chi Square Test was not used because there were many categories with $n < 5$. The significance for the individual questions from Table 7 (p. 50) will now be reported below.
Question 1

- It is easy to determine what information on the board is most important.

Figure 9

*Student Survey Part 1 Q2 Grouping Perceptions Likert Ratings*

![Likert Ratings Chart]

Note: The Fisher Exact test can only take up to 6 categories. To accommodate this restraint, I combined the data for scores of 6 and 7 into one category labeled 6+7. This felt appropriate because student justifications who rated the question a 6 or 7 were nearly identical in having only negative things to say.

The purpose in asking this question on the survey was to see how well students notice and rate *Hierarchy* in the boardwork. Thus, the primary target principle for Question 1 was *Hierarchy*. *Layout* was also grouped with *Hierarchy* for the target principles in this question because the justifications that included *Layout* were using *Layout* in a hierarchal way. Some students referenced *Layout* by talking about how crowded or uncrowded the board felt to them. If the board felt crowded then that made it difficult to distinguish what was important and what was not, but if it felt uncrowded then the student felt they could easily tell what information was important. Note that student quotes below will be accompanied with their Likert scale rating (R) for clarification and reference.

- There wasn't a lot of things written on the board, and what was written on the board was very helpful to understanding the concepts taught in class. (R=2)
• I think that sometimes we write so many different equations for the same thing that it can be a little hard to know which is the best to use or most simplified. Often the same equation will be on the board with two more right beside it. (R=5)

Other students, particularly in Winter semester, referenced a specific section of the board that was always used to summarize important formulas and concepts. They pointed to how that section of the board helped them know what was most important from class.

• Because we have a separate section on the board for definitions and formulas. (R=1)

• We move from exercise to exercise and it’s easy to follow on the board. Also, the board is divided from examples to notes on the far-right side. Easy to understand. (R=2)

As seen in Table 7 (p. 50) there was a significant change in the proportion of students that noticed the target principles from Fall (34%) to Winter (54%), whether positively or negatively. Even when considering the principle of Hierarchy alone, and not including Layout, this produced a significant change as well. Figure 9 shows that not only did students notice the target principle more frequently but also noticed it in a more positive way by giving ratings closer to 1. For the data in Figure 9, a Fisher Exact test produced a significant p value less than 0.05 and an Unpaired t-test on the means produced a significant p value less than 0.01.

Question 2

• The board make it difficult for me to recognize and link crucial information.

The purpose in asking this question on the survey was to see how well students notice and rate Grouping Perceptions in the boardwork. Thus, the primary target principle for Question 2 was Grouping Perceptions. Layout was also grouped with Grouping Perceptions for the target principle in this question because the justifications that included Layout were using Layout in a grouping way. When students had negative things to say about Layout they generally talked
about how it was difficult to connect information because they felt the information was scattered across the different boards and other mediums. These comments are referencing the general organization of the board but noticing a lack of *Grouping Perceptions* being applied.

**Figure 10**

*Student Survey Part 1 Q2 Grouping Perceptions Likert Ratings*

Note: For consistency with the other questions, student responses were reversed so that positive responses were given lower numbers and negative responses were given higher response. For example, if a student had originally given this question a 6 it was changed to a 2 for analysis.

- We had the equations on the chalk board and then on the projector we were following around and for me it is hard to focus and understand when everything is in all different places. (R=3)
- Sometimes the formula or other work was done on a different part of the board when it connected to something on the other side that I had to make sure I caught and connected together. (R=4)

When students had positive things to say about the *Layout* they talked about how the board felt organized because there were sections dedicated to each topic. These comments are referencing the general *Layout* of the board but noticing that *Grouping Perceptions* are being applied.

- There were sections for each topic in its place. (R=2)
- Professor Matthews spaces the information on the board in a neat way and it is organized by topic and examples. (R=1)

As seen in Table 7 (p. 50) there was a significant change in the proportion of students that noticed the target principles from Fall (21%) to Winter (43%), whether positively or negatively. Even when considering the Grouping Perceptions alone, and not including Layout, this produced a significant change as well. Figure 10 shows that not only did students notice the target principle more frequently but also noticed it in a more positive way by giving ratings closer to 1. For the data in Figure 10, both a Fisher Exact test and an Unpaired t-test on the means produced significant $p$ values less than 0.05.

**Layout**

The design principle of Layout did not have a specific question that targeted it, but I wanted to still get a feel for how well students felt this principle was applied. Across the four Likert scale questions, less than 3% of responses in Questions 3 and 4 received the code Layout, whereas more than 20% of responses in Questions 1 and 2 did. To analyze how well Layout was perceived by the students I gathered the Likert scale ratings for each student response that

**Figure 11**

*Student Survey Part 1 Layout Likert Ratings*
received the *Layout* code in Questions 1 and 2 and performed the same Fisher Exact test and
Unpaired t-test on this group of data as I had on the other questions. When gathering the data
from Question 2 I used the reversed scores. For the data in Figure 11, a Fisher Exact test and an
Unpaired t-test on the means produced significant $p$ values less than 0.05.

**Question 3**

- The mathematical representations helped me better understand the topics of the lesson
  (mathematical representations are diagrams, graphs, models, formulas, equations,
  expressions, etc.)

  The purpose in asking this question on the survey was to see how well students notice
and rate the general *Representations* in the boardwork. Thus, the primary target principle for
Question 3 was *Representations*. This code was not combined with any others when creating the
*Target Principle* code. There were no significant changes in how many students noticed
*Representations* in their justifications for Question 3 so this data was left out of Table 7 (p. 50)
and the Fisher Exact test and Unpaired t-test for the data in Figure 12 also produced no
significant results.

**Figure 12**

*Student Survey Part 1 Q3 Representations Likert Ratings*
Question 4

- This mathematical representation helped me better understand the two ways to calculate arc length.

The purpose in asking this question on the survey was to see how well students notice and rate a specific representation from the boardwork, instead of representations in general as in Question 3. Thus, the primary target principle for Question 4 was *Representations*. This code was combined with the *Specific* code as this code was used for justifications that made specific reference to the image that was shown in the question, whereas justifications that made general comments about representations were coded with *Representation*. Since there was a change in content on Days 2 and 3, Table 7 (p. 50) only presents the qualitative data from Day 1. As seen in Table 7 there was a significant change in how many students mentioned *Representations* in Figure 13.

*Student Survey Part 1 Q4 Representations Likert Ratings*
their justification for Question 4 and an even more significant change in how specific the justifications were to the given representation from Fall to Winter.

Since Question 4 is content specific is did not make sense to combine the data from all three days when analyzing the data for both the quantitative and qualitative parts. Thus, Figure 13 presents the individual responses to the Likert scale portion across each day and semester. Even though the content changed significantly on Days 2 and 3, the data for these days are still presented here because other findings can be gleamed from this data which will be presented in the Discussion section. When performing a Fisher Exact test on this data only Day 2 came back as significant with a \( p \) value less than 0.001 and no days had a significant Unpaired t-test for their means.

The Likert scale data for Question 4 across Days 1, 2, and 3 did not remain consistent and presents a very different pattern when compared to questions 1 and 2 in the survey. In the previous questions Winter semester generally saw increases in ratings of 1 and 2 and decreases in the lower ratings indicating an improvement in the application of that design principle. For Question 4, however, the general pattern was an increase in ratings of 1 and 3 and decrease in ratings of 2, 5, and 6+7. But even this pattern did not hold for Day 3 which saw a decrease in ratings of 1 and 5 and increases in ratings of 2, 3, and 4. Brooke made additional changes to her lessons outside of just the boardwork on Days 2 and 3 which I believe effected how well the students grasped the concepts and responded to Question 4. The extent of these changes and their effect on the students’ responses to Question 4 will be given in the discussion section on \textit{Representations}. 

60
Question 5

- Have you noticed any patterns in the way your teacher uses the board from one lesson to the next? Or within one lesson?

The purpose in asking this question on the survey was to see how well students notice Repetition in the boardwork. Repetition as a code across all questions showed to be difficult for students to notice. This code was used if students used language like “always, usually, or often.” This code was also given if a student described a sequence of recurring events. This sequence of events often described Brooke’s tendency to present a problem, let students work on it in groups, and then discuss the solution methods as a class using the board. There was a significant increase in the proportion of students who noticed Repetition in Brooke’s boardwork from Fall to Winter semester.

This code was difficult to code as there were many responses that did not directly use the language described above but, for example, if a student had said “she [always] uses pictures and demonstrates the answers” their response would have received the code but in this instance they did not. For this reason, one could view almost every response to Question 5 as deserving the code Repetition. While there is statistical significance for the target principle Repetition for Question 5, a broader question like “What have you noticed about Mrs. Matthews use of the board in class” might have been a more insightful question.

There is still some interesting significance drawn from this question when looking at which design principles students noticed as repetitious. There was a significant decrease in the proportion of students from Fall to Winter that made mention of patterns in Representations and a significant increase in students mentioning patterns in Layout. More on this will be given in the discussion section.
Question 6

- Which of the following are ways you use the board for learning? Check all that apply.
  - Model for my notes (Notes)
  - Summarize the lesson (Summarize)
  - See the main points of the lesson (Main Point)
  - See examples of worked out problems (Example)
  - Keep track of where we are in the discussion (Discussion)
  - To better understand what the teacher said (Verbal)

For the data in Figure 14, a qualitative approach was taken to analyzing the data where patterns were found in the frequency of each option. The data presents totals across all observations days in Fall semester and Winter semester. The order that the options are listed in Figure 14 (Example, Main Point, Notes, Verbal, Discussion, Summarize) is the descending order for the totals across the combined data from Fall and Winter semester. This order stayed nearly

**Figure 14**

*Student Survey Part 1 Q6 Frequency of Options*
the same from Fall to Winter semester except for Summarize ranking slightly higher than Discussion in Fall semester. Across these semesters it is easy to see that students most frequently use the boardwork to see examples of worked out problems. Second, third, and fourth place (main point, notes, and verbal, respectively) had some variation between individual observation days which resulted in ties when totaling up the data. Consistently in the bottom two were using the boardwork to keep track of where they were in the discussion and to summarize the lesson.

**Students Making Connections**

Making connections was a recurring theme I came across while analyzing my data. This, however, was not a theme I anticipated to emerge from the data. Readers may suspect that Question 2 (The board made it difficult for me to recognize and link crucial information) is asking students about making connections through the wording “link crucial information.” However, most student responses focused on the “recognize … crucial information” portion of this statement. This means that all data from Question 2 is not synonymous with data on students making connections. Across Questions 1–5, though, I tracked how many student responses contained the topic of making connections (13% of responses in Fall semester and 9% in Winter semester). I determined this by one of two ways, they either explicitly referenced making

**Figure 15**

*Students Making Connections Across Survey Part 1 Q1–Q4*
connection by using the word connection (or something synonymous like related, tied to, similarities/differences) or their response showed that they were making connections across the mathematics. I also coded these responses as positive or negative experience trying to make connections. Figure 15 shows the quantitative data from this analysis. A Fisher Exact test on the data from Figure 15 produced no significant \( p \) value.

**Student Survey Part 2**

Questions 1 – 4 asked students what they learned in class that day and what concepts were difficult or easy to understand. Questions 5 – 8 displayed pictures from the boardwork in class and asked what they could recall about that topic, concept, or discussion. Questions 9 – 10 tested students with practice problems like those done in class that day.

Some of these questions were different from day to day because of the change in content from one observation day to the next. Thus, this part of the data was analyzed separately according to the day and not combined across days like in part 1 of the survey. Furthermore, because some of the content in Day 2 and Day 3 changed from Fall semester to Winter semester, most data from those days cannot be directly compared across the semesters and I do not report all results on them below.

**Questions 1 – 4**

These questions did not change from day to day.

- Q1: What did you learn from MATH 111 class on [specific date]?
- Q2: What was your main takeaway from class on [specific date]? What part of the lesson told you this was an important idea?
- Q3: What parts/concepts were easy to understand? Why?
- Q4: What parts/concepts were difficult to understand? Why?
In Questions 2–4 there are two prompts being asked. For example, in Question 3 it asks both “What parts/concepts were easy to understand?” and “Why?” For these questions the second prompt asked in each question was not answered by most students so data on those answers will not be reported or analyzed for lack of a representative sample size.

The content stayed the most similar between semesters on Day 1, unlike Day 2 and 3. The main topics covered in class on Day 1 were: converting between radians and degrees, finding arc length and area of a sector, and calculating angular speed. The only significant difference in content from Fall to Winter was that angular speed was not covered in the 12pm section of Winter semester because Brooke ran out of time. But this topic was only covered for the last 5 minutes of class in the other sections so it did not take up a significant portion of class.

Table 8

Day 1 Questions Q1–Q4 for Fall (n=28) and Winter (n=21)

<table>
<thead>
<tr>
<th>Code</th>
<th>Q1 F</th>
<th>Q1 W</th>
<th>Q2 F</th>
<th>Q2 W</th>
<th>Q3 F</th>
<th>Q3 W</th>
<th>Q4 F</th>
<th>Q4 W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radians to Degrees</td>
<td>64%</td>
<td>29%*</td>
<td>39%</td>
<td>19%</td>
<td>25%</td>
<td>29%</td>
<td>21%</td>
<td>14%</td>
</tr>
<tr>
<td></td>
<td>(18)</td>
<td>(6)</td>
<td>(11)</td>
<td>(4)</td>
<td>(7)</td>
<td>(6)</td>
<td>(6)</td>
<td>(3)</td>
</tr>
<tr>
<td>Degrees to Radians</td>
<td>54%</td>
<td>24%*</td>
<td>21%</td>
<td>19%</td>
<td>29%</td>
<td>19%</td>
<td>11%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>(15)</td>
<td>(5)</td>
<td>(6)</td>
<td>(4)</td>
<td>(8)</td>
<td>(4)</td>
<td>(3)</td>
<td>(2)</td>
</tr>
<tr>
<td>Arc Length</td>
<td>43%</td>
<td>71%</td>
<td>32%</td>
<td>19%</td>
<td>4%</td>
<td>5%</td>
<td>11%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>(12)</td>
<td>(15)</td>
<td>(9)</td>
<td>(4)</td>
<td>(1)</td>
<td>(1)</td>
<td>(3)</td>
<td>(2)</td>
</tr>
<tr>
<td>Area of a Sector</td>
<td>25%</td>
<td>57%*</td>
<td>25%</td>
<td>10%</td>
<td>4%</td>
<td>14%</td>
<td>14%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>(7)</td>
<td>(12)</td>
<td>(7)</td>
<td>(2)</td>
<td>(1)</td>
<td>(3)</td>
<td>(4)</td>
<td>(2)</td>
</tr>
<tr>
<td>Angular Speed</td>
<td>7%</td>
<td>0%</td>
<td>11%</td>
<td>0%</td>
<td>4%</td>
<td>0%</td>
<td>7%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>(0)</td>
<td>(3)</td>
<td>(0)</td>
<td>(1)</td>
<td>(0)</td>
<td>(2)</td>
<td>(2)</td>
</tr>
<tr>
<td>None</td>
<td>18%</td>
<td>5%</td>
<td>0%</td>
<td>14%*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(1)</td>
<td>(0)</td>
<td>(3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0%</td>
<td>0%</td>
<td>29%</td>
<td>5%*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(0)</td>
<td>(8)</td>
<td>(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *p < 0.05

From Table 8 we see that from Fall semester to Winter semester there was a significant change in question 1 with what students remembered as topics for the day. In Fall semester most
students recalled learning about converting from Radians to Degrees (64%) and Degrees to Radians (54%) while in Winter semester only 29% recalled converting from Radians to Degrees and 24% converting from Degrees to Radians. On the contrary, most students in Winter semester recalled Arc Length (71%) and Area of a Sector (57%) as topics for that day but only 43% recalled Arc Length in Fall semester and 25% recalled Area of a Sector.

Question 2, across all days, was also coded for how conceptual students’ responses were, meaning they did more than just name a topic but conceptually explained that topic. This data is given in Table 9 along with the same codes from Day 2 and Day 3. The proportion of students in Fall semester who gave conceptual justifications stays relatively consistent across Days 1, 2, and 3. But in Winter semester there is more variance. Students in Winter semester stated formulas were the most important thing they learned Day 1 but Day 2 and 3 there was a significant increase in their conceptual responses and no mention of formulas those days either semester.

Questions 3 and 4 asked students what they found easy to understand that day and what they found difficult (respectively). From Fall to Winter semester Day 1 Question 4 there was a significant increase in how many students thought none of it was hard to understand and a significant decrease in how many students thought all the content was hard to understand.

Table 9

<table>
<thead>
<tr>
<th>Code</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F(n=28)</td>
<td>W(n=21)</td>
<td>F(n=40)</td>
</tr>
<tr>
<td>Conceptual</td>
<td>7% (2)</td>
<td>10% (2)</td>
<td>15% (6)</td>
</tr>
<tr>
<td>Formulas</td>
<td>14% (4)</td>
<td>52%** (11)</td>
<td>0% (0)</td>
</tr>
</tbody>
</table>

Note: *p < 0.05; **p < 0.01
Questions 5 – 10 Day 1

These questions did change from day to day.

Because the content changed so drastically from Fall to Winter semester on Day 2 and 3, only data from Day 1 is presented here. Because Angular speed was not covered in both sections Winter semester Question 7 had to be thrown out and because of differences in boardwork Question 8 cannot be directly compared both semesters. Only data for Question 8 in Winter semester is presented in Table 10 because its findings are important when analyzing data from the student survey part 1 Question 6. This will be reported on more in the discussion.

Table 10

<table>
<thead>
<tr>
<th>Code</th>
<th>Q5</th>
<th>Q8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>W</td>
</tr>
<tr>
<td>Arc Length</td>
<td>64% (18)</td>
<td>76% (16)</td>
</tr>
<tr>
<td>Same Angle</td>
<td>39% (11)</td>
<td>5% (1) ****</td>
</tr>
<tr>
<td>Two Ways</td>
<td>4% (1)</td>
<td>38% (8) ****</td>
</tr>
<tr>
<td>Formulas</td>
<td></td>
<td>52% (11)</td>
</tr>
<tr>
<td>Main Points</td>
<td></td>
<td>48% (10)</td>
</tr>
<tr>
<td>Concepts</td>
<td></td>
<td>14% (3)</td>
</tr>
<tr>
<td>Summarize</td>
<td></td>
<td>14% (3)</td>
</tr>
</tbody>
</table>

Note: ****p < 0.0001

Question 5 of Day 1 asked students to recall the discussion surrounding a piece of boardwork from class. From Fall to Winter semester, I chose the same topic and showed the boardwork that encompassed that topic (Figure 16). On Day 1 the topic was the two different ways to calculate arc length. Some students focused on the detail that the angles stayed the same, which was not a significant part of the class discussion, others correctly identified the two ways
to calculate Arc Length. In Table 10 we see that from Fall to Winter there was a significant decrease in the proportion of students that focused on the angels being the same and significant increase in the proportion of students that correctly recalled the two different ways to calculate Arc Length. Questions 6, 9, and 10 (Figures 17, 19, and 20 and Table 11) were coded for correct recall of information but showed no statistical significance. Question 8 (Figure 18) from Winter semester will be elaborated on in the discussion.

**Table 11**

*Day 1 Question 6, 9, and 10 Fall (n=28) and Winter (n=21)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Q5</th>
<th>Q9</th>
<th>Q10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F W</td>
<td>F W</td>
<td>F W</td>
</tr>
<tr>
<td>Right</td>
<td>39% 48% 79% 90% 61% 76%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(11) (10) (22) (19) (17) (16)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Half</td>
<td>11% 14% 21% 10% 39% 24%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3) (3) (6) (2) (11) (5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wrong</td>
<td>50% 38% 21% 10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(14) (8) (6) (2) (11) (5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 16

Student Survey Part 2 Day 1 Q5 Data

Q5: What can you remember about the conversation surrounding this piece of the boardwork?

Fall Semester

Winter Semester
Figure 17

*Student Survey Part 2 Day 1 Q6 Data*

Q6: How is finding arc length related to finding the area of a sector?

<table>
<thead>
<tr>
<th>Fall Semester</th>
<th>Winter Semester</th>
</tr>
</thead>
</table>
| \[ S = r \cdot \theta \]  
\[ S = \frac{\theta}{2\pi} \cdot (2\pi r) \]  
\[ A = \pi r^2 \left( \frac{\theta}{2\pi} \right) \]  
\[ \text{Area of a sector} \]  
\[ \% \text{ of circle} \]  
\[ \text{Area of circle} \] | \[ c = 2\pi (2 \cdot 4) = 25.13 \text{ ft} \]  
\[ S = \frac{0.45 \times 25.13}{360} = \frac{2\pi (2 \cdot 4)}{360} \]  
\[ \text{Area of circle} \]  
\[ \% \text{ of radius length} \]  
\[ \text{Area of circle} \]  
\[ \text{Area of circle} \]  
\[ \text{Area of circle} \] |
Q8: How would you describe the purpose of this section of the board?
Student Notes

Student notes were collected during part 1 of the student survey. Though a file submission was required to continue with the survey a few students submitted blank images or images saying they do not take notes. Therefore, the sample size here is different from the sample size for Day 1 on the other questions in the survey. Student notes were analyzed for their content and organization and compared with the boardwork from class. Those findings are reported below.

Content

Student notes were quantitatively analyzed for their content (Table 12). Three areas of content were considered since these three areas were important pieces of the discussion and concepts that day. (1) Recording the two ways of calculating arc length. (2) Using the word (or
symbol) percentage or proportion. (3) Having all the formulas from that day. There were no significant changes found in the content of students’ notes. This data does, however, still give an interesting insight into this research and will be elaborated on in the discussion.

### Table 12

**Day 1 Content Frequency of Students’ Notes**

<table>
<thead>
<tr>
<th>Content</th>
<th>F (n=25)</th>
<th>W (n=21)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two ways of calculating Arc Length</td>
<td>76% (19)</td>
<td>76% (16)</td>
</tr>
<tr>
<td>Concept of percentages</td>
<td>68% (17)</td>
<td>76% (16)</td>
</tr>
<tr>
<td>All the formulas</td>
<td>68% (17)</td>
<td>67% (14)</td>
</tr>
</tbody>
</table>

### Organization

Student notes were also qualitatively analyzed for their organization. There were some noticeable differences in their organization. In Fall semester there was a lack of any recognizably semester as seen in Figures 16 and 17 where S is not labeled as Arc Length. Student notes in Winter semester also mimicked the teacher’s boardwork. Notes in Winter semester largely used the same organization and sectioning as the summary section from the board seen in Figure 18. This occurred whether students wrote personal notes or used the worksheet as seen in Figure 22. Similar organization amongst students notes. Of most notice is that students’ equations were not labeled with what the equation was for (see Figure 21). This mimics the teachers boardwork Fall
Figure 21

Fall Student Notes Example

\[
\frac{d}{360} \cdot 2\pi = \theta \quad \frac{\pi}{2\pi} \cdot 360^\circ = \theta
\]

\[
2.146 \approx 0.91 \
2.4 \cdot 0.46 = 1.08 \
2.9 \cdot 0.46 = 1.305
\]

\[
\frac{\theta}{2\pi} = s \
\frac{s}{2\pi} (\pi \text{ cm}^2) = A
\]

Figure 22

Winter Semester Notes Examples

7.1 Day 1

- rad = deg \cdot \frac{\pi}{180}
- degree = radian \cdot \frac{180}{\pi}

1. a. 3\pi \text{ radians} \quad c. 4\pi \text{ radians} \quad \frac{\pi}{4} \cdot 360^\circ \approx 252^\circ \approx 252^\circ

2. b. 7\pi \text{ radians} \quad d. \frac{\pi}{2} \text{ radians} = 1.57 \text{ radians} \quad 90^\circ

3. a. 52^\circ \quad b. 90^\circ \quad c. 243^\circ \quad d. 24^\circ \quad 0.907 \text{ radian} \quad 1.57 \text{ rad} \quad 4.24 \text{ rad}

4. a. Arc Length \quad \text{Converting Angles}

\[
S = \theta \cdot r
\]

\[
S = \frac{\theta}{\text{circumference}}
\]

\[
\text{Angular Speed}
\]

\[
\text{Area of a Sector}
\]

\[
A = \frac{\theta}{360} \cdot (\pi r^2)
\]

To find arc length

\[
S = \frac{\theta}{2\pi} \cdot (2\pi r)
\]

Area of a sector

\[
A = \frac{\theta}{360} \cdot \frac{\pi r^2}{4} = \frac{\theta}{12} \cdot \pi (\text{area of circle})
\]
Teacher Interview

At the end of collecting all the student data the teacher was also interviewed to gain her perspective on how learning these design principles has impacted herself and how she feels they have impacted her students. This data is qualitative and brief summaries of the findings will be given below but more will be elaborated on in the discussion section.

Impact on the Teacher

Overall, Brooke felt that boardwork planning became an integral part of her approach to lesson planning. Even stating that she doesn’t feel her “lesson plans are fully planned until [she] has planned her boardwork.” Her interview also showed that her views on boardwork broadened. Where beforehand she saw the board as “a vehicle” to simply “get definitions to students quickly” or to “show student work.” But now she sees boardwork as “a tool” and asks herself, “how can the design principles and boardwork together help me accomplish [the five practices from NCTM].”

Brooke also felt that the design principles help her better navigate classroom discussions and especially with *in-the-moment* decision making because her knowledge of her boardwork plan “offloads [her] cognitive load” when deciding what to do with, often unanticipated, student thinking. She said:

I feel like instead of [thinking] oh shoot this is really good thinking, I don't know where to put it … I can just focus on OK I'm looking for these two methods I know where they're going to go, I know what colors I'm going to put them in … I'm not getting stressed about these tiny little things because I feel like I can glance at the board and see my lesson plan in a way already up there … so I would definitely say that it offloads what I have to think about in the moment, especially during the discussion.
Impact on the Students

Brooke also brought up some thoughts on how she felt the design principles in her boardwork impacted her students. She felt the principles she applied to her boardwork helped her students make more connections during her lessons, increased their confidence in her teaching practices, and she overall received less complaints from her students than in past semesters.

- Anytime that I’m able to group things that don't seem connected and highlight the connections I think is incredibly helpful for [my students].
- Students trust the process a little bit more because the boardwork makes it clear that I have a plan and that it's all going to be visually clear how it comes together … they're more confident that it is clear and understand the connections here.
- I've gotten less complaints that I don't make sense or I'm not clear, or that they don't know how to start things … when the boardwork is more organized … I've also gotten less push back on my style of teaching since I have gotten better with boardwork.

Her thoughts on how the design principles helped students better make connections is especially of interest since the topics of making connections and student cognitive load were not explicitly brought up in any of the interview questions. These are things she has noticed without prompting. More on this topic will be considered in the discussion section.

Summary

From these results we see that the application of these design principles produced various significant changes in the data from Fall to Winter semester. There were changes in both how often specific principles were noticed by students and how well students rated that principle in aiding their learning and understanding. We also see how changes to Brooke’s boardwork affected students’ notes, what they remembered from class that day, and how easy or difficult
they felt the material was. Overall, Brooke felt these principles significantly impacted herself and her students. The practice of boardwork planning helped her navigate class discussions and the knowledge of her boardwork plan helped her keep track of the flow of her lesson and better handle unanticipated student thinking. She also felt that the application of these principles helped clarify her boardwork so students could more easily make connections and increased their confidence in her teaching abilities. The interpretation and limitations of these results will be discussed in the next chapter.
CHAPTER 5: DISCUSSION

Introduction

This research has sought to answer the questions of if students notice the quality of boardwork, how applying design principles to boardwork affects students’ learning experience and cognitive load, and how knowledge of design principles influences teacher’s in-the-moment decisions. My data gives strong evidence that yes, students notice the board and they notice it with great detail. It also gives evidence that when the design principles of Hierarchy, Grouping Perceptions, Layout, Representations, and Repetition are applied to boardwork there is an impact on students’ learning experience and cognitive load. And that by learning and applying these principles to boardwork the teacher’s cognitive load can also be lightened in the moment of teaching. I will address each of these points in greater depth below.

Students Notice the Board

The first question this research needs to address is if students notice the quality of boardwork. If students do not notice or appreciate good boardwork, then this research is futile. My data gives strong evidence that yes, students do notice boardwork, whether good or bad, and appreciate the good.

In part 1 of the survey, students rated 4 questions on a Likert scale, each targeting a design principle and afterward justified their responses. Question 5 also elicited a written response but without a Likert scale response. The qualitative data from their responses show evidence that in general students do indeed notice boardwork and notice it with great detail. On average student responses for Questions 1–5 were 23.5 and 25 words in length in Fall and Winter semester respectively. Some responses were even over 100 words. This means students had a lot to say about boardwork.
Students did not only notice obvious aspects of Brooke’s boardwork like graphs and diagrams but small nuances as well. Students noticed that in Winter semester Brooke always had “students write in different chalk colors and [she] always wrote in her own color,” that she “groups similar information” and that “each concept had its own section.” Students even picked up on her general teaching practice of presenting a problem, letting students work in groups to solve it, and then discussing it as a class using the board. Students also noticed when there was a lack of clarity in the boardwork across both semesters. Saying that it was difficult to know “which [equation] is the best” because she writes “so many different equations for the same thing” or that they “get the formulas messed up when they are too close together.” They noticed that when “it is all in the same color and style” it is difficult to see hierarchy and noted that “some stars […] would be nice to know what was important.” These comments show the great amount of detail students noticed in her boardwork both Fall and Winter semester.

Not only do students notice boardwork but appreciate and can tell when it is done well. Especially in Winter semester students left comments that Brooke “does better than most [teachers]” and that she “makes what is on the board easy to read and follow” and this is because “sections are made on the board for the most important information, and she separates student approaches by color so [students] can easily distinguish the work.” The quantitative results from their responses show that with an increase in the teacher’s effort to apply the design principles, there was a significant increase in the proportion of students who were able to identify the principles in the boardwork. This happened for the principles of Hierarchy, Grouping Perceptions, Layout, Specific Representation, and Repetition. Not only did students notice these principles but there was a significant improvement in the Likert ratings for Hierarchy, Grouping Perceptions, and Layout meaning that Winter semester students felt these principles aided their
learning more than Fall semester students. The principles in part 1 of the student survey will now be discussed individually below.

Hierarchy

The principle of Hierarchy was tested for in Question 1 by asking students how they felt the boardwork helped them identify what information was most important. This principle was one of the more difficult principles for students to notice. Only 4% and 11% of total responses in Fall and Winter semester, respectively, alluded to Hierarchy during part 1 of the student survey. There was a significant increase in how many students mentioned Hierarchy in Question 1, across all five questions, and a significant increase in positive scores for Question 1. This means that more students noticed an application of the Hierarchy principle and saw an improvement in its application.

These findings are, however, in opposition with Brooke’s own views of how she implemented this principle. She said that Hierarchy was the most difficult principle for her to implement in her boardwork as she wasn’t sure if she “does that one super well.” Why her students saw an improvement in her application of Hierarchy while she did not can be seen when we look at what students considered as evidence of Hierarchy in Winter semester. In this semester Brooke consistently used a “summary” section in her boardwork. An example of this is shown in Figure 18. This section of the board was outlined with topics beforehand and as they came to important formulas or concepts during the lesson Brooke would fill in the blank areas. In response to Question 1 students consistently mentioned this section of the board as the thing that indicated to them what was most important from class. This is also supported by what students put in their notes. During Winter semester the most common thing, and often the only thing, students wrote in their notes was a copy of the summary section. Brooke, however, saw the
purpose of this piece of the board as summarizing the lesson, not highlighting its most important points. In response to my question “which principles do you feel are most beneficial to your students” she first responded that the summary section is what she thought “[students] would say” but that she wasn’t sure “if that's actually the most helpful thing for them.” Whether or not this section of the board was helpful to students or not is not what is in question here. What we can say is that the repetitious use of a summary section created *Hierarchy* for the students as this section of the board indicated to them what was most important from class each day.

**Grouping Perceptions**

The principle of *Grouping Perceptions* was tested for in Question 2 by asking students how difficult the boardwork made it for them to recognize and link crucial information. This principle, like *Hierarchy*, was difficult for students to notice in the boardwork. Only 3% and 11% of total responses in Fall and Winter semester, respectively, alluded to *Grouping Perceptions* during part 1 of the student survey. There was a significant increase in how many students mentioned *Grouping Perceptions* in Question 2 particularly, across Questions 1–5, and a significant increase in positive scores for Question 2 (see Table 7 and Figure 10). This means that more students noticed an application of the *Grouping Perceptions* principle and saw an improvement in its application.

*Grouping Perceptions* was one of Brooke’s favorite principles to use and in her opinion, the easiest to implement. This was something she said in her interview, but also something I noted when coding for the frequency in which the design principles came up during the interview, *Grouping Perceptions* was at the top. The most common ways she used *Grouping Perceptions* in her boardwork was using color and spacing to create proximity, similarity, and common regions. In the previous section on *Hierarchy*, I spoke about how Brooke felt her
students would say that the summary section was the most useful thing for them. I then asked her what she felt was the most useful thing for students, even if her students didn’t recognize it necessarily. She felt that correcting student misconceptions is the most useful thing for her students, and she does this by using one color designated for the misconception and a second color for the correction. She also felt that in general “grouping things” that don’t seem connected and “highlighting the connections” using color is “incredibly helpful for [her students].” This idea that certain design principles help facilitate student connection making was a recurring, and important, theme as I sifted through the student survey and teacher interview data. More will be said about it in the “Cognitive Load for the Students” section below.

**Layout**

*Layout* was one of the easier principles for students to notice with 12% and 24% of students in Fall and Winter semester mentioning the board’s layout in their responses. There was a very significant increase in the proportion of students who noticed *Layout* across all five questions in Winter semester compared to Fall semester, and an improvement in how well students rated *Layout* in Questions 1 and 2. This means that not only did students notice *Layout* being used more but felt that it improved.

*Layout* was typically the first principle Brooke considered when planning her boardwork and was also one of the principles she noted in the interview as one of her favorites. One common practice Brooke implemented during her lessons in Winter semester was writing up a “skeleton” of her boardwork plan before class started. This meant dividing up and labeling the sections she already wanted to create and drawing any representations or practice problems in the locations she had pre-determined. Students also picked up on this habit in Winter semester, saying that she “always has the practice problems prepared on the board” as well as “several
things that she plans to use during the lesson and then adds to it with important ideas during class.” This “skeleton” of the layout also served as a quick reminder for her during her lessons as she felt she could “glance at the board and see [her] lesson plan already up there.” This made it easier for her to navigate class discussions and handle unexpected student thinking because she knew how to best “budget her board space” and felt that Layout specifically lightened her cognitive load as the teacher. This idea that planning boardwork lightens the teacher’s cognitive load was not an idea I had previously considered and was something she brought up unprompted in the interview. More will be discussed on this topic in the “Cognitive Load for the Teacher” section below.

**Representations**

The principle of **Representations** includes graphs, diagrams, models, formulas, equations, etc. This was the easiest principle for students to recognize in the boardwork with 48% and 34% of students in Fall and Winter semester, respectively, mentioning **Representations** in Questions 1–5 of the survey. This principle was the target principle for both Question 3 and 4. In Question 3 students were asked how they generally felt the representations in class helped their understanding, and in Question 4 they were asked how a specific representation from class helped them understand a specific concept.

Questions 3 saw neither a significant increase in the proportion of students who mentioned **Representations** nor in the Likert ratings but a large proportion of students (above 70% both semesters) mentioned **Representations**. This was much higher than any of the other design principles. I interpret this to mean that in general students notice **Representations** a lot no matter how well they are planned and implemented. The fact that **Representations** is the most obvious principle for students to notice did not surprise me. I would suggest that most teachers’
main purpose in using boardwork is to simply display representations, so students are conditioned to pay attention to them. This certainly was the case for Brooke. In her interview she said that before learning these design principles she saw the board as “a vehicle to get definitions quickly to students … or show student work.” *Representations* was, however, the only principle to see a decrease (and statistically significant as well) from Fall to Winter semester when looking across all five questions. I do not believe this means that Brooke used less representations in her boardwork but rather that students had a greater variety of things to notice about the boardwork because of her additional use of the other design principles. There is evidence for this in the data from part 1 of the student survey.

Across the data for all five questions shown in Table 7 (p. 50) only Questions 3 and 4 targeted *Representations*, and in these two questions there is an increase in this target principle from Fall to Winter semester. The other questions on the survey had significant decreases in how many students mentioned *Representations* and increases for the target principles for those questions. This shows that for the questions that targeted *Hierarchy, Grouping Perceptions, Layout* and *Repetition* they had more they could say in Winter semester about these principles and that is why overall there was a decrease in Winter semester for students mentioning *Representations*.

While Question 3 saw no significant changes, Question 4 on the other hand did see some significant changes. There was a significant increase in the proportion of students that mentioned *Representations* and how specific student justifications were to the given representation, meaning their justification made specific mention to something in the provided boardwork image instead of just a general statement like “graphs always help me learn better.” The Likert ratings for this question (Figure 13), however, tells a different story than what has been seen in other
questions. Since this question is content specific, I decided to analyze the Likert data day-by-day. I wanted to look deeper into how students perceived each representation from each day observed in class. When analyzing more specifically what students had to say about these representations, I realized that changes in what was taught that day and how Brooke chose to teach the concepts affected student responses on the survey more than the changes in the boardwork had. I will discuss the representation chosen from each day of the study below.

Across the 3 days I observed, Day 1 stayed the most consistent in content and style. Both Fall and Winter semester reviewed converting between radians and degrees and taught how to find arc length and area of a sector. The only change is that section 1 of Winter semester did not cover angular speed, but this topic used up less than five minutes of class in the sections where it was taught so it was not a significant topic of the day. Figure 13 shows that students’ perceptions of the representation from Day 1 stayed relatively the same from Fall semester to Winter. This is interesting because when comparing these two representations in Figure 23 I think there are some major improvements to Winter semester’s boardwork. Since the boardwork in Fall semester is all vertically in the same space and same color it is difficult to distinguish that there

**Figure 23**

*Day 1 Boardwork on Arc Length*
are in fact two different calculations on the board. There are also no labels indicating what is even being calculated with the work and no generalizations to explain the concepts at play. In Winter semester, however, Brooke gave a generous amount of space to separate the two calculations, she labeled both with S to indicate arc length, and labeled each piece of the final equation with statements like “% of circle” to generalize what each part of the equation represents. To me this makes the concepts and purpose of the boardwork in Winter semester much clearer.

So why did student ratings not improve significantly? When I looked at how students had justified their responses, I noticed that in Winter semester many of the Likert ratings of 3 and 4 made mention of mistakes that were made in the calculations. You can see in the boardwork of Winter in Figure 23 some numbers are crossed out. This was because in the middle of reviewing one student’s work to better understand the concept another student questioned if the values on the board were correct. The discussion quickly turned into an argument of what numbers were correct or incorrect and the purpose of reviewing their work to understand the concept was lost. Brooke didn’t know which number was supposed to be correct, so the discussion never came to a clear resolution. Some students became too distracted with the computation errors and lost the understanding for the two ways to calculate arc length. Since this mistake was not made in section 2 and most of the 1-ratings came from section 2, I argue that many students who rated the boardwork a 3 or 4 would have given a higher rating if these mistakes in the computation had not been made. Overall students still saw an improvement in the representation from Fall to Winter semester, as evident by the increase in ratings of 1 and decrease in ratings of 5+6+7, but because of the computation error in section 1 there was a greater mixture of opinions in Winter semester hence the increase in ratings of 3+4.
While the ratings in Day 1 stayed relatively similar, Day 2 saw a more dramatic shift. This day was the only day that produced a significant $p$ value with a Fisher Exact test meaning students felt there was a significant difference in the boardwork between Fall and Winter semester. Interestingly, I would say that the representation which stayed the most consistent from Fall to Winter was on Day 2 (Figure 24). Both representations show the graphs of sine and cosine and show the misconception that the lines are straight. The data though says that students in Winter semester felt more confident in their understanding of why the sine and cosine graphs look how they do. There were several changes that Brooke made to her lesson this day which I believe explain the changes in student responses to Question 4 on Day 2.

On Day 2 in Winter semester Brooke decided to not teach the unit circle in class and instead left it as their homework assignment. This gave her more time to spend on the graphs of sine and cosine in class. She used this time to have students reason through the covariational relationship between the angle sweeping out and the vertical/horizontal distances. Brooke noted to me during this lesson how many fewer “circle” graphs she saw in comparison to Fall semester.
Drawing a circle is a common misconception when students graph sine/cosine, so by focusing on the covariational relationship of these quantities students better understood the graph of sine and cosine, but not necessarily because of a change in the boardwork. Because this shift in the data was likely due to a change external to boardwork, the qualitative data from Day 2 was not included in Table 7 (p. 50).

On day 3 Brooke made changes to both the content taught this day and how she approached the concepts. In Fall semester Brooke taught tangent, cotangent, even/odd functions, and right triangle trigonometry, whereas in Winter she cut out even/odd functions and added in secant and cosecant. On Day 3 in Fall semester Brooke banned the term SOHCAHTOA, wanting students to focus on the vertical and horizontal distances but in Winter she also banned the words opposite and adjacent to emphasize the vertical and horizontal relationships even further between right triangle trig and the unit circle. Images of the representations given to the students in the survey are shown below in Figure 25.

**Figure 25**

*Day 3 Boardwork on Right Triangle Trigonometry*
In Question 4 this week students were asked how well these representations helped them understand that sine is opposite over hypotenuse and cosine is adjacent over hypotenuse. The Likert data from Figure 13 (p. 58) shows that there was a decrease in ratings of 1 and an increase in ratings of 2 and 3+4. This is the only day that saw a decrease in ratings of 1 but changes in Brooke’s approach to this lesson I believe explain why. Because Brooke banned the words opposite and adjacent from class discussion students who gave poor Likert ratings in Winter often justified their response saying they felt this question was a “trick question” because they weren’t allowed to use the words opposite or adjacent. A few students even noted that they had never learned SOHCAHTOA so they “don’t feel like [their] understanding of why sine is opposite over adjacent is better. But [they] better know why sine is y/r.” These responses explain why the data for Day 3 saw such a large decrease in ratings of 1 but increases in 2 and 3+4. This change in the content and teaching strategy is also why the qualitative data was not included in Table 7 (p. 50).

Even though the changes in the content produced issues when analyzing the data for Question 4, the student justifications do still give insight into how students notice and rate *Representations*. From Day 1 we learn that computational errors on the board can be distracting for students during the lesson, especially when left without a clear correction. And Days 2 and 3 show that more goes into students’ conceptual understanding than just the representations presented in class. Changes to the amount of time spent on the topic and strategies for approaching that concept also affect how students perceive and understand the representations on the board. This section of the study I believe demonstrates that boardwork is not an end-all-fix-all solution to student learning and comprehension. It is important to plan lessons wholistically,
thinking about how each part of the lesson affects the other, in order to strengthen the connections to the key mathematical ideas we are trying to get across to our students.

**Repetition**

The principle of *Repetition* was tested for in Question 5 by asking students if they had noticed any patterns in how Brooke used the board within one lesson or across lessons. This principle was one of the more difficult principles for students to notice. Only 8% and 14% of responses in Fall and Winter semester, respectively, alluded to *Repetition* during part 1 of the student survey and a large majority of those instances (95%) occurred when responding to Question 5. There was a significant increase in how many students mentioned *Repetition* in Question 5 and across all five questions. This means that more students noticed an application of *Repetition* in Winter semester than Fall semester. This finding, however, is in question because Question 5 specifically asked students what patterns they had noticed so every response could be interpreted as deserving the *Repetition* code. As is, only students who used language like “always, usually, or often” received the code. There was not a Likert scale portion of this question so students’ perceptions of how well *Repetition* was implemented was not measured for.

Though *Repetition* was the target principle of Question 5 we can also analyze what aspects of the boardwork students most noticed *Repetition* being applied to. In Fall semester the design principle that students most noted as repetitious was *Representations* (60%). This was a significantly large portion as responses in Fall semester were not very diverse, *Hierarchy, Grouping Perceptions,* and *Layout* received 0%, 7%, and 11% of responses, respectively, in Fall semester (see Table 7, p. 50). As students described these repetitious representations, they mentioned how Brooke often used the board to work out example problems, show “a step-by-step process,” and “display important equations.” Winter semester, on the other hand, saw more
diversity in what repetitious behavior students noticed. There was a significant decrease in the proportion of students that mentioned *Representations* (26%) and increases in the mention of *Grouping Perceptions* (21%) and *Layout* (47%). Students who talked about Brooke’s repetitious use of *Grouping Perceptions* noted her constant use of color-coding during class. Some even noticed that Brooke “uses a unique color of chalk which helps [them] know what was her work” versus student’s work. Those who mentioned a repetitious use of *Layout* usually had one of two things to say. They had either noticed how she “maps out” the lesson on the board before class starts, or they noticed how the board is “separated by topic” and “always uses the right-hand side of the board for important concepts.”

These findings support what Brooke said in her interview about her favorite design principles being *Grouping Perceptions* and *Layout*. Since she strived to implement *Grouping Perceptions* and *Layout* more her students noticed those principles more. This further supports the claim in this section, that students will notice these design principles if we strive to implement them at greater depths. Students’ shift away from noticing *Representations* and toward noticing *Grouping Perceptions* and *Layout* is also supported by how Brooke’s perception for the use of boardwork broadened from being just a “vehicle” for displaying student thinking to also being a “tool” for helping students make connections. In Winter semester Brooke did more than just display representations, she saw the board as an opportunity to help accomplish one of the *5 Practices* (Smith & Stein, 2018), making connections. She used *Layout* to group the main topics of class and *Grouping Perceptions* to draw similarities between key concepts. Connection making was a recurring theme in this study and will be discussed more in the next section below.
Cognitive Load for the Students

One of the goals of this study was to measure how the application of these design principles on classroom boardwork affects the cognitive load of students. This was measured using self-report measure with the Likert scale portion of Questions 1 – 4 of part 1 of the student survey as well as by measuring student’s recall performance with content specific questions in part 2 of the survey. The Likert scale questions measured students’ self-reported perceptions of cognitive load by having students report how easy or difficult the boardwork made certain tasks. The content specific questions from part 2 of the student survey measured students cognitive load by testing how well students could recall information from the boardwork and comparing the results from Fall to Winter semester.

Self-Report Measures

Overall, the data from Q1 – Q4 of part 1 of the student survey gives evidence that students’ extraneous cognitive load was lightened with the application of the 5 design principles for mathematical boardwork. As seen in Figures 9, 10, and 11, student ratings for how easy it was to determine what was most important (Q1: Hierarchy), how difficult it was to recognize and link crucial information (Q2: Grouping Perceptions), and observations on the organization of the board across Questions 1 and 2 (Layout) saw statistically significant improvement from Fall to Winter semester with an Unpaired t-test for means. This means that students in Winter semester when compared to Fall semester felt the organization of the board lightened their extraneous cognitive load by making it easier for them to recognize what was most important and link crucial information across the board.
Recall Performance

Part 2 of the student survey was designed to measure students recall performance. This part of the survey was given 3 days after the observed class. Because these questions were all content specific to the observed day of class the data from these questions could not be grouped together across all 3 days like I was able to in Part 1 of the student survey. Furthermore, changes in the content taught each observation day also made it so that some survey questions used in Fall semester could not be repeated Winter semester. These complications created small sample sizes which, unfortunately, were too small to find any significance in the remaining quantitative questions. As seen in Table 11 there was generally an increase in the percentage of students who got Questions 5, 9, and 10 correct but these values did not produce a significant \( p \) value. The data here may be capturing something different in students recall and performance abilities with these questions but since the sample size for Day 1 in both semesters was under 30 no statistical significance was produced.

Some qualitative data from part 2 of the student survey does show, however, that students’ extraneous cognitive load was lightened with the application of the 5 design principles. Figure 16 and Table 10 show that changes to the design of *Representations* can impact the concepts students take away and impact what they recall. Significantly more students on Day 1 in Winter semester correctly recalled that the representation was showing the two ways to calculate Arc Length whereas most Fall semester responses thought the purpose of the representation was to show that the angle stays the same no matter the size of the radius. This array of data seems to suggest that the application of these design principles improved students recall performance but further research with a larger sample size is needed to make any correlations between student’s recall abilities and boardwork practices.
Making Connections

Making connections was a recurring theme throughout my data analysis but was not a theme I intended to study. I will discuss here what evidence has emerged from my data on this topic but know that this evidence is limited and needs further study. There are two ways that Smith and Stein describe connection making in a mathematics classroom: (1) connecting previous knowledge to new knowledge and (2) drawing connections across multiple representations. My research suggests that applying visual design principles to boardwork can help students accomplish these two things.

The topic of students making connections first came up when analyzing the teacher interview data. Brooke brought up this idea of students making connections several times throughout the interview without prompt. The very first question I asked her was how she felt her view of boardwork changed after learning these design principles. Her immediate response was that before she thought connection making all happened verbally, by her saying something and students then making those connections in their mind. But now she sees that making connections can be a visual experience.

Later in the interview she also brought up this topic when asked which of the design principles she felt most benefited her students. Brooke felt that the principle of Grouping Perceptions allows her to “group things that don’t seem connected and highlight the connections.” When looking at why the idea of students making connections came up in her interview it was always in conjunction with Brooke talking about the principle of Grouping Perceptions. Brooke felt that Grouping Perceptions aided students in making connections because she could plan her layout so that related concepts were in close proximity, making it
easier to draw connections between them, and she would use color and common regions to highlight similarities and difference between multiple representations.

After having analyzed the interview data and seeing this theme emerge, I also looked at the student data to see if this theme was prevalent there as well. As stated in the results section, across Questions 1–5 of part 1 of the student survey there was no significant difference between positive and negative student comments on making connections from Fall and Winter semester. Of these five questions the one closest to targeting the idea of making connections was Question 2 (the board made it difficult for me to recognize and link crucial information). As seen in Table 7, Question 2 saw a significant increase from Fall to Winter semester in the proportion of students that noticed *Grouping Perceptions* in the boardwork and from Figure 10 there was significant improvement in the average ratings with significantly more students rating the boardwork a 1.

This array of quantitative and qualitative data paints a blurry picture when it comes to how these design principles aid students in making connections. The significant improvement to student response to Question 2 of part 1 of the student survey combined with Brooke’s opinion that the design principles, specifically *Grouping Perceptions*, help make connections more obvious would suggest that the application of these design principles aid students in making connections during class. But since there was no significant difference in the way that students talked about making connections in their response it is difficult to give a definitive answer from this set of data. I believe this data suggests that the application of these design principles to boardwork aids students in making connections in a mathematics classroom, but research more directly aimed at studying this relationship is needed to fully conclude this.
How Students Use the Board in Class

Question 6 on part 1 of the student survey asked students to select all ways in which they use the board in class. They could choose from: model for my notes, summarize the lesson, see the main points of the lesson, see examples of worked out problems, keep track of where we are in the discussion, or to better understand what the teacher said. The data on this question is given in Figure 14 above (p. 60). I was surprised to see that from Fall to Winter semester there were no significant changes in how the students use the board during class, especially that summarizing the lesson still ranked so low given Brooke’s summary section on the board she began implementing in Winter semester. The data from Question 8 on part 2 of the student survey helps clarify some of the data from Question 6 on part 1 of the student survey.

Question 8 asked students what they thought the purpose of this [the summary] section of the board was for. As seen in Table 11, most students viewed this section of the board as either a “formula bank” or where they could find the main points of the lesson. Very few students saw this section as a summary of the day’s lesson. Though Brooke strived to have her board better summarize her lesson in Winter semester, students did not interpret that area of the board the same way she did.

One thing that remains clear from this part of the data is that students largely use the board to see example problems worked out. Though other options on the survey ranked differently from day to day this option remained the top choice every single day Fall and Winter semester. I find this interesting for two reasons. One, it is in contrast to how Brooke intended her students to use the board. Two, it speaks to the teaching culture in the US and shows how difficult it is to change that culture.
Brooke made it clear in her interview that one of her main teaching goals each lesson is to help students see and make connections across the mathematics. She structures her lessons so that students have time to work on a problem on their own, then students are selected and sequenced to present their work for those problems on the board, and a class discussion ensues where the student work is contrasted in order to make connections and deepen understanding. Brooke even described in her interview how previously she thought those connections were “made verbally and that it happened in all the student’s brains but not necessarily anything visual would happen with those connections.” But since learning these 5 design principles she sees how the board can aid students in visually making connections. This description of her general lesson flow and statements from her interview suggest that the purpose of Brooke’s boardwork is to aid students in seeing connections visually. And yet students are still very focused on seeing examples of worked out problems that they can then replicate on their homework. In fact, some students on the survey even justified their ratings according to how confident they felt they would perform on the homework. Some students gave poor ratings because “knowing what [they] needed to know for the homework was confusing” or gave good ratings because “when the formulas were specifically written on the board [they] felt more prepared to do the homework.”

I believe this data speaks to the teaching culture in the United States. In Hiebert and Stigler’s The Teaching Gap, they talk extensively about teaching patterns they found across their countries of study, even in a country as large and diverse as the US. Here they saw a general routine. Teacher’s would begin class with a short review of what they learned the previous lesson. Then the teacher would demonstrate a new skill on the board by walking the class through a few examples. Next students would be given a worksheet of usually 30 or so practice
problems for them to complete. Lastly, students would have time at the end of class to begin their homework (Stigler & Hiebert, 1999, p. 80). In this description of traditional mathematics lessons in the US it is clear what the purpose of the board is, to show example problems of what students can expect on their homework. This common lesson structure has become a part of the teaching culture in the United States and students have been conditioned to expect the board to be a place where example problems are shown. My data supports this and gives evidence for how difficult it is to change student expectations. Even with a significant change in Brooke’s boardwork and teaching practice, the Winter semester students still fixated just as much as the Fall semester students as viewing the board as a place they can see examples of worked out problems. This further solidifies my earlier point in the discussion that improving boardwork is not an end-all-fix-all solution to the problems that we face as educators but just one piece of the puzzle.

**Students’ Learning Experience**

One question this research has sought to answer is how the implementation of these design principle on boardwork impacts students’ learning experience in the classroom. Learning experience is a broad concept that overall encompasses how students feel and act in the classroom. Students’ learning experience was not measured explicitly with any of the student survey questions but was accounted for by analyzing their notes and interviewing the teacher. From this data I found evidence that implementation of these design principles changed how students’ took notes and impacted how they viewed the quality of the teacher’s instruction.

**Student Notes**

In comparing these notes from Fall to Winter semester I found evidence that both the content and organization of the notes were influenced by the boardwork. Fall semester
boardwork was generally disorganized, formulas and concepts were scattered across the board and equations were not labeled with what quantities they found. Consequently, student notes largely mimicked this same structure. Figure 21 shows an example of Fall semester notes. In contrast, Winter semester boardwork was sectioned by topic and a summary with all the key concepts and equations was sectioned off on the board. Student notes largely mimicked this structure as well. Many students copied down exactly the summary section of the board, so they had a list of the key concepts and formulas, and they were all labeled as well. Figure 22 shows an example of Fall semester notes. One of the widely held beliefs in Japanese teaching is that boardwork should serve as a model for student notes. This data gives evidence that students will copy things into their notes exactly as it is written on the board so if the boardwork is clear and well organized then student notes will be too.

**Student Perceptions on Quality of Instruction**

A very interesting, and unexpected, aspect of student’s learning experience that emerged from this data is how the implementation of these design principles influenced the students’ perception of the quality of instruction given by Brooke. Nothing in the student data was used to measure for this idea but was an opinion expressed by Brooke in her interview. She felt that students now “trust the process a little bit more” [meaning her reformed style of teaching]. She felt that the application of these design principles had a direct impact on how her students perceived the quality of her instruction. Brooke said that her boardwork now “makes it clear that [she] has a plan and that it’s going to be visually clear how it all comes together” and in general has gotten “less push back on [her] style of teaching because [she] has gotten better with boardwork.”
She also said that in Winter semester she got less complaints that she “doesn’t make sense or isn’t clear … when the boardwork is more organized.” When I asked why she felt there were “less complaints” she said that usually by this point in the semester (week 7 of 15) she would have had “a very vocal group not happy with … [her] teaching style,” she even likened them to “a mutiny or revolt” but she had not had that yet Winter semester and she believes that “[the addition of the design principles] is probably the biggest difference.” Brooke felt that the implementation of these design principles had a direct effect on how students perceived the quality of her instruction. Having organized and pre-planned boardwork communicated to the students that she has spent time carefully constructing the lesson that day, that she has a plan and goals, and those goals will be clear on the board by the end of class.

**Cognitive Load for the Teacher**

One of the research questions I sought to study was how the application of these 5 design principles impacted Brooke’s in-the-moment decision making during her lessons. In-the-moment decisions are choices the teacher must make in-the-moment of teaching like how to deal with unexpected student thinking or changing plans mid-lesson. In discussing this topic with Brooke in her interview she brought up that she felt planning her boardwork helped “offload” organizational tasks that often became burdensome to her while teaching. In this way, planning her boardwork freed up space in her working memory, thus lightening her intrinsic cognitive load, and allowing her to focus on more important parts of her teaching practice. She discussed two things she felt were easier because of these design principles, in-the-moment decisions and facilitating class discussions.
In-the-Moment Decision Making and Facilitating Class Discussions

In-the-moment decision making most often came to Brooke while facilitating class discussions. In discussing which principles Brooke thought were most helpful when making in-the-moment decisions she brought up three principles, Layout, Grouping, and Hierarchy. Brooke felt that Grouping and Hierarchy are easy principles to implement “spontaneously,” that if she is not emphasizing something enough, she can “go through and underline it (Hierarchy) or use the same color (Grouping)” to highlight connections, but Layout is not something she can design spontaneously. She said that when she tried to make in-the-moment decisions with her boardwork, especially when encountering unexpected student thinking, she would often get bogged down with questions like “do I have space for this,” “how much space does the student need,” or “will I run out of space and now these concepts will be on different sides of the board.” Notice how all of these questions are about board space. These questions would “disrupt [her] quite a bit” while trying to facilitate class discussions. She expressed that these questions are “not the things [she] wants to be worrying about” in that moment, and she feels planning the layout “offloads what [she] has to think about in the moment, especially during the discussion.” By planning her board’s layout Brooke’s intrinsic cognitive load was lightened while having to make in-the-moment decisions during her lessons.

Another way Brooke described Layout lightening her cognitive load is how writing the “skeleton” of her layout plan on the board prior to the start of class served as a “physical reminder” of her lesson plan. She said she feels like she can “glance up at the board and see [her] lesson plan already up there.” This allows her to transition through her lesson more smoothly instead of taking awkward pauses to read her lesson plan. She knows she “needs to make this connection over here because [she] left space for it” or “ask this question next because of what
[she] has written there.” Her boardwork and layout served as a “visual reminder” of the questions she had planned for her students to discuss and the connections she wants them to see.

Summary

The results of this study gives evidence that students do notice the board and they notice it with great detail. The principle of Representations is easiest for students to see but the more obscure principles of Hierarchy, Grouping Perceptions, and Layout are also noticed by students, especially when these principles are intentionally applied by the teacher. This study also gives evidence that when the design principles of Hierarchy, Grouping Perceptions, Layout, Representations, and Repetition are applied to boardwork there is an impact on students’ learning experience and cognitive load. The way the board is designed influences the organization of student notes, the key concepts they take away from class, and how well they recall the conversation surrounding a specific mathematical representation. The data also suggests that the application of these principles improves student performance on free recall questions, but more data is needed to confirm this finding. Lastly, this study provides evidence that when a teacher learns and applies these principles to boardwork their cognitive load can also be lightened in the moment of teaching making it easier to perform some of the more cognitively demanding aspects of teaching like facilitating class discussions.
CHAPTER 6: CONCLUSION

Introduction

The need for this study grew out of my own experiences with poor boardwork in some of my mathematics classes. Most research on boardwork has focused on what the purpose of boards in the classroom should be and few have studied how to design the look of boardwork. This study has expanded on existing research on boardwork by exploring how visual design principles can be implemented with the medium of board space. It has sought to answer three questions: do students notice boardwork and what do they notice about it, does intentionally designed boardwork have an impact on students’ learning experience and cognitive load, and how does the knowledge of visual design principles influence teachers’ in-the-moment decision making. To answer these questions this study used both quantitative and qualitative measures by surveying students to assess their self-reported cognitive load, recall abilities, and learning experience, analyzing student notes as another way of gauging students’ learning experience, and using an interview to assess the teacher’s experience learning and implementing these design principles in her classroom.

This research gives evidence that students do notice boardwork and with intentional use of the design principles they notice even greater detail in the boardwork. Students’ most easily notice Representations but are also able to pick up on more nuanced principles like Hierarchy, Grouping Perceptions, and Layout. Students’ learning experiences do change with the use of these design principles by influencing how they take notes and how they perceive the quality of instruction. And the application of these design principles influences the students’ and teacher’s cognitive load. Making it easier for students to interpret the boardwork and pull out the key
concepts from class and easier for the teacher to facilitate class discussions and handle unexpected student thinking.

**Contributions**

This study provides several important contributions to the field of mathematics education research. It adds to current research by expanding on the purpose of boardwork in the classroom, exploring the connection between Cognitive Load Theory and Gestalt Theory, and looks at ways to reduce the Cognitive Load of students and teachers during class. This study also contributes to the field by introducing new areas of research such as student perceptions, interpretations, and interactions with boardwork and teaching teachers about boardwork best practices through visual design principles.

**Expanding on Current Research**

While most of the current research on boardwork focuses on what the purpose of boardwork can be, it does not detail how to visually accomplish these purposes. This is where my research helps fill in the holes. A large contribution of my research is the framework I created to bridge the gap between the design world and mathematics education world. This framework came from my experience as a graphic designer and mathematics educator and was supported by research I found in my literature review. It bridges this gap by showing how to apply visual design principles like *Hierarchy, Grouping Perceptions, Layout, Representations,* and *Repetition* to boardwork in a mathematics classroom to improve its visual aesthetic and ease the cognitive load of the learner.

Another gap in the research that my study helps fill is the connection between visual design principles and cognitive load. Past studies by Yong et al. (2016) and Moller et al. (2012) suggested that there is a relationship between visual design principles and ease of learning, but
more research was needed to show this is true. My research provides evidence that there is a correlation. Three of the five design principles showed statistically significant improvement in scores by students on part 1 of the student survey meaning they saw an improvement in the use of these design principles and felt they lightened their cognitive load. In addition to this, changes to one of the mathematical representations on Day 1 of observations also significantly improved students’ recall of the conversation surrounding that piece of the boardwork. The data from this study gives evidence that implementing these design principles with boardwork in a mathematics classroom lowers students’ extraneous cognitive load.

This study doesn’t just link design principles with cognitive load for students but also for the teacher. Considering the cognitive load of teacher’s while teaching seems to be a relatively new field of study. The earliest article I could find on this topic is from 2007. These articles look at teacher’s cognition, often comparing experienced teachers with novice teachers, and the varying types of load they experience while teaching (Dessus et al., 2015; Feldon, 2007; Moos & Pitton, 2014). None of these studies, however, have considered the role the board plays in a teacher’s cognitive load. My study gives evidence that, for Brooke at least, learning these design principles and pre-planning her boardwork helped lower her intrinsic cognitive load while teaching. She felt that pre-planning her boardwork and writing up the “skeleton” of her lesson on the board before class made the flow of her lesson much smoother and “definitely offloads what [she] has to think about in-the-moment.” She also felt that knowledge of the design principles, particularly Layout and Grouping Perceptions, aided her in navigating class discussions and unexpected student thinking. She could better focus her working memory on facilitating the class discussion and helping students makes connections instead of being preoccupied with trying to organize the boardwork in-the-moment.
New Areas of Research

This study also explored several new avenues of research in mathematics education. One of the largest of these being how students perceive, interpret, and interact with boardwork in a mathematics classroom. The data from part 1 of the student survey gives strong evidence that students do indeed notice boardwork. They notice the quality of the boardwork, have opinions on what makes boardwork good, and are even able to notice nuanced design principles being applied to boardwork. The qualitative analysis showed many instances where students made very detailed observations about Brooke’s boardwork, picking up on patterns in her boardwork practices, making suggestions for improvement, and identifying specific design principles in her boardwork. Not only do students notice these design principles in boardwork but appreciate and can tell when it is done well. The qualitative and quantitative data from this study shows evidence that Winter semester students felt these design principles aided their understanding of the board more than Fall semester students. All together this study gives strong evidence that students notice boardwork and can tell and appreciate when boardwork is done well.

Another area of research this study contributes to is how students interpret and interact with the board in a mathematics classroom. To my knowledge the closest research on this topic is from Stigler and Hiebert’s (1999) *The Teaching Gap*. Their study detailed pieces of the teaching culture in the US and briefly talked about how teacher’s use public record in the classroom. They share that in the US one of the main purposes of boardwork in the classroom for teachers is to demonstrate example problems. My data shows evidence that students have also adopted this view of the board and that it is a difficult perception to change. In part 1 of the student survey students were asked in Question 6 what ways they used the board during class (see Table 3). From day to day most of these options changed places but across all six
observations in Fall and Winter semesters the option “to see examples of worked out problems” took the number one spot every single day with 90% or more of students choosing this option each day. Even with the changes to the boardwork, students in Winter semester still held a firm view that the board is for demonstrating example problems. This reaffirms Stigler and Hiebert’s notions about the difficulty of changing teaching cultures. US students have spent much, if not all, of their life using the board to view and repeat practice problems and this study shows evidence that their expectations for instruction will not change over the course of a few weeks.

Lastly, an important contribution of this study is the method’s section details on how to teach teachers to use these design principles. This study has shown that someone who has never taken classes on design can learn to use these design principles. Being that the application of visual design principles to boardwork in a mathematics classroom is a new area of study there currently is no research on how to teach these principles to teachers. The methods section of this study outlines one way to go about facilitating this instruction and gives suggestions on changes I would make after having conducted these instructions myself. This contributes new knowledge to the study of boardwork and serves as a springboard for future research.

**Implications**

The results of this study provide implications for both practice and research. In this section I first discuss implications for practice and how the findings from this study can influence teacher practice and teacher education. Then I describe the implications for research and how the findings from this study can influence the research conducted on boardwork design in a mathematics classroom.
Implications for Practice

This research has many implications for teachers, both how they are educated and practices within their classrooms. A large contribution from this study was the framework and how it provides tools for accomplishing boardwork goals. This framework can also be used by teacher’s to evaluate and improve their own boardwork. Once a teacher learns these principles and how to recognize and apply them, they can look at the results of their boardwork efforts and see in what ways each principle is being applied, or perhaps could have been applied. This self-reflection will then allow them to improve their boardwork lesson by lesson. While this study focused on the boardwork efforts of a reformed style teacher, I believe these principles can also be used by those who teach in a more traditional manner as well. No matter ones purpose in using boardwork these principles are meant to aid in accomplishing that purpose. I also believe that while these design principles were chosen with mathematics classrooms in mind, this framework could be applied to other subjects as well but would need future study.

Since this research provides strong evidence that students notice boardwork and appreciate good boardwork, I hope this serves as motivation for teachers to try and improve their boardwork practices. I hope this study makes teachers aware that what they put on the board has an impact on what their students get out of the lesson and makes teachers think twice about the actions they take with their boardwork. Teachers should take a step back from their boardwork and try to imagine how students will interpret it. They can ask themselves “is my message clear?,” “can they easily distinguish what is most important?,” “are my main concepts grouped together?,” “do the representations aid or hinder their understanding of these concepts?” By learning these design principles, I hope that these are the kinds of questions teachers begin to ask themselves.
Another aspect of this study that should motivate teachers to start using these design principles is the finding that they lighten their intrinsic cognitive load. Teachers can offload the boardwork portion of teaching by planning it beforehand. This should lighten their cognitive load when making in-the-moment decisions, allowing them to focus on student responses. I also believe that this finding should motivate pre-service teacher education programs to start incorporating education on boardwork design into their curriculum. There has been some research already on the cognitive load of novice teachers that indicates their intrinsic cognitive load is high. Since this study provides evidence that planning boardwork and applying these design principles lowered Brooke’s intrinsic cognitive load, boardwork planning could be a valuable tool to aid novice teachers their first years of teaching and further research studying this could be of value.

Lastly, this research contributed to knowledge of the teaching culture in the US and showed that by learning and applying these design principles to her boardwork, Brooke’s perception of the purpose of the board changed. Previously she viewed the board as a place to put up example problems and show student work, but now she sees it as a tool for aiding students in making connections across the mathematics. Using the board as a place to show example problems is a strong part of the teaching culture in the US (Stigler & Hiebert, 1999; Wylie, 2012). This research implies that by adopting these practices and learning these design principles that culture could change by showing teacher’s that there are more ways to use the board than just demonstrating example problems.

Implications for Research

This study has a few implications for research as well. The framework of this study provides a new way of analyzing boardwork in the classroom which could be used in future
research on boardwork. It shows that boardwork doesn’t have to be thought of just through the lens of representations but a much wider variety of design principles. Along with the framework, the methods of this study could be used with other design principles. While I have only selected these five design principles to study, this research opens the questions to what other design principles could be applied to boardwork and what are their impact on cognitive load. My study also suggests that analytic measures of CLT like testing recall may be an effective way of measuring cognitive load, though more research with larger sample sizes is needed to confirm this.

**Limitations and Direction for Future Studies**

There are three main limitations to my study that I have seen from analyzing my data and reflecting on my findings. The first comes from limitations on measuring students cognitive load through recall abilities. Since recall abilities are content specific questions, these questions could not be grouped across all three observation days and instead needed to be compared day by day across the two semesters. This created small sample sizes, especially in Winter semester, when analyzing this aspect of my research question. The average number of responses each observation day in Fall semester was 27 and in Winter semester 21. This limited my ability to accurately analyze changes in students recall ability. The data showed improvement in scores by about 15% but because of the small sample sizes this did not produce a statistically significant $p$ value. For this reason, I make minimal conclusions on how these design principles impacted students recall abilities as there was only one instance where significance could be drawn (see Table 10). Future research might focus solely on this aspect of measuring student cognitive load through recall abilities and try to elicit much
larger sample sizes by seeking a larger class than Brooke’s from which to recruit volunteers, making participation mandatory, or providing a greater incentive for those that participate.

Another limitation of this study is the use of self-report measures in the student survey. Having students self-report their perceived cognitive load is very subjective and some might question how reliable this type of measure is. Past studies only focused on an experiment group and used their mean scores on the Likert scale to indicate high or low cognitive load. To minimize this limitation, I used a control group and did not look for my experiment group to average a certain rating by instead looked for significant changes in how students self-reported their cognitive load in the control group versus the experimental group. Furthermore, students were not told which group they were participants of, so the procedure was blinded to minimize subject bias. Since the Likert scale questions in the survey do not rely on the content taught, future studies might look to test the application of these design principles on the same set of students, surveying their cognitive load at the beginning and end of a semester or year. This would, however, make it more difficult to test recall abilities since the content for the observation days would not be the same.

However, a corollary limitation is that there were more changes to her lessons than just her boardwork so the experiment was not perfectly controlled. In helping Brooke plan her Winter lessons I tried to minimize this as much as possible and keep the content the same, but because of differences in the calendar schedule from Fall to Winter semester some content got shifted around, and different teaching techniques were used. I controlled for this limitation by throwing out the data from the content specific questions where I felt this affected the data. This happened on observation days 2 and 3. The significant changes in what was taught those days and how it was taught, aside from changes in the boardwork,
made it so the data from part 2 of the survey could not be directly compared. Instead, I only used data from part 2 of observation day 1 to analyze students recall abilities. Future studies might choose a more tenured teacher than I had. Previous to Fall semester Brooke had only taught one semester of College Trigonometry. During the first few years of teaching, I believe teachers make many changes to their scope and sequence. Future studies might choose a more experienced teacher on the subject to help mitigate the number of changes that occur between the control and experiment groups.

Another area of my results that did not see a significant change was student responses to Question 6 which asked them in what ways they used the board in class. This essentially gathered data on how students view the purpose of the board in a mathematics classroom. The data I pulled from Fall to Winter semester showed no significant changes to the ways in which student view the purpose of the board. In fact, the ranking order in which students chose from the various options stayed nearly identical. Perhaps more time is what is needed to see significant changes in how students’ perceptions on the purpose of boardwork change with the application of these design principles. This could be better researched by performing these same methods but using lessons at the end of two semesters instead of the beginning. This way the experiment group has a full semester’s worth of experimental instruction. Observing lessons at the end of these two semesters could not be done in my study because of the need to stick to a graduation timeline.

Lastly, this study only used one teacher across the experiment which is, of course, a very small sample size. Findings about how these design principles influenced Brooke’s way of thinking and teaching may not be generalizable to all teachers for this reason. This limitation could not be minimized due to the nature of size for a master’s thesis and that the
primary focus of this study was the effect on students. But future studies might replicate part of this study by choosing a group of teachers to first interview about their views on the purpose of boardwork, then teach these design principles and practice implementing them, and finally interview them again to compare how their views may have changed.

Summary

This study was conducted to understand the relationship between cognitive load and the application of visual design principles on boardwork in a mathematics classroom. The need for this study stemmed from my own experiences feeling frustrated by the board’s lack of clarity in some of my mathematics classes. This research addressed this issue by teaching a teacher in a college trigonometry course about five visual design principles, helping them apply it to their boardwork, and documenting the effect it had on them and their students. It found evidence that students do indeed notice the board with great detail and that the application of these design principles impacted both the students’ and teacher’s cognitive load in positive ways. Students in the experimental group found it easier to recognize important information, draw connections across the board, and more accurately recalled the concepts discussed when shown a piece of the board from class. The teacher felt that the practice of planning her boardwork and knowledge of these design principles offloaded what she had to think about in the moment of teaching, making it easier follow her lesson plan, navigate class discussions, and respond to unexpected student thinking. I hope that the results of this study find their way into pre-service teacher education programs and professional development workshops so that more teachers are considering how their boardwork practices impact their students’ learning experience. I also hope that further research will be conducted to better understand the impact of these principles on students’ recall abilities and gain a broader view of the impact of boardwork planning on other teachers.
REFERENCES


Note: The above book had 74 elementary teachers contribute to the writing and Mr. Kamoda was the principal at that school. We do not have the names of the 74 elementary teachers.


https://doi.org/10.1007/978-3-030-03580-8_8


# APPENDIX A: SUMMARY OF STUDENT SURVEYS

<table>
<thead>
<tr>
<th>Student Survey Part 1</th>
<th>Fall Day 1</th>
<th>Winter Day 1</th>
<th>Fall Day 2</th>
<th>Winter Day 2</th>
<th>Fall Day 3</th>
<th>Winter Day 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q1:</strong> It is easy to determine what information on the board is most important. Justify your choice.</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
</tr>
<tr>
<td><strong>Q2:</strong> The board made it difficult for me to recognize and link crucial information. Justify your choice.</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
</tr>
<tr>
<td><strong>Q3:</strong> The mathematical representations help me better understand the topics of the lesson. Justify your choice.</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
</tr>
<tr>
<td><strong>Q4:</strong> This mathematical representation helped me better understand the two different ways to calculate Arc Length. Justify your choice</td>
<td>Same as FD1</td>
<td>From this mathematical representation I feel confident in my understanding of how we create the sine and cosine functions. Justify your choice.</td>
<td>Same as FD2</td>
<td>I am confident in my understanding of why Sine=Opp/Hyp and Cosine=Adj/Hyp</td>
<td>Same as FD3</td>
<td></td>
</tr>
<tr>
<td><strong>Q5:</strong> Have you noticed any patterns in the way your teacher uses the board from one lesson to the next? Or within one lesson?</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
</tr>
<tr>
<td>Q6: Which of the following are ways in which you use the board for learning? Check all that apply.</td>
<td>Winter Day 1</td>
<td>Fall Day 2</td>
<td>Winter Day 2</td>
<td>Fall Day 3</td>
<td>Winter Day 3</td>
<td></td>
</tr>
<tr>
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<tr>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
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</tr>
</tbody>
</table>

**Student Survey Part 2**

<table>
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<tr>
<th>Q1: What did you learn from MATH 111 class on <em>insert date</em>?</th>
<th>Winter Day 1</th>
<th>Fall Day 2</th>
<th>Winter Day 2</th>
<th>Fall Day 3</th>
<th>Winter Day 3</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q2: What was your main takeaway from class on <em>insert date</em>? What part of the lesson told you this was an important idea?</th>
<th>Winter Day 1</th>
<th>Fall Day 2</th>
<th>Winter Day 2</th>
<th>Fall Day 3</th>
<th>Winter Day 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same as FD1</td>
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<td>Same as FD1</td>
<td>Same as FD1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q3: What parts/concepts were easy to understand? Why?</th>
<th>Winter Day 1</th>
<th>Fall Day 2</th>
<th>Winter Day 2</th>
<th>Fall Day 3</th>
<th>Winter Day 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same as FD1</td>
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<td>Same as FD1</td>
<td>Same as FD1</td>
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<thead>
<tr>
<th>Q4: What parts/concepts were difficult to understand? Why?</th>
<th>Winter Day 1</th>
<th>Fall Day 2</th>
<th>Winter Day 2</th>
<th>Fall Day 3</th>
<th>Winter Day 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td>Same as FD1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q5: What can you remember about the conversation surrounding this piece of the boardwork? (image shown)</th>
<th>Winter Day 1</th>
<th>Fall Day 2</th>
<th>Winter Day 2</th>
<th>Fall Day 3</th>
<th>Winter Day 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same question, different image to reflect change in boardwork</td>
<td>Same question, different image to reflect change in topic for the day</td>
<td>Same question, different image to reflect change in boardwork</td>
<td>Same question, different image to reflect change in topic for the day</td>
<td>Same question, different image to reflect change in boardwork</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q6: How is finding Arc Length related to finding Area of a Sector? (image shown)</th>
<th>Winter Day 1</th>
<th>Fall Day 2</th>
<th>Winter Day 2</th>
<th>Fall Day 3</th>
<th>Winter Day 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same question, different image to reflect change in boardwork</td>
<td>How do we find the vertical distance of an object when given theta and sine of theta? (image shown)</td>
<td>Same question, different image to reflect change in boardwork</td>
<td>How is the tangent function related to the unit circle? (image shown)</td>
<td>Same question, different image to reflect change in boardwork</td>
<td></td>
</tr>
<tr>
<td>Fall Day 1</td>
<td>Winter Day 1</td>
<td>Fall Day 2</td>
<td>Winter Day 2</td>
<td>Fall Day 3</td>
<td>Winter Day 3</td>
</tr>
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</tr>
<tr>
<td>Q7: Why did Ms. Matthews draw a box around the equation below? (image shown)</td>
<td>*In your words how would you describe to a classmate when to use one area of a sector equation and when to use the other formula? (image shown)</td>
<td>Why did Ms. Matthews draw the yellow triangle in the first quadrant? (image shown)</td>
<td>*Why did Ms. Matthews draw the yellow and orange arrows on the ceiling fan representations? (image shown)</td>
<td>How do we know the tangent function is odd? (image shown)</td>
<td>*In your own words why does cot(pi/2) = 0? (image shown)</td>
</tr>
<tr>
<td>Q8: Explain how to convert between feet/second and radians/second. (image shown)</td>
<td>*How would you describe the purpose of this section of the board? (image shown)</td>
<td>What do the points (written in black) on the unit circle mean or represent? (image shown)</td>
<td>*Explain in your own words what the sine or cosine function mean or represent? (i.e. what do we put in and what do we get out) (image shown)</td>
<td>Explain in your own words why sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent. (image shown)</td>
<td>Same question, different image to reflect change in boardwork</td>
</tr>
<tr>
<td>Q9: What is 160 degrees as a measure in radians? Same as FD1</td>
<td>What is the vertical distance if I told you theta=1.7 radians and sine(1.7)=0.99? (image shown)</td>
<td>Same as FD1</td>
<td>Evaluate the following, cot(pi/6)</td>
<td>Same as FD3</td>
<td></td>
</tr>
<tr>
<td>Q10: Find the Arc Length of S below. (image shown) Same as FD1</td>
<td>What is the coordinate point for the marked spot below (the red dot) given that theta = 5pi/4. Please explain how you found your answer. (image shown)</td>
<td>*The circle is centered on a coordinate grid. Given that the angle marked below in radians is 5pi/4 what is the coordinate point for the marked spot (the red dot). Please explain how you found your answer. (image shown)</td>
<td>Find cosine of the angle labeled alpha below (image shown)</td>
<td>Same as FD3</td>
<td></td>
</tr>
</tbody>
</table>

Note: Questions marked with an asterisk (*) had to be changed in Winter semester because of the changes Brooke made to her scope and sequence between Fall and Winter semesters.
APPENDIX B: TEACHER INTERVIEW QUESTIONS

General Impressions

Think back to before we started all this, what was your view on boardwork in the classroom? How did you tend to use it or how did you view its usefulness or not usefulness?

How has your view of using boardwork changed by studying these principles? Are some principles easier to incorporate than others? Are some more useful during planning and others in the moment?

Do you anticipate continual use of these principles in your future teaching and lesson planning? Use of all the principles? Only some?

Which of the principles do you find most beneficial for students? For your teaching?

Students

How have you seen the implementation of these design principles in your boardwork affect your students?

Have you seen a difference in the students’ performance on tests and homework from semester to semester, or from the beginning of last semester to the end?

Do you feel the boardwork helped you see how students were struggling?

Teacher

How has learning the principles influenced the way you plan your lessons?

Does thinking about the boardwork principles have an influence on the structure and flow of your lessons? If so, how?

Does your knowledge of design principles influence your in the moment decisions during your lesson? (give specific example from observations)
APPENDIX C: FALL SEMESTER STUDENT SURVEYS

Student Survey Day 1 Part 1
Question 1

Please choose which of the following best describes your experience from class on September 9th according to the following statement: It is easy to determine what information on the board is most important.

1  2  3  4  5  6  7

Very Strongly Agree  ○ ○ ○ ○ ○ ○ ○ Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 2

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The board made it difficult for me to recognize and link crucial information.

1 2 3 4 5 6 7

Very Strongly Agree   Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer

Question 3

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The mathematical representations helped me better understand the topics of the lesson. (Mathematical representations are diagrams, graphs, models, formulas, equations, expressions, etc.)

1 2 3 4 5 6 7

Very Strongly Agree   Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 4

Please choose which of the following best describes your experience from class on September 9th according to the following statement: This Mathematical Representation helped me understand the two different ways to calculate arc length.

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
**Question 5**

Research Study Questions

Have you noticed any patterns in the way your teacher uses the board from one lesson to the next? Or within one lesson?

Your answer

**Question 6**

Which of the following are ways you use the board for learning? Check all that apply.

- [ ] Model for my notes
- [ ] Summarize the lesson
- [ ] See the main points of the lesson
- [ ] See examples of worked out problems
- [ ] Keep track of where we are in the discussion
- [ ] To better understand what the teacher said
- [ ] Other: ________________________________
Student Survey Day 1 Part 2

Question 1

Survey Questions: General

What did you learn from the MATH 111 class on September 9th? *

Your answer

Question 2

What was your main take away from class on September 9th? What part of the lesson told you this was an important idea? *

Your answer

Question 3

What parts/concepts were easy to understand? Why? *

Your answer

Question 4

What parts/concepts were difficult to understand? Why? *

Your answer
Question 5

What can you remember about the conversation surrounding this piece of the boardwork?

Your answer
Question 6

How is finding arc length related to finding the area of a sector? *

Your answer
Question 7

Why did Mrs. Dixon draw a box around the equation below? *

\[ S = r \theta \]
\[ S = \frac{\theta}{2\pi} (2\pi r) \]
\[ A = \left( \frac{\theta}{2\pi} \right) \left( \pi r^2 \right) \]
\[ = \frac{\theta r^2}{2} \]
Question 8

Explain how to convert between feet/second and radians/second? *

- \( \frac{5.2 \text{ ft}}{2.5} \rightarrow 2 \) radians/second
- \( \frac{11.7}{2.0} \rightarrow 4.5 \) radians/second
- \( \frac{11.7}{2.0} \rightarrow 4.5 \) radians/second
- \( \frac{4.5}{2.0} \rightarrow 0.72 \) radians/second
Question 9

What is 160 degrees as a measure in radians? *

Your answer
Recall-Question 1

Please do not use your notes while answering these questions

Question 10

Find the arc length of S below.*

Your answer
Student Survey Day 2 Part 1
Question 1

Please choose which of the following best describes your experience from class on September 9th according to the following statement: It is easy to determine what information on the board is most important.

1  2  3  4  5  6  7

Very Strongly Agree  ○ ○ ○ ○ ○ ○ ○ Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer

Question 2

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The board made it difficult for me to recognize and link crucial information.

1  2  3  4  5  6  7

Very Strongly Agree  ○ ○ ○ ○ ○ ○ ○ Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 3

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The mathematical representations helped me better understand the topics of the lesson. (Mathematical representations are diagrams, graphs, models, formulas, equations, expressions, etc.)

1 2 3 4 5 6 7

Very Strongly Agree  ○ ○ ○ ○ ○ ○ ○  Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 4

Please choose which of the following best describes your experience from class on September 16th according to the following statement: From this Mathematical Representation I feel confident in my understanding of how we create the sine and cosine functions.

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.
**Question 5**

Research Study Questions

Have you noticed any patterns in the way your teacher uses the board from one lesson to the next? Or within one lesson?

Your answer

**Question 6**

Which of the following are ways you use the board for learning? Check all that apply.

- [ ] Model for my notes
- [ ] Summarize the lesson
- [ ] See the main points of the lesson
- [ ] See examples of worked out problems
- [ ] Keep track of where we are in the discussion
- [ ] To better understand what the teacher said
- [ ] Other: __________________________

Student Survey Day 2 Part 2

Question 1

Survey Questions: General

What did you learn from the MATH 111 class on September 16th? *

Your answer

Question 2

What was your main take away from class on September 9th? What part of the lesson told you this was an important idea?

Your answer

Question 3

What parts/concepts were easy to understand? Why? *

Your answer

Question 4

What parts/concepts were difficult to understand? Why? *

Your answer
Question 5

Recall-Question 1

Please do not use your notes while answering these questions

What can you remember about the conversation surrounding this mathematical representation?

Your answer
Question 6

Recall-Question 2

Please do not use your notes while answering these questions

How do we find the vertical distance of an object when given theta and sine of theta?

$\theta = 2$

$\sin(\theta) \approx 0.91$

$\text{vertical distance?}$

$\text{radius} \cdot \sin(\theta) = \text{vertical distance}$

$3(0.91) = 2.73$

$\text{radius} \cdot \cos(\theta) = \text{horizontal distance}$

Your answer
Question 7

Recall-Question 3

Please do not use your notes while answering these questions

Why did Mrs. Dixon draw the yellow triangle in the first quadrant? *

Your answer
Question 8

Recall-Question 4

Please do not use your notes while answering these questions

What do the points (written in black) on the unit circle mean or represent? *

Your answer
Question 9

Recall-Question 5

Please do not use your notes while answering these questions

What is the vertical distance if I told you theta = 1.7 radians and \( \sin(1.7) = 0.99 \) *

Your answer
Question 10

Recall-Question 6

Please do not use your notes while answering these questions

What is the coordinate point for the marked spot below (the red dot) given that theta = 5pi/4. Please explain how you found your answer.

The Unit Circle - Radians

Your answer
Student Survey Day 2 Part 1
Question 1

Please choose which of the following best describes your experience from class on September 9th according to the following statement: It is easy to determine what information on the board is most important.

1 2 3 4 5 6 7

Very Strongly Agree ○ ○ ○ ○ ○ ○ ○ Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer

Question 2

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The board made it difficult for me to recognize and link crucial information.

1 2 3 4 5 6 7

Very Strongly Agree ○ ○ ○ ○ ○ ○ ○ Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 3

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The mathematical representations helped me better understand the topics of the lesson. (Mathematical representations are diagrams, graphs, models, formulas, equations, expressions, etc.)

1 2 3 4 5 6 7

Very Strongly Agree ○ ○ ○ ○ ○ ○ ○ Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 4

Please choose which of the following best describes your experience from class on September 23rd according to the following statement: I am confident in my understanding of why Sine=Opp./Hyp. and Cosine=Adj./Hyp.

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.
Question 5

Research Study Questions

Have you noticed any patterns in the way your teacher uses the board from one lesson to the next? Or within one lesson?

Your answer

Question 6

Which of the following are ways you use the board for learning? Check all that apply.

☐ Model for my notes
☐ Summarize the lesson
☐ See the main points of the lesson
☐ See examples of worked out problems
☐ Keep track of where we are in the discussion
☐ To better understand what the teacher said
☐ Other: ____________________________
**Student Survey Day 3 Part 2**

**Question 1**

Survey Questions: General

What did you learn from the MATH 111 class on September 23rd? *

Your answer

**Question 2**

What was your main take away from class on September 9th? What part of the lesson told you this was an important idea?

Your answer

**Question 3**

What parts/concepts were easy to understand? Why? *

Your answer

**Question 4**

What parts/concepts were difficult to understand? Why? *

Your answer
Question 5

Recall-Question 1

Please do not use your notes while answering these questions

What can you remember about the conversation surrounding this piece of the boardwork?

\[
\begin{align*}
\sin(\alpha) &= \frac{6}{\sqrt{40}} \\
\cos(\alpha) &= \frac{2}{\sqrt{40}} \\
\tan(\alpha) &= \frac{\sqrt{2}}{2} \\
\sin(\beta) &= \frac{2}{\sqrt{40}} \\
\cos(\beta) &= \frac{\sqrt{2}}{\sqrt{40}} \\
\tan(\beta) &= \frac{2}{\sqrt{2}} \\
\alpha + \beta &= 90^\circ
\end{align*}
\]
Question 6

Recall-Question 2

Please do not use your notes while answering these questions

How is the tangent function related to the unit circle? *

Your answer
How do we know the tangent function is odd? *
Question 8

Recall-Question 4

Please do not use your notes while answering these questions

Explain in your own words why sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent.

Your answer
Question 9

Recall-Question 5

Please do not use your notes while answering these questions

Evaluate the following, $\cot(\pi/6)$ *

Your answer
Question 10

Recall-Question 6

Please do not use your notes while answering these questions

Find cosine of the angle labeled alpha below. *

Your answer
APPENDIX D: WINTER SEMESTER STUDENT SURVEYS

Student Survey Day 1 Part 1

Question 1

Please choose which of the following best describes your experience from class on September 9th according to the following statement: It is easy to determine what information on the board is most important.

1 2 3 4 5 6 7

Very Strongly Agree ○ ○ ○ ○ ○ ○ ○ Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 2

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The board made it difficult for me to recognize and link crucial information.

1 2 3 4 5 6 7

Very Strongly Agree 〇 〇 〇 〇 〇 〇 〇  Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer

Question 3

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The mathematical representations helped me better understand the topics of the lesson. (Mathematical representations are diagrams, graphs, models, formulas, equations, expressions, etc.)

1 2 3 4 5 6 7

Very Strongly Agree 〇 〇 〇 〇 〇 〇 〇  Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 4

Please choose which of the following best describes your experience from class on January 13th according to the following statement: This Mathematical Representation helped me understand the two different ways to calculate arc length.

Very Strongly Agree  O O O O O O O Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 5

Research Study Questions

Have you noticed any patterns in the way your teacher uses the board from one lesson to the next? Or within one lesson?

Your answer

Question 6

Which of the following are ways you use the board for learning? Check all that apply.

- [ ] Model for my notes
- [ ] Summarize the lesson
- [ ] See the main points of the lesson
- [ ] See examples of worked out problems
- [ ] Keep track of where we are in the discussion
- [ ] To better understand what the teacher said
- [ ] Other: ___________________________
Student Survey Day 1 Part 2
Question 1

Survey Questions: General

What did you learn from the MATH 111 class on January 13th? *

Your answer

Question 2

What was your main take away from class on September 9th? What part of the lesson told you this was an important idea? *

Your answer

Question 3

What parts/concepts were easy to understand? Why? *

Your answer

Question 4

What parts/concepts were difficult to understand? Why? *

Your answer
Question 5

What can you remember about the conversation surrounding this piece of the boardwork?

Your answer
Question 6

How is finding arc length related to finding the area of a sector? *
Recall-Question 1

Please do not use your notes while answering these questions

**Question 7**

In your words how would you describe to a fellow classmate when to use one Area of a Sector formula and when to use the other formula?

Your answer
Question 8

How would you describe the purpose of this section of the board? *

Converting Angles

\[
\frac{r}{\text{360}^\circ} = \frac{d}{2\pi} \quad \text{degrees} \quad \text{radians}
\]

Arc Length

\[
S = \frac{\theta}{2\pi} \quad \text{radius length}
\]

Area of Sector

\[
A = \frac{\theta}{2\pi} \quad (\pi r^2)\n\]

Angular Speed

\[
A = \frac{\text{360}^\circ}{2\pi} \quad \text{angle of circle}
\]
Recall-Question 1

Please do not use your notes while answering these questions

Question 9

What is 160 degrees as a measure in radians? *

Your answer
Question 10

Find the arc length of $S$ below. *

Your answer
Student Survey Day 2 Part 1
Question 1

Please choose which of the following best describes your experience from class on September 9th according to the following statement: It is easy to determine what information on the board is most important.

1  2  3  4  5  6  7
Very Strongly Agree  O O O O O O O Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer

Question 2

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The board made it difficult for me to recognize and link crucial information.

1  2  3  4  5  6  7
Very Strongly Agree  O O O O O O O Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 3

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The mathematical representations helped me better understand the topics of the lesson. (Mathematical representations are diagrams, graphs, models, formulas, equations, expressions, etc.)

1  2  3  4  5  6  7

Very Strongly Agree  ○  ○  ○  ○  ○  ○  Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 4

Please choose which of the following best describes your experience from class on January 20th according to the following statement: From this Mathematical Representation I feel confident in my understanding of how we create the sine and cosine functions.

Very Strongly Agree ☐ ☐ ☐ ☐ ☐ ☐ Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 5

Research Study Questions

Have you noticed any patterns in the way your teacher uses the board from one lesson to the next? Or within one lesson?

Your answer

Question 6

Which of the following are ways you use the board for learning? Check all that apply.

☐ Model for my notes
☐ Summarize the lesson
☐ See the main points of the lesson
☐ See examples of worked out problems
☐ Keep track of where we are in the discussion
☐ To better understand what the teacher said
☐ Other: ________________________________
Student Survey Day 2 Part 2

Question 1

Survey Questions: General

What did you learn from the MATH 111 class on January 20th? *

Your answer

Question 2

What was your main take away from class on September 9th? What part of the lesson told you this was an important idea? *

Your answer

Question 3

What parts/concepts were easy to understand? Why? *

Your answer

Question 4

What parts/concepts were difficult to understand? Why? *

Your answer
Question 5

Recall-Question 1

Please do not use your notes while answering these questions

What can you remember about the conversation surrounding this mathematical representation?

Your answer
Question 6

Recall-Question 2

Please do not use your notes while answering these questions

How do we find the vertical distance of an object when given theta and sine of theta?

\[
\sin(2) = 0.91 \\
\theta = 2 \text{ rad} \\
y = 3 \text{ inches} \\
0.91 = \frac{y}{3} \\
2.73 = y
\]
Question 7

Recall-Question 3

Please do not use your notes while answering these questions

Why did Mrs. Dixon draw the yellow and orange arrows on the ceiling fan representation?

Your answer
Question 8

Recall-Question 4

Please do not use your notes while answering these questions

Explain in your own words what the sine (or cosine) function represents. (i.e. what do we put in and what do we get out)

Your answer
Question 9

Recall-Question 5

Please do not use your notes while answering these questions

What is the vertical distance if I told you theta = 1.7 radians and sin(1.7)=0.99 *

Your answer
Question 10

Recall-Question 6

Please do not use your notes while answering these questions

The circle is centered on a coordinate grid. Given that the angle marked below in radians is $5\pi/4$ what is the coordinate point for the marked spot below (the red dot). Please explain how you found your answer.

The Unit Circle - Radians

Your answer
Student Survey Day 3 Part 1
Question 1

Please choose which of the following best describes your experience from class on September 9th according to the following statement: It is easy to determine what information on the board is most important.

1 2 3 4 5 6 7
Very Strongly Agree 〇 〇 〇 〇 〇 〇 〇 Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer

Question 2

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The board made it difficult for me to recognize and link crucial information.

1 2 3 4 5 6 7
Very Strongly Agree 〇 〇 〇 〇 〇 〇 〇 Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 3

Please choose which of the following best describes your experience from class on September 9th according to the following statement: The mathematical representations helped me better understand the topics of the lesson. (Mathematical representations are diagrams, graphs, models, formulas, equations, expressions, etc.)

1  2  3  4  5  6  7

Very Strongly Agree  ○  ○  ○  ○  ○  ○  Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 4

Please choose which of the following best describes your experience from class on September 23rd according to the following statement: I am confident in my understanding of why Sine=Opp./Hyp. and Cosine=Adj./Hyp.

\[
\sin(\theta) = \frac{\text{vert. dist.}}{\text{radius}} = \frac{y}{r} = \frac{\text{opposite}}{\text{hypotenuse}}
\]
\[
\cos(\theta) = \frac{\text{horiz. dist.}}{\text{radius}} = \frac{x}{r} = \frac{\text{adjacent}}{\text{hypotenuse}}
\]

1  2  3  4  5  6  7

Very Strongly Agree   O    O    O    O    O    O    Very Strongly Disagree

Please explain in detail, using examples from the board work done in class, why you chose the answer you did above.

Your answer
Question 5

Research Study Questions

Have you noticed any patterns in the way your teacher uses the board from one lesson to the next? Or within one lesson?

Your answer

Question 6

Which of the following are ways you use the board for learning? Check all that apply.

☐ Model for my notes
☐ Summarize the lesson
☐ See the main points of the lesson
☐ See examples of worked out problems
☐ Keep track of where we are in the discussion
☐ To better understand what the teacher said
☐ Other: ____________________________
Student Survey Day 3 Part 2

Question 1

Survey Questions: General

What did you learn from the MATH 111 class on January 27th? *

Your answer

Question 2

What was your main take away from class on September 9th? What part of the lesson told you this was an important idea?

Your answer

Question 3

What parts/concepts were easy to understand? Why? *

Your answer

Question 4

What parts/concepts were difficult to understand? Why? *

Your answer
Question 5

Recall-Question 1

Please do not use your notes while answering these questions

What can you remember about the conversation surrounding this piece of the boardwork?

Your answer
Question 6

Recall-Question 2

Please do not use your notes while answering these questions

How is the tangent function related to the unit circle? *

\[
\begin{align*}
\tan \left( \frac{\pi}{4} \right) &= \frac{\sqrt{2}}{\sqrt{2}} = 1 \\
\tan \left( \frac{\pi}{6} \right) &= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \\
\tan \left( \frac{5\pi}{3} \right) &= -\sqrt{3} \\
\end{align*}
\]

Your answer
Question 7

Recall-Question 3

Please do not use your notes while answering these questions

In your own words why does \( \cot(\pi/2) = 0? \)

Your answer
Question 8

Recall-Question 4

Please do not use your notes while answering these questions

Explain in your own words why sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent.

Your answer
Question 9

Recall-Question 5

Please do not use your notes while answering these questions

Evaluate the following, cot(pi/6) *

Your answer
Question 10

Recall-Question 6

Please do not use your notes while answering these questions

Find cosine of the angle labeled alpha below. *

Your answer