Selecting and Optimizing Origami-Based Patterns for Deployable Space Systems

Diana Stefania Bolanos

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Selecting and Optimizing Origami-Based Patterns for Deployable Space Systems

Diana Stefania Bolanos

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of
Master of Science

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ABSTRACT

Selecting and Optimizing Origami-Based Patterns for Deployable Space Systems

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Master of Science

This thesis addresses the design difficulties encountered when designing deployable origami-based arrays. Specific considerations regarding thickness accommodation, deployment, and parameter modifications are discussed. Patterns such as the Miura-ori, flasher, and hexagon are investigated, with emphasis placed on pattern modification from zero-thickness to finite-thickness.

Applying origami principles to form engineering solutions is a complicated task. Competing requirements may create confusion around which pattern is most favorable for the space array application. Implementing origami into a finite-thickness, engineered system poses challenges that are not manifest in a zero-thickness model. As such, it is important to understand and address the limitations of the pattern before implementing it into an engineered system. A preliminary set of approaches to address and mitigate design difficulties is provided.

This thesis seeks to improve understanding of design parameters, objectives, and trade-offs of origami pattern configurations. Emphasis is placed on finite-thickness models suitable for engineering applications. As a result, engineers and designers should be better prepared to integrate origami principles into space system design.

Keywords: origami-based design, Miura-ori, flasher, deployment, space arrays, design methods, optimization
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NOMENCLATURE

$\eta$  Pattern circularity
$\gamma$  Deployment factor
$\sigma$  Panel circularity
$h$  Miura-ori unit height
$w$  Miura-ori unit width
$\alpha$  Miura-ori unit angle
$m$  Number of unit cells in the vertical direction
$n$  Number of unit cells in the horizontal direction
$A_{hexagon}$  Hexagon deployed area
$\gamma_{hexagon}$  Hexagon stowed area
$A_{Miura-ori}$  Miura-ori deployed area
$\gamma_{Miura-ori}$  Miura-ori deployed area
$r$  Radius of the circle that fully encompasses an $m \times n$ Miura-ori
$A$  Area of the circumscribing circle of an $m \times n$ Miura-ori
$N_{creases}$  Number of creases of an $m \times n$ Miura-ori
$V$  Number of vertices
$\gamma_{1-4}$  Deployment angle of Miura-ori unit cell panels
$l_{min}$  Length of Miura-ori panel in minor direction
$l_{maj}$  Length of Miura-ori panel in major direction
$DoN$  Degree of nesting
$DHO$  Double-hinge offset
$OCS$  Origami-based cylindrical structures
$MA_r$  Rigid mechanical advantage tailored to the Miura-ori
$MA_c$  Compliant mechanical advantage tailored to the Miura-ori
$\delta U_c$  Differential change in strain energy
$\delta W_{in}$  Differential change in input work
$m$  Flasher rotational order
$h$  Flasher height order
$r$  Flasher ring order
$dr$  Flasher separation parameter
$s_f$  Implemented flasher scaling factor
$r_{cc}$  Central hub circumcircle radius
$t$  projected distance between panel nodes as observed from a top view
$n_p$  Number of panels
$\psi$  Deployed diameter to stowed diameter
$\kappa$  Deployed diameter to stowed height
$\zeta$  Stowed height to stowed diameter
$\phi_d$  Deployed diameter
$H_s$  Stowed height
$H_b$  Limiting stowed height
$D_s$  Stowed diameter
$D_b$  Limiting stowed diameter
CHAPTER 1. INTRODUCTION

1.1 Motivation

Origami has influenced product development throughout the past several years. The principles of paper folding have provided novel approaches for engineers to create physical and functional products. From space arrays [3–5] to medical devices [6], origami-inspired systems have diversified the design process for designers around the globe.

Incorporating origami into products introduces the advantage of low friction systems and high ratios of deployed-to-stowed volumes. Space arrays in particular have found a need for such characteristics. However, one consideration when designing origami products is the inherent thickness that comes from converting a paper-thin pattern into a 3D object. This consideration significantly increases the complexity of designing origami products.

Origami-based systems have been categorized into three separate classifications: applied, adapted, and inspired [1]. Systems that have a direct link to origami are classified as origami-adapted. This classification suggests the ability to modify the pattern, accommodate for thickness, and use non-paper-like materials. As a result, most space systems fall under this category.

Deployment is an aspect of origami systems that requires thoughtful evaluation. While benefits of compactness exist for introducing origami-adapted design for systems, deploying the system is another challenge. One of the most effective ways to create deployment is to utilize strain energy. This method avoids the need for electrical actuators and the associated power consumption, weight, and cost. Zirbel presented a specific example utilizing strain energy in the membrane to create a self-deployable array, which showed the potential to achieve strain-energy deployment of origami arrays [7].

A challenge posed by origami design is deciding which parameters will yield the required results. Whether it be to achieve high packing efficiency or reconfigurability, designers are left with the task of deciding which origami parameters will be most suitable for their needs. Various details
such as pattern choice and thickness accommodating techniques produce different outcomes. It is important to accurately and efficiently decide which parameters are best suited for the task at hand.

The objective of this research is to develop tools and metrics to aid in the selection process of origami patterns for deployable systems. This overall objective is decomposed into three subobjectives:

- Develop a systematic framework for selecting an origami source pattern for deployable space array applications
- Explore design challenges specific to the origami pattern and present approaches for mitigating each challenge
- Develop analytical methods for modifying patterns to meet design objectives

1.2 Background

Three important aspects of designing origami-adapted deployable systems are explained below. To develop novel deployable origami-adapted systems, it is important to understand the background of origami patterns, thickness accommodation techniques, and methods used to deploy thick origami systems.

1.2.1 Origami Patterns

A wide variety of origami twists and tessellations exist. From paper cranes to folding furniture, origami’s versatile nature finds application in a broad set of fields. Common patterns used in engineering include: Miura-ori, Square Twist, Tachi Miura, Waterbomb, and many more. Origami patterns come in families, meaning within a specific pattern, internal parameters can be changed to create different variations. Work has been done to investigate the effects of varying zero-thickness crease patterns [8], but little is known on the effects of altering thickened origami pattern parameters such as angles, ratios, and unit-cell orientations. This research will address the tradeoffs that arise as a result of altering patterns and their parameters. Figure 1.1 shows an example of how changing the Miura-ori angle affects the folded state.
1.2.2 Thickness Accommodation

Origami is traditionally known as the art of paper folding. When translating this idea onto a 3-dimensional system, designers need to accommodate for material thickness. Whereas paper can be considered 2-dimensional because of negligible thickness, most real-world systems involve a third dimension: thickness. Adapting origami patterns for 3D-use significantly increases the complexity of the design.

As a result, a variety of thickness accommodation methods have been developed [9]. Some of these thickness accommodating techniques include: hinge shift [10], offset panel [11], and tapered panel [12]. These techniques will play an influential role in deciding which patterns are adaptable to thick origami.

1.2.3 Deployment

Deployable origami is seen in bellows [13], sound barriers [14], and space telescopes [15]. Characterizations between active and passive deployment have been developed to help designers distinguish between two different methods of self-actuation [16].

One common approach for deploying origami systems is to use strain energy [17]. This method of deployment is favorable for space applications because it avoids the need for exter-
nal energy sources or actuators like motors. Strain energy poses as an efficient way to create deployment in space systems [3]. However, other mechanisms such as trusses and booms exist for deploying space systems. These methods may also be considered to converge on an optimal deployment method dependant on the needs of the mission.

1.3 Thesis Outline

This thesis is presented as follows.

Chapter 1 presents the motivation behind this research, as well as pertinent background information useful for understanding the objectives of this work.

Chapter 2 presents a systematic approach for selecting an origami source pattern for deployable space array applications, followed by two case studies used to demonstrate the selection process. This work has been accepted to the ASME 2022 International Design Engineering Technical Conference [18].

Chapter 3 presents the Miura-ori as a favorable, yet challenging, pattern in design of deployable space arrays. Challenges including thickness-accommodation, nesting, grounding and deployment are presented to introduce designers to unapparent complications associated with design of Miura-ori systems. A preliminary set of approaches to address and mitigate design difficulties is then provided. This work was presented and published in the 5th IEEE/IFToMM International Conference on Reconfigurable Mechanisms and Robots (ReMAR 2021) [19], with modifications made for acceptance and publication in Mechanisms and Machine Theory [20].

Chapter 4 focuses on improving understanding of design parameters, objectives, and tradeoffs of origami flasher pattern configurations. Emphasis is placed on finite-thickness flasher models that would enable engineering applications. The methods presented aim to provide clarity on the effects of tuning flasher parameters based on existing synthesis tools. The results are demonstrated in the design of a flasher-based deployable LiDAR telescope where optimization is used to converge on optimal design parameters and the results are implemented in proof-of-concept hardware. This work is submitted to the Journal of Mechanical Design.

Chapter 5 showcases the change in mode shapes between (1) a solid structure versus a hinged structure, (2) different boundary conditions on the same origami pattern, and (3) two completely different origami patterns - Hexagonal and Yoshimura patterns.
Chapter 6 concludes the thesis by highlighting the contributions of this work and discussing future research based on the methods introduced herein.
CHAPTER 2. A PRELIMINARY APPROACH TO SELECT AN ORIGAMI SOURCE PATTERN FOR DEPLOYABLE SPACE ARRAYS

2.1 Introduction

Origami has influenced mechanical product configurations throughout the past several years. The principles of paper folding have provided novel approaches for engineers to create physical and functional products. From solar arrays [3, 21, 22], to medical devices [6, 23, 24], origami-inspired systems have diversified the design process for designers around the globe.

However, designing origami-based space arrays can be an arduous and ambiguous task. It requires a deep understanding of origami behavior and the ability to correlate such behaviors to the application. A key challenge posed by origami-based design is understanding how different pattern parameters yield accompanying results. Whether it be to achieve high packing efficiency or reconfigurability, designers are left with the task of deciding which origami patterns will be most suitable for their needs. Consequently, a process is presented to facilitate origami source pattern selection.

The goal of this paper is to provide engineers and designers a guiding structure for selecting a base origami pattern to later be optimized and adapted for a particular application.

2.2 Background

Deployable space systems have influenced the design of solar arrays [21, 22, 25], reflectarrays [26, 27], and sunshields [15, 28]. One benefit of deployable systems is the high ratio of deployed area to stowed volume, which is a desired characteristic to maintain spatial efficiency in the payload area of the spacecraft. Maximizing this ratio is difficult, as larger deployed sizes pose difficulties stowing the array within the spacecraft prior to deployment. One solution is the use of origami-based design [29].
Figure 2.1: A preliminary design process for origami-adapted design proposed by Morgan et al. [1]. This paper focuses solely on the steps highlighted in green.

Morgan et al. created a preliminary process for origami-adapted design (Figure 2.1). This process provides a strong foundation for designers seeking to create an origami-based system. Morgan et al. categorized origami-based systems into three separate classifications: applied, adapted, and inspired [1]. Systems that have a direct link to origami are classified as origami-adapted. This classification suggests the ability to modify the pattern, accommodate for thickness, and use non-paper-like materials. As a result, most space systems fall under this category. Morgan’s process for designing origami-inspired aerospace mechanisms delves broadly in the design process used to go from origami pattern to origami product. While this process offers a holistic approach for designing origami-based systems, it must be further decomposed into detailed sub-approaches. Meloni et al. also investigated recent applications, design methods and tools used to develop engineering origami-based systems [30]. They note the importance of developing systematic tools used to support designers in the pattern selection process.

As such, the focus of this work will lie within the “Origami Solution” component - particularly on “Find[ing] origami source model.”
2.3 Selection Process

Morgan’s approach for origami-adapted design is presented in Fig. 2.1. Several layers exist of this process, with finding an origami source model being highlighted as a critical step. The goal of this work is to expand on the existing method and facilitate the process for selecting an origami pattern for deployable space array applications. This will be presented in the following sections.

Figure 2.2 presents a design network of events (activities and outcomes) used to converge on an origami source model. Mattson and Sorensen [2] developed this approach as a tool for teams to customize a network specific to the project needs. Nodes represent design outcomes and arrays...
represent design activities. Solid arrows indicate independent relationships that must be sequentially executed, while dashed arrows represent interdependent relationships where the outcome of one activity affects the other. This network provides a guide for engineers and designers to successfully find a suitable origami source model. The following sections provide guided methods for the proposed design network.

2.3.1 Requirements

Once the inherent first step of defining the need for a deployable space array has been established, the next step in the design process involves defining design requirements specific to the application. This step is critical in establishing proper design metrics. For example, the requirements needed for solar arrays vary when compared to telescope arrays. As such, it is important that this application-specific step be defined clearly and early on by the customer. While some requirements may at first seem conflicting, it is the responsibility of the design engineer to ensure pertinent requirements are considered. The outcomes within this stage are mechanical and deployment metrics, as well as guiding principles to define each metric.

2.3.2 Pattern Candidates

Once requirements have been determined, the next step in the design network involves determining pattern candidates. A collection of patterns commonly used in engineering applications can be found in Fig. 2.3. This list can be used as a starting point for this step in the design process.

While several origami creases exist, not all are suitable for space array applications. It is beneficial to explore the history of space arrays, perhaps finding similar applications. This step is critical in gaining an intuition for how to proceed. Although an application with the same requirements may not be found through literature searches, an understanding of similar missions or origami applications may be encountered. Oftentimes it is difficult to correlate desired final outcomes with correct terminology. To facilitate the literature search process, a list of keywords are provided below.
Source Pattern

- Reconfigurability
- Degree-of-freedom
- Packing Efficiency
- Rigid-foldability
- Flat-foldability
- Stowability
- Poisson’s Ratio

Modified Pattern

- Stiffness
- Strain Energy
- Hinges
- Actuation
- Stability
- Thickness accommodation

These words can be used in the literature search process to identify origami applications with similar desirable characteristics. Source pattern refers to elements pertaining to the source pattern before modifications such as thickness, material, or joint selection. Modified pattern characteristics consider final elements needed to create a functional origami array. At the end of this stage, a candidate solution set of patterns should be established.

Figure 2.3: A collection of common origami patterns used in deployable systems.
2.3.3 Analysis

In parallel with creating a candidate solution set comes developing metrics and guiding principles based on the requirements established in Outcome 1. These metrics should provide a starting point for analytical models to be further developed. Once a candidate set of pattern metrics have been established, equations must be created to examine relationships between patterns and their parameters.

The goal of this step is to create metrics that can be transferable across the candidate patterns in order to ensure a fair comparison. As such, it is desirable that non-dimensional relationships are formed. Nonetheless, it may be the case that modifications must take place in order for the metric to be fairly analyzed for each pattern.

While it is favorable to quantify each metric into an analytical equation, some metrics may be subject to binary (yes/no) or intuitive decision making.

2.3.4 Decision

The last steps in the design network involves reducing and converging. Reducing comes once a candidate set of patterns and metrics have been evaluated. Using the equations developed from metrics, each pattern should be analyzed and metric values should be recorded in a systematic manner, such as a decision matrix. This serves as a preliminary reduction method to simplify the final selection process. This process involves utilizing a decision analysis tool to rate and score the pattern candidates, while also aiming to improve the candidates or equations to ensure a fair consideration between patterns is conducted. It is important to note that if a clear decision can be made through ratings, Activity I can be omitted. Outcomes 9 and 10 are interdependent because they rely on each other to ensure the final, and most promising, set of patterns are being contested based on decision analysis tools and pattern refinement. The last step is to select the strongest origami pattern based on the previous analysis.
2.4 Example Case 1 - Deployable LiDAR Telescope

The example case provided will demonstrate the process described through Fig. 2.2 and the aforementioned steps. The application will involve the one-time deployment of a LiDAR deployable telescope sent as a secondary payload.

2.4.1 Requirements

A set of subsystem pattern requirements have been selected to help identify an optimal pattern for a LiDAR telescope application. These requirements may have an effect on other parts of the design process, but they are also pertinent to this step because of their effect on pattern capabilities.

1. The pattern must approximate circularity
2. The pattern must deploy to a 1 meter diameter from a 0.5m x 0.5m x 0.3m volume
3. The panels contained in the pattern must approximate circularity
4. The number of unique panels should be minimized

2.4.2 Pattern Candidates

After conducting a brief literature search and exploring different deployable telescope applications in space, a set of candidate patterns have been established. These patterns include: Flasher, Hexagon, and Square Twist. The Flasher has been examined in work performed by [3] to be used as a solar panel configured in space. A deployable regular hexagon has been analyzed as a potential pattern used in deployable origami reflectarrays [31]. Lastly, the Square Twist pattern has shown potential in providing reconfigurable states of origami antennas, as seen by Wang et al. [32].

2.4.3 Analysis

Using the established requirements, a set of defining metrics and equations have been created and are described in the following sections. It is important to note that at this stage, these metrics should be crease-pattern specific. Meaning, thickness should not be considered. This step
should be iterated on in the future to determine if still applicable once further design decisions such as seed pattern modifications and thickness accommodation techniques are implemented.

**Circularity**

Circularity is a metric used to compare the candidate patterns selected based on Requirement 1. The equation used to analyze this metric is described by:

\[ \eta = \frac{A_{\text{pattern}}}{A_{\text{circumcircle}}} \]  

(2.1)

This equation guides the analysis towards a non-dimensional and simple approach for comparing the area of the pattern to the circumcircle area created by the outermost vertices of each pattern.

**Deployment Factor**

The deployment factor facilitates the measure of Requirement 2. Equation 2.2 compares the area deployment factor between the pattern’s deployed and stowed states. This metric focuses solely on the side of the pattern that faces parallel to the sensor once deployed, and fits parallel to the compartment opening while stowed. This allows for thickness requirements to be neglected at this stage in the process.

\[ \gamma = \frac{A_{\text{deployed}}}{A_{\text{stowed}}} \]  

(2.2)

**Panel Circularity**

LiDAR telescopes exhibit greater optical efficiencies using circular apertures. As a result, one important requirement for this case study is the use of circular-aperture panels. It is desirable to choose a pattern that optimizes individual panel circularity. To quantify this metric, Equation 2.3 establishes a non-dimensional relationship between panel area and its circumcircle area, similar to Equation 2.1. Here, instead of analyzing the fully deployed pattern’s area ratio, each panel’s area is observed with respect to its individual circumcircle area.
\[
\sigma = \frac{A_{\text{panel}}}{A_{\text{circumcircle}}}
\]  \hspace{1cm} (2.3)

Number of Unique Panels

The last metric used to compare patterns is the number of unique panels. For this application, it is desirable that panel uniqueness is minimized as to ensure ease of manufacturing. This directly affects the pattern selection process because some patterns exhibit a variety of different panel geometries, sizes, and dimensions. No equation exists for this metric as it is simply a count of distinct panels within a pattern.

2.4.4 Decision

The last step in the design network is to decide on the source origami pattern. This step may involve several iterations dependant on ongoing changes in requirements. Nonetheless, this example case study uses the metrics and equations developed in the Analysis step to rank the three patterns. Once values were determined for each pattern, scores between one and three were assigned for each pattern’s ability to meet the requirement, with one being the lowest score and three being the highest score. In addition, each metric was subject to a weight value indicating its importance. This decision matrix is recorded in Table 2.1.

Table 2.1: Decision matrix used to compare the results of each pattern’s competency against a set of design metrics. Three represents the highest score. One represents the lowest score.

<table>
<thead>
<tr>
<th></th>
<th>Weight</th>
<th>Flasher</th>
<th>Hexagon</th>
<th>Square Twist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circularity</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Deployment Factor</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Panel Circularity</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Number of Unique Panels</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total Score</strong></td>
<td><strong>14</strong></td>
<td><strong>12</strong></td>
<td><strong>9</strong></td>
<td></td>
</tr>
</tbody>
</table>
The pattern with the highest score was the Flasher. A zero-thickness crease model can be seen in Fig. 2.4. Although it ranked lowest in two of the four metrics, the Flasher’s high level of circularity and deployment factor caused it to outweigh its competitors. At this point in the process, the designer should analyze the results and observe the confidence of the results. If the results are not convincing, it may be necessary to go through the design network steps again and make modifications. For example, perhaps there are overlooked subsystem requirements that may now need to be included in this analysis. Or, weight values may need to be adjusted depending on how important each metric truly is.

Figure 2.4: Paper model prototype of a Flasher crease pattern showed in its deployed (a) and stowed (b) states.
2.5 Example Case 2 - Deployable Reflectarray Antenna

This example case highlights the design process taken to determine a suitable pattern for a deployable reflectarray antenna. The deployable reflectarray will be used for communication between space exploration missions and scientists on earth. Because of this, the pattern needs to be stowable on the side of a CubeSat. CubeSats come in unit cells (U) of 10 cm x 10 cm x 10 cm and can be assembled to create 1U, 2U, 3U, and 6U (this configuration is a 2x3 arrangement of unit cells). These reflectarray antennas are usually secondary payloads accompanying a primary payload. The efficiency and gain of these arrays is dependent on the shape and surface area of the array.

2.5.1 Pattern Requirements

A set of subsystem pattern requirements have been selected to help identify the most optimal pattern for the reflectarray antenna application. The following are the requirements of the pattern.

1. The pattern must stow compactly on the side of a CubeSat
2. The pattern must deploy to as large an area as possible
3. The closer to circular the pattern, the better the efficiency of the array
4. Symmetric patterns are preferred
5. A simple deployment scheme is preferred
6. The deployment must be reliable and repeatable

2.5.2 Pattern Candidates

The hexagon and Miura-ori patterns were selected as candidates for this case as a result of extensive literature searches and keyword identification of similar applications.

The hexagon pattern was selected due to its simplicity, symmetry, and acceptable deployed-to-stowed area ratios. The Miura-ori pattern was selected because of its attractive capabilities in packing efficiency. Although more complicated, the Miura-ori's associated complexities could yield highly favorable results regarding stowing and deploying.
2.5.3 Analysis

The following sections show the metrics used to determine the superior pattern for this application.

Deployment Factor

The deployment factor facilitates the measure of Requirements 1 and 2. Equation 2.2 compares the area deployment factor between the pattern’s deployed and stowed states.

The deployed area of the hexagon pattern is

\[ A_{\text{hexagon}} = \frac{19\sqrt{3}}{4}a^2 \]  

(2.4)

where \( a \) is the length of a side of the middle hexagon. The stowed area is \( \frac{3\sqrt{3}}{2}a^2 \). Substituting these into Eqn. (2.2) yields \( \gamma_{\text{hexagon}} = 3.17 \).

The area of a Miura-ori pattern is dependent on the dimensions of each unit parallelogram. Assuming the height, width, and defining angle of the unit parallelogram of the pattern are \( h, w, \) and \( \alpha \), respectively, and the number of unit cells patterned in the vertical and horizontal directions is \( m \) and \( n \), the total area of a Miura-ori is

\[ A_{\text{Miura-ori}} = mnhw \sin \alpha \]  

(2.5)

The stowed area of the Miura-ori is \( (h + mw \cos \alpha)w \sin \alpha \). This leads to a deployment ratio of

\[ \gamma_{\text{Miura-ori}} = \frac{mn}{(h + mw \cos \alpha)} \]  

(2.6)

With parameters of \( w = h \) and \( \alpha = 82^\circ \), the deployment ratio for a 2x2 Miura-ori is 3.13.

Circularity

The radius of the circumscribing circle of the hexagon pattern is \( \sqrt{3}a \), making the area equal to \( 3\pi a^2 \). This gives a circularity factor of 0.873.
Figure 2.5: The circularity ratio of a Miura-ori where $w = h$ and $\alpha = 82^\circ$. For these parameters, the maximum circularity factor is 0.6277.

The radius of the circle that fully encompasses an $mn$ Miura-ori tessellation is

$$ r(m,n,\alpha) = \begin{cases} \sqrt{\left(\frac{nw}{2} \sin\alpha\right)^2 + \left(\frac{mh}{2} + \frac{w}{2} \cos\alpha\right)^2}, & \text{if } n \text{ is odd} \\ \sqrt{\left(\frac{nw}{2} \sin\alpha\right)^2 + \left(\frac{mh}{2}\right)^2}, & \text{if } n \text{ is even} \end{cases} \quad (2.7) $$

therefore, the area of the circumscribing circle is

$$ A(m,n,\alpha) = \begin{cases} \pi\left[\left(\frac{nw}{2} \sin\alpha\right)^2 + \left(\frac{mh}{2} + \frac{w}{2} \cos\alpha\right)^2\right], & \text{if } n \text{ is odd} \\ \pi\left(\frac{nw}{2} \sin\alpha\right)^2 + \left(\frac{mh}{2}\right)^2, & \text{if } n \text{ is even} \end{cases} \quad (2.8) $$

From Eqns. (2.5) and (2.8), the circularity ratio for the Miura-ori is dependent on the dimensions of the unit cell and the number of each unit cell in each direction. In the case of the reflectarray, an angle of $82^\circ$ was desired. Figure 2.5 shows the circularity ratio of a Miura-ori where $w = h$ and $\alpha = 82^\circ$. For these parameters, the maximum circularity factor is 0.628.

**Pattern Symmetry**

The pattern symmetry of the hexagon pattern exist as one-third of the pattern repeated three times, giving three axes of symmetry. The Miura-ori pattern is symmetric about the vertical axis only when $n$ is even.
**Simplicity of Deployment**

The hexagon pattern opens around the central hexagon panel. In this deployment this hexagon panel doesn’t move. This allows for “relatively” simple deployment by grounding the hexagon panel and actuating any one of the other creases of the pattern using pure rotation.

On the other hand, due to the Miura-ori being a tessellation, the pattern deploys in a symmetric way only if the entire pattern is allowed to slide as one of the creases is rotated. This kinematic motion will require more parts or mechanisms than for the hexagon pattern.

If this is looked at from a numerical standpoint, the number of mechanisms required to symmetrically deploy the pattern can be determined. For the hexagon, one actuator (mechanism) is needed to fully open the pattern. For the Miura-ori pattern, to be stowed compactly and then deployed symmetrically, at least two mechanisms would be needed. The first mechanism would rotate the pattern out-of-plane of the side of the CubeSat. The second mechanism (likely a slider), along with an actuator, would then open the pattern.

**Reliable, Repeatable Deployment**

Reliable and repeatable deployment and motion depends in part on the number of creases in a given pattern. For the hexagon, there are 18 creases. For an $mn$ Miura-ori tessellation, the number of creases is

$$N_{creases} = 2mn - m - n$$  \hspace{1cm} (2.9)

Looking at the case of $m = n$, a quadratic relationship exists for between the number of panels and $m$ and $n$.

**2.5.4 Decision**

Table 2.2 shows a comparison of the parameters and metrics for the hexagon pattern and two cases of the Miura-ori pattern. The first Miura-ori pattern is a $2 \times 2$ array. The other case is a $3 \times 4$ array. This case was chosen as the number of vertices in the pattern is 17, which is as close to possible with the 18 of the hexagon while still maintaining a one degree-of-freedom pattern.
Table 2.2: Calculated metrics of the hexagon and two $m \times n$ Miura-ori patterns calculated assuming a 1U CubeSat.

<table>
<thead>
<tr>
<th>Parameter (Symbol)</th>
<th>Hexagon Twist</th>
<th>2 x 2 MO</th>
<th>3 x 4 MO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$ (cm)</td>
<td>5 cm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha$ (°)</td>
<td>-</td>
<td>82°</td>
<td>82°</td>
</tr>
<tr>
<td>$w$ (cm)</td>
<td>-</td>
<td>10.1 cm</td>
<td>10.1 cm</td>
</tr>
<tr>
<td>$h$ (cm)</td>
<td>-</td>
<td>7.19 cm</td>
<td>3.59 cm</td>
</tr>
<tr>
<td>Deployed Area ($A$)</td>
<td>205.7 cm$^2$</td>
<td>287.6 cm$^2$</td>
<td>431.4 cm$^2$</td>
</tr>
<tr>
<td>Deployment Factor ($\gamma$)</td>
<td>3.17</td>
<td>2.88</td>
<td>5.52</td>
</tr>
<tr>
<td>Circularity ($\sigma$)</td>
<td>0.87</td>
<td>0.56</td>
<td>0.31</td>
</tr>
<tr>
<td>Planes of Symmetry (-)</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Number of Mechanisms to Deploy (-)</td>
<td>1</td>
<td>&gt; 2</td>
<td>&gt; 2</td>
</tr>
<tr>
<td>Number of Vertices ($V$)</td>
<td>18</td>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

The deployment factor of the hexagon sits right in the middle of the two different Miura-ori cases. However, the circularity of the hexagon pattern is much better in both cases than either Miura-ori pattern. Additionally, the hexagon array performs better in having a symmetric and easily deployed system, however these benefits seem to be downplayed by the fact that the deployment factor compared to the number of vertices is lower than both Miura-ori cases. However, the Miura-ori will effectively require more than two mechanisms to deploy the pattern. Based on these metrics, the hexagon pattern was selected as the origami source model for the reflectarray antenna [33]. A stowed and deployed state of the hexagon pattern is shown in Fig. 2.6.
2.6 Conclusion

Selecting an origami source model necessitates structure and intuitive understanding. By following the design network proposed in this paper, it is hoped that readers not only understand the design process, but also gain an understanding of origami applications in space arrays.

As noted in a variety of design process literature, requirements are critical in setting a baseline approach for selecting a concept. This task is no different. However, this stage requires the system requirements to be decoupled into smaller subsystem requirements that pertain only to the origami crease pattern. Factors such as deployment methods, environment, and materials must not yet be considered. If possible, these requirements should not attribute thickness in the system either. This may be difficult because oftentimes, characteristics such as stowability and packaging efficiency depend on thickness. Designers should keep these factors in mind, but not let them bias the exploration of suitable crease patterns, as several techniques exist for pattern modification to achieve desirable characteristics.
Pattern candidates can be observed through literature searches, research exploration, or at a minimum by utilizing the list provided in Figure 2.3. Next, analysis can be divided into two categories: metrics and equations. Requirements lead to metrics, and metrics must be represented by equations. As noted, it may be difficult to create general equations that can be used for every pattern. It is therefore suggested that careful consideration be taken in this step to ensure fair comparisons. If necessary, equations can be altered to fit the context of the pattern.

Lastly, the decision making process should be a result of the analysis performed in the previous step. This step should utilize as much unbiased, quantitative analysis as possible. The tool used in Case 1 to accomplish this task is a decision matrix, while Case 2 compiles the data in a table used to understand the strengths of each pattern. As noted, this step should be iterative. Once each concept has been analyzed and compared, intuitive analysis should then be used to judge the candidacy of the winning pattern result.

The design tools presented in this paper, including Morgan’s design process, a design network, and decision tables and matrices facilitate a systematic approach for selecting an origami source model. As a result of this work, engineers and designers may customize these tools to suit their design needs and application requirements.
CHAPTER 3. CONSIDERING THICKNESS ACCOMMODATION, NESTING, GROUNDING, AND DEPLOYMENT IN DESIGN OF MIURA-ORI BASED SPACE ARRAYS

3.1 Introduction

Deployable space arrays, whether they be used as a solar panel [25], reflectarray [34], sunshield [15], or telescope [35], are designed to achieve a high ratio of deployed area to stowed volume. This is necessary to maintain spatial efficiency in the payload area of the spacecraft. Maximizing this ratio is difficult, as larger deployed sizes pose complications with stowing the array within the spacecraft prior to deployment. One technique to approach this challenge is the use of origami-based design [29].

Traditional origami is created using paper and other materials that effectively have zero-thickness. Origami-based design utilizes origami patterns and principles with engineered materials and applies them to applications such as space arrays.

One origami pattern of particular interest to the aerospace industry is the Miura-ori [36–38]. This pattern is attractive because it can deploy from a relatively small stowed volume to a large deployed area using a one degree-of-freedom actuation system [37], as seen in Fig. 3.1. The Miura-ori pattern has other favorable qualities - such as simplicity of structure resulting from the repetition of its basic unit cell. This allows rows of cells to be added or removed from the pattern for a given design, granting designers the ability to control the deployed to stowed ratio. While these qualities are beneficial for a deployable space array design, other challenges arise. Careful consideration must be made to avoid issues caused by panel thickness [12], deployment [39], and nesting to create a viable Miura-ori based mechanism. The evolution of a Miura-ori origami pattern to an application using such considerations can be seen in Fig. 3.1.

This paper presents the Miura-ori as a favorable pattern to be used in deployable space arrays. Challenges to consider when designing a Miura-ori array are also presented. This material aims to provide high-level discussion points as an introduction to challenges associated with the
Miura-ori that require further analysis specific to the application. It is desirable that engineers and designers utilize the design considerations observed in this paper to better understand the Miura-ori before selecting it as the pattern of choice. A preliminary set of approaches to address and mitigate these commonly unforeseen design difficulties is presented. While the discussion focuses on space array applications, methods can be adapted to aid in the design and development of other Miura-ori applications.

3.2 Background

Deployable origami is seen in bellows [13], sound barriers [14], and space telescopes [15]. One common approach for deploying origami systems is to use strain energy [3, 17]. This method of deployment is favorable for space applications because it avoids the need for external energy sources or actuators like motors. However, other mechanisms such as trusses and booms have also been used in similar applications. All methods should be considered to converge on an optimal deployment scheme dependant on the needs of the mission.

One attractive quality of the Miura-ori is its flat-foldability, meaning that the pattern can be continuously folded from 0 to +/- 180 degrees without requiring deformation of the panels.
Figure 3.2: (a) Angle parameters of the Miura-ori unit-cell. (b) $\gamma_i$ is the deployment angle of each panel and shows the angle of each panel relative to its neighboring panel.

In addition, it is rigid-foldable, which stems from a single vertex that is repeated throughout the pattern [40]. The Miura-ori pattern occurs naturally in the deformation of an elastic plate of infinite length under compression, as it is the lowest energy path in which this plate can deform [38]. Schenk and Guest provide additional analysis and discussion of Miura-ori folded metamaterials [41].

The Miura-ori pattern has been used in applications as diverse as architecture [42], biomedical devices [6], circuit boards [43], robotics [44] and energy absorption on rotorcraft [45]. Folding, in the context of returning the pattern to its flat state for uses in space arrays, is of particular interest. The pattern is often modelled as a single degree of freedom mechanism. However, this is difficult to achieve in physical prototypes because the pattern is overconstrained; panels are not completely rigid and creases are not ideal hinges.

The areas of thickness accommodation, nesting, grounding, and deployment are some of the key challenges in using the Miura-ori in application. These topics are discussed next.

### 3.3 Thickness Accommodation and Nesting

#### 3.3.1 Motivation

Introducing thickness to an origami pattern presents a set of concerns that must be addressed. Because the Miura-ori consists of numerous propagating cells, thickness and nesting
become important factors in the selection, optimization, and design of the pattern. It is important to understand the arising difficulties in order to assess the compatibility of the Miura-ori with the desired application.

To maximize the surface area of a space array in the deployed state, it is generally desirable to reduce panel thickness and use as much of the payload volume as possible (neglecting any mass constraint). The type of array will dictate other desired properties of the deployed state, such as aspect ratio and flatness (considered as a variation of the dihedral angles between adjacent panels). For example, the surface area and aspect ratio of a deployable sunshield are important parameters, while flatness is not. As such, material and form selection for such a mechanism may include thin polyimide films, which can be efficiently stowed in cylindrical form by taking advantage of the compliance of the material with techniques such as slip-wrapping [46]. The compliant nature of the film, residual stresses in creases, and difficulty in uniformly flattening the structure with tensile stress may result in a less-flat deployed state in comparison with thick rigid-foldable mechanisms [46]. Thin compliant films have the greatest potential to maximize surface area and may therefore be the most desirable option. For applications where flatness is paramount, however, thick rigid-foldable structures may be more desirable.
Figure 3.4: Miura-ori with $\alpha = 45^\circ$ (a) and $\alpha = 82^\circ$ (b) shown in open and closed states. Cross sectional views demonstrating nesting are shown for each.

### 3.3.2 Challenges

Lang et al. compiled a review of thickness-accommodation techniques used to convert zero-thickness origami into rigid-foldable structures [47]. Another method proposed by Ku uses an axially volume trimming technique to achieve precise folds [48]. Gu et al. present a PSBL thick-panel origami vertex technique to create a foldable cube preserving kinematic properties [49]. These methods serve as stepping stones to be used for specific thick-origami needs.

A prominent phenomenon related to these thickness-accommodation techniques is referred to as nesting. Fig. 3.3 shows a polypropylene Miura-ori array with $\alpha = 85^\circ$ undergoing nesting complications as a result of using a thick material. This complication is exacerbated as a result of $\alpha$ approximating $90^\circ$. The relationship between Miura-ori parameters and nesting is described in subsequent sections. In general, nesting refers to having folds within folds in the flat-folded
state. When this occurs in thick rigid-folding structures, it requires some form of thickness accommodation. In rigid-folding arrays, nesting may more generally be thought of as portions of pairs of panels resting within portions of other pairs of panels in the flat-folded state, where every fold represents two panels linked at their edges by a revolute joint.

The degree to which an array requires nesting is defined as the maximum number of folds that rest within folds in the flat-folded state. A simple mountain-valley fold is considered a degree-0 fold, as no nesting occurs. A degree-4 vertex is considered a degree-1 fold, as some portion of the interior fold must rest within the exterior fold in the flat-folded state regardless of the parameters of the vertex (see Fig. 3.4). Increasing $\alpha$, increasing the length of the minor fold ($l_{\text{min}}$), or decreasing the length of the major fold ($l_{\text{maj}}$) increases the degree of nesting in integer increments (see Fig. 3.4).

Two types of nesting are discussed. Parameter nesting refers to nesting occurring exclusively due to the choice of the Miura-ori parameters $\alpha$, $l_{\text{maj}}$, or $l_{\text{min}}$. The degree of nesting (DoN) of such a pattern is simply

$$DoN = \text{floor}\left(\frac{l_{\text{maj}}}{l_{\text{min}}\cos(\alpha)}\right)$$

(3.1)

where the floor function rounds to the least integer less than or equal to the original result. This expression represents the maximum number of individual panels nested within the outermost panel. In Fig. 3.4 (a), DoN equals 1, whereas in (b), DoN equals 7.

To introduce the second type of nesting, some terminology must be established. The minor direction is herein defined as the direction of the parallel lines formed by the minor folds of a Miura-ori array in the deployed state. The major direction is herein defined as perpendicular to the minor direction and in the plane of the deployed Miura-ori state. Values $m$ and $n$ refer to the number of panels in a Miura-ori array propagating in the major and minor directions respectively; see Fig. 3.5. Propagation nesting then refers to nesting occurring exclusively due to the propagation of panels in the minor direction. The degree of nesting for propagation nesting is simply $n-1$, which is notably independent of $m$.

In a zero-thickness model, propagation affects nesting by identifying how many nests actually occur within the outermost panel. Eq. 5.1 is independent of $n$ because it simply provides an
upper bound on the maximum number of nests that could occur based on geometric inputs. However, if fewer panels exist in the $n$ direction than needed to reach that limit, propagation nesting identifies how many nests are present.

### Parameter-Nesting Patterns

Two types of parameter-nesting patterns have been identified and are discussed: tapered-panel, and origami-based cylindrical structures (OCSs).

The tapered-panel technique on thick Miura-ori has been investigated by Lang et al. [47]. The tapered-panel Miura-ori retains the kinematics of the zero-thickness model, thereby simplifying analysis of the pattern. The main difference between this pattern and the zero-thickness model is that it cannot fold completely flat; in the stowed state, the panels will rest in some position that is partially unfolded. While tapered panel Miura-ori models enable a much more compact state than untapered models, there is still a notable gap between succeeding minor folds [47], indicating that it is not the most volumetrically-efficient thickness accommodation technique.

OCSs were developed by Wang et al. [50] and are related to Hoberman’s initial formalization of the hinge shift [51]. The basis of these designs is to incorporate axis shifts on every fold.
This is to say that one pair of major and minor folds are shifted relative to the other pair for every quadrilateral unit. Wang et al. join the shifted axis at an angle, enabling curvature such that the propagation of these units flatly folds into a cylindrical or otherwise-curved shape (see [50]). This may be a suitable approach for payloads stowing within a cylindrical volume. A drawback of this pattern is that to continuously fill the volume of a cylinder would require a spiraling flat-folded state, which requires a large multiplicity of unique units, thereby increasing manufacturing complexity. Yet another drawback is that because these units are at an angle and must accommodate for subsequent units, a significant portion of the panels is twice the thickness of the mating portion of the panels. These regions yield that portion of the panel half as efficient as the rest, thereby lowering the overall volume efficiency of the array.

**Propagation-Nesting Patterns**

Three types of propagation-nesting Miura-ori patterns have been identified and are discussed; offset hinge (Fig. 3.5), map fold and modified tapered map fold (Fig. 3.6).

The concept of an offset hinge was formalized by Hoberman in 1988 [51]. Hoberman later generalized a propagation-nesting pattern using the offset hinge technique [52]. A degree-4 vertex and 6-by-6 array using this technique is shown in Fig. 3.5. The models shown in Fig. 3.5 can be propagated indefinitely in the major direction, but can only be propagated $n$ number of units in the minor direction before modifications are needed.

The map fold is a special case of the Miura-ori, in which $\alpha = 90^\circ$. A rigid-foldable map fold was demonstrated by Zirbel in 2014 [53]. The map fold also requires $n$ number of unique units, though it also requires either an elastic membrane [53] or gaps in the hinging to accommodate for thickness and achieve a fully-dense fully-folded form. One negative aspect of the map fold is that it has $m+n-2$ degrees of freedom, and would require at least two external actuators in perpendicular directions to range from folded to unfolded states.

The modified tapered map fold alters the map fold such that the map fold becomes a one-DoF mechanism [53]. This pattern also requires either an elastic membrane or gaps in the hinging to achieve a fully-dense fully-folded form. This pattern requires $n+1$ number of unique units.
One notable drawback of both map fold patterns is the hinge gaps. Such gaps substantially decrease the stiffness of the patterns, and would likely make any form of internal actuation difficult. Such patterns would likely need to be externally actuated.

Propagation-nesting patterns are potentially useful for stowage in rectangular prisms, and may therefore be useful in CubeSat applications. A drawback of propagation-nesting patterns is that the thickness of all the regular panels, as opposed to spacer panels [53], stack in one direction. Depending on the dimensions of the prescribed rectangular prism, this may require a significant decrease in volume efficiency to achieve a desired aspect ratio.

### 3.3.3 Approaches

The challenges of accommodating for thickness, preventing nesting propagation, and maximizing volume efficiency of space arrays are key topics that should be addressed before pursuing the Miura-ori. The following sections build on published work to provide innovative approaches for deployable space array applications.
Figure 3.7: Carbon Fiber prototype of a degree-four vertex undergoing folding motion. The single vertex Miura-ori pattern shows simplicity as a result of minimizing the structure to a single unit cell.

**Single Degree-Four Vertex**

As seen in Fig. 3.4, as the number of panels increase, the amount of nesting follows in accumulation. When the defining $\alpha$ is below 60° and assuming equal side lengths (Fig. 3.4a), a degree-1 nest occurs every other duplex of facets. As $\alpha$ grows beyond 60° (Fig. 3.4b), more facets begin to nest, creating a degree-n nest. Therefore, reducing the amount of facets to a single unit cell is one approach for mitigating nesting to a single degree-four vertex. In addition, this method proves favorable for missions that require the array to stow in thin spaces. As seen in Fig. 3.7, a single degree-four vertex array is capable of deploying to about four times its stowed areal dimension if $\alpha$ is near 90 degrees - often a favorable approach for capturing square apertures. It is important to note that this design still requires the use of a thickness accommodating technique to allow for a single nest of two facets. In Fig. 3.7, the offset hinge method is used which is influence by thickness by requiring an offset measuring twice the thickness of each panel.

This technique highlights a simple and novel approach for modifying the Miura-ori to minimize nesting complications. Rather than intensifying the effects of propagation nesting, a single-degree four vertex is demonstrated as an alternative technique for achieving desirable space array objectives such as large deployed-to-stowed ratios.

**Double-Hinge Offset**

The double-hinge offset (DHO) Miura-ori may be thought of as a special case of the OCS; when $\alpha_i = \alpha_{i-1}$ [50]. This case may be thought of as an OCS array attempting to achieve a cylinder with an infinite radius in the flat-folded state. DHO may also be thought of as a generalization of
Hoberman’s work from 1988 [51]. Hoberman’s original “reversibly expandable three-dimensional structure” was specific to 45°, 90°, and 135° angles, and portrayals of the units exhibited large doubly-thick regions [51]. The DHO seeks to minimize the doubly-thick region, and spans 0 < α < 90°. A DHO degree-4 vertex and 6-by-6 array can be seen in Fig. 3.8.

This method is not amenable to cylindrical payload volumes, but is very amenable to rectangular prism stored volumes as the profile of the flat-folded state is linear, not curved. Furthermore, because α_i = α_{i-1}, the doubly-thick less efficient portions of the panels are minimized, making DHO more volumetrically efficient. One major benefit of the DHO method is that it only requires a single repeating unit cell.

From Fig. 3.8 there are apparent gaps in the flat-folded state at the edges propagating in the minor direction. Left alone, these gaps render any implementation of the DHO method less volumetrically efficient. One method to fill this gap is to slice one of the unit cells along the height of the isosceles triangle that are the interior facets. Doing so creates “1/4 units” and “3/4 units”. Such units can form a degree-4 vertex independent of the repeating degree-4 vertices of the rest of the array, and render the method nearly fully-dense in filling a rectangular prism.
Tapered Gaps

In cases where the allocated stowing compartment of the array is a rectangular prism, tapered offsets can serve as a favorable method of modifying the Miura-ori. This idea follows the method presented by Zirbel in [53], however this time allowing for panels where \( \alpha \) does not equal 90°. Figure 3.9 shows the tapered gap method presented on facets with \( \alpha = 82° \). This technique doubles the gap dimension at each unit cell. In this variation, a combination of gaps and thin membranes were used across the major direction to accommodate for thickness. Thickness influences this design by inherently increasing the length of the gap. As thickness increases, gap spacing increases as well.

The most beneficial aspect of this design is the volume efficiency of its stowed state. The novelty presented here demonstrates how the methods developed by [53] can be tuned, modified, and optimized to achieve similar objectives. This approach demonstrates the feasibility of utilizing an existing thickness-accommodation technique and applying it to the Miura-ori. Furthermore, this method allows for flat-foldability of the structure. In the stowed state, each panel is able to stow parallel to each subsequent panel. This ultimately creates a volumetrically-efficient structure capable of stowing compactly within a rectangular CubeSat.

One potential drawback is the propagating gaps that exist in order to accommodate for stacking of facets. This may not be favorable if rigid panels are necessary across the entire array. Additionally, the lack of fully rigid facets ultimately decreases the stiffness, and thereby flatness, of the array, which may pose issues if deflection angles are critical in the deployed state.

Such growth in gap propagation may also be unfavorable due to the wasted space exhibited in the deployed state. Similar to the tapered map fold, this can be mitigated by filling the gaps with a rigid panel and encompassing the array in an elastic membrane.

Cylindrical

The last approach stems from the aforementioned method of introducing gaps. However, rather than propagating the width of each gap, this method retains a constant gap width measuring twice the thickness of the panels. As seen in Fig. 3.10, by introducing gaps, and therefore removing a hinge of each Miura-ori unit cell, the structure collapses into a cylindrical state. The resultant

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Figure 3.9: (a) Polypropylene Miura-ori with $\alpha = 85^\circ$ and propagating tapered gaps used to accommodate for thickness. (b) Folded state of the Miura-ori without the nesting complications illustrated in Fig. 3.3.

curvature is introduced by removing a constraint hinge and adding evenly spaced gaps within each unit cell. Unlike other techniques, this method is not flat-foldable. In a flat-foldable variation of the Miura-ori, panels on opposite sides of the unit cell come parallel to one another when stowed. As seen in the stowed state of Fig. 3.10, panels do not stow parallel to one another. Rather, the gaps seen in the deployed state are effectively utilized to allow the top vertices of the top two panels of the unit cell to coincide, thus forming a cylindrical stowed state.

This method retains a rigid-foldable structure, therefore not needing to compromise panel rigidity to achieve pseudo curved folding; a novel characteristic yet to be accomplished in rigid-foldable Miura-ori arrays. This method is most favorable when a cylindrical volume is required for the stowed state, such as stowing the structure within a rocket payload.

Another favorable quality of this method is its high deployment ratio. In the example shown in Fig. 3.10, the array is capable of deploying to 7 times its stowed cylindrical diameter. As a reference, if the diameter of the cylinder is 1 meter, then the circumference is 3.14 meters. However, the deployed length of the structure is 7 meters. This is nearly 2.25 times larger than wrapping a solid structure around a cylinder.

Since minimal material is reduced as a result of adding gaps, the reduction in stiffness can be neglected. Concerns may arise as a result of introducing gaps and therefore reducing the
effective area of the structure. Depending on the need for such array, it may be possible to fill the
gaps with a membrane containing the technology used on the rigid facets.

Figure 3.10: Stowed (within a payload fairing) and deployed states of the cylindrical modified
Miura-ori.

3.4 Deployment

3.4.1 Motivation

Origami pattern tessellations are attractive for achieving large apertures for telescopes or
antennas. However, deploying such arrays poses difficulties. In this paper, deployment will refer
to grounding or attaching the array to the base system, and actuating the pattern to and from a flat
state (flat-foldability). This leads to four key considerations:

1. How is the pattern grounded to an adjacent structure?
2. Where is the pattern actuated from?
3. How is the pattern actuated into and out of the flat state?

4. How is smooth deployment achieved?

    As shown in Fig. 3.11, one end of the paper array is held with one hand, serving as ground, while the opposite end of the array is pulled on using the other hand, serving as actuation. Near the end of deployment, the array is pressed flat against a table. Folding the paper pattern back up from this flat state requires the creases to unfurl at specific locations to facilitate a resistance-free folding motion. While this is a relatively simple motion for human arms and hands, this motion becomes difficult to achieve in space.

    When discussing origami-based applications, grounding and actuation methods are often omitted. Specifically for the Miura-ori, Papa and Pellegrino [54] and Cai et al. [55] studied the deployment of Miura-ori patterns in membranes. Papa and Pellegrino [54] noted the membrane initially deployed to an equilibrium position. They then applied corner forces to fully deploy the pattern. Cai et al. [55] suggest that smoothness, meaning how close each panel of the membrane truly forms a flat plane between 3 points of the panel, is best achieved with more attachment points on the edges of the pattern. These studies suggest the need for further consideration on how to actually achieve grounding and actuation of origami-based designs.

    Lynd and Harne [56] looked at the Miura-ori for beam folding in transducers, while Seiler et al. [57] used Miura-ori to create a patch antenna. Both studies provide in-depth analyses on the electromagnetic benefits provided by the Miura-ori in antenna design. This paper will expand on previous work by introducing methods for grounding and actuation in antenna design.
3.4.2 Challenges

Prior work has shown that the Miura-ori pattern can be strongly suitable towards various applications. However, unforeseen challenges remain for implementation of grounding, actuation, and deployed. These are highlighted and addressed below.

Grounding

The Miura-ori pattern is auxetic, meaning that it expands in both of the in-plane directions as it is unfolded (see Fig. 3.12). While this quality is useful for increasing the exposed area, it also requires an actuation mechanism that can expand in two directions.

Orientation of stowing and deployment are interdependent considerations to be made when grounding the array onto a CubeSat. As seen in Fig. 3.12a (i), stowing the array on the x-z plane produces a protrusive system that lacks optimization of space constraints. However, this stowing orientation produces an optimal orientation once deployed using a single degree-of-freedom actuation. As seen in Fig. 3.12a (ii), the array is able to deploy to a flat state symmetric about the CubeSat with a supportive structure underneath.

In contrast, by attaching the array on the x-z plane (Fig. 3.12b (i)), the array stows compactly on the side of the CubeSat. However, as the array deploys, it extends away from the system, making it difficult to utilize the CubeSat as ground and additional support for the deployed pattern.

An additional consideration and challenge associated with grounding a vertex is that all panels and their corresponding vertices rotate with respect to each other during deployment; the system lacks an accessible center panel that remains fixed as all other panels move around it. While there is a vertex that stays in the center of the pattern, this would require a complex mechanical joint with additional hardware. Grounding to a crease line presents a problem as none of the crease lines start on the x-y plane. This would require another complex mechanism to allow for a symmetric deployment.

If instead of grounding a vertex or crease, an entire panel is grounded, the pattern does not deploy symmetrically, as shown in Fig. 3.13. Only one of the corner panels is grounded due to deployment of the other panels.
Figure 3.12: (a) A symmetric Miura-ori pattern (i) stowed on the x-y plane of the gray box representing a CubeSat or other space system. (ii) The deployed pattern in the x-y plane. Attachment would occur at the central vertex. (b) The same symmetric pattern (i) stowed on the x-z plane of the CubeSat. (ii) The deployed pattern from the x-z plane resides in a plane parallel to the x-y plane when fully deployed. Attachment to ground could be via vertex or edge.

In many space-array design situations, deployment occurs through the use of trusses or booms. These systems are capable of deploying the Miura-ori, however, the disadvantage is that they add additional pieces to the overall design, which increases weight and utilizes valuable space. Trusses and booms create an extra mechanism between the array and its compartment, thereby facilitating actuation and grounding. However, this method limits flatness in the center of the pattern, especially if the array is large.

**Actuation**

Designing an effective actuation system depends primarily upon how the pattern is grounded and the desired deployment scheme. Horner and Elliot designed a Miura-ori solar sail utilizing slid-
Figure 3.13: A symmetric Miura-Ori pattern that is grounded to a CubeSat (gray box) by an edge panel in the (a) stowed, (b) partially deployed, and (c) fully deployed states. The darker panel is grounded to the CubeSat. Unlike the deployments from a vertex or edge, as shown in Fig. 3.12, the deployment is asymmetric with respect to the x-y plane.

Figure 3.14 shows a unit Miura-ori as a spherical four-bar mechanism. Barreto et al. [59] describe how the Miura-ori behaves as a spherical mechanism. Butler et al. [60] determined the...
mechanical advantage of origami mechanisms by considering them as rigid, but also incorporated compliance. Using the equation for rigid mechanical advantage provided in [60], and tailoring it to the Miura-ori, the equation becomes

\[ MA_R = \frac{\cos(\alpha)\cos(\gamma_1) - \cot(\pi - \alpha)\sin(\alpha)}{-2\cos(\alpha) + 2\cos(\gamma_1)\cot(\pi - \alpha)\sin(\alpha)} \]

The mechanical advantage of the Miura-ori with different “Miura Angles” (\(\alpha\)) is shown in Fig. 3.15. In the range of desired \(\alpha\) (45° – 89°) the mechanical advantage starts high. For cases close to 45°, the mechanical advantage stays relatively close to 1, meaning that the input force used for folding the array will be transposed on the output joint. However, with higher deployment angles, the mechanical advantage approaches zero. Higher deployment angles correspond to the flat, deployed state of the array.

The plot of mechanical advantage throughout the deployment shown in Fig. 3.15 is for a single unit cell Miura-ori. As the unit cell is multiplied to create an array, the mechanical advantage of the end panel is the initial rigid mechanical advantage raised to the power of the number of unit cell arrays that the end panel is away (\(MA_{R_P} = (MA_{R1})^p\)). This effectively drives all mechanical advantage to zero, especially at the edges of the array, near the end of deployment.

The previous discussion only considers rigid mechanical advantage, meaning zero resistance of the hinges. When considering compliance within the system (Fig. 3.14c, where springs

Figure 3.14: (a) A unit Miura-ori represented as a spherical four-bar mechanism. Facet 1 is the ground link. (b) The rigid mechanical advantage of the mechanism. (c) Incorporation of compliance by modeling springs at each joint. Springs are shown in orange.
Figure 3.15: The mechanical advantage throughout the deployment from folded to flat for different angles of the Miura-ori. The pattern deployment angle is $\pi - \gamma$.

simulate the flexibility of the creases), Butler et al. [60] defined compliant mechanical advantage as

$$MA_C = MA_R \left(1 - \frac{\delta U_c}{\delta W_{in}}\right)$$

(3.3)

Because a portion of the energy used in deployment will be transferred into potential energy in the compliance of the system ($U_c$), the mechanical advantage will decrease even more than that shown in Fig. 3.15, which makes the mechanical advantages closer to zero. As a result, it becomes difficult to precisely actuate to the flat state.

When the pattern is flat, the mechanism reaches a change-point where bifurcation may arise. Bifurcation means that the creases can change orientation from mountain to valley folds, or vice-versa [61]. This would cause the array to fold in an unpredictable direction, or possibly bind and completely prohibit folding. In one-time deployment, this is not an issue. However, for multiple deployments, this challenge must be addressed.
Figure 3.16: (a) A parallelogram four-bar mechanism with one degree of freedom. (b) Additional parallel link adds constraints, but symmetry preserves the single degree of freedom. (c) Misaligned links turn redundant constraints into conflicting constraints and can cause loss of mobility. (d) Removal of redundant constraints does not affect motion of the coupler (shown in the light grey link).

**Overconstrain**

The term binding is used in this work to mean a high resistance to motion. An overconstrained system is one that has more degrees of freedom than is predicted by its mobility equation. For example, the degrees of freedom of the planar linkage system, shown in Fig. 3.16(a), can be calculated as

\[
DoF = 3(n - 1) - 2 \times j_1 - j
\]

\[
= 3(4 - 1) - 2 \times 4 - 0
\]

\[
= 1
\]
where \( n \) is the number of links, \( j_1 \) is the number of lower pairs, and \( j_2 \) is the number of higher pairs. This is defined as an exactly-constrained system, where it is predicted to have 1 DoF and is observed to have 1 DoF.

Now consider the system in Fig. 3.16(b) where a link is added and is parallel to the other vertical links. Here the Gruebler's equation predicts the DoF to be 0, however due to special geometry we know that this system still has 1 DoF. This then becomes an overconstrained system. By adding more parallel links to the system in the same manner, the predicted DoF would decrease, while increasing the issue of overconstrain.

Due to symmetry and repeatability, the mobility of the mechanism is not affected. However, if one link of this was to be slightly misaligned as shown in Fig. 3.16(c), the mechanism will begin to bind at some point in its motion and the stiffness in the system will dramatically increase, causing the mechanism to no longer move. This would then turn the redundant constraint into a conflicting constraint. While potential conflicting constraints, such as the misalignment of parallel links, are not seen during kinematic analysis, they can become apparent once the mechanism is physically constructed. When constructed, the previous ideal kinematic system becomes non-ideal through simple geometric imperfections such as hinge misalignment, tolerances and thermal expansion differences. These changes can turn redundant constraints into conflicting constraints. Conflicting constraints can limit motion by locking or binding, prevent continuous motion, increase mechanism stiffness, and induce undesirable internal loads [62].

Although more complicated, the mobility of a Miura-ori based mechanism can be defined very similarly. The Miura-ori is made up of many degree-4 vertices with repeating geometry. Each vertex can be viewed as a spherical mechanism with \( m - 1 \) degrees of freedom where \( m \) is the degree of the vertex [63]. The larger tessellation can then be seen as a system of linked spherical mechanisms. Symmetry and periodicity give the Miura-ori a unique geometry that allows it to have more degrees of freedom than predicted by its general mobility equation. A Miura-ori-based mechanism is greatly overconstrained, as there are many redundant constraints. Hinge misalignment, too much or too little backlash in the joints, uneven deployment, or poor tolerances can cause the Miura-ori mechanism to bind, cause uneven motion, increase mechanism stiffness, and induce internal loads.
Figure 3.17: a-c) A systems diagram showing the possible configurations of the Miura-ori array, as well as the actuation and grounding mechanisms. For example, the grounding method in c) could be grounding to the panel, as in Fig. 3.13 with the actuation method being internal strain energy.

When designing for space applications, reliability, repeatability, and low-force actuation is important for design success. For a deployable array this requires smooth, defined motion requiring low-force actuation. The overconstrain in a Miura-ori-based deployable array could make these hard to attain.

3.4.3 Approaches

The following sections present novel methods to assist in overcoming the challenges of grounding, actuation, and ensuring smooth actuation and deployment of the array. These methods expand upon previous work and provide a starting point for engineers and designers to modify such approaches depending on specific space array objectives.

Grounding

As discussed, the current methods of deploying Miura-ori arrays utilize trusses or booms surrounding the pattern that expand with the pattern. This is essentially connecting the pattern to a mechanism that serves as a deployment actuator and a grounding scheme. This could also be accomplished with a slider mechanism (trusses, booms, or other sliding mechanism), or with a variation of system architectures, as shown in Fig. 3.17.

If asymmetric deployment is suitable for a particular application, grounding to a corner panel could serve as a constraint method. This allows a potential locking of a crease, as a portion
of 2 panels would be grounded if the grounding structure is large enough. This would lock the entire pattern in the state where the second panel contacts the structure, as the array is single-DoF. Even with compliance in the system, a greater portion of the pattern would be stabilized using this method.

For either case discussed above, increasing stability of the center of the array (as in the first deployment case) and stability of the edge of the array (as in the panel grounding case), hard stop mechanisms that deploy out of the system could be implemented, such as the ones developed by Andrews et al. [64].

**Actuation**

Grounding the structure to a corner, face, or an intermediate mechanism allows for different methods of actuation to be used. Herein, the terms actuation and deployment both convey complete opening of the Miura-ori pattern. To optimize the deployment and minimize required actuation forces, one should select an actuation method that complements the corners, faces, or mechanisms to which the pattern is grounded.

Several mechanisms and techniques are introduced as approaches for deploying complex arrays in space including: internal strain energy [65], external trusses [3], telescoping booms [66], cables [67], magnets embedded between panels [68], or torsional springs on the pattern’s creases [69]. Each method exists on a spectrum of relative strengths and weaknesses. The selected actuation method should be tailored to the requirements and environment of the specific Miura-ori configuration and its real-world application.

Actuation methods exist as either internal (within the volume of the pattern when stowed) or external (outside the structure/folds of the pattern when stowed). These internal and external actuation methods can be further organized by identifying the grounding methods to which they are best suited for (see Table 3.1).

Bowen et al. [70] demonstrate the implementation of magnetic actuation in a waterbomb pattern mechanism. Magnets are oriented on the panels in such a way that repelling magnetic fields cause the waterbomb to fully open. One panel is used as the mechanism’s ground. Similarly, Cowan and Von Lockette have illustrated the feasibility of Miura-ori magnetic actuation [68].
A perimeter truss is a commonly used deployment method for high panel count origami arrays. Zirbel et al. [3] introduced a perimeter truss with serpentine flexures that deploy a flasher pattern solar array. The serpentine flexures exert outward tension, pulling the pattern open, whilst undergoing a 90° torsion and 90° bend [3]. Sun et al. also developed a double-ring deployable truss which showed improvement in stowage, stiffness, and accuracy [71]. Kim et al. [72] showed a deployable truss which can be stored flat, ultimately improving packaging efficiency. As with most trusses, however, the issue of manufacturing complexity still appears.

Several actuation methods can be coupled with various grounding locations to produce a desirable result that satisfy project constraints (Fig. 3.17).

**Achieving Flat-Foldability**

After discussing approaches for attaching and deploying the pattern from a structure, the following discussion focuses on transitioning the pattern to and from a flat state. A possible approach for achieving flatness is to bias the pattern’s minimum energy state to be at or past the desired flat state. In cooperation with this method, thickness accommodation techniques or hard stops can be used to stop the pattern in the desired flat state.

To confront the challenge of transitioning out of the flat state, additional mechanisms could be added to offset the closing force from the neutral axis of the pattern. This would increase the mechanical advantage of the pattern and reduce the required force to actuate the array. Additionally, certain thickness accommodating techniques move hinges away from the neutral axis of the

<table>
<thead>
<tr>
<th>Actuation Method</th>
<th>Grounding Method</th>
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<td>Torsional Springs</td>
<td>Vertices/Int.Mech.</td>
<td>[69]</td>
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<tr>
<td>Embedded Magnets</td>
<td>Panel</td>
<td>[68]</td>
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<tr>
<td>Strain Energy</td>
<td>Panel/Vertices</td>
<td>[65]</td>
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<tr>
<td>Cables</td>
<td>Int.Mech.</td>
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<tr>
<td>Perimeter Truss</td>
<td>Vertices</td>
<td>[3]</td>
</tr>
<tr>
<td>Telescoping Booms</td>
<td>Vertices</td>
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Table 3.1: Internal and external methods of actuation with their complementary grounding methods.
pattern, thereby increasing the moment exerted on the panel. These could be used to offset the forces from the central axis of the pattern and increase the mechanical advantage of the system.

Instead of having the minimum energy state be at the flat configuration, the array could be designed to have the minimum energy state at the stowed configuration. This would bias the array toward a stowed position and prevent the need for an external folding force. However, biasing to the closed state requires a determination of an actuation method for the deployment of the array. The mechanism would also have to resist the loads introduced by the strain energy used to bias to the closed position.

**Overconstrain**

In an overconstrained system, redundant constraints could be removed while still maintaining fully defined motion. Consider the overconstrained mechanism in Fig. 3.16(b). If any of the redundant vertical parallel linkages are removed, the motion of the coupler is still fully defined (as shown in Fig. 3.16(d)). Likewise, in an overconstrained Miura-ori system, redundant constraints could be removed while preserving the same defined motion. Removing redundant joints eliminates the overconstraints, thereby reducing the chance for binding, reducing friction, and reducing joint stiffness. In addition, it decreases the amount of needed hinges, thus simplifying manufacturing and assembly.

A demonstration of this is shown in Fig. 3.18. Membrane joints shown in orange have been severed, thus removing the joint. The figure shows the motion still fully defined throughout the system’s deployment. Brown et al. provide structured approaches to determine possible joint removal locations [73].

**3.5 Conclusions**

While several methods exist for designing origami-inspired tessellations, a framework of methods have been presented to demonstrate the considerations required to effectively design a Miura-ori space array.

Thickness accommodation is critical when engineering origami-inspired mechanisms. A challenge associated with the Miura-ori is nesting. This challenge should be acknowledged es-
especially when thick facets are required. As such, alternative approaches have been introduced to design the Miura-ori for deployable space arrays.

The motion of the Miura-ori makes it an attractive mechanism for space arrays, specifically due to its single-degree-of-freedom deployment from small-to-large sizes. However, these same factors can make it difficult to deploy. Deployment encompasses many different aspects: grounding, actuation, smooth movement, and achieving flat-foldability, which are all inter-connected. As such, deployment scheme (symmetric or asymmetric) affects which deployment mechanisms and grounding to use.

While these concerns may lead to difficulties in full deployment or actuation from a flat state, this paper presents a set of methods to aid in the development of robust Miura-ori deployment.

This work is intended to provide insight to understand and mitigate Miura-ori limitations within the areas of thickness, nesting, and deployment motion. With this foundation, the Miura-ori can be properly modified to suit different needs and may help expand applications of the Miura-ori in aerospace and beyond. Future work should be directed towards expanding the analysis of designing Miura-ori space arrays for a specific application. The introductory discussion points presented in this work can be utilized as a framework for case study analyses.
CHAPTER 4. SELECTING AND OPTIMIZING ORIGAMI FLASHER PATTERN CONFIGURATIONS FOR FINITE-THICKNESS DEPLOYABLE SPACE ARRAYS

4.1 Introduction

Deployable origami-based space arrays improve the potential to achieve large deployed area to stowed volume ratios. This can help antenna, solar, and optical systems meet critical spatial constraints during launch, while maximizing efficiency gains once deployed. Many origami-based mechanisms could provide adequate solutions to meet this requirement. One pattern, the “flasher”, has received notable attention in recent years due to its potential for favorable stowed-to-deployed size ratios.

The flasher has been implemented into a variety of applications, oftentimes requiring different form factors, deployed and stowed dimensions, and stabilization methods. While several tools and mathematical expressions exist for modifying the base pattern parameters [74–77], these methods require deep intuition of the resulting pattern behavior. The parameters of the base flasher pattern are herein referred to as: $m$ the rotational order, or number of central hub regular polygon sides, $h$ the height order, $r$ the ring order, and $dr$ the separation between vertices. An in-depth introduction to these parameters can be found in [78]. A fifth parameter, $s_f$, is herein introduced to denote the scaling factor used to properly scale the pattern to maximize stowing efficiency. This parameter is dependent on stowing constraints. The flasher pattern can be thought of as a template for a family of complex, highly over-constrained spatial mechanisms. It is also sensitive to panel thickness. These and other coupled behaviors mean that selecting and designing a particular flasher configuration can be a cumbersome process.

The early design process for flasher-based deployable systems could be aided by identifying key design parameters and showing their relationships to objectives common to the system performance. It is intended that the heuristics presented in this paper facilitate the design decisions needed to meet case-specific objectives. The design space of flasher configuration possibilities
Figure 4.1: A finite-thickness flasher ($m = 6, h = 2, r = 2, dr = 0.2$) highlighting one gore. Parameters $dr$, $h$, and $m$ (which defines $r_{ce}$) can be varied to tune panel thickness.

is vast. As such, the methods presented in this paper decode inter-dependencies and present the effects of flasher parameters on design objectives.

As parameters are selected and design objectives are met, the following step involves using these zero-thickness values to convert the system into a finite-thickness array suitable for engineering applications. Once deployed, array stability may be an imperative objective to maximize power capabilities. Thus, thick panels provide a means to achieve such objectives by providing a hard-stop to prevent motion. Depending on the boundary conditions of the system, increasing thickness can have an influential role in increasing stiffness. Nonetheless, thickness accommodation techniques must be implemented to account for the transition from zero-thickness to finite-thickness and prevent panel interference.
Our primary focus for this development was a finite-thickness deployable array for a LiDAR space telescope. Our objective is to demonstrate a deployable telescope that can be stowed inside part of an ESPA-class satellite (~ 1 cubic foot) and deploy to 1-2 meters in diameter. ESPA is a commercial standard developed to utilize excess rocket launch capacity by mounting secondary payloads below the primary payload and is an acronym for Evolved Expendable Launch Vehicle (EELV) Secondary Payload Adapter (ESPA). An optical telescope has stringent alignment tolerances compared to radio frequency antennas or solar arrays. This required that we optimize the parameters such as deployed area and stowed volume while assessing the deployed mechanical alignments to evaluate potential optical performance of the deployed system.

This paper presents a novel approach for designing deployable, finite-thickness origami flasher arrays. The concepts are demonstrated in the design of a flasher-based deployable LiDAR telescope.

4.2 Background

The flasher pattern has been studied and proposed for engineering applications. Guest and Pelegrino [79] and Scheel [80] proposed conceptual flasher patterns, where modifications and experimentation have diversified the possibilities of flasher configurations. Aerospace [27, 81, 82], automotive [83], and medical [84, 85] fields have taken advantage of the flasher’s favorable qualities to create systems of varying sizes, materials, and geometries.

Zirbel et al. [78] presented a mathematical model and thickness accommodation techniques to facilitate the transition from flasher concept to prototype. The mathematical developments in this work highlight the complexity of the flasher and its vast configuration possibilities.

Lang et al. [86] expanded flasher possibilities by introducing a single degree-of-freedom cut pattern capable of achieving rigid-foldability and deployment in one motion. Lang furthered flasher design accessibility by creating an open-source tool, Tessellatica [75], where design parameters are input by the user to create fully developed flasher patterns.

Pehrson et al. [65] explored a different approach for achieving flasher foldability and deployment. The design methods proposed utilize strained joint techniques to allow the pattern to bend at each circumferential fold. This method, however, creates a trade off by maximizing deployment energy while sacrificing stability once deployed.
Guang and Yang [87] contributed to feasibility of finite-thickness flasher engineered systems by decomposing the coupled deployment motion of the pattern, creating a system that first rotates each sector, followed by a synchronous fold of each sector to create a fully stowed system. A method of tapering was also introduced to create a parabolic system favorable for solar array and antenna applications. Similarly, Kwok [88] developed a framework to simulate finite-thickness origami which considers 1D bending of mountain and valley folds.

Horvath’s investigation of flasher parameters and application to engineered systems provides preliminary guidance on how parameters - particularly $dr$ - affect flasher configurations [89]. This work will expand on Horvath’s presentation by developing flasher systems with non-constant thickness.

Wang and Stanter [90] propose a bayesian and gradient-descent optimization method where optimal doubly-curved finite-thickness flasher patterns are generated to maximize stowage ratio subject to stowed dimension constraints. This work is as a stepping stone for further analysis into flasher pattern objective trends and optimization models.

4.3 Methods

The purpose of this section is to define, analyze, and optimize flasher design parameters to understand their effects on common deployable space array objectives. Designers are often interested in geometric outputs such as number of total panels, deployed dimensions, and stowed dimensions. The following sections provide context for each design parameter and investigate the effects of varying such parameters. Trends are presented to aid designers in considering tradeoffs.

4.3.1 Design Parameters

Designing the flasher pattern requires five design parameters. The first three will be defined as zero-thickness parameters: rotational order, also equal to the number of sides ($m$), height order ($h$), and ring order ($r$). A detailed introduction to these parameters can be found in Zirbel et al. [78].

The fourth design parameter is a thickness parameter, referred to as $dr$. This parameter is defined as “the desired separation between two nearest-neighbor vertices at the same radial position and the same z-coordinate, normalized to the diameter of the circumcircle of the central
polygon” [78]. $dr$ is most closely associated with panel thickness ($t$). The relationship between the two parameters is defined as:

$$t = \frac{i}{h} \cdot r_{cc} \cdot dr$$

(4.1)

where

- $i = \text{panel height order}$
- $h = \text{total height order}$
- $r_{cc} = \text{central hub circumscribed radius}$
- $t = \text{top view projected distance between panel nodes}$

$t$ is the projected distance between panel nodes as observed from a top view. Figure 4.1 provides different views to demonstrate the beginning of a thickness-incorporated flasher on one gore. Further analysis is provided in subsequent sections.

Now that base parameters have been established, a fifth parameter is introduced herein referred to as $s_f$ - scaling factor. This is considered an implementation parameter which is only relevant when stowed constraints are established. When a base flasher pattern is designed using the first four parameters, it is assumed that $s_f = 1$. However, the structure may be scaled up or down to fit within the stowed constraints. It is important to note that when the pattern is scaled, each dimension is scaled linearly.

These design parameters form the foundation of the flasher pattern. The following sections provide methods used to clarify the design process for deployable, finite-thickness flasher arrays.

### 4.3.2 Number of Panels

The total number of panels is a direct result of $m$, $h$, and $r$ combinations, as described by Eq. 4.2. This pattern characteristic is important when considering array designs because a practicality limit may exist on how many panels can be used for the array application. Different applications may present different limits. In most cases, it is favorable to minimize the number of panels ($n_p$).
From Eq. 4.2, we see \( r \) as bearing the most influence. Trends are further explored in Fig. 4.2. We visually see the effect of varying sides, rings, and height order. Increasing rings causes the greatest influence on number of panels. As a designer, this should be considered when deciding which \( m-h-r \) combination will be most suitable.

### 4.3.3 Deployed and Stowed States

Figure 4.3 provides visual context for the design objectives of interest associated with the flasher pattern. Here we see a scaled representation of how the deployed state is able to achieve a large diameter from a small stowed state.

It is desirable to non-dimensionalize the design objectives and consider the flasher in terms of ratios between deployed and stowed dimensions. For the following analysis, the ratios of interest are defined as:

\[
\psi = \text{Deployed diameter to stowed diameter}
\]

\[
\kappa = \text{Deployed diameter to stowed height}
\]
Figure 4.3: a) Stowed height, diameter, and isometric views of a 4-2-2-0.1 (m-h-r-dr) pattern. b) Deployed incircle diameter of the same pattern. The central hub dimensions are held constant from stowed to deployed.

\[ \zeta = \text{Stowed height to stowed diameter} \]

In most deployable space array applications, it is often desirable to maximize \( \psi \) and \( \kappa \) to achieve the largest deployed diameter while minimizing stowed diameter and height. It is also
Figure 4.4: Deployed diameter to stowed diameter trends. Assuming $s_f = 1$.

often a requirement that the array must stow within a particular form factor. The objective is set to target a specific $\zeta$.

Figures 4.4, 4.5, and 4.6 present non-dimensionalized trends associated with varying the base flasher parameters $m$, $h$, $r$, $dr$. Individual design parameters are varied on each side of each graph. Color shades are used to present objective value results, as a ratio of the central hub incircle diameter, herein assumed to equal 1 unit. $dr$ is incremented across the bottom horizontal axis from 0 to 0.1 in 0.01 steps. Rotational order is discretely varied across the left vertical axis from 3 to 7 sides. Height order ranges discretely from 1 to 3 on the top horizontal axis, and ring order varies discretely from 1 to 2. The ranges used in this analysis provide sufficient intuition on design trends and can be extended to higher order values. For this analysis $s_f$ is assumed to equal 1 due to the general approach being presented; constraints are not established.

Analysis

To better understand the trends observed in Figs. 4.4, 4.5, and 4.6, a simple example will be demonstrated using the analysis shown in Fig. 4.7. We will herein assume $s_f = 1$.

Figure 4.7 shows a decomposed and decoupled analysis of the effects of changing one design parameter individually. Each of the four graphs present the 4-2-2-0.1 pattern and analyze the corresponding parameter at two other levels.
Figure 4.5: Deployed diameter to stowed height trends. Assuming $s_f = 1$.

Figure 4.6: Stowed height to stowed diameter trends. Assuming $s_f = 1$.

Figure 4.4 shows how $\psi$ increases as $h$ and $r$ increase and $m$ and $dr$ decrease. Taking a look at Fig. 4.7d, we visually inspect how increasing $h$ increases deployed diameter at a faster rate than stowed diameter. While both are increasing, it is evident that deployed diameter is growing at a faster rate as a function of increasing $h$. Hence, the trend observed in Fig. 4.4 by increasing $h$ is established.
Next we inspect the effects of increasing $r$. From both Fig. 4.4 and Fig. 4.7b, the ratio of deployed to stowed diameter ($\psi$) demonstrates apparent increase. The rate of increase for deployed diameter exceeds to the rate of increase for stowed diameter.

$\psi$ is seen to increase as $m$ decreases in Fig. 4.4. From Fig. 4.7c, the largest ratio between deployed and stowed diameters is indeed seen at the smallest rotational order. As $m$ increases, this ratio decreases as a result of deployed diameter’s decrease rate overpowering the decrease rate of stowed diameter.

The last analysis for $\psi$ is seen in the effects of varying $dr$. Fig. 4.4 shows $\psi$ increasing by decreasing $dr$. Observing Fig. 4.7a, the following values can be extracted:

$$dr = 0, \quad \psi \approx 6.4$$
$$dr = 0.1, \quad \psi \approx 3.8$$
$$dr = 0.2, \quad \psi \approx 3.0$$

As such, we see that by increasing $dr$, $\psi$ decreases. The same process can be used to understand the trends of Figs. 4.5 and 4.6.

### 4.4 Case Study

#### 4.4.1 Requirements

For this case study, as part of a research collaboration with NASA, specific requirements are established for a deployable LiDAR telescope. The main objective is to maximize deployed optical area of the array, while constrained to a 0.3048m x 0.5588m x 0.6604m (12 in. x 22 in. x 26 in.) prismatic volume for launch. The flasher was selected as a potential candidate for developing a deployable frame structure due to its approximately circular form factor when deployed and its potential for large optical areas as measured by its high packing efficiency - the ratio of the deployed surface area to the stowed volume. Designing a finite-thickness flasher array involves finding an appropriate thickness accommodation technique capable of adapting and folding rigid, thick panels.
Figure 4.7: (a-d) Visual representations showing the effects of varying each parameter level on each objective while holding three of the four parameters constant across each graph. In a), $dr$ is discretely varied from 0-0.2, while $m$, $h$, and $r$ are held constant. The pattern $m = 4$, $h = 2$, $r = 2$, $dr = 0.1$ is held as the base pattern across each graph.

4.4.2 Design Selection

Now that an understanding of parameter effects on deployed diameter and stowed dimensions has been established, a conclusive decision can be made on which pattern will meet the objectives of the telescope application given the constraints.

The coupled nature of these objectives and parameters suggests an optimization approach may be best suited for this analysis. Utilizing Tessellatica as a function evaluator, a gradient-free
binary-encoded genetic algorithm optimization routine was employed. This method was chosen due to its robustness and compatibility with discrete \((m, h, r)\) and continuous \((dr, s_f)\) variables. However, due to the computational expense of evaluating *Tessellatica*, a stepwise regression surrogate model was implemented to reduce computation time. The following optimization statement was implemented.

\[
\begin{align*}
\text{min} \quad & -\phi_d = -f(m, h, r, dr, s_f) \\
\text{s.t.} \quad & H_s - H_b \leq 0 \\
& D_s - D_b \leq 0 \\
& 3 \leq m \leq 7 \\
& 1 \leq h, r \leq 2 \\
& dr \geq 0.1 \\
& s_f > 0
\end{align*}
\] (4.3)

The objective is to maximize deployed diameter (to align with optimization convention, the objective is written as a minimization statement). The first two constraints state that the resultant pattern stowed dimensions (height and diameter) must be less than or equal to the established constraints. For this case, the limiting dimensions were defined by \(H_b = 12\) and \(D_b = 22\). Upper and lower bounds define practicality limits of the flasher pattern parameters. Rotational order must be between 3 and 7, height and ring order between 1 and 2, and \(dr\) shall not be smaller than 0.1 due to thickness requirements. Lastly, scaling factor should be greater than 0 to maximize volumetric efficiency.

The results from the optimization routine show that a 4-2-2-0.2225 flasher pattern with \(s_f = 4.203\) is capable of maximizing deployed diameter while meeting the constraints. The resultant deployed diameter measuring 1.59 meters. Figure 4.8 demonstrates the results of the genetic algorithm output from *MATLAB*. After 57 generations, convergence was achieved with 36 individuals landing on this configuration. Furthermore, Fig. 4.9 demonstrates the tradeoffs between maximizing \(\psi\) and targeting a specific stowed form-factor \(\zeta\). From this analysis we see that if the space allotted for the array to stow during launch is wide and short, the deployed diameter will
Figure 4.8: Convergence of genetic algorithm optimization routine used to maximize deployed diameter. Optimal results show maximum deployed diameter of 1.59.

Figure 4.9: A plot showing tradeoffs between $\psi$ and $\zeta$. The Pareto front is shown in red.

not be much larger than the stowed diameter. However, if the stowage space is tall and narrow in diameter, the array exhibits a large deployed to stowed diameter ratio $\psi$. 
4.4.3 Finite-Thickness Design

Once pattern parameters are selected, the next step involves accommodating for thickness of the panels.

Thickness Accommodation

Accommodating the flasher for finite-thickness panels is a difficult task. The pattern exhibits degree-4, degree-5, and degree-6 vertex interactions. Many panels occupy the same plane in the zero-thickness model, which is impossible to do with thickened material because they would have to occupy the exact same volume. Theoretically, one could “nest” them inside each other, but because none of the panels are uniform shape or size, and because the constraints did not allow for anything to deform or penetrate the center of the panels (so an optical membrane could be included), following the zero-thickness model exactly is impossible, and thickness accommodation techniques need to be considered.

Several of the thickness accommodation techniques reviewed by Lang et al. [47] were evaluated, but because the optical membrane needed to be on the same plane throughout the entire flasher when deployed and be protected from scratching the other panels, some techniques were not feasible. Techniques such as the hinge shift technique were eliminated because the membrane would not align on a single plane, and techniques like the offset panel technique were not selected because they consume too much volume and are difficult to implement in non-axially-symmetric patterns.

Proposed Thickness Accommodation Technique

A modified tapered-panel technique was selected because of its ability to accommodate for thickness in multiple directions at once, while maintaining the flat plane for the optical membrane. The folded state does require a greater stowed volume than the zero-thickness model, but no zero-thickness fold lines occupy the same plane and the additional volume around the fold lines allows for thickness to be added to the panels, as shown in Fig. 4.10. The use of the pattern alteration presented by Lang et al. [47] presented an approach to the flasher pattern, offsetting the zero-thickness fold lines and providing space for non-zero-thickness tapered panels. The frames were tapered and
chamfered to allow the pattern to fold within the anticipated folded configuration. The panels also had the center removed to allow for optical area, creating a border frame-like structure for each panel. In this work, a taper means a cut across the entire panel, and a chamfer is a smaller cut along the edge. These methods are discussed in more detail in the following sections.

Figure 4.10: A 2D demonstration of the tapering process, as used in the prototypes.

![Diagram](image)

Figure 4.11: The tapering process as applied to a single panel of the flasher. The red line shows the modifications from one image to the next. The last cut would be considered a chamfer, but the previous cuts cross the entire panel, and are considered tapers.

**Taper**

The tapering process, as shown in Fig. 4.10, involves thickening both sides of the zero-thickness
Figure 4.12: A demonstration of a chamfer along two adjacent panels.

Figure 4.13: A single section (or gore) of the flasher is demonstrated with the tapered panel approach. Chamfers can be seen on the circumferential folds, while tapers are used on all folds.

line, then removing material to allow the mountain and valley folds to come together without interference. The tapering method shown in this figure is only demonstrated in one dimension, but the flasher pattern requires a more complex tapering process, demonstrated on a triangular shaped flasher panel in Fig. 4.11. This same process of removing thickness in each of the fold directions allows the entire flasher to fold along the original zero-thickness line, preserving the original motion of the zero-thickness origami pattern.

**Chamfer**

Tapers allow the mountain and valley folds to preserve motion along the radial folds, but in order to allow panels to wrap around each other in the circumferential directions, additional material must
be removed to avoid panel interference, and to expose the zero-thickness plane at those locations to maintain the intended zero-thickness motion. Figure 4.12 demonstrates how the chamfers appear in both the unfolded and folded state. The application of these chamfers can be seen in Fig. 4.13, as they are visible on the unfolded state, and help the panels wrap around each other in the folded state.
Final Design

Figures 4.14, 4.15, and 4.16 present the final 4-2-2-0.2225 thickness accommodated prototype. The panels were manufactured using wood and assembled using compliant spring hinges for deployment ease.

The images in Fig. 4.14, 4.15, and 4.16 show the prototype that was created according to project specifications. Fig. 4.14 demonstrates the entire flasher prototype, where Fig. 4.15 demonstrates the compactness of the folding. Fig. 4.16 shows an isometric view where the frames are visible through the sides, illustrating the difficulty of the nesting process. There were some issues involved with the intermediate stages between stowed and deployed states with this prototype, but it was able to successfully fit in the volume allotted and reaches the aperture specified, which was the objective of this prototype.

4.4.4 Next Steps

The next steps for the physical flasher model focus on intermediate stages of deployment, including developing more detailed and analytical models that account more fully for thickness accommodation and energy storage in the springs.

Additional compliance in the system would enable the proper motion because the panels are so inherently stiff that they add stress on the hinges. Because of this, models will need to be developed to determine the ideal locations for this compliance.
Features that ensure precise final deployed position would be valuable for array applications. With compliance being added, the inherent hard stops may not be sufficient, but stability could be achieved after the deployment process is accomplished by using magnets, latches, or locking mechanisms to lock the panels into place.

4.5 Conclusion

The methods presented in this paper contribute to the understanding and development of the flasher pattern. The resulting increased ability to model and design flasher-based arrays will further enable engineering applications. The contributions of this paper are twofold: 1) demonstrate analytical and visual optimization methods used to understand the effects of each flasher parameter on common space array objectives, and 2) introduce a method for accommodating thickness of flasher panels relying on finite-thickness and non-negligible material. A case study of a design of a flasher-based deployable LiDAR telescope was used to demonstrate the methods presented.
5.1 Introduction

Space arrays serve a variety of applications. Several of these include solar arrays [25], reflectarrays [26], and telescopes [91]. While each has a different function, they all share a common characteristic: deployment from a spacecraft. As a result, space arrays need to be able to stow into a small volume and deploy into a large area. One approach for achieving this property is to implement origami-inspired design. By utilizing this ancient Japanese style, space arrays are able to collapse into a small volume and deploy to a large area once in space.

5.1.1 Origami Pattern

While origami-inspired space arrays provide optimal packing efficiency, it is important to understand the limitations posed by introducing more folds to the pattern, specifically regarding vibrations. When more fold lines are introduced, specifically by changing the pattern, different modal behaviors are expected. The geometry of the array is now altered, thereby creating a different modal shape.

Space arrays can reach sizes measuring up to 20 meters in equivalent diameter [92]. Their large size, along with the common requirement to observe flatness, calls for detrimental results if the natural frequencies are low. This phenomena, often observe in large drums, can cause space arrays to vibrate for longer periods and therefore damage its performance. On the other hand, high frequencies tend to not pose much of an issue. As a result, it is crucial to be aware of the exposed frequencies so necessary design changes can be made.
5.1.2 Deployment

Along with choosing which pattern will provide favorable results, it is important to consider the deployment mechanism which drives the boundary conditions that will constrain the array. Trusses are commonly used to deploy space arrays, but the locations at which the truss connects to the array is an alterable design decision. Once a range of exposed frequencies has been determined, the boundary conditions should be wisely chosen to ensure the structure is able to avoid such frequencies.

The purpose of this paper is to showcase the change in mode shapes between (1) a solid structure versus a hinged structure, (2) different boundary conditions on the same origami pattern, and (3) two completely different origami patterns. This paper will focus on the Hexagonal and Yoshimura patterns.

5.2 Methods

Four different FEA studies were performed between two different patterns, each model varying in geometry or boundary conditions. It is important to note that the material chosen for all patterns was 6061 Aluminum alloy. Additionally, it is assumed that the actual deployment mechanism would consist of a truss, rigidly connected to different locations on the array. Lastly, each pattern was assumed to be uniformly 1 inch thick.

As mentioned, four different frequency analyses were conducted. However, only two different patterns were analyzed. The Yoshimura pattern was varied in geometry and boundary conditions three times to investigate the effects of changing different parameters. A standard Hexagon pattern was also analyzed once. A summary of pertinent information regarding each of the four concepts is seen in Table 5.1.

<table>
<thead>
<tr>
<th>Concept</th>
<th>Equivalent Diameter (m)</th>
<th>No. Panels</th>
<th>No. Hinges</th>
<th>No. BC</th>
<th>Collinear BC</th>
<th>Perimeter BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collinear Yoshimura</td>
<td>2</td>
<td>26</td>
<td>32</td>
<td>3</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>Perimeter Yoshimura</td>
<td>2</td>
<td>26</td>
<td>32</td>
<td>12</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Solid Yoshimura</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>12</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Hexagon</td>
<td>1</td>
<td>13</td>
<td>15</td>
<td>7</td>
<td>-</td>
<td>X</td>
</tr>
</tbody>
</table>
5.2.1 Origami Yoshimura

The first pattern analyzed was the Yoshimura. This pattern was selected because of its excellent cubic packing efficiency and high stowed-to-deployed ratio. This pattern is capable of folding rigidly within a cubic space and deploying into a large 6-sided polygon.

Collinear Boundary Conditions

Using the same Yoshimura pattern, the analysis was conducted using collinear boundary conditions. As seen in Fig. 5.1, the structure was fixed at the pink locations, all of which lie on the same line. This boundary condition simulates a structure that deploys using internal strain energy and a truss that is fixed at those locations.

Perimeter Boundary Conditions

The next analysis was performed using perimeter boundary conditions. As seen in Fig. 5.1, the structure was fixed at twelve different locations around the perimeter. Everything else about this pattern was the same as the one used in the collinear conditioned one.

5.2.2 Solid Yoshimura

A third analysis was performed on the Yoshimura, however this time the model did not contain hinges. The purpose of this model was to compare how the same geometry behaves as a solid structure versus as one with hinges. It is important to note that this analysis utilized the same perimeter boundary conditions seen in Fig. 5.1.

5.2.3 Hexagon

The last pattern analyzed was the Hexagon. This pattern is efficient for space applications because of its close to circular shape. In many space array applications, circularity provides the most efficient aperture [93]. Six fixed boundary conditions were included for this model (Fig. 5.2).
5.3 Results and Discussion

The results of the frequency analyses are shown in Table 5.2 and Fig. 5.3. The Yoshimura patterns were relatively similar in frequencies, while the Hexagon was very different.

The equation for natural frequency is given by

$$\omega_n = \sqrt{\frac{k}{m}}$$

where \(k\) is a function of material properties, geometry, and boundary conditions.
The following case studies will compare how changing the geometry and boundary conditions changes the vibration of a deployable space array.

Figure 5.3: Natural frequencies of the Hexagon and Yoshimura patterns seen at each mode.

5.3.1 Case Study 1: Collinear vs. Perimeter Yoshimura

The first study looks at the natural frequencies that result from changing boundary conditions on the same pattern. By fixing the geometry and material, this analysis isolates the variation in natural frequencies as a result of solely changing the boundary conditions. As can be seen in Table 5.2, the natural frequencies are higher for the perimeter Yoshimura. This result shows that by constraining the structure differently, with more fixed points, the system is able to avoid low

Table 5.2: Natural frequencies of the Yoshimura and Hexagon patterns.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>$\omega_{n1}$</th>
<th>$\omega_{n2}$</th>
<th>$\omega_{n3}$</th>
<th>$\omega_{n4}$</th>
<th>$\omega_{n5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collinear Yoshimura</td>
<td>3.3898</td>
<td>22.624</td>
<td>22.91</td>
<td>25.18</td>
<td>50.851</td>
</tr>
<tr>
<td>Perimeter Yoshimura</td>
<td>26.188</td>
<td>43.469</td>
<td>63.84</td>
<td>65.672</td>
<td>68.896</td>
</tr>
<tr>
<td>Solid Yoshimura</td>
<td>5.6043</td>
<td>26.552</td>
<td>31.193</td>
<td>32</td>
<td>61.172</td>
</tr>
<tr>
<td>Hexagon</td>
<td>138.34</td>
<td>140.88</td>
<td>146.13</td>
<td>293.09</td>
<td>304.97</td>
</tr>
</tbody>
</table>
frequency vibrations, therefore avoiding large, drum-like vibrations. The fifth modal shape for both patterns are shown in Fig. 5.4.

The FEA results provide meaningful visualizations to allow the designer to make a critical design decision. While in some cases it is important for the space array to avoid large deflections due to vibrations, as seen in the collinear results, it may not be feasible to include twelve different fixed locations from the truss, as done by the perimeter Yoshimura. The perimeter Yoshimura does a better job at maintaining flatness along the top and bottom edges, making this pattern favorable in flatness-critical situations.

5.3.2 Case Study 2: Solid vs. Perimeter Yoshimura

This study analyzes the change in modes as a result of changing the internal geometry of the Yoshimura pattern. In this case, material properties and boundary conditions are kept the same for both concepts. The variation in natural frequencies can be noted in Table 5.2, and the trends can be seen in Fig. 5.3. The natural frequencies are again greater for the perimeter Yoshimura, despite having the same boundary conditions. One hypothesis for this result is the change in effective length of the structure. Looking at the equation for stiffness of a cantilever beam

\[ k = \frac{3EI}{L^3} \quad (5.2) \]

we see that length would be the geometry that changes between a hinged and solid pattern. In a hinged pattern, the length of the panel is much smaller than that of a solid pattern, which essentially
has a length that spans the entire width of the array. As a result, the hinged, perimeter Yoshimura exhibits a higher stiffness, and therefore a higher natural frequency, as shown by Eq. 5.1.

The modal shapes for both of these concepts follow similar trends. A more interesting analysis is shown in Fig. 5.5

Figure 5.5: Amplitudes of different mode shapes for Solid and Perimeter Yoshimura.

Figure 5.5 shows how the amplitudes vary at different natural frequencies for each pattern. This analysis can be helpful in determining different design tradeoffs. As a result, we see that by adding hinges, the array is able to achieve higher frequencies and avoid low frequencies of vibration.

5.3.3 Case Study 3: Hexagon vs. Perimeter Yoshimura

The last case study investigates the resultant modal shapes and frequencies the come from changing the geometry entirely, besides thickness. This study assumes the same material and similar boundary conditions, but ultimately changes the pattern from a Yoshimura to a regular hexagon. The modal results are see in 5.6. The drastic change in natural frequencies is important to note (Table 5.2). The frequencies are three to five times larger for the Hexagon for the first five modes.

These results show that the Hexagon would be a safer option if vibration at low frequencies is an issue. Its uniform, circular geometry allows for uniform fixed boundary conditions that are
Figure 5.6: Fifth mode shape for Hexagon and perimeter Yoshimura.

capable of maintaining the majority of the structure from vibrating. It should be noted that the Hexagon pattern is half the equivalent diameter of the Yoshimura, and also contains less panels in total (Table 5.1).

5.4 Conclusion

This paper demonstrates three case studies showing the effects of changing geometry and boundary conditions on a deployable space array. While in space, the array is subject to different frequencies; low frequencies being the most detrimental. When deciding which structure to utilize for either telescopes, reflectarrays, or solar arrays, careful consideration should be placed to avoid resonance.

Further work involves validating these FEA results through experimental testing. This would rely on a physical prototype being created and tested. A similar experiment can be seen by Pehrson et al. [65].

From this work, aerospace engineers and designers should have a better understanding on the effects of how changing geometric and boundary parameters affects vibration. While other tradeoffs must also be considered, the effects of material selection, geometry, and boundary conditions should not be neglected in early-stage design.
CHAPTER 6. CONCLUSION AND FUTURE WORK

6.1 Conclusion

The purpose of this work is to develop tools and metrics to aid in the selection process of origami patterns for deployable systems. The methods presented satisfy this objective by demonstrating techniques used to understand, select, and optimize patterns suitable for deployable space array applications.

The design framework outlined in chapter 2 presents a structured process to be used in the selection process of origami-based space array design. This framework should be tuned with details specific to the application. Through this work, designers are introduced to an iterative design process to aid in the design of origami-based space systems.

Origami patterns such as the Miura-ori and flasher are favorable in the space community due to their large deployed-to-stowed ratios. However, understanding and modifying the pattern becomes a difficult challenge. This work presents analytical and theoretical methods to further understand the Miura-ori and flasher patterns. A series of discussions surrounding pattern parameters and metrics are provided to introduce designers to important considerations to be made when designing an origami-based array. This work has shown that parameter tuning and modifications are necessary to for successful design of origami-based arrays.

Lastly, chapter 5 provides a brief vibrational study on two patterns to understand their natural frequencies given a set of boundary conditions, materials, and geometries. This work can be expanded in future studies to understand the effects of varying conditions of a physical origami-based array.
6.2 Future Work

This work has created fundamental ground to continue the investigation of origami patterns and the effects of modifying parameters. Future work should focus on examining a variety of other patterns, such as the Square Twist, Yoshimura, and Waterbomb. These patterns exhibit favorable qualities that, if better understood and modified, may contribute to a variety of applications. In addition, modifications of existing patterns to create new patterns may be a viable approach.

The flasher pattern has been explored in great depth within this work. Future analysis could investigate the effects of tuning stowed constraints. This would expand the design space and create further room for optimization.
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