An Analysis of the Influence of Lesson Study on Preservice Secondary Mathematics Teachers' View of Self-As Mathematics Expert

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AN ANALYSIS OF THE INFLUENCE OF LESSON STUDY ON PRESERVICE SECONDARY MATHEMATICS TEACHERS’ VIEW OF SELF-AS MATHEMATICS EXPERT

by

Julie Stafford-Plummer

A thesis submitted to the faculty of

Brigham Young University

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Of the thesis submitted by

Julie Stafford-Plummer

This thesis has been read by each member of the following graduate committee and by majority vote has been found to be satisfactory.

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As chair of the candidate’s graduate committee, I have read the thesis of Julie Stafford-Plummer in its final form and have found that (1) its format, citation, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

Date

Blake E. Peterson, Chair

Accepted for the Department

Accepted for the College
This research seeks to investigate the influence of lesson study on preservice secondary mathematics teachers’ view of self as mathematics expert. The study acknowledges the commonly held belief that prospective mathematics teachers have that they know and understand secondary mathematics. The purpose in engaging the preservice teachers in lesson study is to dislodge this belief. In particular, this research report focuses on one preservice teacher and her experiences during lesson study. Using the data collected, the researcher reports on the baseline beliefs that the preservice teacher held toward her knowledge of secondary mathematics, her mathematical experiences during the actual lesson study phase of the research and the final status of her beliefs in relation to her secondary mathematics understanding. After assessing the preservice teacher’s beliefs, the report focuses on the moves the preservice teacher makes to protect
her identity as a knower of mathematics. The report details how the researcher probed the subject’s views through a follow-up interview. The researcher discovered during the follow-up interview that the subject was finally able to admit her lack of mathematical knowledge and her desire to not be seen as ‘dumb’ in front of the interviewer. The implications of the study suggest that teacher educators should be sensitive to preservice secondary teachers’ perceptions of their mathematical knowledge and teacher educators should watch for the moves preservice teachers make to shift conversation away from mathematics topics.
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CHAPTER I: INTRODUCTION AND BACKGROUND

The Third International Mathematics and Science Study (TIMSS) revealed gaps in the instructional practices employed in math and science classrooms in the United States, Germany and Japan (Stigler & Hiebert, 1999). The most striking contrast came from the comparison of the U.S. and Japanese classrooms. The discrepancy rested in the methods that the teachers used to present their individual lessons. In particular, the Japanese lessons were more conceptual in nature, whereas the lessons in the United States were classified as “learning terms and practicing procedures” (Stigler & Hiebert, 1999, p. 41). It seemed that the U.S. teachers focused their attention on teaching algorithms rather than providing rewarding mathematical experiences for their students. When compared to their U.S. counterparts, the Japanese students had developed deeper and richer understandings of mathematical concepts (Stigler & Hiebert, 1999). From this comparison, TIMSS researchers realized that U.S. math teachers needed to move toward providing a richer conceptual base for U.S. students. The researchers’ focus then shifted to how Japanese teachers helped their students develop better conceptual understandings of mathematics. After investigation into Japanese practices, the researchers determined that the Japanese teachers enhanced the mathematical experiences of their students through unique teaching methods. These instructional practices became of particular interest to U.S. education researchers and prompted researchers to look closely at how Japanese teachers’ improved their instruction.

While looking at Japanese practice, TIMSS investigators discovered that Japanese teachers improved their teaching as they engaged in a professional development activity called lesson study. As the researchers thoroughly examined the data, they found that the...
Japanese teachers judiciously prepared a mathematical lesson for their students and later reflected on that research lesson. The researchers also noted that the collaborative lesson planning process consisted of feedback and ideas for better teaching strategies, as well as focused the teachers’ attention on student thinking (Lewis, 2000). The TIMSS researchers realized that the teachers’ beliefs about mathematics instruction evolved as the teachers reflected on their practice (Lewis, 2000). This reflection led to not only better lessons, but also improved instructional strategies.

After learning about lesson study, the TIMSS researchers compared the U.S. and Japanese teacher development systems. The researchers found that the U.S. system reinforced a culture of teaching that promoted mathematical skills and that the U.S. system failed to help professional teachers provide their students with enriching mathematical classroom tasks (Stigler & Hiebert, 1999). Other researchers (Cohen & Ball, 1990) acknowledged that U.S. teachers did not receive the support and feedback they needed to implement new teaching strategies effectively. As a result, U.S. teachers had received many reform ideas over the years, which had rarely helped them improve their instruction. Stigler and Hiebert (1999) felt that this led to a system that actually worked against improving teaching, whereas the Japanese system led “to gradual, incremental improvements in teaching over time” (p. 109).

From the cross-cultural investigation, Stigler and Hiebert (1999), advocated implementation of lesson study with U. S. teachers. They envisioned this type of professional collaboration as a way to help U.S. teachers improve their instruction as well as expand students’ experiences with mathematics. They acknowledged that the goal of providing rich experiences for students might never be realized if teachers or education
specialists did not act to improve U.S. teaching methods. They recognized that because of the deeply entrenched culture of U.S. math teaching, putting lesson study into practice would be a formidable challenge. However, even with all of the challenges a few researchers followed Stigler and Hiebert’s suggestion to implement lesson study. These studies gave the education community a look at the impact of lesson study collaboration on U.S. teachers (Fernandez, Chokshi, Cannon & Yoshida, 2001; Lewis, 2000).

Two researchers, Catherine Lewis and Clea Fernandez, studied American style lesson study experiments. They did this in an effort to analyze the effectiveness of the Japanese professional development system with U.S. teachers. The researchers found that through lesson study collaboration, teachers gained knowledge about their students, their subject matter and how to teach it (Fernandez, Chokshi, Cannon & Yoshida, 2001; Lewis, 2000). Lewis (2000) watched the very first pilot research lesson presentation headed by Dr. Clea Fernandez and Dr. Makoto Yoshida at Paterson School #2 in New Jersey. Lewis (2000) recognized that the teachers not only produced a well-refined lesson but had also through “consultation with colleagues, trial in the classroom and critique” (p. 33) improved their teaching methods. Lewis reported that the teachers’ approaches to teaching also changed as they reassessed their own beliefs and understanding about education. The teachers added to their mathematical knowledge and renegotiated their beliefs as they worked side-by-side with other teachers (Lewis, 2000).

In another lesson study experiment, lesson study researchers Fernandez, Chokshi, Cannon & Yoshida (2001) found that lesson study helped the in-service teachers develop their understanding and beliefs about mathematics and instruction. The investigators reported that the lesson study discussions centered on mathematical content and how to
teach it. The lesson study activity motivated the teachers to think about how they typically presented area of triangles to students and forced the teachers to reassess how they personally viewed the concept and formula for area of a triangle (Fernandez, Chokshi, Cannon & Yoshida, 2001). The researchers found that lesson study impacted the issues that the teachers believed to be important and relevant to discuss when they met together in collaboration and the plan that they formed to teach the topic. This gave evidence to suggest that in-service teachers reflected upon their beliefs about mathematics and pedagogy during lesson study, and that this reflection helped the in-service teachers in their day-to-day pedagogical decision-making processes (Fernandez, Chokshi, Cannon & Yoshida, 2001). Even though only a few researchers have reported on their lesson study research findings, the studies cited here have been significant and positive thus far. Catherine Lewis and Clea Fernandez’s research findings help to support the assumption that lesson study could help U.S. teachers develop better conceptual understandings, which would improve mathematics instructional practices.

Researchers studying prospective secondary mathematics teachers have showed that preservice teachers need to develop their mathematical understandings as well as pedagogical content knowledge (Cooney, 1999). However, preservice teachers do not feel that it is necessary to focus on content and didactic knowledge. In particular, Peterson and Williams (2001) discovered that conversations between student and cooperating teachers often revolved around administrative issues. The preservice teachers centered their attention not on instructional practices or how to provide rewarding mathematical experiences for students but on disciplinary or managerial issues (Peterson & Williams, 2001). Possibly the teacher candidates’ did not reflect on their
instructional practices because they did not believe that it was relevant or important for them to do so. In their book, *The Teaching Gap*, Stigler & Heibert (1999) thought lesson study might also encourage teacher candidates to reflect upon their beliefs about education and help improve instruction. Since the experiments with in-service teachers had been a success, preservice teacher researchers centered their attention on using lesson study to improve teacher education for future mathematics teachers. This particular study sought to contribute to the research about lesson study with preservice teachers.

*Uniqueness and Significance of Research*

A research goal, in the area of teacher education, is to develop methods that improve the quality of teacher education for preservice teachers. A part of that goal is to help prospective teachers develop better teaching strategies. Because of this, it is imperative that new ideas are tried and tested in order to determine the effectiveness of those new ideas. With this in mind, the goal of this particular research was to investigate whether lesson study could improve teacher education. This method of lesson planning was new to teacher candidates, and the researcher hoped to gather information about the effectiveness of lesson study in promoting different cognitive processes about mathematics and math teaching. The objective of the study was to determine whether lesson study could be used to improve mathematics teacher education.

Unlike previous studies that only observed in-service elementary teachers in lesson study, this project focused on preservice secondary teachers. Stigler and Hiebert (1999) commented that they did not know of any teacher education programs that utilized lesson study for prospective teacher development. Based on this statement this research was possibly one of the first of its kind to engage preservice teachers in lesson study.
More specifically, the purpose of this research was to test the theory that lesson study could promote deeper discussions about mathematics, and change preservice teachers’ beliefs about mathematics. The researcher hoped that as the preservice teachers discussed deep mathematical ideas that the prospective teachers’ would realize their lack of mathematical knowledge and seek for greater understanding. The principal investigator hoped that this increase in mathematical knowledge would lead to a heightened awareness of the preservice teachers’ view of their own mathematical knowledge. The confrontation would hopefully lead the preservice teachers to acknowledge that they were not experts in their field. More specifically, this study addressed the following question: How does lesson study influence prospective mathematics teachers’ view of self as mathematics expert?

*Design of Study*

The participants in this study were preservice secondary mathematics teachers who attended a large, private, religious institution in the western United States. They were enrolled in a teacher education course just prior to student teaching. While engaging in the research, the teacher candidates shared their impressions of mathematics and mathematics teaching through interviews, conversations during lesson planning and a report. To assess whether a change in beliefs or knowledge occurred, the principal investigator monitored the preservice teachers’ beliefs throughout the semester. Data was collected through a survey, field notes, interviews (initial & exit), dialogue journals, audio taped conversations, videotaped lessons and a final written report. The researcher compared beliefs reported during the initial interview, during the lesson study experience, and those expressed at the end.
This research report focuses on the impact of lesson study for preservice secondary mathematics teachers. The chapter that follows discusses the relevant literature and the framework used for this research. The third chapter focuses on the organization and design of the study, while the fourth details the analysis of the data. The last chapter of the report focuses on the implications of the research, and gives suggestions for further research.
CHAPTER II: LITERATURE REVIEW

This research is about preservice mathematics teacher beliefs. In particular, this chapter focuses on teacher candidates’ views of mathematics and their view of their knowledge of mathematics. It is often difficult, however, to discern where beliefs start and knowledge ends; therefore, this paper recognizes the two as inexplicably connected and constitute a person’s overall conception of a construct. Furthermore conceptions could be described, as a person’s own beliefs about a topic intertwined with actual knowledge learned (Thompson, 1992). The researcher investigates the beliefs and knowledge of prospective teachers with this understanding.

Alba Thompson (1984) gives a valid reason for first addressing the beliefs and knowledge of preservice teachers when she said that “any attempt to improve the quality of mathematics teaching must begin with an understanding of the conceptions held by [mathematics] teachers” (p. 106). Without this understanding it would be difficult to start working on the problem. After these beliefs have been delineated, it is then possible to address how lesson study could shape teacher candidates’ beliefs about math teaching. Teachers’ beliefs are particularly important because their orientation toward the subject matter “contribute[s] to the ways in which teachers think about their subject matter and the choices they make in their teaching” (Grossman, Wilson & Shulman, 1989, p. 27). So finding ways to enrich the conceptions that prospective teachers have in relation to mathematics will influence their ability to make better pedagogical decisions. This impacts not only their beliefs about mathematics but also their pedagogical content knowledge. The following paragraphs outline the conceptions prospective mathematics teachers’ hold.
Framework for Beliefs and Knowledge

The beliefs and attitudes that prospective mathematics teachers hold are complex as well as rigid in nature. Teacher candidates’ beliefs are so closely intertwined that they are difficult to unravel and investigate. In a 1990 study, McDiarmid focused on preservice teachers’ beliefs during early field experiences. He admitted that teacher education courses typically did not confront preservice teachers with their beliefs or create situations that challenged their knowledge. He said “most prospective teachers complete their teacher education programs without having examined the bases for their most fundamental beliefs about the teacher’s role, pedagogy, diverse learners, learning, subject matter, and the role of context” (McDiarmid, 1990, p. 13). With this in mind, McDiarmid created a situation that he hoped would cause conflict in the minds of his preservice teachers in relation to their beliefs about mathematics teaching. He followed his own prospective teachers as they observed Deborah Ball teaching a lesson on positive and negative numbers. He had them then compare and contrast their own beliefs and knowledge about positive and negative numbers, to what they saw in the lesson. He then had them teach someone else the topic and write about their experience. The purpose of the task was to help the preservice teachers face their beliefs and knowledge, and then modify any inconsistencies they found in their beliefs or knowledge. After assessing some of the preservice teachers’ responses, McDiarmid acknowledged that the prospective teachers showed little sign of change and that their belief systems had remained intact. That meant that their knowledge of positive and negative numbers also remained unchanged. It seemed that the preservice teachers needed to recognize as well as believe that their knowledge was incomplete or inaccurate before they would allow a
McDiarmid (1990) went so far as to say that belief/knowledge systems are much like spider webs and that “the strands constituted a web of remarkable resilience; severing one strand barely diminishes the overall strength of the whole” (p.18). This strong organized web of beliefs seems to be a result of many years of previous experiences in the context, the classroom.

Prospective mathematics teachers’ experiences before teacher education seem to shape a belief system that is resistant to change. “Preservice teachers, unlike novices in other professions, have [as students] spent many years in the work context” (Geddis & Wood, 1997, p. 619). Because of this many teacher candidates feel that because of their experiences in the classroom they have learned important knowledge and information such as how to control a classroom, how to teach the subject and so forth. This leads to a conflict as preservice teachers embark on becoming teachers. In part, they feel that they do not need to learn anything new since they are so familiar with the math classroom (Weinstein, 1989). “Students begin teacher education programs with their own ideas and beliefs about what it takes to be a successful teacher. These preconceptions are formed from thousands of hours of observation of teachers, good and bad” (Clark, 1988, p. 7). Preservice teachers may have a difficult time sorting through the good and bad examples of mathematics teaching because their experiences have cultivated deeply entrenched beliefs about mathematics. Prospective teachers may also be blind to the fact that their view of mathematics is incorrect and may need some alteration. Consequently, prospective mathematics teachers’ beliefs are resistant to change.

In another study by McDiarmid (1993), he found that preservice teachers did not change their views or knowledge on certain ideas. To gather data, McDiarmid used
questionnaires, interviews, and observation guides to monitor the beliefs systems of the teacher candidates over the course of their teacher education programs. The kinds of topics that the preservice teachers were asked to respond to included statements like: “In general, boys tend to be naturally better at math than girls” or “To be good at mathematics, you need to have a kind of ‘mathematical mind’” (McDiarmid, 1993, pp. 121-2). He found that the “teachers rarely appeared to change their minds completely—that is, disagreeing with statements at the end of their program with which they had initially agreed. Rather, they appeared to become more or less convinced about a given issue” (McDiarmid, 1993, p. 141). From McDiarmid’s findings, it appears that preservice teachers do not drastically change their views of mathematics and mathematics teaching issues during teaching education. This study highlights the fact that teacher education course contribute very little to the modification of belief systems. Thompson (1992) summarized the results of Collier & Shirk when she said, “that prospective teachers’ conceptions are not easily altered, and that one should not expect noteworthy changes to come about over the period of a single training course” (p. 139). Lerman (as cited in Thompson, 1992) similarly found that teachers tended to assimilate their experiences rather than changing their entire belief systems.

This research acknowledges that teacher candidates possess strong and rigid belief and/or knowledge systems, and that these systems are 1) formed by past experiences and 2) resistant to change. This understanding of belief and knowledge systems must be taken into account to appreciate where the prospective teachers are coming from and their background. This will influence the kinds of comments the teacher candidates make about their own beliefs and knowledge of mathematics and mathematics education.
Beliefs and Knowledge of Mathematics

Prospective teacher conceptions of mathematics involve personal beliefs about mathematics as well as knowledge of mathematics. Belief and knowledge systems influence preservice teachers’ notions of what constitutes the best way to teach mathematics. Hersh (as cited in Thompson, 1992) reflected on how one’s view of mathematics influences pedagogical decisions:

One’s conceptions of what mathematics is affects one’s conception of how it should be presented. One’s manner of presenting it is an indication of what one believes to be most essential in it….The issue, then, is not, What is the best way to teach? But, What is mathematics really all about? (italics in original, p. 127)

It is important then to look at people’s orientations toward mathematics and their knowledge base. Ernest (1988) outlined three views of mathematics.

First of all, there is a dynamic, problem-driven view of mathematics as a continually expanding field of human creation and invention, in which patterns are generated and then distilled into knowledge. Thus the sum of knowledge. Mathematics is not a finished product, for its results remain open to revision (the problem-solving view).

Secondly, there is the view of mathematics as a static but unified body of knowledge, a crystalline realm of interconnecting structures and truths, bound together by filaments of logic and meaning. Thus mathematics is a monolith, a static immutable product. Mathematics is discovered, not created (the Platonist view).
Thirdly, this is the view that mathematics, like a bag of tools, is made up of an accumulation of facts, rules and skills to be used by the trained artisan skillfully in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts (the instrumentalist view). (p. 10)

In light of Ernest’s descriptions, there are teachers with these three orientations. (1) Teachers with the problem solving perspective see past the mathematical symbols to a field of study that focuses on mathematical ideas and concepts. For these teachers, mathematical procedures or formulas have only been developed to simplify a commonly repeated process. These teachers use the common, taken-as-shared symbols to express natural phenomena to others. The term “problem solving” to these teachers means solving problems that are non-trivial and lack a clear path to the solution. (2) Teachers with the Platonist view, see mathematics as truths existing in the world, where these truths are not created but only uncovered. For these mathematics teachers the field of mathematics is axiomatic in nature. Therefore, the purpose of “problem solving” is to discover how these truths fit together within the entire system of truths. (3) Teachers with the instrumentalist perspective see mathematics as a collection of tools. Unlike the Platonist perspective, these instrumentalist teachers fail to see the connection between the tools or mathematical skills and see the tools as procedures devoid of any real meaning. These teachers then view “problem solving” as using the mathematical procedures to solve large amounts of trivial problems.

In her study involving mathematics teachers, Alba Thompson (1984) discovered these three previously mentioned orientations toward mathematics among three
professional teachers. One teacher Kay agreed, “mathematics is continuously expanding its content and undergoing changes to accommodate new developments” (Thompson, 1984, p. 113). This idea fits with the first view that Ernest (1988) described as the problem-solving view. Another teacher, Jeanne, commented that “mathematical content is fixed and predetermined, as it is dictated by ideas present in the physical world” and that “its topics are interrelated and logically connected within an organizational structure” (Thompson, 1984, p.110). This second teacher’s view fits with Ernest’s Platonic view. The third teacher Lynn, viewed mathematics as “a static collection of facts, methods, and rules necessary for finding answers to specific tasks” (Thompson, 1984, p. 116) and was representative of Ernest’s instrumental view. (It should be noted that a teacher can hold any combination of the three views, however, what is reported here seemed to be the dominant view). This shows that all three perspectives exist in mathematics teaching and that teachers’ beliefs and knowledge about what constitutes mathematics are not uniform.

If professional teachers possess different conceptions of mathematics, then it is most likely that preservice teacher views of mathematics vary from one prospective teacher to the next. Wilson (1994) reported that some preservice teachers were aware of other orientations toward mathematics besides procedures but struggled to make sense of them. This seemed to be connected to their knowledge and understanding of the mathematics itself. In the study one particular teacher candidate, Molly, was interviewed about her knowledge and views of functions. Wilson felt that Molly’s conceptions were similar to a beginning student’s knowledge of functions, meaning that her knowledge was underdeveloped. This was evidenced in Molly reporting that she felt more comfortable working with concrete examples rather than worrying about theoretical ones. Apparently
her knowledge of functions influenced the way in which Molly actually viewed them. Molly acknowledged that there were different branches of abstract mathematics but liked the math that she considered to be “solid and down to earth” (Wilson, 1994, p. 352). She viewed mathematics as “a collection of ‘concrete’ procedures to be applied in isolated contexts to obtain correct answers to well-defined problems” (Wilson, 1994, p. 361). This is exactly in line with what Ernest (1988) described as the instrumental view and knowledge of mathematics, and points to the fact that preservice teachers may possess this perspective about mathematics.

In another study, Ball (1990b) found that the instrumental view of mathematics was widespread among preservice math teachers. Ball’s study consisted of interviewing nine secondary mathematics majors about their knowledge of division of fractions, division by zero and division of algebraic equations. Each subject was given tasks related to these three areas and probed about their interpretations. With the first task, the subjects were asked to represent $1 \frac{3}{4} \div \frac{1}{2}$ as a story problem. Eight out of the nine secondary majors were able to produce the correct numerical answer and only five were able to generate an appropriate example (Ball, 1990b). This focus on mathematical rules was also seen with the second task. In particular, Ball reported that when the nine subjects were questioned about dividing a number by zero, five out of the nine gave an answer that represented their rule-based knowledge. One of the subjects said that in a teaching situation he would “tell them that this is a rule that you should never forget that anytime you divide by zero, you can’t. You just can’t. It’s undefined, so…you just can’t” (italics in original, Ball, 1990b, p. 138). The other four secondary majors gave justification for their division of zero answers. The fact that close to half of the subjects...
focused on the underlying meaning for the first two numeric expressions was positive. However, the preservice secondary teachers’ responses to the third interview task pointed to a strictly procedural knowledge of mathematics. The third task required the subjects to make sense of an algebraic expression. The expression \( \frac{1}{x^2} = 5 \) was quickly solved using procedural knowledge (Ball, 1990b). All nine of the teacher candidate subjects failed to provide an adequate explanation of the procedure that they used to solve the problem. The conclusion Ball (1990b) drew was that the “prospective teachers’ knowledge of division seemed founded more on memorization than on conceptual understanding” (p. 141). The types of facts the subjects had memorized were the commonly taught procedures or rules found associated with the topic of division. Ball (1990b) went on to suggest that this kind of knowledge was something that the subjects “had been taught in school” (p. 142). So for the prospective teachers to respond in such a manner was actually not surprising. This suggested that that the majority of preservice secondary math teachers probably possess an instrumental view of mathematics.

The principal investigator anticipated that the subjects in this research would possess the same conceptions that subjects in the literature held. Granted, all three views of mathematics may emerge, however, from the research it seems that the instrumental view of mathematics dominates prospective math teachers’ perspective of mathematics. The researcher approaches this study with this understanding.

Beliefs and Knowledge of Mathematics Teaching

Skemp (1978) introduced two perspectives that teachers hold toward mathematical understanding and how those perspectives influence teachers’ approaches to mathematics teaching. These two views of understanding seem to be closely tied to
teachers’ views of mathematics as well as their knowledge (Skemp, 1978). Researchers like Grossman, Wilson & Shulman (1989) agreed with Skemp that a teacher’s conception or knowledge of mathematics make an impression on a teacher’s attitude toward teaching mathematics. Thompson (1992) also agreed that there was a relationship between conceptions and practice, however, she further stated that the nature of the relationship was more complex than researchers realized. In addition, Thompson commented that factors like the political or social climate of a school might contribute to a teacher’s instructional practice. Because of this, it is difficult to plainly state that instructional practices are completely inline with teacher beliefs. Other factors like lack of knowledge might also play a role in how a teacher teaches mathematics. A teacher may recognize the value of conceptual teaching, but may not be able to teach conceptually because he or she may not possess the conceptual knowledge needed to teach in such a manner (Ball, 1990a). Skemp addresses this issue with his statements about beliefs and knowledge concerning understanding. Because of this, it is important to look at Skemp’s views of understanding and how the two views could possibly shape instructional styles.

The first understanding Skemp (1978) described was an instrumental understanding and closely resembled Ernest’s (1988) instrumental view of mathematics. People with this view of mathematics saw the discipline as a collection of tools to solve mathematics problems. The instrumental understanding could easily be described as learning the rules of mathematics devoid of meaning, something Skemp (1978) called learning “rules without reason” (p. 9). He even commented that this kind of understanding should not be considered understanding at all (Skemp, 1978). Skemp said that this kind of learning did not require the processing of symbols in a meaningful way
but focused on memorizing information. The kind of teaching that evolves from this view of mathematics would then be called instrumental teaching. Skemp points out that the reason teachers teach in such a manner is directly tied to the kind of understanding the educators are trying to achieve. Teachers with instrumental understanding as their goal, give students “instrumental explanations” (Skemp, 1978, p. 9). Teachers then teach in such a manner that focuses on instructing students about formulas, algorithms or other tools to solve problems. These teaching moments lack explanations about the deeper mathematics behind the symbols and center on surface idea.

Skemp’s (1978) second description of understanding comes from what he called relational understanding. This kind of understanding reflects what Ernest (1988) called his Platonist and problem-solving views of mathematics. That is because mathematical understanding then consists of recognizing relationships and connections between mathematical ideas and being able to solve problems with this knowledge. Skemp points out that those teachers who seek to develop relational understanding within their students could classify their teaching as relational teaching. This teaching style stems from teachers possessing the ability to help their students make mathematical connections. Teachers who are able to teach relationally have a rich knowledge as well as understanding of mathematics, and seek to help their students develop better mathematical understanding.

Skemp (1978) suggested that most teachers in the U. S. teach mathematics in a style that reflects an instrumental view of mathematics and understanding. This is possibly a result of teachers possessing a rather inadequate understanding of relational mathematics. Because of their limited perspective, these instrumental teachers lack the
ability to help their students develop relational understanding. Teachers who seek for relational understanding, however, seem to be more flexible and can also help students with instrumental understanding because they posses both instrumental and relational understanding. Often relational teachers know the instrumental math techniques but they also recognize the deeper mathematical meanings behind the symbols (Skemp, 1978). Because a large majority of teachers do not aim for relational understanding in mathematics, it is important to watch this trend among the research participants.

**Belief of Self as Mathematics Expert**

Deborah Ball (1990a) conducted a study to uncover prospective teachers’ views about mathematics. The study consisted of 35 mathematics majors who were planning on teaching at the secondary level. Ball questioned the participants “about teaching, learning and the teacher’s roles, and [their] feelings or attitudes about mathematics, pupils, or self” (Ball, 1990a, p. 451). In particular, the questionnaire assessed the teacher candidates’ knowledge of slope. The typical response was that the topic of slope could easily be explained and that the best explanation consisted of giving students the rule or formula (Ball, 1990a). Because Ball was simultaneously studying elementary teacher candidates, she recognized that in comparison the secondary candidates felt more confident about their secondary knowledge and did not seem to hesitate when discussing slope. Ball concluded from her research that there were three assumptions that preservice mathematics teachers bring with them to teacher education. These include: “1) that traditional school mathematics content is not difficult, 2) that precollege education provides teachers with much of what they need to know about mathematics, and 3) that majoring in mathematics ensures subject matter knowledge” (Ball, 1990a, p. 449).
seems that secondary teacher candidates enter teaching with the feeling that their mathematics background has prepared them sufficiently to understand as well as teach secondary mathematics.

Most preservice mathematics teachers have the attitude that secondary school mathematics is not difficult and that their earlier years in mathematics classroom have given them the skills necessary to teach mathematics. This is a result of the training that they have had in their mathematics classrooms over the elementary and secondary years (Geddis & Wood, 1997). Since most teachers teach instrumentally, students receive instrumental lesson. The next generation of math teachers then view math instruction as involving the teaching of procedures or rules. This attitude can be seen in the kinds of decisions that preservice teachers make when asked about math pedagogy. This was exemplified in a Siebert, Lobato, & Brown (1998) study when a prospective teacher, Antonio, was questioned about his approach to the teaching of division of fractions. What resulted was an explanation of the procedure invert and multiple. In another study, Wilson (1994) described a preservice teacher named Molly and highlighted Molly’s pedagogical views in relation to functions. Molly stated that “she [believed] that it [was] sufficient for students to know only how to correctly apply procedures” (Wilson, 1994, p.353). Because of this, Molly chose to present functions without a conceptual explanation. She described a function as a grinder, where the number goes in and out comes another number. This exemplifies Molly’s view that mathematics teaching involves computational techniques and therefore easy to teach. This fits with Ball’s (1990a) assessment that most teacher candidates do not see secondary mathematics as difficult and have learned enough math to teach it.
Ball (1990a) reported on the prevalent preservice mathematics teacher belief that it is not necessary to worry about the math content. Ball’s surveys revealed such sentiments as: “Teaching the material is no problem. I have had so much math now” (italics in original, Ball, 1990a, p. 449). It seems that most preservice teachers live with the false idea that the mathematics they have learned will be adequate to help them teach mathematics. There are also examples from the literature that typify prospective mathematics teachers’ assumptions that they have acquired adequate mathematical knowledge during their college years. In a study that focused on a preservice teacher named Ms. Daniels, the researchers realized that the conceptions Ms. Daniels held were not sufficient to help her teach mathematics conceptually (Borko et al., 1992). In a teaching situation Ms. Daniels was asked about the concept of division of fractions, and she responded by presenting a multiplication of fractions example. She realized her example was not answering the question even though she had learned it in a college methods course, and she was still unable to give a division of fraction example. She did not have a conceptual knowledge base of the topic enough to share it with her students (Borko et al., 1992). So even though preservice teachers have been in mathematics classes during their college years, this may not have provided them with enough knowledge about their subject. However, teacher candidates continue to hold onto the view that once they complete a bachelor’s degree their knowledge of math is sufficient. Conant observed, “that the acquisition of an undergraduate degree does not necessarily mean that [a preservice teacher] has developed a depth and/or breadth of subject matter knowledge” (as cited in Grossman, Wilson, & Shulman, 1989, p. 24). Anne Meredith’s 1993 study findings once again support the claim that prospective mathematics teachers
believe that they have learned enough mathematics. With the use of Likert-scale
questionnaires and open-ended questions, Meredith was able to tease out this assumption.
Most preservice teachers “reported their mathematical knowledge as being the one aspect
of teaching about which they felt most confident” (Meredith, 1993, p. 331). Apparently
preservice teachers feel that they have learned enough mathematics during their time in
college to help them in their endeavors as teachers.

In summary, most preservice secondary mathematics teachers feel that they are
experts when it comes to knowledge of their field. This assurance comes from their
beliefs about what mathematics is and how to teach it. The researcher expects that the
preservice teachers in this study will also possess many of the typical perspectives that
math education researchers have seen in their work. It is probable that most of the
prospective mathematics teachers interviewed will report that they too feel comfortable
with secondary mathematics because of their experiences in precollege and college
mathematics classrooms and therefore, expect to have minimal difficulty teaching it.

**Coherence Theory**

While discussing beliefs and knowledge, it is important to note that coherence
theory is an integral part of the framework for this research. This theory about an
individual’s beliefs states that people tend to make comments and other statements that
are consistent within their own understanding of the world. This does not mean that the
comments may be logical to another individual but that they only make sense to the
subject. “The ‘coherence’ at issue in the coherence theory is a matter of a proposition’s
relation to other propositions—not its ‘coherence’ with reality or with the facts of the
matter” (italics in original, Rescher, 1973, p. 32). Oftentimes a subject’s beliefs may
disagree with truth (Alcoff, 1996) or with another belief, however, the challenge is to see how they fit within the subject’s interpretations of their existence. That means that even when several statements seem at odds with one another, to the subject they are very much consistent and in agreement. The purpose of this research is to make sense of those statements, which at first glance may seem to be contradictory.

Reflection

In a 1984 research paper, Thompson reported on three teachers and their belief systems. One particular teacher, Kay, stood out as possessing the most lucid of the three belief systems that Thompson described. The teacher may have developed that coherence as “a result of her greater reflectiveness upon her actions, her beliefs, and the subject matter” (Thompson, 1984, p. 123). That means that there is something to be said about reflection and how it contributes to teachers’ perceptions of mathematics and how to teach the subject. Thompson (1992) said “beliefs systems are dynamic in nature, undergoing change and restructuring as individuals evaluate their beliefs against their experiences” (p. 130). Therefore, the nature of reflection and how it influences prospective teachers’ conceptions of mathematics and teaching is rather significant. Because of this, the key elements and benefits of reflection are discussed below.

John Dewey, an early educator in the United States, gave his own conception of reflection. One of the ways that Dewey described reflection was as “a specialized form of thinking” (Grimmett, 1988, p. 6). So reflection is an activity that requires conscious thought about one’s actions or conception of ideas. Dewey (1933) stated that the process of reflection involves two factors, they include: 1) “forming the idea or supposed solution” according to the conditions and 2) testing the idea “by acting upon it, overtly if
possible, otherwise in imagination” (pp. 104-5). In this way, an investigator can determine whether an idea is practical or if it needs to be changed. These two factors seem to be integral in the process that leads to development of practice.

One of the great benefits of reflection is the increased understanding of mathematics and subject matter pedagogy. In a 1993 study with math student teachers, Anne Meredith reported that they “viewed reflection on teaching tasks as fundamental to the transformation of their own mathematical understanding” as well as “central to the development of pedagogical content knowledge” (p. 336). Because reflection helped the teacher candidates improve upon their mathematical and pedagogical understandings, the prospective teachers saw the value of reflecting on their practice.

In summary, what this means for mathematics teachers is that if they do not reflect they will never grow mathematically and will remain stagnant in their knowledge of their discipline. Mathematics teachers that learn to reflect on their knowledge and instructional practices will be able to modify and improve their understanding and knowledge of mathematics and mathematics instruction. Dewey (1904/1965) asserted that teachers should continue to learn about their subject and “unless a teacher is such a student, he may continue to improve in the mechanics of school management, but he can not grow as a teacher, an inspirer and director of soul-life” (p. 151). Reflection will lead to a greater understanding of the mathematics and improve prospective teachers’ subject knowledge, but will also help them develop more cohesive strategies for teaching.

Lesson Study

With teacher reflection viewed as a way to improve understanding, this discussion comes full circle to the suggestions made by Stigler and Hiebert in their book, *The*
Teaching Gap. They suggested reflection as the way to improve the quality of mathematics teaching in the United States. The particular reflective activity that they promoted was lesson study. This form of reflection was the teacher development method discovered in Japan during the TIMSS research. Interestingly enough it possesses the key elements that Dewey (1904/1965) sets forth as integral to reflection, including 1) ideas stemming from observations and 2) action taken upon those ideas. Lesson study involves reflecting on observations in order to implement improvements to teaching.

Lesson study is patterned after the scientific method because it utilizes observations to make conjectures about teaching. This is done as teachers collectively form a hypothesis about the best way to teach a topic and then set out to test their theory. The testing of their conjecture is done in front of students, and several group members who record observations. The group then sets out to collectively determine whether the lesson was a success and what parts need improvement. After much deliberation and conscious thought, the teachers alter their hypothetical lesson plan and re-teach it. Once again the teachers monitor the student reactions and discuss possibly changes. In this way the teachers are able to test their hypothesis and determine whether the lesson truly did promote mathematical connections for their students.

Beliefs and Knowledge of Student Learning

In the process of participating in lesson study, teachers increase their knowledge of student thinking and learning. Initially this is sought after as group members think deeply about their students and the learning goals they have for them. The teachers then envision the level they want their students to attain. To accomplish the task of creating a learning goal, the teachers determine the kind of students they want to have in their
classroom and discuss the gap that exists between the current level and the goal level (Fernandez, Chokshi, Cannon, & Yoshida, 2001). Then the research lesson is created around that goal. The teachers discuss the prior knowledge students should bring with them to the lesson and talk about how specific problems could challenge student thinking. The group then creates a list of possible student responses to the focus problem. During the actual presentation of the lesson, the teachers collect data on student ideas and thinking. Later, as the teachers reflect on the lesson, they discuss their observations and new knowledge about student thinking. In this way, each teacher is developing his/her own understanding and knowledge of student learning.

This added knowledge and experience helps shape the beliefs teachers hold about student learning. During the lesson study phase, the teachers are able to bounce ideas off one another about how children learn. Oftentimes the teachers may not have accurate perceptions of how children learn. “The research lesson system increases the likelihood that teachers will hear opposing points of view, rather than hear only from likeminded colleagues” (Lewis, 2000, p. 18). Having a group with differing opinions about how children learn helps the teachers to reflect upon their own beliefs. As the teachers evaluate and reflect upon their experiences in relation to their beliefs, they are better able to decide whether their beliefs are consistent with student thinking.

Beliefs and Knowledge of Mathematics

One of most powerful results of lesson study is that a teacher’s understanding and knowledge of the content is enriched. The teachers talk not only about the math concept in terms of student understanding but they also talk about the ways in which they understand the topic (Stigler & Hiebert, 1999). Because lesson study facilitates and
promotes mathematical conversations it leads to greater understanding of math content on the part of the teachers. The discussion focuses on the whys of the problem: why does it work this way?, or why is it that way? Catherine Lewis (2000), a lesson study researcher, admitted that she also learned new content as she observed other teachers engage in lesson study. Teachers involved in the actual development of the lesson and those that only observe the proceedings also learn. This leads to teachers filling in the gaps they have about the math topic, which deepens their mathematical understandings.

In the process of gaining new mathematical knowledge, it is possible that teachers’ beliefs about mathematics evolve. As teachers make new content discoveries they are organizing and reorganizing their beliefs (Thompson, 1992). Teachers’ beliefs about the discipline seem to evolve as they fill in their mathematical gaps and straighten out some of their fragmented knowledge. In a study conducted by Fernandez, Chokshi, Cannon, & Yoshida (2001), this change seemed to occur. The lesson study group worked on a core curriculum topic. This enabled the group to reassess their beliefs about the way in which they viewed the mathematics curriculum. “Working on core topics was a powerful process because it allowed the American teachers to discover gaps, repetitions, and variations in how they were building children’s understanding of core topics across grades” (Fernandez, Chokshi, Cannon, & Yoshida, 2001, p. 9). Fernandez, Chokshi, Cannon, & Yoshida also commented that the teachers developed a better sense of the vision of the mathematics because of their lesson study experience. This evolution was a product of serious reflection as the teachers placed their developing math knowledge against their beliefs. This was only possible after the teachers struggled to make sense of their new mathematical knowledge within their old belief system.
Lesson study in the United States

If good teaching requires reflection like Dewey espoused, then lesson study is worth looking into for the U.S. school system. Fernandez, Chokshi, Cannon & Yoshida (2001) feel that the idea of lesson study is to focus the teachers’ attention on how their practice can be improved as the teachers watch their ideas take shape in an actual classroom lesson. “This exploration is particularly powerful because it is coupled with a period of sustained reflection, which encourages teachers to contemplate their ideas and deepen their thinking about important issues they encounter in the course of their work” (Fernandez, Chokshi, Cannon & Yoshida, 2001, p. 3). Lesson study then becomes a form of professional development for math teachers. This development has typically been denied teachers (Stigler & Hiebert, 1999) because of the current education environment. The only way that teachers will become better educators is to provide opportunities that further the development of teachers’ skills. In The Teaching Gap, Stigler and Hiebert (1999) commented that one way to improve teaching would be to introduce teacher development programs that bear a resemblance to lesson study. One benefit of lesson study is that it increases teachers’ knowledge and mastery of the subject they teach, which then feeds their ability to make better pedagogical decisions. Stigler and Hiebert (1999) agree that the knowledge of mathematics will be increased through mathematical discussions and these conversations would be directly related to better teaching strategies.

The introduction of lesson study could have a profound influence on in-service teachers but also on preservice teachers. It is important that preservice teachers also have experiences where their mathematical understandings can develop. Since “the subject matter preparation of teachers is rarely the focus of any phase of teacher education” (Ball,
1990a, p. 465), this emphasis on helping preservice teachers develop their mathematical knowledge is vital. It is especially significant because prospective teachers lack the knowledge they need to teach mathematics for understanding (Ball, 1990a). Preservice teachers are unable to communicate deep conceptual ideas because they themselves do not have the cognitive understanding to do so. This development is important because without better understanding themselves, they will not be able to teach mathematics for understanding. It then becomes imperative that teacher candidates engage in some form of professional development so that they can gain the content knowledge they need to teach it. With the research that Cooney (1999) has done, he felt that there was a “need for teacher education to address making connections within the mathematics the teachers would eventually be teaching” (p. 166). Lesson study does this as it combines learning about the mathematics with the teaching of it.

Summary

Since lesson study is a reflective activity it has the potential to change prospective teachers’ beliefs and attitudes toward subject matter knowledge or mathematics teaching. The research to be conducted will implement lesson study, analyze the effects of lesson study and determine whether lesson study is an effective method for improving mathematical discussions among math teachers. With all of this in mind, the researcher focuses on the following question: How does lesson study influence preservice teachers’ view of self as mathematics expert? If lesson study can make teacher candidates more aware of their lack of content knowledge, then it could be an avenue toward the improvement of the quality of mathematics teaching in the United States. The theory supporting the proposed research is that collaborative lesson planning efforts increase and
enrich discussion of deep mathematical concepts, which will lead to reflection and an increased understanding of mathematics, and thus improve the quality of mathematics teaching by preservice mathematics teachers.
CHAPTER III: RESEARCH DESIGN AND METHODOLOGY

Participants and Setting

The participants were preservice secondary mathematics teachers enrolled in the university’s fall 2001 semester secondary math methods course. As part of the major, they were enrolled in not only math methods, but also education and math courses. The course instructor was from the math education department.

In addition to the preservice teacher subjects, public school teachers and students participated in the study. The two teachers were selected based upon their teaching style and their belief in teaching mathematics according to the National Council of Teachers of Mathematics Standards. To locate these teachers, recommendations came from teachers at the university and secondary level. The public school students participated based upon their enrollment in one the public school teachers’ classrooms.

Class Structure

The Teaching Methodology and Instruction course consisted of individual and collaborative lesson planning/teaching situations. Each teacher candidate individually prepared two lessons to present to his/her peers during September through November. Each time a preservice teacher presented, class members critiqued the lesson and discussed possible lesson improvements. In mid-October, the preservice teachers learned about lesson study and started working on a research lesson. Over a nine-week period, the teacher candidates focused their lesson on a particular secondary math topic and a learning goal. The teacher candidates then taught the same lesson twice, once to their peers (in early November) and once to public school mathematics students (in early December).
Data Collection

Instruments

Several instruments were utilized in the data collection process. These included a survey, field notes, interviews (initial & exit), dialogue journals, audio taped conversations, videotaped lessons and a final written report.

Survey. The preservice secondary mathematics teachers received the survey during the first class session in September. (See Appendix A.) It was given as an assignment and the prospective teachers returned it during the next class period. The Likert-scale survey consisted of 12 statements that intended to promote reflection on the part of the preservice teachers about the teaching and learning of mathematics.

The survey played a part in selecting which preservice teachers would be used in the research. It was also used to assess the eight participants’ feelings about their knowledge of secondary mathematics and how secure they were about teaching high school mathematics. Depending upon the survey responses, each group included two preservice teachers who felt comfortable with their math knowledge and two that did not.

Field notes. The researcher observed the class proceedings and watched for preservice teachers who felt comfortable speaking in class. The purpose in watching for this characteristic was to ensure that the subjects selected would later provide verbal data.

Interviews. Since two groups were followed throughout the semester, eight subjects were interviewed. The one-hour preliminary interviews helped record the subjects’ baseline beliefs about mathematics and the teaching of mathematics. To accomplish this, the prospective teachers were asked to clarify their responses to the survey as well as describe what they considered to be a typical classroom scenario. An
open-ended section of the interview probed the teacher candidates’ understanding of division of fractions. This determined their level of understanding concerning the topic and what they thought was important to teach about division of fractions. These interviews occurred during the third and fourth weeks of the semester.

The exit interviews were similar to the initial interviews. However, these final pieces of data focused on uncovering a change in beliefs. The subjects were questioned about comments they had made 1) during the first interview, 2) during the lesson study experience and 3) on the Likert-scale survey. Once again the preservice teachers were questioned about a mathematical topic, slope, and asked to couch their understanding of the topic in terms of a teaching situation. These exit interviews occurred during the middle of December. (See Appendix A for both initial and exit interview questions.)

**Dialogue journals.** Two initial journal questions, in conjunction with the Likert-scale survey formed a baseline perspective of each subject. The subjects’ responses to the questions: “what is mathematics?” and “how do you think you learn mathematics best?” revealed the preservice teachers’ most dominant view of mathematics and helped the researcher create lesson study groups.

Another aspect of the dialogue journals involved the preservice teachers responding to journal prompts. They were required to answer five questions during the course of the lesson-planning phase of the collaboration. The dialogue journals constituted a running dialogue between the principal investigator and the members of the two groups that were followed throughout the semester. This provided a progressive view of the teacher candidates as they engaged in lesson study. (See Appendix A for open-ended questions and journal prompts.)
Audio taped conversations. After learning about lesson study, the groups started meeting near the end of October to work on their research lesson. The meetings were approximately two hours in length and were audio taped. The first meeting involved a professional secondary teacher. This in-service teacher helped the preservice teachers solidify a goal for their research lesson and provided the teacher candidates with information about the type of students the teacher had in his or her classroom. The public school teacher also gave the preservice teachers a lesson topic that was to be taught during the first week of December. The teacher candidates then planned their research lesson around the teacher’s goal. After about three weeks, one of the members of the group taught the lesson to their preservice teacher peers. Following the lesson presentation, the group met again privately to debrief and discuss the lesson. The preservice teachers explored what they could do to improve the lesson and then re-taught the revised research lesson to the public school students. The final meeting consisted of the preservice teachers once again discussing possible lesson revisions and suggestions for future teaching of the topic.

Videotaped lessons. The researcher collected two videotaped teaching episodes from each subject. The first lesson was particularly helpful in creating a picture of the subject’s initial views about mathematics whereas the second provided evidence of the subject’s views of mathematics during lesson study.

The researcher also collected two other videotaped teaching episodes. The first tape consisted of the preliminary research lesson presented to the preservice teacher peers. It was during this early lesson that the other three members of the group stood around the classroom and observed the students’ reactions to the lesson. The members of
the group did not interfere with the lesson or try to teach it but wrote down what they observed through field notes. After the lesson was over the preservice teacher who taught explained what he or she felt about the lesson. Next, the other members of the planning group shared their feelings and then discussion was open to the entire class. The second group videotape consisted of the revised lesson presentation to the secondary school math class. Once again, one preservice teacher taught while the other members of the group observed the public school students’ responses to the lesson.

Reports. The lesson study groups completed a two-part final written report during the middle of December. The preservice teachers recorded, in written form, the evolution of their research lesson as well as some revision and improvement ideas. Some of the group’s instructional decisions and student work were also highlighted in the group report. The final lesson plan was included in the appendix, but the bulk of the report outlined of the evolution of the lesson, for example, what the preservice teachers learned, and tips they had for future improvement of their research lesson. The personal essay portion of the report recorded any personal reactions that the preservice teachers wanted to share and how their lesson study experience might have been improved.

Data Analysis

Formation of Lesson Study Groups

Preliminary analysis of the survey, the two open-ended journal questions and the initial interviews helped in the formation of the lesson study groups. The responses were analyzed for the prospective teachers’ most dominant view of mathematics and how confident they felt with their secondary mathematics knowledge. From the open-ended journal question analysis, it was determined that the prospective teachers’ views of
mathematics tended toward either a procedural or conceptual perspective. Specific responses to the question: “what is mathematics?” signified their individual orientation. The Likert-scale survey and initial interview analysis helped determine which prospective teachers felt comfortable with their mathematical knowledge. In particular the response to the statement, “I would consider myself an expert when it comes to secondary mathematics” on the survey and their reactions during the interview determined whether the individual displayed a sense of confidence with their secondary knowledge. Of course it was not possible for the preservice teachers to fit rigidly in just one category, but for the sake of convenience each subject was classified according to his or her most dominant view of mathematics or feeling toward their math knowledge. The specific mathematics codes that came from this method of analysis were conceptual (C) or procedural (P), and the secondary mathematics knower codes were confident (+) or not confident (-). The four possible combinations were then C+, C-, P+ and P-. Where possible, a subject of each type was placed in a group. This ensured that the way each preservice teacher viewed mathematics and his or her knowledge would be somewhat different from the other members of the group.

**Final Analysis**

Partway into the analysis, the researcher made the decision to only write one case study. This was a result of 1) the researcher’s interest in one particular subject’s lesson study experience and 2) the exhaustive nature of compiling several case studies. The purpose in writing the case study was not to make a general statement about how preservice teachers would respond to lesson study but to provide a descriptive picture of one subject’s beliefs and her encounter with lesson study. In particular, the researcher
focused on two specific beliefs during the rest of the analysis. The two beliefs were, 1) the subject’s view of mathematics and 2) the subject’s view of her mathematical knowledge. The following section of the paper elaborates on how the data sources helped the researcher create an image the subject’s conception of these two beliefs.

*Interview data.* In order to capture the subject’s most dominant beliefs or attitudes toward mathematics and education, the researcher transcribed the audiotaped interview data. After the transcription, the principal investigator read the interview data to pinpoint the subject’s beliefs about what constituted mathematics and the subject’s view of her secondary mathematics knowledge. The researcher noted statements that tended toward a particular orientation, and looked for evidence that either confirmed the researcher’s assumptions and weighed that against the seemingly contradictory statements. The principal investigator then made a comparison between the initial and final interviews in an effort to determine the subject’s most dominate views on those two ideas and to identify issues that repeatedly surfaced. The researcher then used the data collected from the initial and exit interviews to create a follow-up interview. The principal investigator determined that it would be wise to question the subject even further about particular issues and check the subject’s responses to statements the researcher felt fit the subject. In this interview, the researcher once again looked for the admission of lack of mathematical knowledge, positive responses of change in beliefs about math knowledge and if the subject attributed this change to lesson study. (See Appendix A for follow-up interview protocol.) Looking for these commonalities or discrepancies among comments accomplished the task of identifying emerging themes.
Journal and report data. The principal investigator used the dialogue journal and final report data to locate comments that shed light on the subject’s belief system and captured specific examples of reflection. The researcher read the journal entries and final report in the same manner as the interview data, looking for comments that either confirmed or disconfirmed the researcher’s previous conclusions. However, in addition to scanning the journal data for initial beliefs, the researcher also looked for evidence of reflection, new feelings of uncertainty or a realization of lack of knowledge. Since the subject responded to the prompts during the lesson study experience, the entries recorded incidences that were still fresh in the subject’s mind. The researcher also read the final report in order to locate experiences that pointed to the subject renegotiating her knowledge and her own perception of that knowledge.

Conversation data. The emerging themes identified during the interview process provided key issues to focus on during analysis of the conversations. To draw on this rich data source, the principal investigator transcribed the conversations. In order to differentiate between speakers’ voices, the researcher attended every group meeting. The transcriptions led to an analysis that uncovered episodes that revealed how the preservice teacher thought about mathematics. The principal investigator watched for the teacher candidate’s attitude toward mathematics, the way the prospective math teacher conversed about mathematics and what she desired to include in the group lesson plan. The researcher also paid close attention to how the subject handled herself during mathematical conversation. It was expected that the groups’ interaction would be sufficient to cause some kind of perturbation of belief and knowledge systems. So, the principal investigator was interested in the way the subject reacted when she did not
know a particular math idea. The researcher looked for incidences where the teacher candidate struggled and whether the subject ever admitted not knowing the content.

*Lesson plan and videotaped teaching data.* The researcher utilized the lesson plans and the videotaped lesson presentations to confirm assumptions made from analyses of the interview and journal response data. The researcher felt that these two data sources were a strong indicator of the preservice teacher’s beliefs and knowledge about secondary mathematics. In order to identify how the preservice teacher individually developed the mathematical ideas, the principal investigator monitored the individual lesson plans and teaching situations. The researcher also closely examined the group data (i.e. videotaped teaching, the lesson plans, group conversations) in order to support researcher’s interpretations of the individual data. The videotaped lessons were then used as data to triangulate what was gleaned from the audio taped conversations. These lessons added a piece of evidence to support the understanding of what occurred in the lesson-planning phase of the project.

*Summary.* In conclusion, each of the data sources aided the researcher in creating a picture of the subject’s belief system. The initial interview data and open-ended question responses provided a snapshot of the subject’s beliefs before lesson study; the conversation data, the journal entries, the final report and the lesson plans/teaching episodes provided information about the evolution of the subject’s beliefs over the course of the semester; and the final interview provided another look at the prospective teacher’s views after lesson study. In this way, the different data pieces helped to develop and confirm the researcher’s conclusions.
CHAPTER IV: RESULTS AND ANALYSIS

This chapter focuses on the results of the research through a case study. In particular, the case study reported here centers on Janica, one of the eight subjects from the study. The first section outlines Janica’s background and how she came to be a math education major. The next section focuses on Janica’s beliefs about mathematics and her experiences during lesson study. The last half of the case study details how Janica fits within current math education research and provides a look at the overall findings.

Background of Subject

Janica was selected based upon her answers during the interview process and her rich and varied background. Outside the Teaching Methodology and Instruction course, Janica worked with two math education professors and was enrolled in graduate level mathematics courses. Typically math education majors did not take many of the classes that Janica was enrolled in. These factors seemed to point to a successful pursuit of mathematics. Because of these reasons and her experiences during lesson study, Janica was selected to be the focus of this case study.

While investigating Janica’s background, during the interview stage of the research, a pivotal occurrence surfaced. This unpleasant incident actually contributed to her later success in mathematics. Janica remembered a routine evaluation she received while in the 5th grade. As she recalled, the mark in mathematics was a B+, which greatly disappointed her. Janica remembered thinking that she was not a B+ student and that she was determined never to get anything less than an A again. This powerful event caused Janica to strive to do better not only in 5th grade math but also in math classes thereafter. It was not until later, however, that she realized she truly liked mathematics.
Janica’s drive to excel continued on into her high school years. During high school Janica enrolled in the top math classes and received excellent grades. In Janica’s high school the math classes were uniquely combined with the advanced physics classes. Janica remarked that the physics teacher and math teacher often conversed and shared ideas about lesson plans. Janica especially appreciated this connection between the two subjects because it illustrated how mathematics could be applied in the world of physics.

Janica’s success in the physics/math combination classes in high school influenced Janica into college. Early on in her college career Janica planned on majoring in physics. She remembered heading down the physics path because she saw the applicability of the mathematics when she studied physics. However, she eventually decided that she would prefer studying math and abandoned her pursuit of physics.

About half way through her college years, Janica took a break from college. This break eventually led to a change in her career path. Janica spent several months away from home engaged in missionary work. As a missionary, Janica developed a strong affinity toward teaching, and she began to cultivate thoughts of changing to another university. Once she returned from her missionary work, she enrolled in a private religious university and added the math education major to her mathematics major requirements. At the new school, she began work with math education professors. Many of her job responsibilities required her to code data, read articles about educational systems of other countries, transcribe mathematical conversations, and work with preservice elementary math teachers. Janica found these new experiences, and her introduction to the world of math education, enjoyable. These experiences helped her learn about the challenges of teaching.
Janica’s work experience had introduced her to lesson study and the beliefs of preservice teachers. She felt that she was familiar enough with lesson study that she could recommend the practice to the other members of her class. Because of her familiarity with lesson study, Janica approached the project with enthusiasm, and was excited about engaging in a lesson study experiment.

When asked about her future, she commented that she would probably come back and start work on a Masters in Math Education. She referred to her experiences with the math education professors as the motivation for returning and pursuing the degree. However, her current plan was to complete her classes successfully, one of which included the Teaching Methodology and Instruction course.

The Case Study: Janica

The next three sections of the paper follow Janica through her lesson study venture. In particular, the descriptions center on Janica’s beliefs prior to lesson study, the rationale for the grouping of the subjects, Janica’s own reflective experiences during lesson study and her beliefs following collaboration.

Janica’s Baseline Beliefs

The initial interview assisted in uncovering Janica’s beliefs and knowledge of mathematics. Several themes emerged during the interview to create a baseline perspective. These themes are discussed below.

Initial view: What is mathematics?

After reading Janica’s statements and listening to her view of mathematics, it appeared that Janica might have held either a problem solving or Platonist perspective of mathematics. One of the first signs that Janica’s perspective might have been one of
these views of mathematics came from a response to an in-class writing assignment. The instructor asked the teacher candidates to answer a couple of open-ended questions about mathematics. One of those questions was: What is mathematics? Janica reported:

Mathematics is the study of numbers and the manipulation of numbers. Not only that, but it is an explorative science that builds from basic concepts to more complex ideas to abstract ideas. The basic building blocks of mathematics are numbers and manipulatives with numbers or symbols representing numbers or functions. Mathematics is used to solve day-to-day problems and more universal problems. All sciences use mathematics as a way to represent and manipulate data from their respective fields.

From Janica’s response there was a hint that Janica held mathematics as a problem solving or a Platonist field of study. First of all, Janica wrote that mathematics dealt with “complex ideas”, meaning the field of mathematics could convey someone’s advanced thoughts. Janica referred to this problem solving view when she wrote that mathematics was “used to solve day-to-day problems and more universal problems” or an entity that described natural phenomena. On the other hand, Janica also commented that “symbols representing numbers” were “the basic building blocks of mathematics.” This statement closely fit with the Platonist view of mathematics. From this perspective, the rules of mathematics were part of a connected body of related truths that built off one another. With these statements in mind, the first two views could easily have described Janica’s perception of mathematics. However, as the researcher looked closer at Janica’s comments, the researcher decided that Janica’s words conveyed a view of mathematics that was neither problem solving nor Platonist in nature. Janica’s written response to the question, “what is mathematics?” focused on the use of variables and the manipulation of
symbols to solve problems. Janica’s answer implied that mathematics consisted of using the tools of mathematics to solve problems. This meant that Janica’s view seemed to be more inline with Ernest’s (1988) instrumental description of mathematics.

The instrumental view of mathematics was consistent with Janica’s math experiences prior to university study. In high school she had utilized her newly acquired math skills to complete physics assignments. Janica often found that she would manipulate symbols to arrive at answers for specific situations. This perception, that math procedures were used to solve real-world problems, became a common theme for Janica. She repeatedly referred to the idea that mathematics should be couched in terms of real-world contexts. The type of problems Janica was referring to, though, were physics problems that could be easily solved with algorithms. For her, mathematics was a collection of ways that aided the sciences in solving problems quickly. This description of mathematics tended to be instrumental.

There were several other experiences that supported the assumption that Janica’s prevailing view of mathematics was instrumental and not one of the other two categorizations. Comments made during the initial interview only served to support the idea that Janica held the third view of mathematics. The first example occurred when the interviewer asked Janica how she would describe mathematics to a friend who knew nothing about mathematics. Janica depicted mathematics as “a way to represent everyday situation[s]… maybe apply the variable…to something and use it, or manipulate it into something else.” Another part of the interview required Janica to describe what would be important for students to know or understand about fractional division. Janica’s response reinforced the idea that mathematics was about rules without
meaning. She mentioned that it would be important for students to know how to “just
invert the fraction and multiply.” Later on she was asked how she might go about
drawing a division of fractions problem. In response to this question, Janica shared an
example of cutting a 5-inch string into half inch pieces. This model highlighted maybe a
possible use of mathematics to solve a problem. However, when Janica was questioned
about her idea, the researcher discovered that she had recently found the idea in a book
and Janica’s explanation had not originated with her.

Janica’s first assignment to teach a mathematics lesson to her peers highlighted
her instrumental view of mathematics. The topic Janica ended up teaching was about
division of both positive and negative integers. An element of that assignment was to
identify what the teacher called the Big Mathematical Idea (B.M.I.). The B.M.I. Janica
submitted was, “Quotient of 2 integers with the same signs (pos & pos or neg & neg) are
positive and quotient of 2 integers with different signs are negative. This is just applying
rules of multiplication.” The mathematical idea she considered to be significant centered
on the multiplication and division integer rules. Janica’s actually teaching of the lesson
gave physical evidence of her rather narrow and instrumentally based knowledge of
integer division. Janica’s lesson sought to help the students achieve instrumental rather
than relational understanding of the mathematics topic. She attempted to show the
following patterns: If the integers in the problem shared the same sign, then the answer
would be positive, or If the integers had different signs then the answer would be a
negative integer. She even used the procedural method of solving equations in effort to
explain integer division. Partway into the lesson, however, students began to question
her statements and why the procedures worked. Reflecting on the presentation, Janica
wrote that she felt the “lesson was a flop” because she “assumed that the class knew more” than they did, and she personally did not make the connection that fractions were another way to describe division. Janica realized that she was not prepared or flexible enough for the concerns the students had. This dramatic first teaching moment seemed to support the idea that Janica held an instrumental view of mathematics.

Initial view: Mathematical expert

The initial interview revealed Janica’s confidence when it came to secondary mathematics. In the interview Janica commented about her Kindergarten through high school mathematics experience. After listening to Janica talk about high school, the researcher felt that Janica’s high school experiences contributed to Janica feeling comfortable and strong when it came to the subject. The few undergraduate experiences Janica had with secondary mathematics had also contributed to Janica feeling good about her secondary mathematics knowledge. Below are a few of her quotes and experiences that support the idea that Janica felt secure in her mathematical knowledge.

Janica’s confidence in secondary mathematics seemed to be connected to successful high school experiences with math. She said in the initial interview that “it always seemed to come easier to [her]” and she “got it,” meaning high school math. She remembered that she learned mathematics quickly and felt comfortable with it. She even recalled helping other students with the math lessons after she had mastered them. When asked about the topic division of fractions, she commented that she was “never confused with or never really had a problem with it.” In particular, the idea that division with a number less than one gave a larger answer was something that came natural to her. As she looked back to those high school years, Janica’s memory did not reveal any dramatic
struggles when learning mathematics. These positive high school math memories seemed to influence Janica’s beliefs about her current mathematical knowledge.

In the initial phase of the research, the researcher sensed that Janica’s confidence and self-assurance during high school carried over into her current beliefs about her secondary mathematics knowledge. Janica’s answer to a particular interview question revealed this confidence. The two-part question drew attention to Janica’s feelings toward college and secondary mathematics and gave the researcher a sense of the weight of Janica’s responses in relation to one another. The issue put to Janica was: On a scale from 1 to 10, how would you rate your knowledge of 1) college mathematics and 2) secondary mathematics when compared to other preservice math teachers. After the researcher asked the question, Janica quickly responded to the part pertaining to college mathematics. She reported that she did not “feel very confident with [her college] math abilities because [she had been] gone” and that “the last calculus class was like 4 years ago.” Janica went on to comment that it had left her feeling “very uncomfortable with like the beginning [college] math classes.” This comment highlighted the fact that Janica recognized her weakness in university math. With that in mind, the researcher found Janica response to the subsequent part of the question about high school math interesting. Her reply about her knowledge of secondary mathematics seemed to give a different perspective when compared to college mathematics. She said that, “with secondary education it should be fine until [she] got up to calculus.” Her notion of “fine” implied a feeling of being comfortable with secondary school subject matter. The ratings she went on to assign each type of mathematics seemed consistent with her explanations. When it came to her knowledge of college mathematics, she said that she would give it “probably
like a 7, and then high school mathematics, probably an 8 or a 9.” Later she even admitted that she would change the 7 to a “6... on the college” math, showing some lack of confidence with university math. From these statements, the researcher sensed that Janica felt strong in secondary but not college math.

An experience Janica had while observing a high school Algebra 1 class seemed to convey the notion that Janica did not think secondary mathematics was difficult. The topic for that day was multiplying negative fractions. Several students had questions about their work and Janica stepped in to help the students. Janica expressed her feelings that “it wasn’t difficult at all” but that “for them in the beginning [it] might be hard.” Janica recognized that from the student perspective the ideas might have been difficult to initially grasp. However, Janica felt that the topic, multiplying negative fractions, fell into the category of material that did not seem to be difficult. The researcher felt that Janica’s statement did not clearly explain why Janica felt the topic would be considered easy. One guess was that Janica was familiar with the typically taught method to multiply fractions, multiplying the denominators and numerators, and then extended this notion to fractions involving negatives. Janica’s comfortable feeling was probably tied her successful background in secondary mathematics and it seemed logical that she would perceive high school mathematics as lacking in difficulty.

In summary, the researcher felt that Janica’s view of her secondary math knowledge was one of confidence and self-assurance. This confidence stemmed from Janica’s positive high school mathematics experiences and her undergraduate opportunities working with students.
Formation of Lesson Study Group

The perspectives and common themes that emerged during the initial phase of the project laid the groundwork for the formation of the groups. The researcher handpicked each group based upon the preservice teachers’ responses to questions. The principal investigator used the following two criteria, the way math was viewed and the way the preservice teachers viewed their knowledge of mathematics, to form the groups.

Once the researcher understood where Janica’s beliefs stood in relation to mathematics, she organized the lesson study group. The researcher selected the other members, Cindy, Robin and McKenna, according to their initial responses. The researcher felt Cindy and Robin had stronger and more conceptual problem solving views of mathematics than the other two subjects. Janica and McKenna held more of a procedural approach to problem solving. The biggest difference the researcher saw between Cindy and Robin was that Cindy did not perceive herself as an expert in secondary mathematics whereas Robin felt confident in her secondary math knowledge. The researcher also considered the other two subjects’ confidence with secondary math. Janica reported a high level of confidence with secondary mathematics, whereas McKenna was not as sure. Each member brought something different to the group, which provided for rich mathematical and pedagogical discussions. (Table 1 summarizes the distribution.)

<table>
<thead>
<tr>
<th>Confidence (+)</th>
<th>Problem Solving: Conceptual (C)</th>
<th>Problem Solving: Procedural (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robin (C+)</td>
<td></td>
<td>Janica (P+)</td>
</tr>
<tr>
<td>Confidence (-)</td>
<td>Cindy (C-)</td>
<td>McKenna (P-)</td>
</tr>
</tbody>
</table>

Table 1. Formation of the group
Janica’s Experiences During Lesson Study

Janica’s lesson study group prepared a lesson to introduce logarithms to pre-calculus students. The secondary school teacher assigned this topic because her class would be studying logarithms during the first week of December. The group’s reflective experiences were a result of their conversations about logarithms. These episodes were captured with audio recording equipment for later investigation.

After investigating the data, the principal investigator noticed that there were several experiences that challenged Janica’s knowledge of exponents and logarithms. The purpose of the following section is to outline several of those episodes. Since the researcher felt that it would be impossible to highlight all of the experiences, only three of those incidences are discussed below.

Episode 1

During the first official planning meeting there was one episode that challenged Janica’s current knowledge of exponents and powers. The group was discussing the idea that a number (the base) can be raised to a rational number. They had for the most part focused on raising the base to a whole number, but then Janica suggested that they raise “2 to the one and a half power.” The discussion then expanded to focus on other powers. Robin wondered, “What is 2 raised to the 2 and a ½ power?” All the members of the group thought about the question and attempted to make sense of it. Cindy simplified the expression to “4 square root of 2” and Janica thought about it as “2 to the, to the 5 halves, that’s like the square root of 2 to the” 5th. Their attention focused for a moment on the interesting idea. Janica then commented, “I actually, when, when I said that you can raise it to the power, one and a half, I was like, wait a second, can you?” The talk that
ensued went into uncharted territory for Janica and she wondered if the question was even answerable. This episode showed that even Janica was uneasy about the answer and it pushed her to serious contemplation. This passage highlighted Janica’s uncertainty when it came to the topic of exponential functions and powers.

*Episode 2*

Another experience during the next group meeting showed that Janica often consulted other sources when she was unsure. As a part of the lesson the group wanted to have the students prove the property, \( \log ab = \log a + \log b \) however, as the group proceeded with the proof, they found it to be more difficult than they anticipated. The group then focused their attention on trying to make sense of the property so that they could understand it. As they proceeded Janica had determined that “you’d have to say log of a is x and log of b is y. And show that log of a times b is x y”. However as Janica continued she ran into some trouble.

J: times, its plus, that’s what I said the first time and then I thought maybe I’d said it wrong.

R: Well, x plus y right?

J: no, cuz its x,… x times y, but, that’s why I kept getting stuck. Right?

M: Its 10 to the x to the y

J: Yeah, that’s it. (laughs) or 10 to the y, to the x.

M: So that’s log,

J: So then…if we take the log again…

M: You could put log b in for y…

J: Oh, yeah, I think that’s probably how you do it.
Janica started to feel like she understood the proof. However as the group continued to investigate this logarithm property they were not sure and some members of the group vocalized their frustrations. Cindy commented, “Maybe, we don’t know how to do this” showing that she was unsure of how to figure out the proof and McKenna showed her own lack of certainty when she asked, “So is this even gunna work?” McKenna did not even know if a proof could ever be found. Janica felt more comfortable with the development of the property but then finally turned to the book and asked “Did they develop it anywhere?” Her dependence upon the book once again illustrated that Janica questioned her own knowledge.

In this particular passage, there were several different reactions to the experience. Janica finally turned to the book but two other members made specific comments about their uncertainty. Cindy admitted she might now know how to figure out the proof and then McKenna questioned whether the problem would ever work. Janica did not express any feelings related to these sentiments nor did she even discontinue working on the task at hand. Only later when Janica asked how the book developed the properties did she even show a sign that she possibly did not know how to figure out the connection of the two properties. Janica then went on to say that she “didn’t read through the chapters very well” and that she knew she was “a slacker.” For her it seemed that since she had not prepared well enough, that was to blame for her struggles.

The conversation continued on as Janica attempted to make sense of the book’s development of the proof. As she looked in the book Janica realized that the group really was not that far off.
J: If $a$ and $b$ are both positive, we can write $a = 10^m$. Oh, we’re on the
right track. And $b = 10^n$, so $\log a = m$ and $\log b = n$, and the
product of $a$ times $b$, can be written. Oh, OK, hee, hee. We’re just using the $x$’s and $y$’s
but it should have been like…the other way around.

Janica continued to follow the book’s proof. She talked out loud so the group could also
follow along. At the end she commented that she did not “know why [they] didn’t think
of that, anyway.” Janica continued to make sense of the property as the group once again
acknowledged their own frustrations.

M: uh-hu, this is tough. I’m nervous for the, uh…

C: Ah, so we put this to the $5^{th}$ power, so $5$ to the log base $5$ of $a$, $5$ to the $m$ so
you can get $a = 5$ to $m$. I get logarithms! Finally. (laughs)

J: So this point if they see $a = 5$ to $m$, they need to have enough
understanding to realize that that’s umm, $m = \log \text{base } 5$. OK. And now that
they take the inverse.

C: These properties may be harder than we’re thinking they are. Cuz we’re
having a hard enough time with them.

J: Just because this is an introduction. So they’re not gunna have any background
or practice.

During this episode, Janica’s group members verbally admitted that following the
proofs of the properties were more difficult than they had anticipated. That for them as
the teachers they would have difficulty, however, at no time does Janica share the same
feelings. Instead Janica turned to the book to figure out the proof.
Episode 3

This last episode showed Janica reflecting on her own knowledge of numbers and exponents. Another member of the group, Cindy, questioned the group, “Well, can’t a be written as 10 or 5 or whatever to some power?” At first Janica dismissed the comment and then finally stopped and turned her attention toward Cindy.

J: Well what did you mean when you said any number can be written as a base to a power?
C: Like any number, like 200 can, or 100 can be written as base 10 to the power of 2. Or...
J: There’s like 2.
C: 2 can be written as base 10 to some decimal power, see what I’m saying?
R: 100.
C: What? Well she. What do you mean? 2 where as in, 2 instead of a hundred? K.
J: Or pi.
C: pi?
J: Or any irrational number?
C: Can’t they?
M: hmm?
J: Uh-uh. (Negative intonation.)
C: Not all of them?
J: I don’t think so. Can they? (looks at observer in room)

The fact that Janica turned to the researcher for the answer showed how she did not trust her own knowledge and the knowledge of her group members. It is also
important to note here that Cindy was considered to have the least knowledge about logarithms. She proclaimed at the first meeting that in high school she had moved right before logarithms and thus never learned about them. The group members, especially Janica, recognized this and even mentioned in the final interview, “Cindy didn’t know very much about logs.” It may have been particularly difficult to learn from Cindy, the one who was considered to have the least knowledge of the topic. Even after hearing Cindy’s statements, Janica continued to try and make sense of the idea. The discussion finally ends with a comment by Janica, “I’ve never thought of it as actually a base to a power and that’s why, since I’ve never heard it, that’s why I don’t know if it would be true or if it is true.” Since Janica had never heard this before, she struggled to determine the validity of the statement and to accept the truth of Cindy’s statement.

Conclusions

All three of the episodes point to the fact that Janica struggled with her knowledge of exponential and logarithmic functions. However, her reactions in the face of difficulty during the second and third episodes were very different than her group members. In the second episode, Janica turned to the book for help. In the third episode, Janica turned to the graduate student researcher for the answer. This seemed to point to a trend in Janica’s behavior. When Janica did not have the answers, she trusted the knowledge of the book or a graduate student, rather than trust her group members’ knowledge.

Janica’s Beliefs Following Lesson Study

After reading the journal prompts, the researcher suspected that there was a change in Janica’s knowledge of logarithms. The researcher compared the statements Janica made during the final interview with those written in the dialogue journal. Below
is a discussion of Janica’s typical journal responses and how those statements relate to comments Janica during the final interview.

*View of Logarithms*

*Journal prompts.* Analyzing the journal prompt responses led to some interesting conclusions about how these reflective experiences had influenced Janica. In the first journal (following the first meeting) Janica spoke about the comfortable feeling she felt among group members. She shared that they “weren’t ashamed to ask questions if [they] didn’t understand something.” This showed that Janica grew comfortable with the lesson study group and talking about mathematics. In the same journal she felt that she had “definitely gained a deeper understanding of logs because [they] were forced to justify every step together.” The group worked intently on understanding the material themselves, so that they would provide a better lesson for students. This helped her to understand more about the topic as they delved in deeper. It seemed that the challenging experiences influenced Janica’s perception of logarithms.

*Final interview.* The final interview confirmed many of the statements about logarithms that Janica expressed in her dialogue journal. She mentioned that as a group they had “had disagreements, but those [led them] to understand logs on a more in-depth level.” That by talking through the mathematics of logarithms they “all understood logs much, much, much better than [they] did coming into it.” Janica plainly stated that the group interaction enriched and deepened her knowledge of the secondary topic. Collectively, the comments made during collaboration, the journal entries and the final interview comments served to confirm the conclusion that Janica’s understanding of logarithms increased throughout her lesson study experience.
Global View of Mathematics

Although Janica’s understanding and knowledge of logarithms may have changed, the researcher realized that Janica’s global view of mathematics did not change. Even the challenging lesson study experience was not enough to change Janica’s global perspective of her knowledge. The principal investigator thought this was the case because Janica’s final interview comments and her global view of her knowledge seemed to be consistent with those conveyed in the first interview.

Final view: What is mathematics?

After looking at Janica’s final interview comments, the researcher felt that Janica had not adopted the Platonist or the problem-solving view of mathematics. She still saw math as “a method that you learn” meaning that math constituted methods to be memorized. Her view of math lent itself more to Ernest’s (1988) view of mathematics as instrumental. She said that math was learned best when “learning a method and then just practice using it” and that the best way to accomplish this was through “applications.” Her perception of mathematics was consistent with her previous beliefs and the beliefs she brought with her from high school. She also commented that “mathematics [was] like the foundation for everything that we do. Like physics, chemistry, everything, they’re just applying mathematics or this science, like a new language in order to understand their fields.” This was consistent with her earlier comments and fit with the Janica’s mathematics exposure during the high school years.

Also during the latter part of the final interview, Janica talked about her view in terms of teaching mathematics. This episode highlighted the fact that even after all of Janica’s experiences her perception of mathematics remained constant. Janica shared her
feelings about her first lesson presentation to her peers. She talked about how a typical lesson might consist of “present[ing] like the rules and then hav[ing] you apply them.” She went on to admit that that was “basically what [she] did.” When Janica was questioned about the lesson and whether she saw a problem with it, she said that “It’s probably OK in certain instances and in this case it might be OK. Just because hopefully [students’ will] see the connection between multiplication and division…and then that’ll help also when they learn about solving equations.” Couched in terms of a teaching situation, mathematics to her was not related to problem solving or developing relational understanding but was more about seeing the appropriate rules to apply and developing instrumental understanding.

*Final view: Mathematical expert*

When looking at Janica’s view of her knowledge of mathematics, the researcher felt that Janica’s beliefs also lacked change. After looking at the final interview, the researcher felt that Janica had assimilated her experience. One of Janica last comments was that the “beginning material [was] going to be easy because it’s always, you know, just background and the foundation material, so of course that [would] be fine.” This comment showed how Janica felt secondary mathematics was easy and that secondary mathematics seemed basic to her. Janica’s statement highlighted how her confidence in the area of secondary mathematics seemed to remain constant over the course of the study. Because of these reasons, it appeared that lesson study did not seem to move Janica’s view of her ability in the area of secondary mathematics.
Analysis

After some preliminary analysis, the principal investigator noticed that Janica’s belief system was not altered during the lesson study experience. This lack of significant change was consistent with the works of Collier and Shirk (as cited in Thompson, 1992). The fact that Janica may have assimilated the experience was also consistent with research carried out in Britain. Lerman (as cited in Thompson, 1992) suggested that in situations of inquiry, teachers take new knowledge and fit it within their existing framework instead of changing it. Because Janica was thought to assimilate her knowledge during lesson study, she seemed to be similar to subjects in other studies. Janica did not present any evidence to suggest that she acknowledged her lack of mathematical knowledge. Through Janica’s comments and actions, she quietly defended her position as a knower of mathematics. The following section of the paper discusses Janica’s belief system and makes some conclusions about the ways in which Janica was able to preserve her identity.

_Janica’s Mathematical Belief System_

Current research highlights the fact that Janica’s beliefs about mathematics were comparable to beliefs held by other preservice math teachers. Ball (1990a) noted that it was natural for secondary majors to perceive mathematics as “a collection of arbitrary rules to be memorized” (p. 460). This commonly held conception was also prevalent in Janica’s belief system. Janica viewed mathematics as the procedures that could be used to solve real-world problems. For her, this provided the impetus for learning mathematics. This view of mathematics was also consistent with Cooney’s (1999) research with prospective secondary math teachers, who often displayed “a strong
computational orientation” (p. 165). The description of mathematics that Janica gave was consistent with typical preservice secondary teachers but did not fit within Ernest’s (1988) depiction of the problem solving or the Platonist view of mathematics.

Janica’s notion of what constituted mathematics spilled over onto her outlook of her knowledge of secondary school mathematics. Her strength in procedural knowledge sustained Janica’s view of herself as one that was good at mathematics. This view was also held by students in a study by Meredith (1993) who “reported their mathematical knowledge as being the one aspect of teaching about which they felt most confident” (p. 331). Janica also expressed this same sentiment. Like the subjects in Ball’s (1990a) research, Janica was a good student who received good grades in college. Because these secondary math majors were considered above average in high school math and then later had majored in the subject, the preservice teachers believed that they knew the subject matter (Ball, 1990a). Cooney (1999) agreed that this was a common assumption but gave a convincing argument why this assumption was not necessarily true. In his research, Cooney (1999) determined that “often preservice teachers have a poor understanding of school mathematics – having last studied it as teenagers with all the immaturity that implies” (p. 165). So even though teacher candidates, like Janica, felt that their knowledge of secondary mathematics was ‘fine’ their mathematical knowledge lacked depth. Furthermore, even though Janica experienced success in college mathematics, this did not signify a thorough understanding of secondary mathematics.

Ways That She Protected Herself

As has been pointed out in the cited episodes, Janica did have areas of mathematical weakness. As the principal investigator looked deeper, she realized that
there were several common responses or actions Janica used to divert attention away from mathematical knowledge. Unknowingly Janica used these methods to keep from revealing her weakness when it came to secondary mathematics. Siebert, Lobato & Brown (1998) reported similar actions by a preservice teacher named Antonio. The interviewer questioned Antonio about his beliefs about division (similar to the questions put to Janica). Siebert, Lobato & Brown reported that Antonio’s responses pointed to a possible ‘move’ away from having to reorganize his beliefs about division. Janica also used her own ‘moves’ to divert attention away from her weakness in mathematics. One of the ways that she did this during the lesson study collaboration was by acting as the scribe. In addition, during interviews she was able to keep from revealing any information about her math ability by using three different methods. In response to a new idea, Janica’s first ‘move’ was to comment that it was a good review, or she forgot the topic. The second way that she protected her identity was by saying that the class structure was the culprit when it came to her inability to identify mathematical ideas. The third way was by emphasizing that the reason she was having trouble was because of her lack of pedagogical knowledge.

* I’m the scribe. During the lesson study collaboration Janica was able to regulate the pace and direction of the conversation because of her role as the scribe. By acting in this capacity, Janica was able to mediate the kinds of concerns the group discussed and the order in which the mathematics was placed in the lesson. In one episode, Janica started writing the group’s ideas down in a lesson plan format. As Janica wrote the group would continue to talk, but Janica struggled to write down all the group members’ comments. The group was focusing on how to get the students to discuss the multiple
ways of representing the inverse or find the inverse. As the discussion ensued, many of the individual comments and ideas were lost as Janica tried to make sense of the lesson plan on her own. Or if it seemed that Janica did not agree with a specific idea she would not write down the comment. She would then think of how she would ask the question and see if there was a group consensus. This led to her being able to focus on the math that she felt was important and necessary to discuss. Unknowingly, Janica was able to manage the depth of the mathematical conversation because of her role as scribe.

*I’ll need to review.* One of the first indications that Janica was unknowingly diverting attention came during the interviews. Janica’s comment that this was a good review or that she forgot the little details of a topic, focused the attention away from her knowledge of secondary mathematics. One particular experience highlighted Janica’s need to review. In the interview, Janica attempted several times to draw a picture of 3 divided by 2/3rds. At first Janica drew an incorrect model. However, after a moment of thought she utilized her previous string example to draw a picture. (The string example came from a textbook.) When Janica was asked to explain her model, she struggled to describe her drawing. After a moment of thought she turned and said, “Thanks, this is a good review.” For her, the reason she had trouble communicating her ideas effectively was related to her lack of recent experience with the topic.

In the final interview Janica was questioned about her confidence with secondary mathematics. The question “Does it bother you to know that you’re…maybe not as complete in your knowledge…?” was directed to her. Janica’s response revealed once again the tendency to point at forgetting the mathematics and the need for review. She commented that “[she had] forgotten a lot of mathematics, especially the mathematics
that [she] learned in high school, the stuff that [she would] be teaching.” This idea of forgetting the mathematics seemed to her like a typical first year teacher reaction. She commented that she “would have to…and every teacher [did she was] sure when they’re starting out to teach a new subject, just having to review the material and really understand what’s going on.” Later in her final interview, she commented that she realized that she did not “know a lot of mathematics, and [she’d] forgotten a lot” and that “it [would] come back and [she’d] be able to teach it.” For Janica it was more an issue of forgetfulness, and through the process of reviewing the material it would all come back to her. It was not as if she did not know the material, it was simply that she forgot it.

*Class structure to blame.* The second way Janica was able to deflect the focus away from the math was blaming the structure of the class for her lack of ability to identify mathematical ideas. One of the first indicators of this defense was when she blamed the teachers for not helping her recognize the main idea in the mathematics. “Early on [she] thought everybody was kind of confused with the big mathematical idea. What exactly [the professor] wanted, what he thought was the big mathematical idea.” She thought she had to play the game of trying to figure out what the teacher wanted her to write rather than realizing that she struggled to spot the main mathematical ideas.

Janica also blamed the materials for her inability to summarize the big mathematical idea. She went on to say that when she would write her lesson plans that she would try and pin down the main idea but then would run into a problem. “That’s one of the problems with the way the class was set up” she said, “a little bit with the text that we got, that, when I was like writing some of my lessons I didn’t know what the big mathematical idea should have been.” She felt that since they did not have the extra
information in the teacher’s edition or previous chapters that it was hard to isolate the point of the lesson. In this particular instance, she blamed the books for her struggle to come up with the big mathematical idea. In a small way, she held the teacher and the materials responsible for her lack of ability to spot mathematical ideas.

*That’s a pedagogical question.* Another way that Janica kept from revealing a lack of knowledge in relation to secondary mathematics was explaining that the reason that she might not have an answer was that it was a math education question. Janica often made this comment because she felt that ideas about how to teach mathematics were new to her. She believed that she was learning a lot about how to teach mathematics because of her recent change to the mathematics education major. Since the idea of teaching mathematics was so new to her, Janica depended upon the book to help her make pedagogical decisions. One particular experience during the initial interview highlighted this point. Janica was asked to write down everything that she would want her students to know or understand about division of fractions. Her response indicated that she would probably have to consult a book. She said “its hard just off the top of my head, its easier if I have like a book so that I can see, you know, what students are going to need to know, and apply it.” This dependence on the book was couched in terms of making curricular decisions and focused the attention on her personal knowledge of pedagogy. After being asked about how she would teach slope, Janica responded in a similar manner. She said, “asking a question like that is sort of, I mean, not…its sort of dumb because I don’t have a background in teaching up to that point and I don’t know exactly what’s gunna be after.” Rather than drawing on her personal knowledge of slope, Janica claimed that her lack of teaching experience inhibited her from providing a
description of the topic. She even commented that the lesson study group members lacked mathematical pedagogical knowledge since they were not teachers yet. Janica said that this was part of the reason the group struggled to create a lesson plan. She said that they were “inexperienced preservice teachers” and that “there’s only so much we know, there’s only so much we know about pedagogy…about how to present the mathematics to students.” Janica felt that if they would have had someone more experienced then the process of creating a lesson may have gone quicker but then again she admitted that it “might be good because that, that gives [them] sort of a fresh approach on how [they could] teach the lesson…” In her eyes there was good and bad to the inexperience. However, Janica’s reference to inexperience means inexperience in teaching mathematics, not lack of mathematical knowledge.

**Findings of the follow-up interview & their connection to research**

Janica’s pattern of focusing the attention away from the mathematics motivated the researcher to conduct a follow-up interview. The purpose of the follow-up interview was to make sense of Janica’s belief system and to help answer a few questions about Janica’s personal views about her mathematics knowledge. The researcher felt that during interviews Janica would answer questions indirectly and that her statements would often appear to contradict one another. Janica described herself as being good at mathematics, and that she felt comfortable with her secondary mathematics knowledge. However, when Janica was asked a mathematical question she would shy away from answering the question directly. Instead, Janica would provide an explanation for why she could not complete the task successfully. Was it possible that Janica did not actually see herself as an expert? If Janica did not see herself as an expert, how come she often
spoke of feeling comfortable with secondary mathematics? Was Janica protecting herself from having to reveal a weakness in her mathematical knowledge? The researcher recognized that according to coherence theory Janica’s statements and actions must be consistent within her belief system (Rescher, 1973). Because of this, the principal investigator felt it was necessary to probe Janica further about her beliefs, and try to assess the nature and intent of Janica’s responses. The questions appeared difficult to answer, but the researcher felt that the follow-up interview would provide insight into Janica’s thinking.

The follow-up interview revealed how Janica’s beliefs were consistent within her own mind and shed some light on why the attention was often shifted away from mathematical conversation. Near the end of the follow-up interview Janica shared that the first interview experience with the task of drawing $3 \div \frac{2}{3}$ made Janica feel uncomfortable. She mentioned that she was grateful that she had studied the book example a few days before and was able to share it during the interview. She even thought to herself, what would I have done if I did not have the example from the book to use? When asked why she felt uncomfortable, Janica revealed that she would have felt “dumb” in front of the researcher. The fact that Janica had recently read the book’s example saved Janica some embarrassment. She had been worried about revealing her secondary mathematics weakness.

Researchers considered Janica’s reaction to the division of fractions task typical for American students. Stevenson and Stigler (1992) called attention to this reaction in their book *The Learning Gap*. They reported a striking cultural difference between students in Japan and the United States in this area. The difference seemed to come from
a cultural belief about the ability of students. In Japan, if a student was not successful in solving a problem teachers felt that the child had not worked long enough on the task, whereas in the U.S., teachers felt that it was related to their innate abilities (Stevenson & Stigler, 1992). The idea of assessing a child as lacking innate ability was not considered in Japan as much as in the U.S. Stevenson and Stigler gave an example of how oftentimes children may be put into groups according to ability. For example, the Robins are the better students and the Bluebirds struggle in a subject area. Stevenson and Stigler (1992) pointed out that sorting students “may enhance the self-image and pride of children who happen to be Robins, but Bluebirds may be stigmatized…” (p. 109). For Janica this idea of being categorized was very real all throughout her education and came through in the initial interview and a journal entry as well. In a journal entry Janica shared that she would “begin to feel inferior/superior depending on [her] level of knowledge and experience with the particular subject.” From this comment it was easy to see that Janica felt some sense of worth associated with her knowledge or lack thereof with regard to mathematics.

A few days after the follow-up interview, Janica pointed out a quote to the researcher. Janica shared that the quote represented the feelings she had during the first interview. The following quote from the NCTM 2000 Principles and Standards seemed to sum up Janica’s first interview experience.

Many students have developed the faulty belief that all mathematics problems [can] be solved quickly and directly. If they do not immediately know how to solve a problem, they will give up, which supports a view of themselves as incompetent problem solvers. (p. 259)
Janica identified with this quote because it fit well with her initial interview experience. During the interview, she felt she should be able to provide an answer speedily if she was to be considered bright in mathematics.

Janica’s identification with the quote fits with current research and also the feelings the researcher had suspected about Janica’s beliefs about mathematics. In his study with 10th grade geometry students, Schoenfeld (1988) stated that often students feel that they must solve mathematics problems quickly if they truly understand the material. However, the kind of mathematics that was to be solved swiftly consisted of similar problems where the numbers were often changed (Schoenfeld, 1988). This type of mathematics was considered to lack conceptual understanding and focused solely on mastering rote procedures. This seemed consistent with Janica’s view of mathematics.

The follow-up interview confirmed the conclusions that the researcher made about the way Janica viewed mathematics. After the initial interview, the principal investigator determined that Janica probably had a procedural view of mathematics. During the follow-up interview, the researcher described the different perceptions of mathematics and asked Janica if she could identify with any of them. Ernest’s (1988) first description was that mathematics was constantly being discovered, the second view saw mathematics as a dynamic discipline, changing and evolving over time and the last view, instrumental, described mathematics as a bag of tools used in order to solve problems. After describing these three views of mathematics, Janica was asked once again if she could identify with any of these views of mathematics. Janica said that the view of mathematics that most closely categorized her knowledge was that of “instrumental.” She said that her knowledge consisted of a package of tools used to solve problems. “Cuz, that’s what it’s
always been. In a class you learn the tools and you solve problems.” So from her perspective, her accumulated knowledge consisted of learning the rules of mathematics. If she did not remember the rules, she said that it was because she needed to review. In Ball’s (1990a) research with secondary education majors she too found the same characteristic response. “If they could not figure something out, they assumed they were ‘rusty’” (Ball, 1990a p. 464). This idea of needing to review was a common theme seen not only in Janica but also in secondary math majors Ball studied. These comments confirmed the researcher’s assumption that Janica’s view was more aligned with Ernest instrumental description of math.

Even with the sense of confidence that surrounded her mathematical knowledge, Janica freely admitted that she was not as confident when it came to teaching mathematics. Janica saw mathematical and pedagogical knowledge as separate and distinct knowledge pieces. In research conducted by Cooney (1999), he found a similar perception that content knowledge and knowledge needed for teaching are not integrated. Janica agreed with this idea of separation. She felt that her knowledge of secondary mathematics was fine but that she would not be able to describe how to teach it because of lack of teaching experience. Cooney’s research findings also agreed with Janica’s statements about preservice teachers’ lack of pedagogical knowledge. Oftentimes teacher candidates do not have the kind of knowledge needed to be able to teach secondary mathematics. Janica believed that she lacked the knowledge necessary to make good pedagogical decisions. In her mind, her knowledge of secondary mathematics, even though she felt comfortable with it, did not influence the kinds of decisions that she made when teaching the subject matter.
Janica went on to say that she wanted to teach differently than she had been taught. Janica said that she saw a difference between the kind of knowledge she had about mathematics and the kind that she would be expected to use when teaching. Ball (1988) said that other preservice teachers recognized that they learned to teach from the way they were taught and that a deeper understanding of mathematics would be necessary to teach conceptually. Skemp (1978) agreed that for students to develop conceptual understanding, the teacher must aim to develop that understanding within his or her students. However, teachers that only possess an instrumental understanding will be unsuccessful in helping their students develop conceptual or relational understanding (Skemp, 1978). Janica recognized this discrepancy because she admitted that she lacked the kind of knowledge she needed to teach conceptual mathematics. Her first teaching episode was also very instrumentally oriented and fit with Janica’s comment that her view of mathematics was instrumental. Showing that in order for Janica to be able to improve her teaching she first needed to develop her own conceptual understanding.

The follow-up interview highlighted the fact that Janica had truly reflected on her knowledge of mathematics and recognized that it was an area of weakness. It is unclear whether lesson study was the motivator for this reflection but it points to the idea that preservice teachers can reflect on their knowledge and beliefs. In a study conducted by Meyerson (as cited in Thompson, 1992), Meyerson reported that preservice teachers could change their beliefs; and the essential element was the preservice teacher’s realization that he/she felt uneasy while solving problems. Janica also reported feeling uncomfortable and recognized this emotion. Her ability to pinpoint why she felt uneasy could represent the potential for growth and later change.
Summary

After analyzing the initial data, the researcher realized that Janica reported a sense of confidence when asked about her secondary mathematical knowledge. This was consistent with Ball’s (1990a) research with prospective math teachers. She too found that preservice teachers commonly assumed they knew math, however, the mathematics they were referring to consisted of procedural, rule-bound mathematics and not conceptual mathematics. This was also true of Janica’s description of her own math knowledge.

As she entered into her lesson study experience, Janica reported a process of learning about logarithms. However, when questioned about her knowledge of mathematics during the final interview, she still held tightly to the idea that she was comfortable with her secondary mathematics knowledge. This held constant from her initial interview. This confirmed the suspicion that Janica’s belief system did not change even after reflecting with her lesson study group. It seemed that Janica’s belief system had remained stagnant during the process of lesson study.

Further investigation of the data revealed trends in Janica’s actions and her comments during interviews. She had a fear of looking ‘dumb’ in front of her peers and the researcher. With this fear, certain natural defense mechanisms were used in order to protect against revealing any weak areas in her mathematical knowledge. It appeared that Janica had pushed the attention away from her mathematical knowledge and unknowingly pointed to other aspects of her knowledge that she felt were weak. These actions were significant because they helped to paint the picture that Janica really did have a weakness in her conceptual knowledge.
Janica hid her true feelings during many of the interviews. Upon further questioning, Janica admitted that she would have felt dumb if she had appeared to struggle with her mathematical knowledge. Also it became clear after a follow-up interview that Janica truly was afraid of being seen as incompetent when it came to mathematics. For these reasons, Janica used several ‘moves’ to protect her identity as a knower of secondary mathematics.
CHAPTER V: DISCUSSION AND IMPLICATIONS

Discussion: Assessing the Research Question

The focus of this study addressed the following question: How does lesson study influence prospective mathematics teachers’ view of self as mathematics expert?

Through the lesson study experience and from statements made during interviews, the researcher felt that Janica opened her eyes to her weakness in relation to logarithms. In more than one instance during collaboration, Janica looked to outside sources to help her make sense of the logarithm material. For example, Janica would often search the textbook for answers or turn to the researcher observer when she was unsure about an idea. These actions painted the picture that Janica was still grappling with her understanding and knowledge of logarithms. Furthermore, the trying experiences opened Janica’s eyes to the fact that her knowledge of logarithms was incomplete and underdeveloped. Janica commented in the final interview that her knowledge of logarithms grew as she engaged in lesson study collaboration. Janica felt that lesson study forced her to rethink how she viewed logarithms and helped her redefine her fragmented knowledge of the subject. As a result of her experiences, Janica was able to openly admit that the topic of logarithms was an area of difficulty for her.

Even though Janica’s view of logarithms changed, Janica’s lesson study experience did not change her global view of mathematics. Janica’s experience may have highlighted her lack of knowledge in a specific area but did not force Janica to feel uncomfortable with her secondary mathematical knowledge as a whole. This means that Janica more or less assimilated the knowledge gained through collaboration into her own mathematical world. From Janica’s experience, the researcher felt that engaging
preservice teachers in rich discussions could change prospective teachers’ beliefs about a particular topic but not an entire mathematical belief system. One experience with lesson study could not change Janica’s global view of mathematics and the same would probably be true for other preservice teachers.

The follow-up interview shed more light on Janica’s feelings about her mathematical knowledge. During the follow-up interview, Janica admitted that she was aware of her lack of secondary mathematics knowledge. This attitude had been in direct opposition to what the researcher had heard Janica attest to before. Janica further explained that she feared that the researcher would think she was ‘dumb’ if she struggled with mathematical questions. This sentiment seemed to present the idea that Janica closely associated her view of herself with the type and amount of knowledge she had about mathematics. After looking at the data again, the researcher realized that Janica had used several ‘moves’ to divert attention away from her mathematical knowledge. These ‘moves’ had been the means of protecting a very sensitive issue and not just a way to change the subject. The follow-up interview helped the researcher realized that Janica’s ‘moves’ had been used to preserve her self-worth.

Implications for teacher educators

The results of this research provide possible implications for teacher educators and the following paragraphs outline how the findings could influence their decisions.

Implication 1

One of the most profound findings of this research is that preservice secondary mathematics teachers closely associate their mathematical knowledge with their view of themselves. As a result teacher candidates may feel hesitant to talk about math topics
with teacher educators or peers because they run the risk of exposing a weakness in secondary mathematics. Any attempt to uncover this weakness in mathematics, might challenge preservice teachers’ views of themselves. These experiences may be detrimental to the ego of a preservice teacher if an observer notices that he/she struggles during challenging tasks.

The first implication for educators is that teacher educators need to be sensitive to the fact that preservice teachers may closely associate their self-worth with their math knowledge. This means that prospective secondary math teachers need to be in situations where they can enhance their knowledge of mathematics without feeling that their self-worth is being challenged. Preservice teachers need to feel that if they struggle with mathematical ideas, others will not perceive them as dumb. The responsibility then falls to the teacher educator to create a classroom environment that fosters learning and allows preservice teachers to feel comfortable taking risks.

Implication 2

The second implication of this research is that a secondary teacher candidate might use certain ‘moves’ to focus the topic of conversation away from mathematics in order to preserve his/her image as one who knows secondary mathematics. These actions may be a result of how closely the prospective teacher links his/her identity with his/her knowledge of mathematics. In some cases, preservice teachers may change mathematics conversations into discussion about pedagogy, management or other teaching issues in an effort to keep from revealing that they struggle with mathematics. Teacher educators should look for these and other moves when discussing mathematics with prospective secondary mathematics teachers.
Implication 3

The third implication is that it is not clear that lesson study changes prospective secondary mathematics teachers’ beliefs about themselves and their knowledge. It may be possible, however, that the interviewing process may help preservice teachers open up and freely discuss the state of their secondary mathematics knowledge. This stems from Janica’s own admission that the interviewing process was uncomfortable for her and caused her to reflect on her secondary knowledge. For the teacher educator this may mean that in order to help preservice secondary mathematics teachers realize and acknowledge their weakness, interviews may be needed to highlight gaps in mathematical understanding. Once the areas of weakness are uncovered then the teacher candidates may be willing to talk with teacher educators about the mathematics deficiency openly. In doing so, the teacher educators may help prospective teachers close the gaps in their mathematical understanding.

Limitations of the Study and Future Research Suggestions

With any study, there are certain limitations that must be addressed. In this study there were several areas that may have possibly limited the project. In hindsight, the limitations of the study provide ideas for possible future research. Below are a few ideas.

Suggestion 1

One of the ideas not attended to, because of lack of time, was that of the group dynamics and the interactions between members of the group. Future researchers could look at how the group members worked together and how their interactions with one another changed over time. Researchers could look at how hesitant the teachers were to work with one other in the beginning, and how comfortable they felt with their group
members at the end. The principal investigator in this study realized that from the initial interviews that many of the teacher candidates would rather have worked independently than in a group. Assessing whether the subjects changed their opinion about collaboration from beginning to end would also be another question worth investigating.

**Suggestion 2**

Once again because of lack of time, the researcher chose not to investigate what the preservice teachers focused on during their observations of other research lessons. Often teachers in the U.S. focus on teacher mannerisms or lesson delivery rather than on the development of the mathematical lesson. With lesson study, the research lesson is a product of group collaboration, and not just one teacher’s impression of how to teach the topic. The researcher feels that it would be interesting to identify whether the preservice teachers recognize the lesson as a culmination of the group efforts or whether their observations still reflect a focus on the teacher and not on the idea development. The purpose of the research in this area would be to determine whether the preservice teachers changed the focus of their observations and how that change was achieved.

**Suggestion 3**

One of the main complaints received from the preservice teachers was that they lacked access to ‘real’ students. Because the first research lesson was presented to other preservice teachers and because the subjects were not directly working with children, they felt that this hindered their ability to consider student thinking. They felt that they struggled to determine the best way to present mathematical ideas to students and how students would think or respond to certain math questions because of this. This may have been the case but it was decided that since the focus of the research was on the preservice
math teachers’ understanding of the math topics, that this would not hinder the study. If anything, this particular fact highlighted that the preservice teachers needed more experience with the mathematics, since they struggled to make sense of it even without having direct access to children on a daily basis.

As far as a suggestion for further research, it would be interesting to investigate student teachers as they engage in lesson study. This would give researchers an opportunity to not only assess the effectiveness of lesson study with inexperienced teachers but it would also give the teachers a specific group of children to work with. The problem of not having direct access to children would then be eliminated.
REFERENCES


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APPENDIX A

Instruments

Protocol 1: Likert-Scale Survey
Protocol 2: Initial Interview Questions
Protocol 3: Final Interview Questions
Protocol 4: Open-Ended Questions and Journal Prompts
Protocol 5: Follow-up Interview Questions and Statements
Protocol 1: Likert-Scale Survey (1 strongly disagree---5 strongly agree)

1. I think that the mathematics that I will be required to teach in the secondary schools will not be difficult but rather easy to teach.

2. I think it would be important for me to share ideas about mathematics and teaching with other math teachers.

3. I feel that my current knowledge of advanced or college mathematics is not complete and rather incomplete in some areas.

4. I would consider myself an expert when it comes to secondary mathematics.

5. It is possible to anticipate the types of responses that students will have when learning a new concept.

6. I consider mathematics to be a discipline that is discovered rather than created.

7. When I think about teaching mathematics in the schools, I worry more about the mathematical content rather than about management problems.

8. I consider the textbook to be the authority when it comes to difficult math concepts.

9. I feel that my teacher education training has prepared me sufficiently to teach junior high or high school mathematics.

10. I do not think that it is so important for me as a teacher to investigate the errors that students make when they do their homework.

11. It is necessary to have a good understanding and knowledge of mathematics to be a good math teacher.

12. I feel comfortable answering any mathematical question that I might be asked during a lesson.
Protocol 2: Initial Interview Questions

1. What math or math education classes are you currently taking? What math or math education classes do you plan to take?

2. What kind of work experience (if any) do you have in the area of math education?

3. What do you remember about your K-12 math experience?

4. How do you feel that you learn math best?

5. If a friend of yours, who knew nothing about mathematics, asked you what mathematics is all about, how would you describe it to him/her?

6. On a scale of 1 to 10 (10 being high), how would you rate your knowledge of mathematics when compared to other prospective mathematics teachers? First of college mathematics and second of secondary school mathematics.

7. How do you feel about teaching secondary school mathematics? How do you feel about your ability to teach secondary mathematics?

8. What do you think the role of a math teacher is? Give an example.

9. What are some characteristics of an excellent math teacher?

10. What do you think your strengths are as a math teacher?

11. Have you had opportunities to teach or tutor students on math topics?

12. How do you think most students in general learn math best?

13. Suppose you had to teach division of fractions to a class of high school students, what would you do if you didn’t know anything about the topic? (If they say collaborate.) How long do you think it would take to collaborate on the topic?
Division of Fractions

At some time during your teaching career you will have the opportunity to teach the topic: division of fractions.

1. Write down everything that you would want your students to know or understand about division of fractions. Be specific.

2. Some students will already know something about division of fractions, how would you go about finding out what they know?

3. How would you teach this topic? What would be a specific example that you would use to teach this topic?

4. (If they give a symbolic justification) Could you draw a picture of $3 \div \frac{2}{3}$ to justify the invert and multiply rule?
Protocol 3: Final Interview Questions

1. After all that you have experienced this semester how do you feel about your mathematics teaching?

2. What are the specific things your group worked on during the semester?

3. On a scale of 1 to 10 (10 being high), how would you rate your knowledge of mathematics when compared to other prospective mathematics teachers? First of college mathematics and second of secondary school mathematics.

4. How do you feel about teaching secondary mathematics? What did you say during your first interview about this?

5. In one of your conversations you said _______________, can you explain why you said that? What was your intent in making that comment?

6. Describe what mathematics is.

7. Do you feel that you have had any new insights about mathematics or mathematics teaching during the semester? If so, what are they? What do you think promoted those new insights?

8. Has your view of mathematics changed? If so, what do you attribute this change to? Give a specific example that shows how your view has changed.

9. Think for a moment about the topic that you focused on during your collaboration, what sticks out in your mind about the kinds of things you focused on while preparing your lesson? Would you say that your group focused on procedures or conceptual ideas? Give an example of a particular instance that exemplifies this.
10. Do you feel that you have developed any particular strength during your collaborative lesson-planning time?

11. Do you think that lesson study would be useful for you as a professional teacher? Why or why not? Or do you think that it is more work than it is worth? Why or why not?

Division of Fractions

During the initial interview you made some comments about how you would present division of fractions. These were some of your comments.

1. What do you think of how you planned to present this topic during the initial interview?

2. Do you think that you would change how you would teach this topic? If so, how would you teach this topic now? What would be a specific example that you would use to teach this topic?

3. You shared an example of $3 \div \frac{2}{3}$. Could you draw a picture of $3 \div \frac{2}{3}$?
Protocol 4: Open-ended Questions and Journal Prompts

Initial in-class questions

A. What is mathematics?

B. How do you learn mathematics best?

5 weekly journal question

1. Describe your impressions of the time you spend in your lesson study collaboration. Describe your group and how you fit into it.

2. What is your lesson study goal? How do you plan on achieving that?

3. What contributions do you feel you make to your lesson study group? Give a specific example.

4. Do you like working collaboratively on lessons? Why or why not?

5. Give a percent of how much you talk about: mathematics, pedagogy, classroom management, or other topics (please list)? Explain why you gave each topic its percent.
Protocol 5: Follow-up Interview Questions and Statements

1. What is it about mathematics that you like? What is it about mathematics that you do not like?

2. On a scale of 1 to 10 (10 being high), how would you rate your knowledge of secondary school mathematics when compared to other prospective mathematics teachers?

3. How do you feel about teaching secondary mathematics?

4. In one of your conversations you said ________________, can you explain why you said that? What was your intent in making that comment?

5. How did you feel about your interactions with other members of your group?

6. How did you feel about your knowledge of mathematics when compared to members of those in your lesson study group?

7. How easy was it for you to talk about mathematics with your group members? Explain.
Respond to the following statements:

1. The reason that we do mathematics is to find the algorithms necessary to solve real-world problems.
2. Mathematics is the study of manipulating numbers rather than the study of quantity and the relationship between those quantities.
3. Mathematics is linear and in order to learn something new, one must know the previous procedures. That is why math teachers should teach it step-by-step.
4. It is important for students to master a skill before they are taught another one.
5. I have a handle on the mathematics needed to teach.
6. Teaching mathematics means teaching all the different step-by-step ways to learn it.
7. I think it is important to go step-by-step through a concept.
8. People look up to me because I know mathematics.
9. Typical school math content is not difficult.
10. K-12 math education gives math teachers with what they need to know about mathematics.
11. Majoring in mathematics will ensure knowledge of the subject matter.
12. I use the book to help me determine how a math topic should be taught.
13. I see myself as knowing mathematics.
14. Lesson study did not change the way I view mathematics.
15. Lesson study did not change the way I view my knowledge of mathematics.
16. I never learned the type of mathematics that I know I should teach.
APPENDIX B

Consent Forms

Protocol 1: Preservice Teacher Consent Form

Protocol 2: Public School Teacher Consent Form

Protocol 3: Public School Student Consent Form
CONSENT TO BE A RESEARCH SUBJECT  
Preservice Teacher

The purpose of this research is to study prospective mathematics teachers as they engage in lesson study. The study will focus on your beliefs about mathematics and teaching, as well as the topics discussed during collaboration. You have been selected based on your enrollment in Mthed 377, and your responses to the initial survey questions. The study will begin in late September and commence in December 2001. Julie Stafford, a graduate student in the BYU Mathematics Education Department, is conducting the research.

As a participant in the lesson study group, it is anticipated that you will:

1. Participate in 2 one-hour interviews, one initial and one exit interview. Both of these interviews will be recorded with the use of audio equipment.

2. Record the conversations that occur during your weekly 1-2 hour lesson study planning sessions and permit the investigator to observe the first group meeting.

3. Let the principle investigator videotape both the presentations of the research lesson and the individual lessons.

4. Allow the principal investigator to copy written work (i.e. journals, final reports...).

You may feel discomfort from either being audio or video taped, and/or from reflecting on teaching. However, this should not be any different than what has occurred in other secondary education classes. The possible benefits for you, as a participant, are to learn about lesson study and how it could be used to promote reflection on teaching situations.

Participation in this research is voluntary. You have the right to refuse to participate and the right to withdraw without any penalty to your grade. Strict confidentiality will be maintained. To provide for this all audio taped interviews and conversations will be locked in an office and pseudonyms will be assigned to any written or transcribed data. This will ensure that no identifying information will be disclosed in reports of the research. Dr. Peterson will only have access to data that is part of the course. All data will be saved for three years, and then destroyed.

If you have any questions concerning research, you can reach Julie Stafford in TMCB XXX, by phone XXX-XXX-XXXX or by email XXXXX@XXX.XXX.

If you have any questions regarding your rights as a participant in this or any research project you may contact Dr. XXXXX XXXXX, Chair of the Institutional Review Board, Brigham Young University, Provo, UT 84602; phone XXX-XXX-XXXX.

I have read, understood and received a copy of the above consent, and desire of my own free will and volition to participate in this study and accept the benefits and risks relating to the study.

(Signed) ______________________________ Date __________________________
CONSENT TO BE A RESEARCH SUBJECT

Public School Teacher

The purpose of this research is to study prospective mathematics teachers as they engage in lesson study. The study will focus on prospective teachers’ beliefs about mathematics and teaching, as well as the topics discussed during collaboration. Teachers at the university and secondary level have recommended you for participation in this study. The study will begin in late September and commence in December 2001. Julie Stafford, a graduate student in the BYU Mathematics Education Department, is conducting the research.

As a participant in the lesson study group, it is anticipated that you will:

a. Participate in a one-hour meeting with four preservice math teachers. This meeting will be recorded with the use of audio equipment and the principal investigator will observe. This meeting will not be held during class time.

b. Allow the preservice teachers to present their research lesson during the first week of December. Let the principle investigator videotape the preservice teachers’ presentation of the lesson. The lesson will only last one class period.

The only anticipated discomfort may come from being audio taped, and/or from reflecting on teaching. The possible benefits for you, as a participant, are to learn about lesson study and how it could be used to promote reflection on teaching situations.

Participation in this research is voluntary. You have the right to refuse to participate and the right to withdraw at any time. Strict confidentiality will be maintained and pseudonyms will be assigned to any written or transcribed data. This will ensure that no identifying information will be disclosed in reports of the research. All data will be saved for three years, and then destroyed.

If you have any questions concerning research, you can reach Julie Stafford by phone at XXX-XXX-XXXX or by email at XXXXXXX@XXX.XXX.

If you have any questions regarding your rights as a participant in this or any research project you may contact Dr. XXXXXXXX XXXXXX, Chair of the Institutional Review Board, Brigham Young University, Provo, UT 84602; phone XXX-XXX-XXXX.

I have read, understood and received a copy of the above consent, and desire of my own free will and volition to participate in this study and accept the benefits and risks relating to the study.

(Signed) ______________________________  Date __________________________
CONSENT TO BE A RESEARCH SUBJECT

Public school student

The purpose of this research is to study prospective math teachers as they engage in a collaborative lesson planning activity called lesson study. You have been selected based upon your enrollment in your math class. Julie Stafford, a graduate student in the BYU Mathematics Education Department, is conducting the research.

As a participant you will:

1. Let a preservice math teacher present a math lesson.
2. Allow the preservice teacher to be videotaped in your math class.
3. Be observed by prospective mathematics teachers and allow observers to hand copy your work.

It is expected that the discomfort from participating in this study will be minimal. Any discomfort that you may feel may be a result of having outside observers in your classroom. This experience will be similar to having a substitute for one day or having your teacher be observed by another teacher. Discomfort may also come from having the video camera in the classroom. However, the camera is there to videotape the teacher and not you or your classmates. The benefits from this research will allow preservice math teachers to learn about lesson study and how it could improve the way they teach mathematics.

Participation in this research is voluntary. You have the right to refuse to participate and the right to withdraw without any penalty to your grade. Strict confidentiality will be maintained. To provide for this the videotape will be locked in an office and no information about individuals in the class will be given out. Videotapes will be saved for three years, after which they will be destroyed.

If you (or your parent/guardian) have any questions concerning the above-mentioned research now or at any time during the study, you can reach Julie Stafford by phone at XXX-XXX-XXXX or by email at XXXXX@XXX.XXX.

If you (or your parent/guardian) have any questions regarding your rights as a participant in this or any research project you may contact Dr. XXXXX XXXXX, Chair of the Institutional Review Board, Brigham Young University, Provo, UT 84602; phone XXX-XXX-XXXX.

I have read, understood and received a copy of the above consent, and desire of my own free will and volition to participate in this study and accept the benefits and risks relating to the study.

(Student signature) ______________________________ Date ______________________

(Parent/Guardian signature) _________________________ Date ____________________