Parametric Models of Maize Stalk Morphology

Michael Alan Ottesen

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Parametric Models of Maize Stalk Morphology

Michael Alan Ottesen

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Science

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ABSTRACT

Parametric Models of Maize Stalk Morphology

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As the most produced grain crop world-wide, 5% of corn is lost due to stalk lodging (above-ground structural failure of the stalk near the roots). Current modeling methods lack the ability to manipulate the stalk architecture. In contrast, parameterized models enable advanced analyses such as sensitivity and optimization studies. This thesis advances previous work on a parameterized cross-sectional model of maize stalk morphology and investigates the validity of a parameterized three-dimensional model. The parameterized cross-sectional model is based upon previous work that approximated the cross-section of maize stalks using an ellipse plus principal components. Validation of the cross-sectional model was done by evaluating the structural response in four loading cases: axial tension/compression, bending, transverse compression, and torsion. 2D prismatic extrusions of specimen specific cross-sections were tested under the load conditions and compared against 2D prismatic models of the parametrized cross-sections. Analysis of the 2D prismatic model consisted of a parameter sensitivity analysis to determine influential morphological features, and a load bearing analysis to quantify the proportion of load borne by each material tissue. Validation of the parameterized 3D model was completed by comparing the structural response of the parameterized 3D model against empirical test data. A comparison against CT-based finite element models was also done to quantify the level of predictive discrepancy caused by geometric parameterization. The elliptical 2D prismatic model responded with less than 5% error for axial tension/compression, bending, and transverse compression, suggesting that the ellipse model is sufficient for analyses and 3D parameterization. The 2D prismatic model maintained an error less than 10% under a torsion load. The parameter sensitivity analysis revealed that ellipse parameters are significantly more influential to stalk strength than material or finer geometric details. In the load bearing analysis, the rind bore a median of over 90% of the load in axial tension/compression, bending, and torsion, but less than 10% of the load in transverse compression. The parameterized 3D model validates yielding correlations with empirical test data resulting in $R^2$ values of 0.82 in flexural stiffness, and for critical buckling a value of 0.71. Comparison with CT-based models resulted in very strong correlations with $R^2$ values of 0.81 in flexural stiffness, and for critical buckling a value of 0.87. The parameterized 3D model validates and can be used in future studies.

Keywords: maize, corn, stalk, biomechanics, bioengineering, modeling, parameterization
ACKNOWLEDGEMENTS

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Next, I would like to acknowledge the students and other faculty who have helped me to complete this thesis work. Ryan Larson, the student who preceded me in the Crop Biomechanics Lab, has taken extra time to train and guide me as I embarked on this project. I am grateful for his time training me. Joseph Carter, an undergraduate researcher, gathered much of the data included in the results of this thesis. I express my gratitude to him for helping gather results data. Christopher Stubbs of the University of Idaho met with me several times to share his expertise in finite element methods. I am grateful to him for taking the extra time to assist me in that process.

Finally, I express my deep gratitude to my wife Tanisha. Her unwavering support throughout my schooling and the preparation of this thesis has carried me to this point.
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CHAPTER 1. INTRODUCTION

1.1 Background

Maize is a multi-billion-dollar global industry that contributes to thousands of common goods throughout the world. The United States alone produced over $84 billion of corn in 2021, making it the most produced crop (USDA, 2018). The stalks of these plants occasionally collapse in a failure process called stalk lodging (see Figure 1-1). Lodging kills the plant and prevents the grain from being harvested. This results in a loss of roughly 5% of crops each year (Duvick, 2005). By comparison to other multi-billion-dollar industries, this failure rate is extreme. Other industries such as automotive or electronics aim for six-sigma production which corresponds to a failure rate of less than 4 out of one million occurrences. The 5% failure rate in maize stems is more than 10,000 times more common than the six-sigma standard held in other industries. If maize stalks were engineered to resist lodging, production rates would be significantly increased, even if production area is held constant.

Stalk lodging has been studied for several decades. Some of these studies focus on environmental effects and confounding variables associated with crops that are subject to lodging (Thompson, 1972). Many looked at plant stalks as structural members, and even began analyzing the correlation between lodging and stalk morphology (Zuber & Grogan, 1961; Cloninger, 1970; Zuber & Kang, 1978; Schullgasser & Witztum, 1992). It wasn’t until 2014 that maize stalk
research began to directly correlate geometric and material properties to stalk lodging (Robertson et al., 2014).

The failures that occur in corn stalks can be modeled using engineering techniques and analysis. Maize stalks are vertical cantilever beams with a complex structural architecture. Failure of the stalk is generally caused by wind and results in localized buckling failure in the stem. Previous models have shown that the morphology of the plant stem is more influential than material properties with regards to resisting these failures (Von Forell et al., 2015; Stubbs et al., 2022). Previous models include test data from stalks in 3-point bending and CT-based finite element models of stalk segments. Methods such as these are observational with regards to stalk
shape and lack the ability to optimize stalk structure. A different approach is needed to fully understand the influence of maize stalk features.

The goal of this research was to develop a morphological model of the maize stalk that is sufficiently flexible to enable optimization analysis. Such a model consists of several input parameters that can be altered individually, each of which independently controls a separate physical feature of the stalk. This controllability will help to rapidly determine the physical features of the stalk that are most influential to resist lodging. Stronger stalks will also support the development of newer higher yielding varieties which are not currently possible.

### 1.2 Modeling Background

To understand the modeling background, one must first understand maize stem physiology. Maize stalks are made up of two material tissues: rind and pith. The rind is a high-density material that encompasses the outer layer of the stalk cross-section, and the pith is a low-density material that encompasses the outer layer of the stalk cross-section, and the pith is a low-density

![Figure 1-2: CT scan of maize stalks. A) Longitudinal cross-section; B) transverse cross-section; C) the same transverse cross-section, showing segmentation boundaries between the rind and pith.](image-url)
material that fills the center part of the stalk. Longitudinally, maize stalks follow a repeating pattern of nodes followed by long internodal regions (see Figure 1-2). The nodal region contains a thicker rind than in the internodal region. The internodal region commonly exhibits a groove in the cross-section where the ear of the corn grows.

The first studies to model stalk lodging outside of the corn field tested bare stalks in 3-point bending (Robertson et al., 2016, 2017). These studies provided valuable insight to the most common failure modes of maize stalks, as well as the location of failure. Several failure modes and locations were identified from these tests. The overwhelming majority of failure modes involved Brazier buckling and occurred in the 4 cm region just above the node. This result was very informative. Since the failure location is known, it is not necessary to model the entire stalk. Instead, only a small segment of the stalk needs to be modeled.

The first computational models of maize stalks used CT scans of maize stalk segments to define their geometry. These models provided an extremely high level of geometric fidelity (Von Forell et al., 2015; Stubbs et al., 2022). These scans modeled approximately 100 mm of the stalk, spanning from the center of one internode to the center of the adjacent internode. The process used to create these models is shown in Figure 1-3. These smaller models reduce the computation time while still capturing the essential mechanics of the failure region (Robertson et al., 2016, 2017). The structural response of these CT-based computation models was validated against the 3-point bending test results (Stubbs et al., 2022). These computational models provide accurate results while giving researchers full access to the precise geometry of the failure region. The limitation with these CT-based models lies in the complexity of model generation and reliance on CT scanning. Each computational model undergoes a complicated and time-consuming process (see Figure 1-3). Because the model geometry is derived from CT scan data
and involves a point cloud of thousands of data points, the geometry of each model is static, and meaningful manipulation of the geometry is not possible with this modeling approach.

Figure 1-3: The process used in creating computational models of a maize stem. CT scans were processed to create finite element models using the steps shown. (C. J. Stubbs et al., 2022)

Parameterized Stalk Model

<table>
<thead>
<tr>
<th>2D Prismatic Model</th>
<th>Parameterized 3D Model</th>
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<tr>
<td>1. Creation</td>
<td>1. Creation</td>
</tr>
<tr>
<td>2. Validation</td>
<td>2. Validation</td>
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<tr>
<td>3. Analysis</td>
<td>3. Analysis</td>
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</table>

Figure 1-4: General overview of parameterized maize stalk model.
Laboratory 3-point bending tests and CT-based computational models have been useful in letting us know that this system can be modeled effectively. However, CT-based finite-element models are limited in geometric exploration because they rely on the existing corn stalks at our disposal. The development of a parameterized maize stalk model began in 2019 to fill these gaps in modeling limitation (Larson, 2020). The parameterized maize stalk model was divided into two stages: a two-dimensional cross-sectional model and a three-dimensional model. Each stage involved model creation, validation, and analysis (see Figure 1-4). The creation of the 2D prismatic model was previously completed (Larson, 2020). Additionally, a single loading case (transverse compression) was used to perform model validation and a sensitivity analysis.

![Diagram of loading cases](image)

**Figure 1-5: Loading cases applied to the 2D prismatic model**

While a preliminary validation study was completed, it used a loading case that differs substantially from the actual loading experienced by the maize stalk. In the field, the maize stalk experiences a complex stress state that is the combination of axial tension/compression, bending,
and torsion. A more thorough validation and analysis would therefore involve these additional loading cases. The simplest approach would be to extrude Larson’s parameterized cross-sectional model to form prismatic models of the maize stalk under these loading cases, as shown in Figure 1-5.

### 1.3 Purpose and Scope

The primary purpose of this thesis was to perform validation and analysis of the 2D and 3D parameterized maize stalk models. As shown in Figure 1-6, my work included further validation and analysis of the 2D prismatic model, as well as validation of the parameterized 3D model. The parameterized 3D model creation was a collaborative effort between several individuals and is not detailed in this thesis. Analysis of the parameterized 3D model (sensitivity and optimization studies) is to be done in future studies that will rely upon the validation work described in this study.

<table>
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<tr>
<th>Parameterized Stalk Model</th>
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<tr>
<td><strong>2D Prismatic Model</strong></td>
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<tr>
<td>1. Creation</td>
</tr>
<tr>
<td>2. Validation</td>
</tr>
<tr>
<td>3. Analysis</td>
</tr>
</tbody>
</table>

Figure 1-6: Overview of parameterized model with author contributions highlighted.

To complete the validation of the 2D prismatic model, the model was subjected to the additional load cases that were not previously evaluated: bending, axial tension/compression, and torsion. Validation was performed by varying the level of geometric fidelity. For each loading case the geometric fidelity was systematically varied, and structural stiffness predictions were
compared to the model prediction when the actual cross-sectional shape was used. The analysis phase of the 2D prismatic model was divided into two parts: a sensitivity analysis and a load bearing analysis. The sensitivity analysis determined the influence of each model parameter on the predicted structural stiffness. This approach provides valuable insight to the important cross-section features that resist lodging. The load bearing analysis quantified the load borne by each material tissue: pith and rind. The results from this study help provide insight on the roles that each tissue plays in resisting deformation (a precursor to failure). Both the sensitivity and load bearing analysis were evaluated for three new loading cases: axial tension/compression, bending, torsion. Although it was completed previously, the transverse compression loading case was also included to provide a complete understanding of the maize stalk.

Validation of the parameterized 3D model involved comparisons between empirical test results and predicted structural responses of corresponding models. Test results and CT scans were performed previously (Robertson et al., 2016, 2017). Specimen-specific 3D parameterized models were created from this data and compared to the actual response of corresponding specimens. Two structural responses were used: flexural stiffness, and critical buckling load (i.e., the maximum moment applied to the specimen before failure).

Recall from the introduction that CT-based specimen-specific models have previously been created and validated (Stubbs et al., 2022). Parameterized 3D models were created for each of these same specimens and the results were compared to CT-based models. This is a valuable comparison because CT-based models and parameterized models differed only in their geometry (all material properties were identical between each corresponding pair). This provides further information and insight into the influence of the parameterization process.
This thesis thus involved critical work in support of the parameterization process. Future studies will be able to perform advanced sensitivity and optimization studies with the assurance that the parameterization process provides a valid, highly accurate approximation of the structural performance of the maize stalk.
CHAPTER 2. METHODS FOR PRISMATIC MODEL SIMULATIONS

This chapter focuses primarily on the methods associated with the 2D prismatic model. Section 2.1 describes the existing model used in this work. Section 2.2 covers the validation process of the cross-sectional mode, while sections 2.3-2.4 explain the 2D analyses performed. Finally, section 2.5 defines the sampling used in the studies.

2.1 Existing Models Utilized

A previous study described the process of parameterizing the maize stalk cross-section (Larson, 2020). In that study, the maize cross-section was first approximated using an ellipse.

Figure 2-1: Top left: Illustration of an actual cross-section. Top right: Cross-section and corresponding ellipse approximation with major and minor diameters (interior boundaries excluded for clarity). \( R \) is the distance from the geometric center to the real stalk shape, and \( e \) is the distance from the center to the ellipse approximated shape. Bottom: Enlarged views depicting geometric convergence with added principal components (PCs).
Next, the non-elliptical geometric features were captured using principal component analysis (PCA). By adding additional principal components to the ellipse, greater and greater geometric fidelity was possible. Recall that the cross-section of a stalk exposes the ear groove which is a notch in the ellipse cross-section that allows space for the ear of the corn to grow (see Figure 2-1). Using a pure ellipse approximation for the cross-section ignores this groove feature entirely. By adding principal components, the cross-sectional model was able to capture the ear groove as well as other minor geometric details as shown in Figure 2-1.

In (Larson, 2020), the loading case investigated was transverse compression (see Figure 1-5). The transverse compression loading case was chosen because Brazier buckling is strongly influenced by the degree of transverse ovalization (Schulgasser & Witztum, 1992). The plane-stress assumption was used to model relatively short prismatic extrusions of this cross-section (see Figure 2-2). The accuracy of the parameterized cross-sectional model was assessed by creating a reference model which was a prismatic extrusion of an actual maize cross-section (from CT scan). Next, each cross-section was approximated with varying levels of geometric fidelity, each of which corresponded to a separate model with identical material properties as the reference model. The influence of geometric approximation on the structural characteristics of

![Specimen Cross-Section Slice](image1.png) ![2D Prismatic Model](image2.png)

Figure 2-2: Left: real specimen cross-section slice. Right: 2D prismatic model extrusion.
the cross-section was assessed by comparing the predictions of each parameterized approximation model with those of the reference model. Finally, a sensitivity analysis was performed to determine the influence of each parameter of the cross-sectional model on transverse stiffness. All transverse compression simulations were performed via finite element analysis.

In summary, Larson (2020) developed a method for parameterizing the maize cross-section and then assessed the relationship between geometric fidelity and transverse stiffness. The main shortcoming of this approach is that it relied upon a single loading condition. Additional load cases that affect maize stalks include axial tension/compression, bending, and torsion. Validation of the parameterized cross-sectional model is needed for each of these loading conditions to provide further insight to geometric accuracy and parameter importance.

2.2 Validation of the Parameterized 2D Models

The purpose of this part of the research was to more fully determine how the geometric fidelity of the maize cross-section influences the accuracy of the corresponding structural response. Three additional loading cases were investigated: axial, bending, and torsion. Using essentially the same comparison method as done previously (Larson, 2020), each approximate model was compared to the corresponding reference model to assess the degree of discrepancy introduced by the geometry (see Figure 2-3). The following sections describe the models and methods used to perform this validation study.
Figure 2-3: Diagram showing models used in 2D validation. Left: CT scan of stalk cross-section. Middle: Edge detection shape derived from CT scan. Right: Ellipse approximation of the edge detected shape.

2.2.1 Loading Cases and Models

All four loading cases utilized a geometry which was a prismatic extrusion of the selected cross-section. In other words, these models had no axial variation in their cross-section. These models are therefore referred to throughout the thesis as “prismatic models”.

Table 2-1: Structural stiffness models used for each loading case. $K$ is the structural stiffness for a given loading condition, $E$ is the elastic modulus, $A$ is the cross-sectional area, $I$ is the area moment of inertia of the cross-section, $G$ is the shear modulus, $J$ is polar moment of inertia, and $p$ and $r$ represent the pith and rind respectively.

<table>
<thead>
<tr>
<th>Loading Case</th>
<th>Structural Stiffness Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Tension/ Compression</td>
<td>$K = \frac{E_p A_p + E_r A_r}{L}$</td>
</tr>
<tr>
<td>Bending</td>
<td>$K = \frac{48(E_p I_p + E_r I_r)}{L^3}$</td>
</tr>
<tr>
<td>Torsion</td>
<td>$K = \frac{G_p J_p + G_r J_r}{L}$</td>
</tr>
<tr>
<td>Transverse Compression</td>
<td>Finite element model</td>
</tr>
</tbody>
</table>
The structural response under axial, bending, and torsional loading were based upon analytic equations. Analytic loading cases required the numeric calculation of the relevant geometric quantities such as cross-sectional area (axial), area moment of inertia (bending), and polar moment of inertia (torsion). The structural stiffness of each prismatic model was calculated using the analytic equations listed in Table 2-1.

Structural response was calculated for an approximated shape beginning with a pure ellipse, then sequentially including principal components. Each of these models’ response was compared to the response of a model created directly from an actual maize cross-section. Because this study focused on relative differences between the actual and approximated geometries, results are reported as error in structural response. As such, calculated stiffness is measured as a ratio of stiffnesses where the length of the prismatic models canceled out and thus had no influence on the corresponding results.

2.2.2 Material Properties

Each model consisted of two tissue types: rind and pith. Maize tissues are well-approximated as transversely isotropic (Stubbs et al., 2018, 2019). Transversely isotropic materials require five independent material properties (Cook et al., 2008). In general, an application of this material model to the maize stalk would require five independent material properties for each material (pith and rind). However, because each loading case used in this study each activated only a single stress state, each structural model used in this study was dependent upon just two material properties (one for each tissue type). This can be seen in the equations provided in Table 2-1.
Table 2-2: Material properties used for each loading case in the structural models.

<table>
<thead>
<tr>
<th>Loading Case</th>
<th>Material Properties</th>
<th>Property Sources</th>
</tr>
</thead>
</table>
| Axial Tension/Compression  | $E_{rind, k} = 11 \pm 2$ GPa
$E_{pith, k} = 0.33 \pm 0.06$ GPa | Al-Zube et al., 2017
Al-Zube et al., 2018 |
| Bending                    |                                           |                                                       |
| Torsion                    | $G_{rind, k} = 8 \pm 2$ GPa
$G_{pith, k} = 0.25 \pm 0.06$ GPa | No measurements currently available. Estimated from wood literature (Green et al., 1999) |
| Transverse Compression     | $E_{rind, k} = 8.07 \pm 3.3$ GPa
$E_{pith, k} = 0.259 \pm 0.1$ GPa | Stubbs et al., 2019 |

The material property distribution used in this study are listed in Table 2-2. Axial loads used the longitudinal modulus values measured for maize stems (Al-Zube et al., 2018; Al-Zube et al., 2017). Shear moduli for maize stems have not been measured, thus these values were approximated from the wood literature (Green et al., 1999). Material properties used in transverse compression models were found through experimental measurements (Stubbs et al., 2019). All material properties used in this study were selected from the material properties listed in Table 2-2.

2.3 Sensitivity Analysis

To quantify the influence of each model parameter, a series of local sensitivity analyses were performed. A normalized sensitivity approach was used since this approach allows comparisons across input/output pairs of different units (Robertson et al., 2016). The local
sensitivity analysis was performed by changing one parameter at a time by 10% and then computing the normalized sensitivity as finite difference numerical derivative, as shown here:

\[ S = \frac{(y_{\text{new}} - y_{\text{ref}})/y_{\text{ref}}}{(x_{\text{new}} - x_{\text{ref}})/x_{\text{ref}}} \]  

(2.1)

In this equation, \( x_{\text{ref}} \) and \( y_{\text{ref}} \) represent the input and response of the reference case while \( x_{\text{new}} \) and \( y_{\text{new}} \) represent the input and response values from the modified case. The sensitivity can be interpreted as the percent change in output divided by the percent change in input. A total of 10 parameters were evaluated for each load case: major diameter, minor diameter, rind thickness, modulus of elasticity of both the pith and rind, and the first five principal components.

### 2.4 Load Bearing Analysis

A load bearing analysis was performed to gain additional insight regarding the role of the rind and pith tissues in resisting deformation under each of the four loading cases. This was done by determining the proportion of load borne by the rind and pith. The derivation of the load borne by each tissue was similar for all three analytic loading conditions. Thus, in the interest of brevity, the derivation for axial is described, and the remaining computational load cases follow the same process using their respective stiffness relationships.

First, we begin with structural stiffness for a two-material structure under axial tension/compression where \( K \) is structural stiffness, \( E \) is the transverse elastic modulus of the cross-section, \( A \) is the surface area, and \( L \) is the length of the prismatic extrusion.

\[ K_{\text{axial}} = \frac{E_p A_p + E_r A_r}{L} \]  

(2.2)

We can distinguish the tissue load contributions by evaluating the force supported by each tissue type. The force is defined by the stiffness multiplied by the displacement of the
material as shown in equations 2.3 and 2.4 where F is the force output of the sample, \( \delta \) is the displacement, and subscripts \((r \text{ and } p)\) represent rind and pith.

\[
F_{Pitch} = \left( \frac{E_p A_p}{L} \right) \delta 
\]

(2.3)

\[
F_{Rind} = \left( \frac{E_r A_r}{L} \right) \delta 
\]

(2.4)

Next, the proportion of load borne by each tissue is defined as the individual force of a tissue divided by the total force response of both materials combined as in Equations 2.5 and 2.6.

\[
Load_{Rind} = \frac{F_r}{F_p + F_r} 
\]

(2.5)

\[
Load_{Pitch} = \frac{F_p}{F_p + F_r} 
\]

(2.6)

Finally, this proportion of load borne can be simplified to the known values by substituting equations 2.3 and 2.4 into equations 2.5 and 2.6.

\[
Load_r = \frac{E_r A_r}{E_p A_p + E_r A_r} 
\]

(2.7)

\[
Load_p = \frac{E_p A_p}{E_p A_p + E_r A_r} 
\]

(2.8)

The derivation of load borne for the bending and torsional loads follow the same pattern as illustrated for the axial load. Table 2-3 shows the load bearing derivation for all load cases.
Table 2-3: Load bearing equations for each loading case.

<table>
<thead>
<tr>
<th></th>
<th>Axial Tension/Compression</th>
<th>Bending</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stiffness</strong></td>
<td>( K = \frac{E_p A_p + E_r A_r}{L} )</td>
<td>( K = \frac{48(E_p I_p + E_r I_r)}{L^3} )</td>
<td>( K = \frac{G_p J_p + G_r J_r}{L} )</td>
</tr>
<tr>
<td><strong>Load Contribution by Tissue</strong></td>
<td>( F_{\text{Pith}} = \left(\frac{E_p A_p}{L}\right) \delta_{\text{Axial}} )</td>
<td>( F_{\text{Pith}} = \left(\frac{E_p I_p}{L^3}\right) \delta_{\text{Trans.}} )</td>
<td>( F_{\text{Pith}} = \left(\frac{G_p J_p}{L}\right) \theta )</td>
</tr>
<tr>
<td></td>
<td>( F_{\text{Rind}} = \left(\frac{E_r A_r}{L}\right) \delta_{\text{Axial}} )</td>
<td>( F_{\text{Rind}} = \left(\frac{E_r I_r}{L^3}\right) \delta_{\text{Trans.}} )</td>
<td>( F_{\text{Rind}} = \left(\frac{G_r J_r}{L}\right) \theta )</td>
</tr>
<tr>
<td><strong>Proportion of Load Borne</strong></td>
<td>( \text{Load}_{\text{Rind}} = \frac{F_r}{F_p + F_r} )</td>
<td>( \text{Load}_{\text{Pith}} = \frac{F_p}{F_p + F_r} )</td>
<td></td>
</tr>
<tr>
<td><strong>Simplified Form</strong></td>
<td>( \text{Load}_r = \frac{E_r A_r}{E_p A_p + E_r A_r} )</td>
<td>( \text{Load}_r = \frac{E_r I_r}{E_p I_p + E_r I_r} )</td>
<td>( \text{Load}_r = \frac{G_r I_r}{G_p J_p + G_r J_r} )</td>
</tr>
<tr>
<td></td>
<td>( \text{Load}_p = \frac{E_p A_p}{E_p A_p + E_r A_r} )</td>
<td>( \text{Load}_p = \frac{E_r I_r}{E_p I_p + E_r I_r} )</td>
<td>( \text{Load}_p = \frac{G_p J_p}{G_p J_p + G_r J_r} )</td>
</tr>
</tbody>
</table>

The transverse compression load bearing contributions were found using the finite element rather than computational equations. The creation of the cross-section finite element models was done previously (Larson, 2020). This analysis used the cross-section slices from Group 2 (sampling groups are defined in section 2.5, see Figure 2-4) due to the computational complexity of running finite element models. The load contribution of the rind was calculated by removing the pith from the model and comparing the response to a model with both the rind and pith included. The load contribution of the pith was therefore calculated by subtraction of these
known values. With this approach, the boundary conditions applied to the edge of the rind remained consistent. This method may produce imprecise results as the removal of the pith may influence the deformation mode of the cross-section.

2.5 Cross-section Sampling

Two sampling groups were used in this study. First, CT cross-sections used in this study were drawn from thirteen sample points ranging from 40 mm above the node to 40 mm below the node. This set of cross-sections is referred to as Group 1 (see Figure 2-4). As seen in this figure, more points were sampled near the node to provide higher fidelity where the maize stalk fails most frequently (Robertson et al., 2015). Group 1 included 12,740 unique cross-sectional images: thirteen sample points for each of the 980 stalks in the data set. Thus, Group 1 represents the full population of stalk samples. Group 1 was used for the load bearing analysis that involved axial tension/compression, bending, and torsion loading cases. Group 2 consisted of 70 randomly selected stalks from the total of 980. The cross-sections were sampled from 5 of the 13 cross-sections of Group 1. This resulted in a total of 350 stalk cross-sections. Group 2 was used for the validation of the 2D prismatic model, sensitivity analysis, and the transverse compression load case in the load bearing analysis. Figure 2-4 illustrates the scan region, the associated sampling points, and provides representative cross-sectional images.
Figure 2-4: Illustration of the CT slice locations and group sampling used in this study.
CHAPTER 3. RESULTS FROM 2D PRISMATIC MODEL SIMULATIONS

This chapter covers the results from the 2D prismatic model validation and analyses. First, section 3.1 reviews the results from the cross-sectional parameterization validation. Then sections 3.2 and 3.3 report results from the sensitivity and load bearing analyses respectively.

3.1 Cross-section Structural Response Results

Results suggest that high geometric accuracy can be obtained with relatively few geometric parameters. The relationships between geometry and mechanical response to loading cases was evaluated using just the first several principal components. Overall, the response of approximate models was highly accurate, even when using approximated models. The charts of Figure 3-1 depict the error distributions for various geometric approximations for each of the four loading cases. In these charts, relative error was defined as the percent difference in structural response between the approximate model and the corresponding model based on the original cross-section.

As seen in Figure 3-1, as additional principal components were added to the ellipse, the mechanical response quickly approached the response obtained when using the original cross-section. The ellipse alone provided better than 95% accuracy (errors less than 5%) for the axial, bending, and transverse compression cases. For the torsional loading, the ellipse alone was 90% accurate. In each case, the addition of principal components progressively reduced error, with
error levels within 1% at 5 principal components for axial, bending, and transverse loads, and within 1% after 6 principal components for torsional loads.

Figure 3-1: Convergence patterns showing the distributions of relative errors obtained with models consisting of various numbers of geometric components. Error is defined as the percentage difference between the approximate model and the original maize cross-section. Each box used Group 2 sampling (see Figure 2-4) containing 350 cross-sections. On the x-axis, $E$ means “ellipse” and $PC$ means “principal component”.

3.2 Loading Case Sensitivity Results

The influence of each model parameter on the four types of mechanical response was quantified by computing normalized sensitivities. This approach non-dimensionalized all results, thus facilitating comparisons between parameters as well as across loading cases. The output of
interest was the force/deformation stiffness of each loading case. Sensitivity results are shown in terms of absolute values with negative sensitivities indicated by a (-) symbol. These results are shown in Figure 3-2. Note that each panel is split into three parameter groups: ellipse parameters, material properties, and principal components.

The most consistent finding was that principal components had minimal influence on the structural responses. The greatest influence of principal components was for the torsion loading. This is because the polar moment of inertia is very sensitive to minor changes in the geometry of the outer rind tissue. However, even for the torsional case, most sensitivity values were below 5%. This indicates that a 10% increase in the principal component scaling factor would result in only a 0.5% change in the response. In contrast, a 10% increase in the major diameter ($a$) would increase the torsional stiffness by approximately 18%.

Across all loading cases, the most influential parameters belonged to either the ellipse or mechanical tissue properties groups. For axial, bending, and torsional loading, the influence of the rind modulus was many times more influential than the modulus of the pith tissue. However, in transverse stiffness, the pith tissue was more influential than the modulus of the rind. In fact, one important role of the pith is to allow the maize stalk to resist cross-sectional ovalization thereby increasing the critical buckling load (Karam & Gibson, 1994).

As expected, the influence of geometric parameters ($a$, $b$, and $t$) varies according to the different loading cases. For example, bending stiffness is most sensitive to the minor axis ($b$), while torsion is highly sensitive to both radius values ($a$ and $b$), etc. The rind thickness was found to have the highest influence on transverse stiffness, with a mean sensitivity of 97%. This is approximately the same as a 1:1 influence. The next most influential parameters were major diameter and the Young’s Modulus of the pith tissue. The Young’s Modulus of the rind and the
minor diameter had relatively low sensitivity values (0.3 and -0.08, respectively). Transverse compression exhibited notably broader distributions than the other loading case. This was found to be caused by strong nonlinear relationships between the morphology of the cross-section and the resulting sensitivities.

![Normalized sensitivity results for each loading case.](image)

**Figure 3-2:** Normalized sensitivity results for each loading case. Horizontal lines within each box represent 25th, 50th, and 75th percentiles. Whiskers tips indicate 95% coverage for each distribution.

### 3.3 Load Bearing Analysis Results

The load bearing analysis on maize stem cross-sections reveals the proportional load held by each maize stem material. Table 3-1 displays the results from the analysis. In the three analytical loading cases: axial tension/compression, bending, and torsion, the rind bore the most, contributing median values of 91.81%, 96.41%, and 96.17% of the load respectively. Contrarily, the pith contributed much less of the load bearing for these three cases by contributing just
8.19%, 3.59%, and 3.83% respectively. Contrarily, a transverse compression load is borne most by the pith. Results from the finite element models revealed that the rind only carried a median of 7.86% while the pith contributed to 92.14% of the load (see Table 3-1).

Table 3-1: Load bearing analysis results.

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Rind (median)</th>
<th>Pith (median)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Tension/Compression</td>
<td>91.81%</td>
<td>8.19%</td>
</tr>
<tr>
<td>Bending</td>
<td>96.41%</td>
<td>3.59%</td>
</tr>
<tr>
<td>Transverse Compression</td>
<td>7.86%</td>
<td>92.14%</td>
</tr>
<tr>
<td>Torsion</td>
<td>96.17%</td>
<td>3.83%</td>
</tr>
</tbody>
</table>

Notably, there was little variance in axial tension/compression, bending, and torsion load cases (see Figure 3-3), suggesting that the rind bears most of the load no matter the cross-section shape. Results had much larger variance for the transverse compression load indicating that variance in the cross-section morphological features influence how much load is borne by each material tissue.

Next, the influence of the cross-section features was evaluated by comparing the load bearing contributions to the amount of rind in the cross-section (see Figure 3-4). The ratio between rind thickness and minor diameter was used to quantify the amount of rind in the cross-section. More most of the corn stalk, this ratio is between 0.08 and 0.2, but in rind-heavy regions near the node, the ratio is much larger. The load contribution did not change a significant amount.
for any cross-section in axial tension/compression, bending, or torsion. Contrarily, in transverse compression there was a strong correlation between the amount of rind present, and the loading contribution. This result helps explain the high variance found in transverse compression by indicating that the rind plays a very minor role in resisting transverse compression when the rind presence is small but plays a much larger role in resisting transverse compression when the rind presence is larger.

Figure 3-3: Percent load borne by both pith and rind for axial tension/compression, bending and torsion using ~12,000 cross-sections. Percent load borne by transverse compression also included using 350 cross-sections.
Figure 3-4: Load bearing contribution of the rind against the amount of rind in the cross-section.
CHAPTER 4. VALIDATION OF PARAMETERIZED 3D MODELS

This chapter contains both methods and results that correspond to the validation of the parameterized 3D models. Methods are contained in Sections 4.1-4.3. Results are contained in Section 4.4.

4.1 Existing Models and Data Utilized

As shown in Figure 4-1, all 3D models were created to represent relatively short segments of the entire stalk. This decision was motivated by three factors. First, failure typically occurs near a node (Robertson et al., 2015). Second, in three-point bending tests, failure occurred exclusively near the loaded node (Robertson et al., 2017). Third, this approach reduces the computational effort by minimizing the amount of stalk that is modeled. This approach has been previously validated (Stubbs et al., 2022)

4.1.1 Empirically Measured Test Data

In previous studies, a set of 1000 stalks representing five varying hybrids were tested under 3-point bending loads (Robertson et al., 2016, 2017). The tested segments were approximately one meter long. From the 3-point bending tests, flexural stiffness and critical buckling load results of each stalk were collected. In addition, CT-scanning was performed to obtain information on the internal and external features of each stalk. The CT scan data was used to parameterize the cross-sectional and 3D models of the maize stalk (Larson, 2020; Ottesen et al., 2022).
Figure 4-1: Diagram illustrating interaction between all models. Top: real stalks under 3-point bending load. Middle-left: X-ray CT scans of stalk segments taken from real stalks. Bottom-left: CT-based specimen-specific model derived directly from CT scans. Middle-right: parameterization process showing 2D ellipse model and parameterized 3D model derived from CT scans of stalk cross-sections. Bottom-right: parameterized specimen-specific model derived from parameterized 3D model.

4.1.2 CT-Based Specimen-Specific Finite Element Models

A previous study described the process and validation of specimen-specific models based directly on CT-scan data (Stubbs et al., 2022). Model creation consisted of converting CT scan data to point clouds, from point clouds to surfaces, from surfaces to solid bodies, and from solid bodies to finite element models (see Figure 1-3). These models were subject to flexural stiffness
tests and linear buckling analysis. As depicted in Figure 4-2, these models have very high geometric fidelity. Validation of these models was obtained by demonstrating that the structural response of these models correlates well with the empirical test data (Stubbs et al., 2022). However, these CT-based models have two major limitations. First, these models are very time consuming to create, requiring approximately 45 minutes to create a single model. Second, once generated, the CT-based models are essentially static: manipulation of the model geometry is not possible.

Figure 4-2: A CT-based specimen-specific finite element model.

4.1.3 Parameterized 3D Model

Results from the prismatic ellipse model (see section 3.1) strongly support the notion that multiple stress states are effectively captured by a simple ellipse. The parameterized 3D model simplifies the complex geometry of the maize stalk by assuming that each cross-section is an ellipse. This model simplifies the cross-section but incorporates important axial variation (see
Figure 4-3 for an illustration of this concept). A parameterized 3D model is shown in Figure 4-1. Recall that the prismatic model has no axial variation in the cross-section. The parameterized 3D model thus simplifies the transverse shape of the maize stalk but captures the axial variation in shape. Axial variation is important to capture because a failure analysis revealed that failure virtually always occurs just above the node (Robertson et al., 2015). This suggests a strong connection between axial geometry and stalk strength.

![Figure 4-3. Isometric view of elliptical parameter variation traced on a real stalk segment](image)

The parameterized 3D model is defined by three axial profile paths: one for the major diameter, one for the minor diameter, and one for the rind thickness. The essential features of each profile are captured using distinctive features of each profile which are referred to as landmarks. Each landmark is defined by two Cartesian coordinates, as shown in Figure 4-4. The path between each pair of landmarks is based on empirical patterns devised from a principal component analysis of the relevant data. Altogether, the shape of the parameterized 3D model is defined by 51 parameters (see Table 4-1), each of which can be independently adjusted/controlled.
Table 4-1: List of all geometric coefficients for the parameterized 3D model.

<table>
<thead>
<tr>
<th>L or E</th>
<th>Landmark vs. Empirical Eigenfunction</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, B, or T</td>
<td>Major Diameter vs. Minor Diameter vs. Rind Thickness</td>
</tr>
<tr>
<td># 1-6</td>
<td>Landmark numbers (eigenfunctions have two digits indicating the surrounding landmarks)</td>
</tr>
<tr>
<td>Z or Y</td>
<td>Z direction (along stalk) vs. Y direction (A, B, or T value) on profile paths</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Major Diameter (A)</th>
<th>Minor Diameter (B)</th>
<th>Rind Thickness (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Param. number</strong></td>
<td><strong>Parameter</strong></td>
<td><strong>Param. number</strong></td>
</tr>
<tr>
<td>1</td>
<td>LA1Z</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>LA1Y</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>LA2Z</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>LA2Y</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>LA3Z</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>LA3Y</td>
<td>23</td>
</tr>
<tr>
<td>7</td>
<td>LA4Z</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>LA4Y</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>LA5Z</td>
<td>26</td>
</tr>
<tr>
<td>10</td>
<td>LA5Y</td>
<td>27</td>
</tr>
<tr>
<td>11</td>
<td>LA6Z</td>
<td>28</td>
</tr>
<tr>
<td>12</td>
<td>LA6Y</td>
<td>29</td>
</tr>
<tr>
<td>13</td>
<td>EA12</td>
<td>30</td>
</tr>
<tr>
<td>14</td>
<td>EA23</td>
<td>31</td>
</tr>
<tr>
<td>15</td>
<td>EA34</td>
<td>32</td>
</tr>
<tr>
<td>16</td>
<td>EA45</td>
<td>33</td>
</tr>
<tr>
<td>17</td>
<td>EA56</td>
<td>34</td>
</tr>
</tbody>
</table>
Figure 4-4: Cross-section parameter variation plots with all 51 geometric parameters labelled. Black dots indicate landmark features, and gray lines indicate empirical eigenfunctions between landmarks. Interpretation of parameter numbers is in Table 4-1.

4.2 Finite Element Model

The finite element method was used to evaluate the structural response of the parameterized 3D model. The finite element models were generated using Abaqus/CAE 2020 by defining the appropriate assembly, material properties, mesh, boundary conditions, and analyses performed which are described in the following sections.
4.2.1 Assembly

The assembly of the parameterized finite element 3D models is made of 2 bodies (rind and pith) tied to each other. The circumferential outer surface geometry of the pith body exactly matches the inner surface of the rind body. These surfaces interact through a fixed constraint.

4.2.2 Material Properties

Pith and rind tissues are both characterized by fibers which run longitudinally through the stalk. Both materials were therefore modeled as transversely isotropic materials. This requires two elasticity moduli, two shear moduli, and one Poisson’s ratio for each tissue type (Stubbs et al., 2019). The most important material property in this system is the longitudinal modulus of the rind ($E_{ref}$), which was estimated using a method involving flexural test data combined with Euler-Bernoulli beam theory and CT-scan data (Al-Zube et al., 2018). All other material properties were estimated based on previously reported ratios between pairs of material properties. The modulus of elasticity in the transverse direction ($E_3$) was assumed to be two orders of magnitude smaller than in the longitudinal direction ($E_1 & E_2$). The transverse shear modulus ($G_{12}$) was three orders of magnitude smaller than the reference elastic modulus ($E_3$), and the remaining axial shear moduli ($G_{13} & G_{23}$) were one order of magnitude smaller than in the longitudinal direction. Finally, all moduli for the pith body were all 2 orders of magnitude smaller than the moduli for the rind body. Poisson’s ratio ($\nu$) was assumed 0.3 for both tissues (see Table 4-2) (Al-Zube et al., 2017; Al-Zube et al., 2018; Stubbs et al., 2019).
Table 4-2: Material property relationships used in parameterized finite element 3D models.

<table>
<thead>
<tr>
<th>Material Property</th>
<th>E1 &amp; E2</th>
<th>E3</th>
<th>G12</th>
<th>G13 &amp; G23</th>
<th>ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rind</td>
<td>$E_{ref} \times 10^{-2}$</td>
<td>$E_{ref}$</td>
<td>$E_{ref} \times 10^{-3}$</td>
<td>$E_{ref} \times 10^{-1}$</td>
<td>0.3</td>
</tr>
<tr>
<td>Pith</td>
<td>$E_{ref} \times 10^{-4}$</td>
<td>$E_{ref} \times 10^{-2}$</td>
<td>$E_{ref} \times 10^{-5}$</td>
<td>$E_{ref} \times 10^{-3}$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

4.2.3 Mesh

The mesh of each parameterized finite element model was broken into three sections: top end, node region, and bottom end. The node region spans 10 mm with the fixed boundary condition in the center. The top and bottom ends were meshed using a hex mesh (hexahedron elements) due to the simplicity of the geometry and advantages in bending analysis. The nodal region was meshed using a Tet mesh (tetrahedron elements) to account for the more complex internal geometry of this region (see Figure 4-5).

Figure 4-5: Parameterized 3D model mesh in ABAQUS 2020
4.2.4 Boundary Conditions

The boundary conditions for the parameterized finite element 3D model match the shear and moment loads that were present upon the modeled segment during physical testing. Tests consisted of 3-point bending configurations, as shown in Figure 4-6. Because the finite element models only account for a portion of the stalk, the relevant shear and moment loads were applied to the end faces of the finite element model. These boundary conditions were obtained from shear and moment diagrams corresponding to each test. A diagram of the physical test along with shear and moment diagrams are shown in Figure 4-6.

![Figure 4-6: Stalk testing boundary conditions for real stalk and parameterized models.](image)

Using the measured test values from previous research (Robertson et al., 2016, 2017), the applied loads for each finite element model was calculated using the equations in Table 4-3. The
center point (plus nodes within a radius of 2 mm) of the finite element models was assumed fixed to simulate the anvil contact from testing.

Table 4-3: Boundary condition equations for parameterized finite element models.

<table>
<thead>
<tr>
<th>Boundary Cond. Location</th>
<th>Bottom Face</th>
<th>Top Face</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stalk Moment Failure</td>
<td>$M_{fail} = \frac{L_{bot}L_{top}}{L_{bot} + L_{top}} P_{fail}$</td>
<td>$F_a = \frac{M_{fail}}{L_{bot}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_b = \frac{M_{fail}}{L_{top}}$</td>
</tr>
<tr>
<td>Force</td>
<td>$F_a = \frac{M_{fail}}{L_{bot}}$</td>
<td>$F_b = \frac{M_{fail}}{L_{top}}$</td>
</tr>
<tr>
<td>Moment</td>
<td>$M_a = \frac{L_{bot} - l_{bot}}{L_{bot}} M_{fail}$</td>
<td>$M_b = \frac{L_{top} - l_{top}}{L_{top}} M_{fail}$</td>
</tr>
</tbody>
</table>

4.2.5 Finite Element Analyses Performed

Structural response was measured in two ways for each model: flexural stiffness and critical buckling load. The flexural stiffness of the model was obtained using a static structural analysis. Critical buckling loads were obtained using the ABAQUS linear buckling analysis. Both methods have been previously validated for this purpose and more information is available in (Stubbs et al., 2022).

4.3 Validation of the Parameterized 3D Model

The validation of the parameterized 3D model includes two different comparisons. The first comparison is a direct validation between empirical test data and the parameterized model (see Figure 4-7). The second comparison involved the CT based model versus the parameterized model. Recall that the CT based model was validated for both flex and buckling analyses in a previous study (Stubbs et al., 2022). Validation was quantified using linear regression (coefficients of the fit line as well as the $R^2$ value).
4.3.1 Stalk Sampling

A unique set of specimen-specific stalks was sampled for each validation comparison. The CT based specimen-specific versus parameterized specimen-specific comparison used 12 samples that were previously evaluated (Stubbs et al., 2022). The direct validation used 62 specimen-specific stalks, 12 of which were the ones used in the CT-based comparisons, and 50 that were randomly selected from the dataset of 980 CT scanned maize stalks.

Figure 4-7: Validation comparisons used to validate the parameterized 3D model.
4.4 Validation Results for Parameterized 3D Model

Results indicate that the parameterized 3D model accurately captures maize stalk morphology. The structural response of the parameterized model was evaluated and validated with four different comparisons. Each comparison showed strong correlation relationships.

The response of the parameterized 3D model was compared against CT-based specimen specific models and empirical test data (see Figure 4-7) by calculating the $R^2$ value of each respective line of best fit. The $R^2$ values are an indicator of the strength of the correlation between each comparison pair. Both flexural stiffness and critical buckling loads were evaluated for each comparison. All comparisons possess strong correlations with all $R^2$ values above 0.7.

Figure 4-8: Parameterized 3D model validation results.
As expected, the comparisons between the CT-based models and the parameterized 3D models had a stronger correlation than with the empirical test data, and the empirical test data vs. parameterized linear buckling had the lowest correlation (see Figure 4-8).

Additionally, each comparison follows closely with a 1:1 correlation as illustrated in Figure 4-8. The best-fit slope and offset coefficients are reported in Table 4-4. A 1:1 correlation implies that the structural response of the models is the same, not just strongly correlated. The best-fit linear regression line provides further evidence of this insight. Notably, the CT-based models tended to match the results of the parameterized models more closely than the empirical data. This result was expected since the only difference between these models is the parameterized geometry. The flexural stiffness comparison with empirical data followed closely with the 1:1, but stiffer stalk samples exhibited more variance. The Buckling results with the empirical data showed a strong correlation but is heavily shifted away from the 1:1 correlation. This result was expected since parameterized 3D models were subject to a linear buckling analysis, which overestimates the critical load.

Table 4-4: Slope and offset coefficients of the best-fit lines and R² values from parameterized 3D model comparisons.

<table>
<thead>
<tr>
<th>Validation Comparison</th>
<th>Slope Coefficient</th>
<th>Offset Coefficient</th>
<th>R² values</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT vs. Parameterized - Flexural Stiffness</td>
<td>0.71</td>
<td>4.29</td>
<td>0.81</td>
</tr>
<tr>
<td>CT vs. Parameterized - Critical Buckling</td>
<td>0.90</td>
<td>-0.12</td>
<td>0.87</td>
</tr>
<tr>
<td>Empirical Test Data vs. Parameterized - Flexural Stiffness</td>
<td>0.68</td>
<td>6.79</td>
<td>0.82</td>
</tr>
<tr>
<td>Empirical Test Data vs. Parameterized - Critical Buckling</td>
<td>1.64</td>
<td>4.05</td>
<td>0.71</td>
</tr>
</tbody>
</table>
CHAPTER 5. DISCUSSION AND CONCLUSION

5.1 Discussion

The development and validation of both the 2D and 3D parameterized models provide valuable benefits in studying the structural response of maize stems. These models combine simplicity with controllability to enable studies that were not previously possible. Results from the 2D prismatic model indicate that the simple ellipse provides very good accuracy within an extremely simple model. Analysis of prismatic models provided interesting insights to the influence of certain geometric and material features of the maize cross-section. The simplicity of the ellipse was critical to the success and validation of the parameterized 3D model, which uses the ellipse as the common feature for each cross-sectional shape. The parameterized 3D model validated nearly as strongly as CT-based models and provides much more control over geometric features of the maize stalk. Of course, these models also have limitations that should be carefully considered moving forward.

5.1.1 2D Prismatic Model

One purpose of this thesis was to validate and analyze a parameterized 2D model of the maize cross-section as a first step towards a parameterized 3D model. This is important for two reasons. First, current models rely on specimen-specific geometry, which cannot be readily manipulated (Von Forell et al., 2015; Stubbs, 2019; Stubbs et al., 2020). Second, parameterized
models enable a much greater range of future and more advanced analyses such as optimization of the maize stalk morphology, sensitivity analyses, etc.

The ellipse provided remarkably accurate estimations of structural stiffness in axial, bending, and transverse loading cases (see Figure 3-1). For each of these cases, the ellipse alone was able to predict structural stiffness with errors of less than 5%. For the case of torsion, the ellipse exhibited structural discrepancies of less than 10%. The conclusion that the ellipse provides excellent accuracy in a very simple model is further reinforced by the results of the sensitivity analysis Figure 3-2, which show that the ellipse parameters exert far more influence on the structural response than any of the principal components. Finally, these results agree with the prior empirical observations which suggested that the ellipse provides an effective approximation of the maize cross-section (Robertson et al., 2017).

Since the ellipse is defined by just three parameters: major diameter, minor diameter, and rind thickness, prismatic models of simple loading cases can be described by just five parameters: three for the ellipse, and two for the tissue. This provides a very compact and convenient way of parameterizing the cross-section while still providing relatively high levels of predictive accuracy. This was an important finding as the small number of cross-sectional parameters supported the development of a parameterized three-dimensional maize stalk model.

The load-bearing analysis of prismatic models provided new insights on the roles of rind and pith tissues. Both material tissues (rind and pith) were found to provide important contributions to the stalk under certain load conditions. The rind contributes more than 90% of the structural support under axial, bending, and torsional loads. Meanwhile the pith is the primary contributor to the resistance of transverse compression (see Table 3-1). This result suggests that the rind generally provides the most structural support under normal conditions, but
the pith is critical to resisting the cross-sectional ovalization which causes buckling failure. The proportion of rind in the cross-section had little effect on the axial, bending, and torsion loads, but had significant effects on the transverse compression load. The amount of rind present in the cross-section possesses a positive correlation to the load contribution in transverse compression loads (see Figure 3-4). Interestingly, the rind thickness is slightly greater than normal in the regions of the stalk where failure is most common (D. J. Robertson et al., 2015) (see Figure 4-4). While more research is needed, one possibility is that this is an adaptive response to prevent buckling failure.

5.1.2 Limitations of 2D Prismatic Model

First, the simplified loading conditions used in this study differ from those experienced by an actual stalk. Simplified structural models were intentionally used in this study because they enabled a comprehensive evaluation of many different model configurations (~12,000 different models). Although simple models were used, results from each loading case indicated that the ellipse provides a favorable balance between predictive accuracy and model complexity. The principle of linear superposition indicates that (at least for small deformations) structural stresses are additive and do not interact. Thus, although the loading cases used in this study are simplistic, they represent important components of more complex loading situations.

The most significant geometric limitation in this study is the assumption of constant rind thickness. An alternative (and more accurate) approach would be to decompose the interior boundary of the maize stalk using a separate ellipse and additional principal components. This approach was not used because it would have doubled the total number of geometric parameters. And given the high accuracy of the results, this approach would not have significantly increased predictive accuracy (in most cases, the ellipse plus 5 principal components produced models with
errors less than +/- 1%), so it seems unlikely that doubling the number of geometric parameters would prove worth the expense.

Another limitation in this study includes the indirect load-bearing calculation for transverse compression loads. The load bearing contributions of rind and pith tissues under transverse compression was calculated by comparing the cross-section response of a rind-only cross-section against one that included both rind and pith. Then, the load contribution of the pith was calculated by subtraction of these known values. This method was used out of convenience but neglects important interactions between the rind and pith. Material tissues act independently in axial, bending, and torsion loading cases, but for transverse compression, the rind and pith interact with each other, and their influences are not independent. Excluding the pith from the model eliminates that interdependence. More accurate results could potentially be calculated through a more detailed stress analysis. However, it seems unlikely that this approach would cause a major change in the results obtained through the approach used in this study.

Additional (but relatively minor) limitations include the use of several simplifying assumptions. For example, tissues were modeled as transversely isotropic and linear elastic. The plane-stress assumption was invoked in transverse compression. All simulations were static in nature and did not include any dynamic effects. However, since the primary goal was to develop a useful and accurate geometric model, not to investigate the actual mechanics of transverse ovalization, these assumptions are justified and appropriate.

5.1.3 3D Parameterized Model

The structural response of the parameterized 3D model correlates with empirical test data and results from previous maize stalk studies (Robertson et al., 2016, 2017; Stubbs et al., 2022). This indicates that the parameterized 3D model is an accurate representation of maize stalk
structure. This is significant for three major reasons. First, the parameterized 3D model is validated to be used in the future as a computationally inexpensive replacement to other testing methodologies. Second, the parameterized 3D model allows the maize architecture to be manipulated. Finally, each instance of the parameterized 3D model can be created approximately 100 times faster than any previous testing method.

The parameterized 3D model is a computationally inexpensive maize stalk model that provides accurate structural response in flexural stiffness and is closely correlated to the actual critical buckling load. As shown in section 4.4, the correlation between the parameterized 3D models and physical tests produced $R^2$ values of 0.82 for flexural stiffness, and 0.71 for critical buckling (see Table 4-4). As expected, the comparisons between the CT-based models and the parameterized 3D models had a stronger correlation than with the empirical test data, and the empirical test data vs. parameterized linear buckling had the lowest correlation (see Figure 4-8).

These validation results suggest that this model can be used as an accurate replacement. A comparison between CT-based specimen specific models (Stubbs et al., 2022) help us quantify the level of predictive discrepancy caused by using the parameterized model as opposed to a fully detailed CT-based model. For flexural stiffness the correlation between these models was $R^2 = 0.81$, and for the buckling, the value was 0.87. These values indicate that the parameterized 3D model is more than 80% accurate.

The primary reason for the creation of the parameterized 3D model is the capability to manipulate its architecture. The model is defined by 57 individual parameters. Each parameter is defined by a single coefficient and is independent of the other coefficients. Of these, 47 are used to define the geometry, and 10 define the material properties. Parameter independence allows each measurable feature to be changed individually without influencing the others. This level of
control cannot be obtained in a physical experiment and provides many new analysis capabilities. The capacity to manipulate stalk architecture fulfills the original purpose of the parameterized model by enabling new studies such as parameter sensitivity analysis and optimization studies (Ottesen et al., 2022). This parameterized 3D model is a powerful tool capable of analyzing maize stalk structure in greater depth.

Finally, the parametric nature of the 3D model allows for automated model creation. Because the parameterized 3D model is defined by 57 coefficients, recreating a new model to match a desired stalk is repeatable and the process can be automated.

5.1.4 Limitations of Parameterized 3D Model

The most obvious limitation on the parameterized 3D model is the simplified cross-section shape. The cross-section is assumed to be an ellipse with a constant rind thickness which ignores the ear groove as well as finer geometric details. This assumption introduces a small amount of error in the model’s structural performance. However, if improved structural performance is desired, the stalk geometry can be improved in the parameterized 3D model by implementing the principal components described in previous work. For example, the first principal component could be added to the ellipse. The 1000 CT-scans would be re-analyzed to obtain the axial profile of the scaling of this principal component. The axial profile would then be parameterized (as described in section 4.1.3), and the 3D model would be updated accordingly. Based on the analysis performed on major diameter, minor diameter, and rind thickness, this would introduce approximately 12-17 new geometric parameters to the parameterized model. While this approach would increase the accuracy of the model’s geometric fidelity, it would not significantly improve its structural performance. This is because adding the first principal component improved predictive accuracy of prismatic models by only 2-3%.
Adding all 5 principal components may improve predictive accuracy but would require adding ~75 additional model parameters. Given that the R² values between parameterized and CT-based models are both in the range 0.8 – 0.9, it seems unlikely that the massive increase in model complexity would be worth a marginal increase in accuracy.

In terms of finite-element analysis, a potentially more accurate analysis, such as a non-linear buckling analysis, would better simulate realistic conditions and would yield different results than a linear buckling analysis. This was not done due to the increased complexity and computational time. Because the linear buckling results already show a good correlation (R² value of 0.71) to the empirical test data, increased accuracy in model validation analysis was determined to be unnecessary.

5.2 Conclusion

Evaluating the prismatic model under multiple load conditions further confirmed that the ellipse approximation provides a favorable balance between model complexity and predictive accuracy. Under axial tension/compression, the ellipse model performed with less than 5% error. Under bending loads, the ellipse cross-section performed with less than 3% error. Finally, under a torsional load, the ellipse model performed with less than 10% error. Validating the ellipse under each of these loads builds increased confidence using the ellipse in a 3D model.

A load bearing analysis revealed which tissues contributed the most structurally for each of the prismatic model loading cases. The rind was found to be the most influential in axial tension/compression, bending, and torsion loads by bearing a median value more than 90% of the load in each case. Interestingly, the pith provides the most structural strength in transverse compression by contributing a median value of 92% of the load.
Although the 3D parameterized model utilized a significant simplification of the cross-sectional shape of the maize stalk, the results of this model compared very favorably to empirical test data (see Figure 4-8) with all $R^2$ values above 0.7. This comparison is a direct validation that tells us that the model performs in a manner that is very similar to real corn stalks. The parameterized 3D model also performed similarly to CT-based finite element models (see Figure 4-8) in both flexural stiffness and critical buckling yielding $R^2$ correlation values over 0.8. This comparison isolated the geometric change manifest by the parameterization and suggests that the parameterization of the geometry exhibits some error in structural performance, but that the additional error is not a major drawback on the method.

The data presented in this thesis strongly support the conclusion that these models are appropriately accurate for future modeling and analysis studies. The future studies will allow for synthetic stalk generation enabling parameter sensitivity studies and optimization analysis. Additionally, parameterized models can be generated using a computer program. This approach drastically reduces the time spent on model creation and enables the evaluation of thousands of possible model configurations. In summary, the parameterized 3D model will be a powerful tool platform for performing studies that will help us better understand and optimize the structural architecture of the maize stalk.

5.3 Research Contributions

The work on the 2D prismatic model in this thesis resulted in the completion of a journal article detailing the entirety of said model in the *Biosystems Engineering* journal (Ottesen et al., 2022). With the analysis of the parameterized 3D model still unfinished, the work from this thesis will be integral to future studies of the maize stalk morphology. It is anticipated that
another journal article will result from the parameterized 3D model once a sensitivity analysis and optimization study is complete.

Further, the research presented in this thesis provides a tool that will be instrumental in informing the maize crop industry which morphological maize stalk features are most influential to stalk strength. By breeding future stalks that exhibit high-strength features, the maize stalk lodging rate can be reduced. Thus, this research fulfills a role that will contribute to saving billions of lost industry dollars and improving the strength of maize stalks for future generations.


