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PROOF TECHNIQUES IN SEVENTEENTH CENTURY CHINESE MATHEMATICS

Jean Claude Martzloff

Introduction

According to D. B. Wagner "Early Chinese attempts at proofs of mathematical propositions are not well known in the West." Concerning later Chinese attempts at proofs, the situation is still worse since historians generally maintain that "the historical period of indigenous mathematical accomplishments came to an end with the arrival of Jesuit missionaries at Peking in 1601" and that "from this time onwards [i.e. circa 1600] the Chinese mathematical literature become voluminous, but though still somewhat isolated, it is a part of the world literature." These statements are doubtlessly oversimplified, since indigenous mathematics were to be still practised with no reference to Western mathematics until approximately 1900, and perhaps even later. It is therefore the purpose of this paper to attempt a broad comparison between Western and Chinese seventeenth century mathematical proof techniques. We restrict our investigations to elementary geometry and numerical analysis.

Geometry

Native Chinese geometry is "non-demonstrative in character," for Chinese mathematical argumentation can never be analysed in terms of postulates, definitions, theorems, etc. This, however, does not amount to saying that it is impossible to find proofs (or "derivations") in Chinese mathematics.

After the translation of Euclid's *Elements* (1607), Chinese interest in geometry was aroused. Chinese scholars such as Du Zhigeng, Yang Zuomei, Chen Houyao, Fang Zhongtong, Mei Wending, etc. produced works on geometry. Of course, proofs of the Euclidean type are easily found in some of their books. Nevertheless, on the whole, their line of thought is not "Euclidean". The converse of theorems, reductio ad absurdum, and proofs of "obvious facts" are always lacking. An interest in absolute, topological, non-numerical geometry is wanting. In spite of all these seemingly negative facts, 17th century Chinese math-
ematicians were able to derive new mathematical results (i.e. previously unknown in China), mainly in the field of elementary metric geometry.”

Actually, there exist “proof techniques devices” often used by Chinese mathematicians. By way of illustration, let us give a few examples of such devices:

a. The method of “geometrical algebra.”
b. The method of decomposition.
c. The method of similitude.

In the method of “geometrical algebra”, geometrical drawings are used as “visual heuristic devices” leading to the discovery of algebraical identities between lengths, areas or volumes.

In the method of decomposition (or dissection), a given figure, A, is suitably decomposed into a finite number of pieces which can be reassembled to give a new (and perhaps simpler) figure B. As a general rule, such a method is avoided in Western tradition, but examples of its use are well instanced in China.

The method of similitude is used much in the same way, as well in China as in Europe.

The reader will get some taste of the preceding methods in the appendix, where we shall compare the “Chinese” solution of some problems with their “European” (Jesuit) counterparts.

**Simultaneous linear equations**

From the Han dynasty onwards, Chinese mathematicians had always been interested in solving systems of linear algebraic equations. Linear equations were arranged in matrix form, and elimination was performed according to the so-called Gaussian method, which is “the simplest of all methods for solving linear systems from the computational point of view.” In seventeenth century China, this method was still in use, but its rationale was not properly understood and mathematical books were often swarming with inaccuracies. Therefore, Mei Wending undertook a reconstitution of the whole procedure. His attempt was successful and much admired by his contemporaries.

It should be pointed out that the “European method” for solving linear systems of equations, which used the cumbersome “rule of double false position” (regula falsi duplicis positionis), was considerably more troublesome than the “advanced” Chinese procedure (cf. appendix, section 2).
Quadratic equations:

The Jesuit description of European arithmetic was included in the *Tong wen suan zhi* [Combined Cultures Mathematical Indicator] (1614), a treatise not only based on a work by Clavius, but also on Chinese autochthonous mathematics. For example, in chapter 7, the problem of solving numerically quadratic equations was founded on Yang Hui’s methods, which were derived from geometrical diagrams. Without any doubt, these procedures were inferior to those advocated by Qin Jiushao, Zhu Shijie and Li Ye, for their geometric foundation hindered further progress.

Conclusion

Chinese seventeenth century geometrical proofs were based on *a priori*, “visible,” assumptions. General heuristic methods were never explained for their own sake, but simply applied in various situations. “Algebra” was often geometrically founded and remained essentially “rhetorical.” In the field of numerical analysis, Chinese methods, although non-symbolic, were sometimes considerably more advanced than Jesuit techniques.

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Appendix

1. Geometry
   
   The gnomon property:

   ![Diagram](image)

   The rectangles (1) and (2) have the same area, therefore $GH \cdot EH = EF \cdot BE$; $GH/EF = BE/EH$; etc. (similitude is avoided). This property was known in Yang Hui’s time (perhaps independently of *Euclid* I-43) and, in Chinese context, is very often preferred to similitude.  

   a. Application: to inscribe a square in a given triangle (as shown in fig. 2).
Euclidean solution (fig. 2): construct on AB the point D such that AD/DB = AB/BC (the construction follows from Elements VI-10; draw DE//BC. The triangles ABC and ADE are similar (VI-4, corollary), hence BC/AB = DE/AD. By "ex aequali," BC/BC = DE/DB, therefore DE = DB, and since DE//BC, DBFE is a square, as required.

Mei Wending's solution (fig. 3)

Construct MB//AC, MA//BC, construct the square ACNJ on AC and draw JB. JB cuts AC in E, then EC is a side of the required square. Proof: (rectangle; "=" means equal in area)

\[
\begin{align*}
\text{MF} & = \text{EH} \quad \text{(gnomon property in ACMB)} \\
\text{MF} + \text{AF} & = \text{EH} + \text{AF} \\
\text{ME} & = \text{AH} \\
\text{ME} & = \text{EN} \quad \text{(gnomon property in JNBM)} \\
\text{AH} & = \text{EN} \\
\text{CH} \cdot \text{AC} & = \text{CN} \cdot \text{EC}
\end{align*}
\]
Since $AC = CN$, it follows that $CH = EC$ and $CHFE$ is the required square.

In his text, Mei Wending never refers to the "gnomon property", but proves it each time he needs it.

**Chen Jinmo’s solution** (fig. 2): bisect the angle $B$, then $BE$ is a diagonal of the required square.

b. *A problem on the right-angled triangle:* Let $x, y, z$ be the lengths of the sides of a right angled triangle ($x^2 + y^2 = z^2$). Given $z$ and $x + y$, to find $x, y$.

**Prestet’s solution** (abridged):

\[
\begin{align*}
    x^2 + y^2 &= z^2 \quad (1) \\
    z &= a \quad (2) \\
    x + y &= b \quad (3)
\end{align*}
\]

\[
z^2 = (b-y)^2 + y^2 = b^2 - 2by + 2y^2 \quad \text{or} \quad 2y^2 - 2by - (a^2 - b^2) = 0
\]

Solve the quadratic. This is the same as our school algebra.

**Du Zhigeng’s solution** (fig. 4)

The diagram yields:

\[
z^2 = 4S + (y-x)^2 \quad \text{(inner square)}
\]

hence

\[
2z^2 = 8S + 2(y-x)^2
\]

moreover

\[
(x+y)^2 = 8S + (y-x)^2 \quad \text{("big"square)}
\]

hence

\[
2z^2 - (x+y)^2 = (x-y)^2
\]

\[
\sqrt{2z^2 - (x+y)^2} = x - y
\]
\[\frac{1}{2} \left[ (x + y) + (x - y) \right] = x\]
\[\frac{1}{2} \left[ (x + y) - (x - y) \right] = y\]

2. Elementary linear numerical analysis

Solve

\[x + 73 = 2(y + z) \quad (1)\]
\[y + 73 = 3(x + z) \quad (2)\]
\[z + 73 = 4(x + y) \quad (3)\]

a. Clavius' solution: \(^{30}\) Start with 4 triplets \((x, y, z)\). Solution of (1): [Clavius chooses \((1, 2, 35); (1, 5, 32), (3, 2, 36), (3, 23, 15).\] Let \(e_i, e_2, e_3, e_4\) be the errors in equation (2), i.e., the values of \(y + 73 - 3(x + z)\), which would be equal to zero if the triplets were simultaneously solution of both (1) and (2):

\[y_i + 73 - 3(x_i + z_i) = e_i \quad (i = 1, 2, 3, 4)\]

(here \(e_1 = -35, e_2 = -21, e_3 = -42, e_4 = +42\))

Applying the rule of double false to find two new triplets verifying (1) and (2) simultaneously:

\[x_{ij} = \frac{x_i e_j - x_j e_i}{e_i - e_j}\]

one finds the triplets \((1, 10\frac{1}{4}, 26\frac{3}{4})\) and \((3, 12\frac{1}{2}, 25\frac{1}{2})\). We have now two triplets, both solutions of (1) and (2). In the same way, we determine the new errors in equation (3) and we apply the rule of double false position to find one new triplet solution of the whole system, namely \((7, 17, 23)\).

This cumbersome method would be almost unworkable with four unknowns, nevertheless, it was the nec plus ultra technique explained in Jesuit works to handle linear systems.

b. Mei Wending's solution\(^{31}\) (abridged)

Arrange the numbers in columns:

\[
\begin{pmatrix}
1 & -3 \\
-4 & 1 \\
-4 & -2 \\
1 & -2 \\
-73 & -73 \\
-73 & -73
\end{pmatrix}
\]
By elementary row operations, reduce the system to the triangular form:

$$\begin{pmatrix}
0 & 0 & -3 \\
12 & 5 & 1 \\
7 & 9 & -3 \\
365 & 292 & -73
\end{pmatrix} \rightarrow \begin{pmatrix}
0 & 0 & -3 \\
0 & 60 & 1 \\
73 & 35 & -3 \\
1679 & 1825 & -73
\end{pmatrix}$$

Apply the backward substitution method. Hence $x = 7$, $y = 17$, $z = 23$.

Notes


4. "As for Ch’ing mathematics, the most important feature was the revival of autochthonous mathematics and the study of old procedures" (U. Libbrecht, *Revue Bibliographique de Sinologie*, 10 (1964), p. 452). See also: Li Yan, *Zhong suan shi luncong*, vol. 2, (1955), bibliography of Qing period Chinese mathematical works.

5. In the sequel, "Western mathematics" refers only to Chinese adaptations of European mathematical writings.


7. Here, "demonstration" means "argumentation in terms of explicitly stated definitions, axioms, theorems and rules of inference, whereas "proof" (or derivation) refers only to the vague concept of "persuasive argument", whatever it may be.


12. In his *General Treatise on Geometry (Jihe Tong jie)*, ca. 1690, Mei
Wending asserts in a very clear way that the real content of Euclid/Clavius’s Chinese version of the Elements is in fact nothing else but the Chinese theory of right triangles (gougu). In order to prove his claim, Mei was obliged to pass over axioms, definitions, absolute and topological geometry, etc. in silence. Nevertheless, he was quite successful in his creative reworking of metrical geometry. Towards the end of the 17th century, he concluded that Euclidean geometry had been devised to build up the stereometry of regulars polyhedras and other related three dimensional objects (the cube-octahedron and the icosiododecahedron). In these days, such a theory was entirely new in China, and only tiny scraps of information about it were available through chinese adaptations of Europeans works. Mei’s works served as a basis for the great mathematical encyclopaedia of the Kangxi reign, the Yuzhi Shu li jing yun (1723) [The Principles of Mathematics, Profound and Hidden, Compiled by Imperial Order].

13. Since Tannery, Zeuthen and others, the concept of “geometric algebra” has been used to describe the methods of Elements, book II, in relation with the notions of “application of areas” and quadratic equations. What we label here “geometric algebra” in Chinese context is slightly different from what it is in Greek context, and means only “an heuristic technique to discover ‘algebraic’ algorithms by means of geometrical drawings.” Cf. Appendix.

14. “Suitably” means “with no overlapping pieces”. Cf. Hilbert, Foundations of Geometry, La Salle, Illinois, 1971, ch. 4, p. 60, theory of plane area. The figures A and B are called “equidecomposable” and have both the same area (or volume). This method enables us to derive the area of an arbitrary polygon knowing the area of the rectangle.

15. For example: Liu Hui’s derivation of the length of the side of a square inscribed in a right triangle; the pythagorean proposition; the sum of the squares of the first \( n \) natural numbers. See: Li Yan and Du Shiran, Zhongguo gudai shuxue jianshi, vol. 1 (1963), p. 92; Xu Chunfeng, Zhong suanjia de jihexue yanjiu, Peking (1952), p. 4 sq.; Du Zhigeng, Shuxue yao, Juan 4, p. 41b.


20. Takeda Kusuo, Dō bun sanji no seiritus [The formation of the Tong wen suan zhi], Kagakūshi Kenkyū, 30 (1954), pp. 7-14.

22. Cf. the notices in the Dictionary of scientific biography (Ch. Coulston Gillispie, ed., New York). Li Ye, Qin Jiushao and Zhu Shijie’s methods for solving polynomials were akin to Ruffini-Horner’s well known techniques.


26. Mei Shi congshu jiyao, ch. 21, p. 3a (cf. supra, note 18)

27. Ibid. p. 3b. On Chen Jinmo, see Siku tiyao, ch. 44.


29. Shuxue yao [The Key of Mathematics], juan 6, p. 5b sq.

30. Tong wen suan zhi tong bian (cf. supra, note 20), juan 4, p. 223; Clavius, Epitome . . . (supra, note 19), ch. 23, p. 243. For detailed account of Clavius’s solution the reader is referred to the book quoted in note 21, p. 215 sq.

31. Mei Shi congshu jiyao, ch. 11, p. 8b sq.

Glossary

a. 陈知耕
b. 杨作枚
c. 陈厚耀
d. 方仲通
e. 梅文鼎
f. 同文算指
g. 幾何通解
h. 御製数理精藴
i. 数学鑰