Laser Forming of Compliant Mechanisms and Flat-Foldable Furniture

Daniel Calvin Ames
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Laser Forming of Compliant Mechanisms and Flat-Foldable Furniture

Daniel Calvin Ames

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of
Master of Science

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ABSTRACT

Laser Forming of Compliant Mechanisms and Flat-Foldable Furniture

Daniel Calvin Ames
Department of Mechanical Engineering, BYU
Master of Science

Compliant mechanisms are useful for improving existing machines and creating new ones that were not previously possible. They also help us to think of new methods and technologies needed to both improve existing systems as well as manufacture systems that have not been done before. The purpose of this thesis is to show novel implementations of compliant mechanisms into folding systems, and to show new methods for fabricating such mechanisms with nontraditional materials and on difficult scales. Folding systems are shown in furniture applications with chairs, stools, and childcare furniture applications as results of research into how such structures could be created with compliant mechanisms to be deployed from a flat state. Compliant mechanisms are also shown to be folded by a laser into simple mechanisms and into a potentially more complex parabolic reflector. Small-scale flexible (or compliant) mechanisms are valuable in replacing rigid components while retaining comparable motion and behavior. However, fabricating such mechanisms on this scale (from 0.01 to 10 cm thick) proves difficult, especially with thin sheet metals. The manufacturing method of laser forming, which uses a laser to cut and bend metal into desired shapes, could facilitate this fabrication. However, specific methods for designing mechanisms formed by lasers need to be developed. This work presents laser forming as a means for creating compliant mechanisms on this scale with thin sheet metal. The unique challenges for designing mechanisms to be laser-formed are explored, and new adaptations of existing designs are fabricated and discussed. The design of basic “building blocks” and features are developed for several mechanisms: a parallel-guided mechanism, a cross-axis flexural pivot, a LET joint array, a split-tube flexure, and a bi-stable switch. These mechanisms are shown to perform repeatable behavior and motion comparable to existing non-laser-formed versions. The further possibilities for fabricating compliant mechanisms with laser forming are explored, as advanced applications can benefit from using lasers to create compliant mechanisms from thin sheet metal. One such possible system is a parabolic reflector, which is useful for making solar collectors and antennas. Such shapes have been developed in various patterns and typically manufactured out of rigid components. Applications for these systems could benefit from paraboloids that can fold up and be deployed into a final shape. This work presents a conceptual method for designing a flat-foldable paraboloid and a means for its fabrication using laser forming.

Keywords: compliant mechanisms, laser forming, origami-inspired, folding furniture
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CHAPTER 1. INTRODUCTION

Compliant mechanisms are useful for improving existing machines and creating new ones that were not previously possible. They also help us to think of new methods and technologies needed to both improve existing systems as well as manufacture systems that have not been done before. There are difficult materials to fabricate compliant mechanisms with, and challenging scales to machine them on. The purpose of this thesis is to show novel implementations of compliant mechanisms into folding systems by demonstrating their potential in nontraditional materials and difficult scales. New methods for fabricating such mechanisms are also presented, as the designs are fabrication-driven.

The research is divided into two key components: flat-foldable furniture and laser-formed mechanisms. While seemingly disparate, both areas advance the existing technologies of compliant mechanisms and their fabrication by introducing novel techniques for mechanism design.

1.1 Flat-Foldable and Compliant Furniture

Furniture is a design space full of potential for innovation. Specifically, origami-inspired design can be used to create furniture that is space-saving, cost-effective, and versatile. Principles of compliant mechanisms and deployability can be used to make pieces of furniture that differentiate themselves from the rest of the market by their design and use. For this research, several specific areas were explored as possible innovation spaces; folding wooden chairs and deployable furniture used in childcare spaces.

1.1.1 Wooden Lamina-Emergent Torsional Joint Chair

Wood, while one of the original compliant materials, is not often used in furniture for its flexibility. Wood is easy to machine and flexible enough to be used as a compliant material. Chapter 2 will show the development of a wood chair that is folded from a flat sheet using lamina
emergent torsional (LET) joints as replacement hinges. Simple LET joint arrays are explored in wood as an option for strong but simple stools as well as the basis for more complex folding furniture.

1.1.2 Deployable Childcare Furniture

The childcare furniture design space is very tightly regulated, and therefore a difficult design space in which to innovate. With this in mind, furniture could be developed using origami-inspired design to create safe and space-saving furniture that help in child development and is easy to use for childcare workers. The work in chapter 3 shows the development of a “safe space” for children which can be used in a childcare setting. Several different safe space designs are proposed and their engineering principles are explored.

1.2 Design of Laser-Formed Compliant Mechanisms in Thin Sheet Metal

Thin sheet metal can be difficult and costly to machine and use for compliant mechanism applications, and yet can be a good solution for applications requiring metal for its conductivity, reflectivity, and cleanliness. The technology of laser forming is a method for processing metal sheets into desired shapes, but has been primarily implemented for rigid shapes. New design methods for applying this technology need to be created to fabricate compliant mechanisms. For this work, the foundational principles for laser forming compliant mechanisms are explored and a concept for an advanced application is shown.

1.2.1 Laser Forming of Compliant Mechanisms

The foundational principles of fabricating compliant mechanisms in thin sheet metal is explored through the design and creation of five distinct “building-block” mechanisms. Each mechanism demonstrates different hurdles that have to be overcome for laser forming to work as a fabrication technology. Taken as a whole, the mechanisms show that the key principle to understanding how to create compliant mechanisms this way is through the reorientation of features to both limit and allow deflection. The work shown in Chapter 4 paves the way for the design of increasingly complex compliant mechanisms fabricated using laser forming.
1.2.2 Flat-Foldable Deployable Parabolic Reflectors

Knowing the basics of laser forming compliant mechanisms, new concepts for folding deployable systems out of thin sheet metal can be created. One such concept is for a flat-foldable parabolic reflector. In order to be flat-foldable, the reflector can be discretized into individual “petals” that use the geometry of an Euler spiral to be able to lie flat. Parabolic reflectors are useful for a variety of applications, including solar energy collection and antennas. However, such reflectors can be unwieldy or difficult to transport (like in a CubeSat). Having a flat-foldable antenna could allow for space-efficient transport and simple, strain energy-induced deployment. Chapter 5 outlines the basic principles of geometry for such an antenna and shows potential configurations.
CHAPTER 2. FLAT-FOLDABLE FURNITURE – LET CONSTRUCTS

2.1 Introduction

Furniture can be improved by origami-inspired design to solve issues of space and manufacturability. Creating something from a flat workpiece to be folded after-the-fact can be easier to manufacture than 3D objects with features on multiple different planes, as the manufacturing process can mostly happen within a two-dimensional plane. Being able to create an essentially two-dimensional shape which can then be folded into a stable state offers many possibilities for transport and deployment.

The key technique for creating flat-foldable wooden furniture is to create load-bearing LET joint arrays that would replace traditional joints and hinges in the presented systems. Two flat-foldable wooden furniture items are presented: a round stool and a flexible chair. Both of these furniture items apply novel applications of LET joint arrays and show nontraditional loading conditions for such arrays.

2.2 LET Joint Structures

2.2.1 LET Joint Stool

Stools are the simplest form of furniture. As such, they clearly demonstrate the basic behavior of LET joints in arrays and their viability in wooden furniture.

Concept Overview

LET arrays have been employed to various degrees as hinge substitutes, but are rarely seen as load-bearing members due to their inherent flexibility. However, loading the array perpendicular to the direction of the array (i.e., the side/edge of the array) allows the structure to both maintain
its shape and to carry a load. This load-carrying ability depends on the boundary conditions at the ends of the arrays (which, ideally, are fixed to prevent unwanted motion due to the loading).

To test this concept, a simple stool was created out of plywood with a solid circular top and a LET array as a supporting structure. The wood was cut using a laser cutter, and the array was curved into a cylindrical shape and the ends were fixed together, thus providing a stable structure for relatively high loads. The described configuration, shown in Figure 2.1, shows the torsional motion of the beams for each array member. This concept is further explored in the later example of a LET array chair.

The stool is flat-foldable in that it deploys from a sheet of plywood into its final, stable state. While its stowed state is not particularly transportable—it cannot be folded smaller than the initial sheet of plywood—it does qualify as flat-foldable. It is stable in that its base has constant contact with the floor, except under cases where the load placed on it creates extreme torsional loads: i.e., if the sitting person twisted their body, twisting the top of the stool and each of the torsion bars connected to it, the base may lift off the floor, thus causing the stool to lose stability. Because this is a unique case of loading on plywood, several loading tests were performed on the stools,
including force-deflection measurements when the stool is loaded as if it was being sat upon. The tests will be described and analyzed further below.

A new composite material was also developed using a plywood LET array in order to see how the technology could be strengthened and improved. This involved filling the pre-existing gaps in the LET array with polyurethane in order to strengthen the array and prevent catastrophic failure under the application of heavy loads. Arrays with and without the polyurethane fill were tested; data from these tests is presented below.

**Testing**

Little data on the mechanical properties of laminated wood has been published [1]. Data generally focuses on the behavior of different pure woods directly under compression, tension, torsion, or laminated wood sheets under shear or bending. Our furniture application uniquely utilizes plywood, which alternates grain as it layers wood and therefore greatly alters the mechanical properties from pure wood, so we ran several of our own tests.

In order to determine viable loads for the stool, several different plywood LET arrays were put under compression with an Instron compression tester. The LET arrays were two feet long and each was linked end-to-end to form a cylinder, with the torsion bars of the array oriented perpendicular to the circular face of the cylinder.

Four arrays of different sizes were tested in order to compare and contrast behavior. Two were made of the same type of plywood, one of which was 1.5 feet tall, and the other of which was 1 foot tall. The third was made of a higher-quality plywood with a finer grain and was also about 1.5 feet tall. The fourth was made of the same finer-grained plywood as the third, was filled with polyurethane, and was about 1.5 feet tall.

A distributed load was placed on each of the arrays while in its curved configuration, and the maximum force was measured when the array began to fail. Figure 2.2 shows the results of this testing as a relationship between the force and the displacement for each of the unfilled arrays. Note that the peak of each curve was the maximum force experienced by the array before failure. The short, tall, and refined-grain wooden arrays failed at about 1550lbf, 1500lbf, and 2950lbf respectively. Figure 2.3 shows that the polyurethane-filled LET array failed at about 4500lbf.
It is impressive just how much force any of these arrays can withstand until failure; each vastly exceeded expectations of the applied load of just body weight. Interestingly, using the same plywood, the short and the tall array had a similar maximum force, while the array using the higher-quality plywood was able to withstand almost twice the load. The best-performing array, however, was the polyurethane-filled array, which failed at over 4500lbf, significantly outperforming any of the non-filled arrays. This proves that reinforcing a wooden array with a rubbery, elastic material like polyurethane significantly strengthens the array.
Benefits

This stool configuration is rather simple in its geometry, but can sustain a deceptively large force when loaded. Since the array does not undergo meaningful plastic deformation when bent into the curved configuration, the array can be flattened out again when the stool is not in use. This saves significant space and is quick and easy to do depending on how the ends are fixed.

Limitations

LET arrays can be complicated to manufacture because of their potential for intricacy, depending on the relative dimensions of the array. Additionally, if improperly handled, the array may bend too much, causing the plywood to fracture and fail. Designing such an array to achieve the desired motion is also more time-consuming and more difficult than using traditional pin joints because of its complexity and lack of repeatable usability.

2.2.2 Reversible Single-Sheet LET Array Chair

Chairs are the next most complicated piece of furniture after stools, so the LET array for the wooden stool was adapted to form a more complex, flat-foldable, wooden chair.

Concept Overview

The heart of the reversible single-sheet LET array chair is multiple arrays of LET joints replacing traditional hinges or joints. Further, the goal was to develop a design concept that was monolithic. A single sheet of polyurethane-composite plywood was therefore selected as the sole material.

Before developing a design, tests were run to see if a plywood LET array could withstand a compressive load about the bending axis (along the torsion bars [2]). A set of LET arrays were cut out of plywood and loaded along the torsion bars in a tensile test. The failure load results are shown in Figure 2.4. Both of the more inexpensive plywood samples failed at approximately 1500 lbf, whereas the array cut from the higher-quality plywood did not fail until loaded with approximately
3000 lbf. These data show that a simple LET array can adequately hold a compressive load in a furniture application.

As this design was developed, numerous prototypes were created at various scales. Figure 2.5 shows the first three full-scale prototypes. Throughout the creation of the iterations, the cutting process was refined to reduce fraying of the plywood. Aesthetics were of significant focus, with the back angle, solid panel design, and back locking mechanism adjusted with each iteration.

A concept was developed that could be cut from a single sheet of plywood using a computer numerical control (CNC) mill. Figure 2.7 shows the base pattern for this design. This pattern was cut out of a 4 ft. by 8 ft. sheet. The brown is wood that was kept, while the white is
wood that was removed. A zoomed-in view of the LET array is given to show the finer details in higher resolution.

Figure 2.8 shows the final version of the single-sheet LET array chair. One side has been painted to show its reversible nature.

This design is a natural progression from the LET joint stool; it involves the same principles, modified in a way that allows for the stool to include a robust back. This design is both flat-foldable and stable; similar to the LET joint stool, it can fold flat into a single sheet of wood, and also has significant testing to prove that it is stable under various loads.

Benefits

The most appealing benefit of this design concept is that it can be fabricated from a single sheet of plywood in a single manufacturing process. This will lower costs while preserving the
desired functionality. Another benefit is that the design is bidirectional, with two stable, deployed states, thus allowing the design to be reversible. Additionally, the design could be applied to other materials with minimal alteration.
Limitations

Since the size of the design is tied to the manufacturing method, the size of the chair is dependent on the size of the manufacturing bed and the size of available sheet material. For example, the prototypes of the design were shorter than desired because of the specific CNC mill that was used.

Unlike the other foldable furniture concepts developed, whether or not a compact stowed state is feasible is still under investigation. This would require that some of the LET arrays be able to curve 180 degrees, which would likely require a longer LET array than those currently included in the design.

There are many trade-offs that are made with this particular design of the LET chair. While it can be milled from a single sheet of wood, the negative space left behind by the chair pattern could be repurposed or the chair redesigned such that minimal wood waste is generated. Also, while the key thrust behind the design was to make the chair flat-foldable to show the implementation of origami-inspired principles, the resulting flattened shape can be cumbersome to wield and not ideal for transport and actuation by a single person.

2.3 Conclusion

Origami-inspired furniture made of wood is shown to be both flat-foldable and structurally sound. By using LET joint arrays in nontraditional configurations, both a stool and a chair were developed to be functional and rigid while only consisting of a single workpiece. Also, the pieces were able to be fabricated on a two-dimensional plane. Furniture like this can save space when not in use while being simple to manufacture. More furniture could be developed that implements the principles demonstrated in creative ways.
CHAPTER 3. FLAT-FOLDABLE FURNITURE – CHILDCARE

3.1 Introduction

Having access to quality childcare is one of the most effective ways to secure a safe and healthy society [3]. Developmentally appropriate education for young children helps them grow into healthy, resilient, responsible, and contributing citizens. Childcare space can help facilitate this nourishment and growth. However, space for childcare facilities can be costly. Furniture that transforms and folds flat can save money, provide multi-purpose functionality, and ultimately improve childcare facilities and thereby the children they serve.

Origami-inspired structures have been developed for a wide variety of industries and technologies in both simple and complex applications [4], [5]. Origami has been used as inspiration because of the creative ways in which flat surfaces can be folded and unfolded into unique shapes and structures. In addition, floor space can be costly; having furniture that can transform and fold flat can save money and/or provide multi-purpose functionality.

By design, furniture needs to be designed for the setting it will be in, and in doing so it acts as an extension of the space [6]. Every space has its own specific requirements that need to be carefully considered when furniture for that space is designed. At times, specific technologies or methods need to be developed in adapting furniture for a designated setting or for a designated audience.

Childcare settings have very specific uses, users, and requirements. Such requirements create different needs for the furniture designed for those spaces. It is very important for childcare spaces to be safe and secure environments, therefore regulations that have been put in place by governmental regulating bodies and institutions are more stringent than the requirements for furniture in other situations.

A key requirement of childcare furniture is simplicity, both in design and function. Having simplicity allows for ease of use and maintenance. This is an important concept to implement
in childcare furniture design, especially when using origami-inspired design, which can often be quite complex.

By using techniques for flat-folding furniture and origami-inspired engineering principles, durable, sturdy, and safe furniture can be created to improve childcare settings.

3.2 Fundamental Design Considerations for Childcare Furniture

Designing furniture in a childcare setting can be difficult because of many health and safety restrictions put in place by law. Moreover, engineers and product designers have to consider the wants and needs of a variety of children in different age ranges and sizes. Furniture items designed for the childcare industry must be easy for a single adult to use, must be easy to keep clean, take up minimal space when not in use or otherwise be used very often, be able to withstand daily use and abuse from children in many age ranges, and be safe for a child to use.

3.2.1 Material Design Considerations

Products used by children must adhere to strict regulations to be approved by appropriate governing bodies—failure to do so can result in civil and or criminal penalties [7]. These groups include the Consumer Product Safety Commission (CPSC), the National Institute of Standards and Technology (NIST) from the U.S. Department of Commerce, the Environmental Protection Agency (EPA), and more [8]. All materials used in any type of furniture must be deemed safe, typically through third-party testing, before they may be distributed. These regulations deal largely with the materials being nonhazardous. References [8] should be reviewed for a more comprehensive treatment on the topic. Additional standards may be developed by individual states.

Along with what governing bodies declare as safe for childcare product materials, preferences of childcare workers should be used to select an appropriate material. In an effort to compile a list of these preferences, administrators and workers from two childcare facilities were interviewed and asked to describe which products they currently use were most valuable to them. The traits and properties of these products were then extracted and compiled into the list below. Both facilities were largely in agreement on several points.
Nonhazardous

As already noted, the utmost care must be taken to prevent children from coming into contact with any hazardous materials. Toxic finishes and coatings are not allowed by the United States Consumer Product Safety Act [9]. If a materials’ surface has a finish on it, the finish must be deemed safe. Several third party certification systems, such as the GREENGUARD Environmental Institute (GEI), exist to create a standard for qualifying materials in educational environments. Such standards may be useful in determining which materials should be considered for products. Materials that contain formaldehyde, mercury, or lead paint should not be used [9].

Comfort and Cleanability

Materials that are both comfortable and easy to clean should be chosen. These two consideration will encourage children to interact with the design and encourage childcare workers to use it more frequently. All surfaces that can be accessed by children must be able to be sanitized with nonhazardous cleaners. If all or a portion of the design has cloth, ensure it is both easily removable by an adult and washable.

Durability

Childcare administrators and workers reported that several favorite furniture pieces they had purchased had to be replaced often due their fragile construction. A more durable design would be worth the extra investment and lower follow-up costs. Products will likely suffer more abuse by children than by adults and should be designed to withstand such.

Visibility

Visibility in products, particularly those that could conceal children, is crucial for safety. This characteristic was heavily emphasized in both facilities. Transparency can be achieved through the material’s properties (e.g., transparent acrylic) or through design (e.g., cutting a hole in the side of an opaque tent). In childcare facilities, it is absolutely critical that workers see the children at all times.
3.2.2 Functional Design Considerations

Frequently, states will create a Child Care Licensing Committee to define functional requirements that products must meet for use within their respective state [10]. Functional characteristics can be described largely by the following parameters: motion, stability, and usage.

Motion

The motion of deployable furniture may cause harm if not designed properly. Pinch points can be identified as any point wherein any portion of a person’s body may be caught between parts of a machine or material. Pinch points are of particular consideration when designing deployable children’s furniture because moving parts are inherent to the design. These points are typically most accessible during the deployment of the furniture item. As such, small children should be kept away while the furniture is being deployed. Additionally, coverings over compliant or rigid joints can help discourage children from inserting their fingers into small spaces.

Stability

A stable design is one that remains in the same structural state when no intentional motion is used to change it. For example, a small folding step stool would not be expected to close while a user is on top of it. Similarly, no furniture design should allow state changes unless it is willful of the user. Design stability also includes the ability to bear the appropriate amount of weight.

Usage

Other questions that may influence design will likely arise after reading through the Child Care Licensing documentation for a state. For example, can the furniture item be considered a “play structure?” If so, additional regulations will apply as the height of the furniture item is now governed by the age group that is expected to interact with it. How much does the furniture item extend in length/height when deployed? Will it extend into other “use” areas? If so, then it must adhere to pertinent regulations of that area as well. Familiarization with these documents
will ensure that critical dimensions, loads, and other functional requirements/recommendations are considered during the design process.

3.3 Fundamental Mechanical Design Principles for Deployable Childcare Furniture

There are several mechanical design principles that can be used or adapted in order to better create deployable, origami-based childcare furniture. These fall under several categories: space accommodation, materials, folding methods, and joints. Specific ways in which to implement mechanical principles under these fields must be considered as they apply to this unique problem.

3.3.1 Space Accommodation

A key aspect of this research is to save space consumed by large and unwieldy furniture that cannot be condensed. Origami-inspired systems can be beneficial by drastically increasing the ratio of deployed volume to stowed volume and are thus useful for space-strapped childcare facilities. Since these are real-world systems and not idealized paper-thin pieces of art, certain accommodations must be made for the folding and storing of origami-inspired furniture to work. Many different types of materials are found in a childcare setting, including wood, foam, fabric, and plastic. However, as with any folding mechanism, if the material being used has a thickness greater than a sheet of paper, some technique is needed in order to accommodate for thickness using.

Simplicity in childcare furniture is essential. This is because both children and teachers/caretakers have to be able to effectively and correctly use the furniture items. Child safety is also always needed. Several thickness accommodation techniques are preferable because of their seemingly simplistic characteristics. Some thickness accommodation techniques possible for a childcare setting are 1) the tapered panel technique, 2) the offset panel technique, and 3) the membrane technique. All of these various techniques allow for a more cohesive folding process to take place. It is imperative that someone who is unfamiliar with a given furniture item which uses any of these techniques finds it easy to figure out the folding pattern in order to maintain usability. These techniques also allow for a safer product because of the minimization of parts and pinch
points. The following list contains a brief explanation for several useful thickness accommodation techniques:

1. Tapered Panel Technique

   The tapered panel technique is a thickness accommodation technique that preserves the motion and kinematics of a zero-thickness origami pattern. It can be used on many complex patterns and has proven to be a viable option for many designs. To design a tapered panel origami pattern, equal thickness is added to both the top and bottom of a zero-thickness pattern. Then two tapers are cut into opposite edges from the point of zero-thickness to a certain distance away from the edge determined by the desired angle of the folded materials. (For more information see [11] and [12].)

2. Offset Panel Technique

   The offset panel technique also preserves the kinematics of a zero-thickness origami pattern and can be used in arbitrary origami patterns. In the offset panel technique, an origami pattern is chosen and a plane is chosen where all the joints will be. Thickness is decided and added to each panel and self-intersection is addressed. This thickness accommodation technique allows all panels to rest planar and flat against one another in the folded state, but in the unfolded state, the panels lay in planes offset to the joint plane. (For more information see [11], [13] and [14].)

3. Membrane Technique

   Membrane has been explored to act as a jointing technique in certain mechanisms. But in origami-inspired designs, membranes can also act as a thickness-accommodating technique. This is simply done by adding thickness to the zero thickness model and adding gaps for any valley fold in the pattern. In the membrane technique, the kinematics and motion of a mechanism is not preserved, although the design complexity is relatively simple. (For more information see [11], [15]) and [16].)
3.3.2 Material Selection for Deployable Furniture and Joints

Selecting appropriate materials for deployable furniture is critical to overall stability, safety and ease of use. Young’s modulus of elasticity, strength, environment, density, susceptibility to creep and stress relaxation, and predictable fatigue life are material properties that become increasingly important when compliant joint are used in a design [17]. As discussed by Dr. Larry Howell, “... materials for compliant mechanisms are chosen to maximize flexibility rather than stiffness” [17].

Well-suited materials could be corrugated polypropylene, rotomolded plastic, or injection molded plastic. Such options could provide the strength, weight, and flexibility required for full-size folding furniture items. Metals such as steel, stainless steel, aluminum, titanium, and others could be possible options if metal is required [17].

3.3.3 Folding Methods

Origami is the art of paper folding, and it has been around for hundreds of years. The art form has influenced engineering design in space arrays [18], shelters [4], emergency management [19], architecture [5], biomaterials [20], and more. Most forms of origami consist of a flat sheet of paper intricately folded into a three-dimensional shape, and many of the techniques found in origami are only possible because of the unique characteristics of a sheet of paper. As such, when using diverse materials in engineering designs inspired by origami, many issues such as stress/strain, curved folds, distributed folds, kinematics of motion, dynamic behavior, external forces, nonflat/non-Euclidean surfaces, significant cutting, and thickness must be considered [11].

Furniture used in a childcare setting often benefits from being simple, compact, and durable. There are many reasons for this. Childcare spaces are often small, so furniture needs to be designed with space in mind. Children and caretakers need to have the ability to rigorously use furniture in this setting. Designs that consume a lot of space are undesirable in many childcare settings because space costs money, and education centers and daycare centers are underfunded in many areas of the world.

Because of the need of more space and simple designs, childcare furniture could be greatly impacted by integrating origami into its design. Integrating simple folding methods into furniture
can create compact designs that are intuitive to fold. Simplicity can be achieved in folding patterns by decreasing the amount of folds in a design and minimizing vertices, especially large degree vertices. Often, complicated origami models and designs are used to solve engineering problems, but there is no need to implement those very complex designs in a childcare setting. In fact, highly complex designs with complicated folding patterns could make the design less desirable to a caretaker because of its lack of intuitive use and high quantities of pinch points.

### 3.3.4 Joints for Origami-Inspired Systems

Furniture joints come in a large variety of shapes, sizes, materials, and configurations. Joints used specifically for origami-inspired furniture can range as well, and can be completely dependent on the folding pattern, loads carried, and whether or not it is a compliant or rigid system. This paper defines joints as a system of rigid or flexible components that connect rigid panels together and allow for motion of panels relative to each other.

Flexible, compliant joints could be made of a typically rigid material like plastic, but structured in such away to allow for bending motion. The LET joint is an example of this and has been demonstrated in numerous configurations [21], [22].

Creating joints for folding systems is fairly difficult, as accommodations must be made for them and care must be taken as to what kind of joint methods must be used for the given application. For furniture used in a childcare setting, even more consideration must be given for joint selection, as furniture must satisfy more requirements in such a setting than otherwise.

Safety is of the utmost importance in a childcare setting, but joints can be inherently quite dangerous for children. One of the big dangers with joints is the presence of pinch points. With typical hinges like pin joints, the moving parts present a hazard for children to get their fingers stuck or to be exposed to a lot of friction. A membrane joint could be more desirable as a safety measure, as membranes typically consist of soft fabrics. However, membranes often do not have the same sort of structural integrity that pin joints have. That being said, membranes could be used in a hybrid configuration with pin joints to use the benefits of both types of joints. For example, a pin joint could be covered by a membrane such that the pinch points are hidden while the structural rigidity is maintained.
Another important consideration for designing joints for a childcare setting is the fact that childcare furniture needs to be able to withstand rugged use. Children tend to be more aggressive with their furniture than adults, and subject such objects to a larger variety of forces from a variety of directions. Joints must be designed in a way such that, unless they are being deliberately actuated, they should be fixed in place, even when undergoing thorough use by children of various sizes and energy levels. Not only is this problem configuration-dependent, but material-dependent as well. Standard pin joints are quite reliable, and can be subjected to large amounts of force without failing. In contrast, membrane joints have variable responses, as their strength is dependent on the properties of the membrane material itself. Compliant, flexible joints could be useful because of their low parts count, ease of fabrication, and maintainability. However, flexible joints like “living hinges” and LET joints can be fragile depending on their thicknesses and configurations. Compliant flexures could be enhanced/protected by membranes in a hybrid setup in order to have the best of both worlds. Figure 3.1 shows a hybrid joint design where a traditional LET joint is augmented by an added adhesive membrane, known as a membrane-enhanced lamina emergent torsion (M-LET) joint.

![Figure 3.1: A membrane added onto a LET joint to enhance and protect joints, known as an M-LET joint](image)

3.4 Principles Applied to Functional Furniture Designs

3.4.1 Hidey-Spaces

Just as adults cope with emotion in differing ways (running, reading, quiet time, etc.), children also need space to do likewise. Many childcare facilities utilize “safe spaces” as a place where
children may go when they experience strong emotions or sensations. These spaces allow children to learn coping mechanisms and to develop healthy emotional routines. While these products are invaluable, they often occupy a large amount of space, even when the product is not in use. A design that acts as a deployable, rigid safe space for children will help facilities conserve valuable space when a safe space is not in use. Childcare facilities often lack furniture options that are functional and stowable. By applying the principles previously discussed to a safe space this industry can improve childcare spaces by using safe, functional, and deployable pieces of furniture. The following two furniture items are innovated designs for a safe space in a childcare setting. Aptly named “hidey-spaces,” they provide examples of how folding principles can be implemented into childcare furniture design.

**Hidey-Cube**

A product currently on the market and seen in use at childcare facilities consists of a rigid, plastic cube with holes on the sides where children can enter. While useful as a safe space, such a product takes up a lot of valuable space when not in use and can be costly to purchase. By using origami-inspired techniques to improve the existing product, a new, collapsible safe space for children shown in Figure 3.3. This design is identifiable by its foldable sides which allows the structure to have a compact storage size. Being able to collapse a piece of furniture like this allows the deployed volume to be larger than a doorway, which is what currently limits the sizes of comparable products on the market. The folding method presented for the Hidey-Cube allows for a drastic reduction in volume and eliminates the need for assembly/disassembly when moved in and out of spaces. The wood prototype shown in Figure 3.2 uses a simple Sarrus fold to collapse the structure into a flat state that is only five panels thick. The bottom panel folds up parallel to the panel without a hole. The top remains open, and the three remaining sides have large holes in them to allow the children access.

The exploded view of the wood Hidey-Cube, shown in Figure 3.4, serves as a reference for the parts that were involved in its assembly. The thickness of the panels is accommodated by thin wood rectangular prisms, which can be seen in Figure 3.4.

Compliant joints can also be implemented into the Hidey-Cube along the middle of the two sides that fold inward, and can serve as substitutions for the pin joints used in Figure 3.3. However,
to maintain the thin profile of the folded structure, acrylic can serve as a potential material. Acrylic is also useful as a material for childcare furniture because of the visibility it provides the childcare workers. Figure 3.5 shows the Hidey-Cube constructed from acrylic and compliant joints on the sides. The joints chosen are the M-LET joints shown in figure 3.1. These joints provide adequate flexibility while limiting undesired motion. No breakage in the flexible joints was noted. Figure 3.6 shows how the folding happens with the flexible joints. The completely folded Hidey-Cube is shown in Figure 3.7.
The Hidey-Tent is an extremely simple design that implements folding principles into child-care furniture and also create a safe space for children. The idea is to reduce the structure to the simplest shape possible that could still be folded. This results in the four-panel triangular shape. The design utilizes a traditional triangular tent shape and is intended to fit only one child. The prototype consists of four plywood panels with hinges at each vertex to allow it to stow in a flat, compact form. One panel has a circular hole to allow for visibility, while the openings on each end enable a child to climb in and out. Wood was chosen for its durability and ability to be cleaned.
It is also relatively easy to machine. Figure 3.8 shows how the Hidey-Tent appears in its deployed state.

Thickness accommodation techniques were required in order for the Hidey-Tent to fold completely flat. The technique chosen was the offset-panel technique. This involved adding 3D-printed prisms to the top edge that are each the width of a single panel. This allows the bottom panels to fold upwards and for the folded system to be completely flat. Figure 3.9 demonstrates the configuration of the folded Hidey-Tent.

### 3.4.2 Application in Future Designs

Origami principles are seen in several furniture applications already. There are many different kinds of folding tables and chairs that utilize some of the folding principles discussed. One example is an origami-inspired bench designed by blackLAB architects inc. [23].

Often, as seen in this origami-inspired bench, current origami-inspired designs are used to create an appealing aesthetic in certain interior designs. Some are also used for space saving purposes, but very little has been done to further innovate origami-inspired furniture for practical use, especially in a childcare setting. Innovating current furniture, by applying origami principles
Figure 3.5: The deployed Hidey-Cube constructed from acrylic and M-LET compliant joints could be greatly beneficial in a childcare setting, as well as in any other setting, so as to create products that are easy to use, space-saving, educational, and cost-effective.

Some furniture items in a childcare setting that could be further improved by applying origami principles in their design are: sensory tables, sensory spaces, children’s chairs, art racks, stepping stools, changing tables, and climbing toys/play stations.
Figure 3.6: The semi-folded Hidey-Cube constructed from acrylic and M-LET compliant joints

The origami-inspired principles demonstrated with both the Hidey-Cube and the Hidey-Tent work as effective kinematic structures for childcare furniture. Folding methods and thickness accommodation techniques can be used to create furniture that is intuitive to use and durable enough to withstand a rugged use. As discussed previously, safety is of the utmost importance. Information is provided in order to discover various requirements for a furniture item to be legally
Figure 3.7: The completely folded Hidey-Cube constructed from acrylic and M-LET compliant joints

...licensed to use in a childcare setting. Additionally, several types of joints have been discussed which accommodate for a variety of different designs. Quite possibly, creating compliant joints out of a material different from the material used for the rest of the furniture (e.g. polypropylene, carbon fiber, etc.) could improve the flexible behavior and strength. Such hinges could be fabri-
cated separately from the body of the furniture piece and then “slotted-in” to the structure to form a finished object.

3.5 Conclusion

Not only is childcare furniture an important design space, but also a difficult area in which to innovate. However, by applying principles of origami-inspired design and compliant mechanisms, new techniques and strategies for the development of furniture for childcare spaces have been developed and can be applied to future designs.

Principles of origami-inspired joint design, thickness accommodation, material consideration, and folding methods were demonstrated in several configurations for secure spaces for childcare settings. The methods for creating these secure spaces could be generalized to other types of furniture with careful thought and engineering. The Hidey-Tent and the Hidey-Cube showcase the offset-panel technique used to maintain a flat form when folded. The M-LET joint is shown to be a promising technique for pin joint replacement in a simple folding piece of furniture that, while not load-bearing, maintains the desired kinematics of the noncompliant version.
By creating childcare furniture that is both functional and space-saving, childcare facilities can save money and increase the versatility of their spaces. This could thereby help provide improved care and help to foster growth in young children.
CHAPTER 4. LASER FORMING OF COMPLIANT MECHANISMS

4.1 Introduction

Compliant mechanisms are being designed with increased complexity requiring more precise and flexible solutions to achieve desired performance [24], [25]. They have been used as a solution for systems which require simplicity in form while retaining complex motion, but such mechanisms are challenging to form and assemble on small scales [26]. Many such mechanisms require feature precision on the mesoscale (0.1–5 mm) and low macroscale (5–100 mm), which can be difficult to create [27]. Metals are used in compliant mechanisms for their various desirable properties (e.g. electrical/thermal conductivity, durability, recyclability, and cleanability).

Metal 3D printing is a current method of fabricating metal compliant mechanisms, but the technology can result in component anisotropy, high production costs, and a discontinuous manufacturing process that present challenges in some applications [28]. However, there is some research starting to be done on improving additive manufacturing for metal compliant mechanisms [29]. Outside of additive manufacturing, metal is difficult to machine on the mesoscale, with electrical discharge machining (EDM) being one of the few viable options. EDM has been shown to successfully manufacture extremely small metal compliant mechanisms [30]. EDM has also been used to manufacture bi-stable compliant mechanisms on the macroscale out of bulk metallic glass [31]. However, EDM requires expensive and complex machinery with high tool wear ratio and low surface quality [32]. Additional advances are needed to fabricate compliant mechanisms and flexible systems on the mesoscale in thin sheet metal.

Laser forming is a fabrication process involving the controlled deformation of a workpiece through the use of a laser to introduce plastic thermal stresses and is typically used on large scales and very simple geometries. However, this process has recently been explored on the mesoscale to fold rigid structures [33]. But without moving components, the ability for these structures to
form more complex mechanical systems are limited. Fortunately, laser forming has potential to fabricate more than just rigid shapes and structures.

This work develops laser forming as a method for creating simple compliant mechanisms and shows how this fabrication method could be useful in manufacturing such mechanisms in sheet metal on the mesoscale. To achieve this goal, the research focuses on the design, fabrication, and force-deflection testing of three simple compliant mechanisms (or “building blocks”). The building blocks of these designs will be broken down into features and discussed. The building-block mechanisms will then themselves serve as the basis of two more complex mechanism applications. This will demonstrate the first usable compliant mechanisms created by laser forming.

4.2 Background

This research space requires an understanding of the current state of the art of compliant mechanisms and laser forming, especially how they each relate to thin sheet metal. The field of compliant mechanisms is quite broad and covers a variety of sizes (micro to macro) and many different kinds of materials. Compliant mechanisms designed to be fabricated in a flat plane have been explored to various degrees in plastics and metals. Much research has been done on laser forming as well, including different kinds of metals with sheets of various thicknesses.

4.2.1 Compliant Mechanisms

Compliant mechanisms consist of flexible members in mechanical systems and can serve as replacements for traditional mechanical components. A variety of materials and structures have been used to design compliant mechanisms, though metals and polymers are common materials used to develop compliant components. The principles of compliant mechanisms can be used to change existing mechanical designs using rigid-body replacement [17] or to develop innovative new solutions to mechanical problems. Compliant components can enable the folding of materials into different configurations [34]. Such mechanisms are a way to achieve mobility with flexible members without the need for mechanical joints and have been applied at both the micro and macro scales.
Compliant mechanisms have been more recently used to approximate origami systems because of their flexibility [35]. As such, flexible members can be used for many possibilities when they are substituted for typically rigid members so they can be utilized in unique ways to achieve complex behavior [36]. Techniques for fabrication of such systems are valuable because all components are folded from a flat configuration into a desired shape.

The current methods of manufacture for metal compliant mechanisms are 3D printing (e.g. sintering, powder metallurgy, etc.), EDM, stamping, milling, and manual cutting and bending. Each method has inherent advantages and disadvantages. 3D printing metals is time consuming, expensive, and is ineffective for production of thin, smooth structures. EDM was demonstrated by Miller et al. [30] to create thin, compliant mechanisms. While EDM does not require hard tooling, it requires large moving systems and electrically conductive metal to work. Stamping is tedious because many progressive dies must be made for each unique shape. Milling can be precise if automated, but is difficult to manufacture thin features with without destroying them with the high forces. Another complex manufacturing method for metal compliant mechanisms was proposed by Hayes et al. with the lost mold-rapid infiltration forming process (LM-RIF) [27]. Manual cutting and bending by hand is imprecise, labor-intensive, and impractical for large scale production. Traditional fabrication tends to perform better on parts with a lower aspect ratio (overall size to minimum feature), while compliant mechanisms typically require thin dimensions of high aspect ratio (overall size to compliant feature thickness).

Current fabrication processes for compliant mechanisms do not require the motion of the features on the workpiece to move them to their final position.

**Single-Plane Mechanisms**

Sheet metal that is machined in such a way that complete compliant out-of-plane mechanisms are formed are only starting to be explored. However, compliant mechanisms have been created so that after fabrication they can fold out of a single plane and are called “lamina-emergent mechanisms” (LEMs) [37]. These mechanisms are manufactured in a single plane (typically with polymers, but also with sheet metal), and their final manufactured shape is also still in-plane. Only when actuated do these mechanisms move out of their original plane.
While mechanisms like LEMs that can bend out-of-plane after manufacture have been examined, manufacturing sheet metal compliant mechanisms to have geometries out-of-plane has not readily been explored. Because of the difficulty in fixturing thin sheet metals to prevent damage during fabrication, traditional manufacturing methods for metal compliant mechanisms are not robust enough to create intricate flexures without compromising the structural and mechanical integrity of the piece.

### 4.2.2 Laser Forming

Laser forming is the specialized use of a laser to induce thermal stresses in metal in ways that cause the metal to bend. Final geometries are achieved by the combined deformation of all of the laser actions [38]. This technology was first described and developed to create rigid shapes out of metal by Geiger et al. [39]. Laser forming is a very complex process with many parameters and effects, but such aspects have been studied in depth since the method was first introduced [40]. Laser forming has been used to fabricate features on a large scale in the shipbuilding and automotive industry [41], and more recently methods have been developed to form more complex structures with features on the low end of the macroscale.

Two key methods of laser forming exist, 1) the temperature gradient mechanism (TGM), and 2) the buckling mechanism (BM) [42]. Both methods use a laser to induce thermal stresses in the metal. The thermal gradient mechanism introduces just enough stress in the piece to cause a crease to form that bends the metal as the metal cools. The buckling mechanism is based on creating a lateral thermal gradient (a heated membrane region), that pushes outward and causes the membrane to buckle. The buckling direction is random in the absence of other factors, and can go up or down [43]. It is possible to control this direction consistently by introducing a pre-strain, but it is difficult to replicate a consistent pre-strain in a more complex pattern. For this reason the TGM was chosen as the method of laser forming for this work.

Laser forming is a nontraditional manufacturing method that is used to shape materials in ways that are difficult otherwise. Much research has been done to bend metal foams into difficult shapes using laser forming [44], [45], [46]. The technology of laser forming has also been used on polymers to achieve complex origami-inspired shapes because of the sequential nature of the
method [47]. Such work is useful in understanding how origami and kirigami-inspired shapes can be achieved with laser forming.

Past work by several of the authors demonstrates laser forming used on thin sheet metals with added complexity, combined forming mechanisms, and hands-free processes [33]. Techniques for large scale bends were adapted in that work to a small scale with 0.0762 mm thick stainless steel. Rigid structures such as a six-sided cube and a paper-airplane-like shape were formed with both temperature gradient mechanism laser forming and buckling mechanism laser forming [48]. This research provides the groundwork for laser forming mechanisms designed to bend and flex.

4.3 Building Block Design and Fabrication

To explore the possibilities laser forming offers for the fabrication of compliant mechanisms, three simple existing compliant mechanisms were selected and developed for the laser-forming technique (the parallel-guided mechanism [49], cross-axis flexural pivot [50], and a lamina emergent torsional (LET) joint array [51], [52]), along with two more advanced mechanisms (split-tube flexure [53] and bi-stable switch [54]), as shown in the top row of Figure 4.1. These five particular compliant mechanisms were chosen because they can be used as building blocks to create more complex mechanisms and varied nature from each other. The parallel-guided mechanism can demonstrate fixed-guided beam behavior restricted to a single axial degree of freedom perpendicular to the compliant beam, the cross-axis flexural pivot can demonstrate fabrication features that have a single rotational degree of freedom by layering compliant features through folding, and the LET joint array can show augmentation of torsional beams to restrict undesired motions. The more complex mechanisms, the split-tube flexural pivot and the bi-stable switch, can demonstrate the forming of curved flexible shapes and actuatable systems respectively. These mechanisms and the features that comprise them can serve as building blocks for later, more complex designs and were modelled after existing designs manufactured in other mediums.

The particular desired behaviors for each mechanism are shown in the bottom row of Figure 4.1 for the three simple mechanisms and the two more complex mechanisms. The parallel-guided mechanism in Figure 4.1f undergoes an applied displacement, the cross-axis flexural pivot and LET joint array (Figure 4.1g and Figure 4.1h) undergo an applied moment, the split-tube flexure is
Figure 4.1: Existing simple compliant mechanisms and their expected motion. a) static parallel-guided mechanism. b) static cross-axis flexural pivot. c) static LET joint array. d) static split-tube flexure. e) static bi-stable switch. f) displacement of parallel-guided mechanism. g) displacement of cross-axis flexural pivot. h) displacement of LET joint array. i) displacement of split-tube flexure. j) displacement of bi-stable switch.

subjected to an applied radial displacement (Figure 4.1i), and the bi-stable switch is moved to its second stable state with an applied force (Figure 4.1j).

4.3.1 Mechanism Design

The building block mechanisms were designed such that they would fit within a 4.5 by 4.5 centimeter square, as that was roughly the area within which the chosen laser cutter maintained a solid focus. However, larger sizes and areas could be feasible if using lasers with a larger workspace in which they are focused. Two-dimensional patterns were drawn and laser parameters were assigned to individual lines so the laser cutter would know which segments needed to be cut and which needed to be folded. Laser forming is a subtractive manufacturing method, thus care is needed when designing a mechanism to be cut and formed out of a single workpiece. Designing patterns for such a process is similar to the art of Kirigami, in which a single sheet of paper is cut
Figure 4.2: Orientation of features for parallel-guided mechanism.  a) rigid segment formed by orienting two panels perpendicular to each other.  b) compliant flexures formed by orienting narrow segments perpendicular to rigid segments at the ends.  c) base of mechanism formed by orienting the entire mechanism perpendicular to the base of the workpiece

and folded into shapes. However, instead of hands folding paper, the laser forming process uses a laser to fold a sheet of metal.

**Reorientation**

The behaviors of compliant mechanisms depend on feature dimensions and feature orientation about a bend axis to perform intended functions (assuming set material properties). For instance, a flexure that has a thickness larger than its width will be much more difficult to bend than the other way around. Because the thickness of the sheet metal is set as constant, other parameters must be changed to achieve desired behavior.

Laser forming can cut and bend features of a workpiece into configurations that have desired stiffnesses. This can be controlled by the orientation of individual mechanism features. For instance, rigid features can be formed by bending two metal segments into an L-beam shape. Also, compliant flexures can be formed by having the ends of the metal segment perpendicular to rigid sections, so that the flexure can be formed by the laser at the two ends, but the flexure itself is simply a thin bending beam. Through techniques like these, the stiffnesses of individual mechanism features can be defined in the design. The left side of Figure 4.2 shows how a pattern was designed
for it to be laser formed. The laser was programmed to either cut or fold the designated lines, and become the folded features shown on the right side of Figure 4.2.

4.3.2 Laser Forming Setup

The laser used was a Full Spectrum 1060nm Nd-YAG engraving laser. Laser cuts were made with 50 percent power at 500 mm/s for 400 loops. To laser form 90 degree folds, the laser was run with 10 percent power at 30 mm/s for 5 loops, repeated 20 times. Studies have been made by Sentoku et al. on calculating input parameters to achieve specific laser-formed angles [55] using a CO2 laser. However, since an Nd-YAG laser was used here, only estimates and iterative design were used to achieve the desired angles. The stainless steel workpiece was 0.0762 mm thick and was held down by a brass frame and magnets, for heat dissipation and stability respectively. The fixture is shown with the laser-formed LET array in figure 4.3. The results of this fabrication process are shown in section 5 for each mechanism.

Figure 4.3: Fabrication fixture for laser forming
4.4 Cutting and Forming Process

The process of laser forming is highly sequential, with the laser following a single path at a time. Each feature fabricated can be formed by the laser in specified orders. Thought must be taken in deciding the order in which the features of a mechanism are formed. Figure 4.4 shows the process of cutting and forming used to cut, orient, and reorient individual features to fabricate a complete laser-formed cross-axis flexural pivot.

4.4.1 Approach for Experimental Tests

The force-deflection relationship for each of the mechanisms was tested using an Instron 3300 tensile tester with an Interface SMT 1-1.1 S-Type load cell affixed. This allowed for millimeter-sized displacements to be applied and the resultant force to be measured. Magnets held the pieces in place to provide a secure ground and to inhibit undesired motion. Each mechanism was tested repeatedly and multiples of the same mechanism were tested to account for variation between each fabricated piece. A force-displacement curve for each mechanism was developed based on the tests. Figure 4.5 shows the setup for the test system.

4.5 Fabrication and Testing Results

The results for the fabrication and testing for the three building block mechanisms are shown followed by the fabrication results for the two more complex mechanisms.

4.5.1 Building-Block Mechanisms

Three mechanisms: the parallel-guided mechanism, cross-axis flexural pivot, and LET joint array, were fabricated and tested to show force-deflection behavior. These mechanisms demonstrate different desirable properties of compliant mechanisms that can be fabricated with laser forming.
Figure 4.4: Sequential cutting and forming of cross-axis flexural pivot. a) Step 1 - cut panels. b) Step 2 - cut edge of flexures. c) Step 3 - form panels. d) Step 4 - orient flexures. e) Step 5 - cut panels. f) Step 6 - cut rigid segments. g) Step 7 - orient rigid segments through forming. h) Step 8 - form panels. i) Step 9 - cut base edges. j) Step 10 - form base to bring both mechanism sides together.
Figure 4.5: Testing fixture of laser-formed parallel-guided mechanism
Parallel-Guided Mechanism

Parallel-guided mechanisms allow for in-plane motion while the flexures remain parallel with each other. The resulting design for a laser-formed parallel-guided mechanism maintains typical components for such a mechanism, including rigid top and bottom segments with flexures on the right and left sides. The most important consideration for designs like this is for the forces applied to not be distributed through the laser-formed creases.

To create compliant segments with desirable behavior and isolated the motion, the flexible members were oriented to be perpendicular to the direction of motion. Sections at the top and bottom of each flexure were connected to the rigid sections as space is needed for the laser forming to occur. The triangular shapes that share an interface with the flexures shown in Figure 4.6a accomplish this purpose and it is these rigid sections from which the flexures are formed. These techniques demonstrate a key aspect of creating laser-formed compliant mechanisms; features can be oriented by the laser to tune their stiffnesses by creating desired cross-sections.

The completed mechanism shown in figure 4.6c demonstrates a successfully laser-formed piece. The laser was able to cut where specified and fold the flexures to their expected angle. With simple creases like these, the metal is not able to fold past 90 degrees, as the laser is directly above the workpiece so it could not make focused contact with the metal if material is in the way. Figure 4.6b shows how the mechanism behaves when loaded as intended.

The experimentally tested mechanism exhibits repeatable behavior over multiple tests, with the mean shown in figure 4.7. As expected, the first actuation of the mechanism after fabrication was an outlier in the data because of stresses being relieved in the system, so that data is not shown. However, for each subsequent test the mechanism displayed a nearly identical force response.
Multiple parallel-guided mechanisms were fabricated and tested, with results being consistent with each other. Figure 4.6b shows how the system behaves when loaded. This motion is similar to the expected behavior shown in Figure 4.1f.

Because the creases were subjected to concentrated heat from the laser, the device failed at the fold when over-actuated. However, further process development could understand, quantify, and mitigate the damage. It was therefore important to ensure that the force was distributed almost solely through the flexures.

Analytical models shown by Howell [17] for this kind of mechanism can be used to predict motion for idealized parallel-guided mechanisms, and do not take into account the unique attributes of laser-formed mechanisms.
Cross-Axis Flexural Pivot

The cross-axis flexural pivot is useful because it can simulate a pin joint but with limited rotation, depending on the boundary conditions and material stresses [56]. The design of the laser-formed cross-axis flexural pivot shares many of the building blocks with the laser-formed parallel-guided mechanism. However, the key difference is that the flexures have to overlap and cross each other to perform as desired. To do this, the mechanism design was essentially created by cutting and forming two mirrored sections, and then bringing them together to interlock with each other and therefore act as a single mechanism. The fully formed mechanism can be seen in Figure 4.8c. Note that the rigid segments at the top of both halves overlap each other to join the sections together. To increase efficacy, these segments should be fixed together (e.g., welded, taped, etc.), though the mechanism still works desirably even with the imposed constraints seen in Figure 4.8b. This motion is similar to the expected behavior shown in Figure 4.1g.

The ability to orient flexures to overlap and cross shows that it is possible to form a mechanism that performs a single rotational degree of freedom even though seemingly disparate features are used. The fact that the rigid sections at the top of the mechanism have been oriented to face each other helps the whole piece maintain its shape with or without loading.

The mechanism was tested by adhering a string to the rigid segments and the measurement device. The string was then pulled and the resultant forces measured. The curve in Figure 4.9 demonstrates the relationship between the applied displacement and the measured resultant force.

Analytical models shown by Jensen et al. [56] for this kind of mechanism can be used to predict motion for idealized systems, though they do not take into account the unique attributes of laser-formed mechanisms.
Lamina Emergent Torsional Joint Array

Lamina emergent torsional (LET) joint arrays enable increased bending via the torsional motion of individual beam members. They are made up of multiple LET joints (developed by Jacobsen et al. [57]) in series and in parallel. The shape is straightforward to cut and fold using lasers because, unlike the mechanisms shown previously, individual flexures do not need to be formed. Instead, the laser forming was done at the base in order to orient flaps out-of-plane for each of the holes. These flaps reinforce the structure from undesired axes of motion, while still allowing for motion in the desired bending direction. The flaps can also be shortened or removed in areas where reduced torsional stiffness is desired, thus “tuning” the stiffness of the mechanism. Figure 4.10a shows how this mechanism looks once formed. Care must be taken with the laser
Figure 4.10: Laser-formed LET joint array. a) static, b) actuated, c) isometric

settings, as cutting and forming so many small shapes so close to each other concentrates heat, and warping occurs more easily.

The array was able to be actuated as intended as demonstrated in Figure 4.10b, which compares to the expected behavior shown in Figure 4.1h. The curve shown in Figure 4.11 for the force-displacement relationship demonstrates the average force response for 10 repeated tests on a laser-formed LET joint array. Very little plastic deformation occurred, and the results were consistent with one another.

Mathematical models developed by Pehrson et al. [51] can be used to predict motion for idealized LET joint array systems.

4.5.2 Complex Mechanisms

Two mechanisms, the split-tube flexure and the bi-stable switch, were fabricated to see how principles learned from three building-block mechanisms could be implemented to form more complex shapes. While the force-deflection relationship was not measured, these mechanisms still exhibit repeatable and expected behavior for such systems.

Split-Tube Flexure

Split-tube flexures allow for torsional motion while keeping its shape in both compression and bending [58]. Segments of the tube were folded by the laser starting from the outside and working inwards. On one end of the tube, a panel was folded perpendicular to the base of the
workpiece to act as a handle for actuation. As can be seen in Figure 4.12a, the cylindrical shape was achieved.

To prove repeatability, the mechanism was actuated by hand as shown in Figure 4.12b. The tube was able to be twisted both to the right and the left, demonstrating desirable expected behavior shown in Figure 4.1i.

Analytical models shown by Howell [17] for this kind of mechanism can be used to predict motion for idealized split-tube flexures, although this would be for a perfect curve, and not for a curve made of discretized segments. Although the method shown for achieving curvature for this mechanism accomplishes the desired function, smoother gradients in the shaping could be achieved with laser forming as demonstrated by Cheng et al. [59].

Figure 4.11: Measured force-displacement relationship of LET joint array
Bi-Stable Switch

Bi-stable switches can move between two stable states of the mechanism, and the flexures allow for the mechanism to move between those states [60]. Such switches can be used in complex configurations [61]. Using flexures designed similarly to those in the parallel-guided mechanism and cross-axis flexural pivot, the bi-stable switch demonstrates that the method of laser forming can be expanded for mechanisms with a variety of behaviors. The flexible members were folded up perpendicular to the plane in a way similar to previous mechanisms shown for the forces to not be distributed through the creases, but rather through the flexures. Figure 4.13a shows the implementation of these techniques in a laser-formed bi-stable switch.

While the mechanism was able to achieve repeated actuation between the bi-stable states, significant plastic deformation occurred to accommodate the motion shown in Figure 4.13b.

The design for this mechanism was inspired by principles demonstrated by Opdahl et al. [54] and Zirbel et al. [60]. The models they developed can be used to compute force-deflection relationships for idealized versions of bi-stable mechanisms.
4.6 Lessons Learned

The mechanisms developed performed desirably, as repeated motion was achieved for all five mechanisms, with force and deflection able to be measured for the parallel-guided mechanism, cross-axis flexural pivot, and the LET joint array. There were several key challenges with this technology: 1) designs are limited to being formed out of a single sheet, necessitating careful orientation of features, 2) intended forces must take care to not apply undue stress to the laser-formed creases, 3) the laser can cause warping while cutting because of its high thermal output, and 4) the temperature gradient mechanism of laser forming was more reliable than the buckling mechanism.

At the moment, the single sheet design constraint proves difficult to form with the laser for anything more than simple mechanisms. However, multiple sheets could be laser formed and joined together (via welding, adhesive, locking, etc.) to create mechanisms which are more complex, and have multiple-degrees of freedom. While many modern rapid prototyping methods use an additive manufacturing process, the laser forming method is subtractive; care must be taken in designing compliant mechanisms so that they can be “subtracted” out of a single sheet. Applying principles of orientation shown in this work allows the ability for both rigid and compliant features to be formed out of a workpiece of a single thickness.

The laser-formed creases are inherently weaker than the rest of the sheet, as the metal has been heated and cooled repeatedly [62]. Designs must be designed to limit forces and stresses distributed through any of the creases. Post-processing thermal treatment such as annealing could be used to regain any material hardening at the fold.

Thermal factors must be considered when created laser-formed compliant mechanisms, as the laser will heat up the metal in different ways depending on the laser power, laser speed, repeated passes, and metal used. If more heat is applied to the workpiece than can be dissipated, then the metal will warp to accommodate for the added heat [63]. This warpage can prevent the laser from cutting or forming if the warped section of the workpiece is no longer in the focal range of the laser. Even if the laser can successfully cut and form the workpiece, the resulting mechanism may not function as desired because the flexures or rigid sections could be too warped to behave as intended.
The temperature gradient mechanism (TGM) enables the metal to be bent into “valley folds.” Which is to say, in origami parlance, that the metal can only be folded up towards the laser and not the other direction (mountain folds). The buckling mechanism can be used for downward folds, but it is much more difficult to consistently achieve folds this way. Authors have demonstrated that pre-stresses would need to be applied to the metal to get it to tend away from the laser. These factors limit the types of shapes which can be created, as designs must take into account the use of solely upwards folds if TGM is the only forming mechanism implemented.

4.6.1 Application

The building blocks of laser-formed compliant mechanisms can be implemented in a variety of ways, with each other and with other rigid components. The flexures developed for the parallel-guided mechanism, for instance, can be stacked in an array as shown in Figure 4.14.

Laser forming of compliant mechanisms could be useful when manufacturing in locations with limited machinery, as laser metal cutters/engravers can be relatively affordable and compact. Being able to prototype or create replacements for complex metal systems can be a boon in a variety of situations, such as in the field, at a research lab, or at any kind of company where small, metal, compliant components could be useful. As a rapid prototyping method, the techniques presented have the ability to be an accurate and cost-effective solution.

Other applications could include small tunable antennas, locking mechanisms, and large scale approximations of microelectromechanical systems (MEMS). The design space for applications is vast, with a variety of practical uses.

4.7 Conclusion

The fields of laser forming and compliant mechanisms have been successfully joined with the demonstration of five compliant mechanisms cut and formed using only a laser. This research demonstrates that compliant mechanisms can be laser formed and exhibit desirable repeatable behavior. Simple flexures can be laser formed, oriented, and combined in more complex shapes to achieve a variety of force-displacement behaviors. Five compliant mechanisms were cut and formed and the techniques for designing such mechanisms were explored. The ability to create
compliant mechanisms out of thin sheet metal can be useful in replacing polymer components of similar size because of the unique properties of metal. Also, such mechanisms could be faster to fabricate with laser forming than with traditional methods.

The next step is to implement the principles of this research in designing and fabricating increasingly complex mechanisms with real-life applications. Different types of metals could be explored, like spring steel, brass, and aluminum. Further analytical modelling can be done to predict behavior of laser-formed mechanisms.
CHAPTER 5. CONCEPTUAL STUDY OF A FLAT-FOLDABLE DEPLOYABLE EULER SPIRAL PARABOLIC REFLECTOR

5.1 Introduction

A need exists for parabolic reflectors that can be efficiently created, flat-foldable, and easily deployed, as many reflectors are completely rigid or difficult to stow and deploy. The general shape of a parabolic reflector can be difficult to stow or transport due to its volume and curvature. For applications such as antennas and solar reflectors, the ratio of stowed to deployed volume can be important for transport and multipurpose use.

By their nature, curves can be difficult to compress. Many flat-foldable shapes consist of flat panels and discrete angles instead of continuous curves [64]. Origami-inspired systems can be used to create mechanisms that can be deployed after being folded flat [65]. While not paper, such systems can draw from the methods for folding and unfolding nearly zero-thickness materials on the macro scale. Using thin sheet metal, a system could be created that could fold into a nearly two-dimensional plane, and be opened up into a three-dimensional parabolic reflector.

To achieve the flat foldability of a metal parabolic reflector, unique geometry is required for compliant curves to lie flat. The sides of the parabolic reflector can be discretized and approximated as Euler spiral sections. Such geometry is designed to allow mechanisms to lie flat and stow strain energy [66].

Forming Euler spiral-based curves out of sheet metal could be possible through means of laser forming, where the laser causes the metal to bend by inducing thermal stresses [48]. Laser forming has been used to create rigid shapes, compliant mechanisms, and nearly-continuous curves [33], [67], [68]. Laser forming can be a fairly efficient way to create the curves for the petals of the paraboloid and approximate a continuous curve closely without the need for expensive stamps or dies.
The parabolic reflectors developed with these methods could be used for a variety of applications: solar reflectors, antennas, and on-site/space manufacturing methods. This work explores a concept for an Euler spiral parabolic reflector (ESPR) through possible geometries and configurations.

5.2 Background

To understand the approach to making a flat-foldable paraboloid, it is important to understand the background for the geometry of the Euler spiral, parabolic reflectors, the technology of laser forming, flat-foldable compliant systems.

5.2.1 Euler Spiral

The geometry of the Euler spiral was developed by James Bernoulli and is defined by the linear relationship between arc length and curvature [69]. Advanced use of the Euler spiral for its mechanical benefits was demonstrated by Yellowhorse et al. [66] and Ynchausti et al. [70]. Such work shows that by using the Euler spiral, shapes can be made that are designed to lay flat when a force is applied at the free end.

5.2.2 Folding Parabolic Reflectors

Parabolic reflectors come in a wide array of shapes and sizes and can function in a variety of roles, including as solar collectors and antennas. Such systems have been turned into folding reflectors by many researchers to make more compact systems. For example, a shape-memory polymer parabolic reflector was developed to be thermally actuated to “fold” into its final shape from a flat state by Jape et al. [71]. Also, Sessions et al. developed a self-folding reflector using discretization through a tessellation [72]. In a unique configuration, a flat-folding parabolic reflector was developed to fold into a stowed case as shown by Jang et al. [73]. It is important to note that parabolic arrays can be discretized into individual panels while still retaining reflecting capabilities, as demonstrated by Hijazi et al. [74].
5.2.3 Laser Forming

The fabrication method of laser forming employs the heat from a laser to induce thermal stresses in a material that causes it to crease and bend. [42]. It is a relatively complex process, but has been quantified into several mechanisms which have been enumerated [39], [42], [75]. Curves can be closely approximated with laser forming as shown by Kim et al. [68] [76].

5.2.4 Folding Compliant Systems

Flat-folding stiffeners have been developed by Yellowhorse et al. [66] and use the Euler spiral model to achieve a shape for a compliant flexure that can completely flatten. Other kinds of folding compliant systems have been developed and use various folding methods like the split-vertex [77], the sandwich, the symmetric bird’s foot [78], and the Sarrus folding techniques.

5.3 Design Methodology

Designing a parabolic reflector that can be folded flat and deployed requires the intersection of several different design methodologies. In order for the structure to fold flat, specific geometries must be used to allow this to happen; in this case the Euler spiral geometry is used. However, the Euler spiral has generally been examined using its two-dimensional profile. This case requires a more parabolic shape in both the two-dimensional and three-dimensional spaces. The shape also cannot be completely idealized, as the realities of fabrication and real-world deployment must be considered.

5.3.1 Euler Spiral

The design of an Euler spiral is meant to allow a material to lay flat when pressed. For this purpose, a shortened Euler spiral was used where the curve does not spiral back on itself. To create a paraboloid that can fold flat, the curves of the paraboloid could be discretized into Euler spiral segments, where each individual segment can fold flat. When the system is unfolded, the spiral segments can pop back into their original shape, thus releasing the stowed strain energy from being compressed. While the shape of an Euler spiral is ideal for a folding configuration, the shape is not
achieved with an exact parabolic equation. However, the equation for a parabolic approximation of the Euler spiral curve was shown by Yellowhorse, et al. [66] and was demonstrated to have reasonably low error when compared with the original spiral equation. The error would not be large enough to impact the desired deflection behavior of the curve.

For this work, the petals of the antenna will consist of intervals on the Euler spiral. Research on Euler spirals focuses solely on mechanisms of constant width whereas this application requires the spiral geometry to have a varying width. Depending on the dimensions of the paraboloid, the width could vary a lot or a little along the length of the arc.

5.3.2 Designing for Parabolic Reflectors

Parabolic reflectors require both a reflector and a collector. For the purpose of this work, only the reflector component will be considered. In order for the array to reflect properly, the focal point will need to be aligned with the collector. An equation for a parabolic profile is required to create a reflector with a proper focal point.

There are two key difficulties with designing a parabolic reflector with Euler spiral approximations: 1) it is not possible to have a perfect paraboloid and still have feasibility to create and fabricate from a piece of sheet metal, 2) the curves of the petals need to be tangential to the flat bottom of the reflector in order for the petals to lay completely flat, but adding that extra space at the bottom displaces the petals outward and rather than causing a focal point, causes a focal “ring.”

5.3.3 Designing for Fabrication

While a variety of techniques might exist for fabricating the ESPR shape, the method of laser forming could prove to be a reasonable method for cutting and shaping the reflector. Methods for forming curves proposed by Kim et al. [68], [76] are possible options for discretizing the petals of the reflector into narrow, laser-formed panels while retaining the general shape of the curve. To form an ideal parabolic curve, an infinite number of laser passes would be required so that there is basically zero discretization. However, the aforementioned authors show the curve geometry “chopped-up” into straight sections as an infinite number of laser passes would be unrealistic. While
5.4 Geometrical Results

The approach to creating a flat-foldable deployable antenna using Euler spirals requires unique geometries for the parabolic shaping. The ESPR is the result of these geometries and is shown in Figure 5.1 in two states that will be described mathematically in the following sections. The left side of the figure shows the parabolic reflector in its unstrained, final, formed shape. This is the shape that would theoretically be the final form after the fabrication process, with the petals formed into their respective curves, and the edges of them touching, but not fixed together. The right side of the figure shows the flattened, strained state of the reflector if all of the petals were pressed down. Also, this flattened state would be how the reflector would appear during the fabrication after the cutting of the panels but before the forming of them.

The shape of the ESPR can be defined by several parameters: $l$: the arc length of each Euler spiral petal, $L$: the arc length of the circumcircle of the paraboloid, $R_i$: the radius of the incircle of the polygon at the base, $R_t$: the radius of the circumcircle at the top of the paraboloid, $w_1$: the width of each side of the symmetric polygon and the short end of the petal, $w_2$: the width of the long side of the petal, $\psi$: the angle from the centerline of the petal to the edge of the petal, $\theta$:
the angle from the centerline of one petal to the centerline of another, $b$: the length of the the petal along the x-axis, $d$: the height of the petal, and $n$: the number of petals and sides to the center polygon.

The parameters for the reflector can be chosen in a variety of ways. A logical method starts with choosing the desired amount of petals. Doing so informs what shape the inner polygon will be as well as the fidelity of the paraboloid. It is then important to pick the radius of the incircle of the inner polygon. This can be a sub-measurement of the total desired size of the shape including the length, height, and arc length of the petal. The dimensions shown in the equations that follow can be understood in Figures 5.2 and 5.3 which show the flat state of the system and the deployed/formed state respectively.

The equation for the parabolic approximation of an Euler spiral, as shown by Yellowhorse et al. [66], is defined by 5.1, where $d$ and $b$ are linear dimension of the spiral height and length respectively. To set the shape of the profile, one of the following parameters must be chosen: $d$, $b$, or $L$. The other two parameters can be calculated from the chosen one. The relational equations are shown by Yellowhorse et al. [66].

$$y = \frac{d}{b^2} x^2$$  \hspace{1cm} (5.1)

In order for the Euler spiral to lay flat without exceeding the yield strength of the material, the base curvature is defined by 5.2 [66], [70]. Using this value to calculate the necessary geometry of the spiral depending on the chosen material is shown by Yellowhorse, et al. [66].

$$\kappa_0 = \frac{2\sigma_y}{NE_i}$$  \hspace{1cm} (5.2)

Once the profile of the Euler spiral petal is calculated, the geometry of the reflector can be calculated with equations 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, and 5.10.

$$\theta = \frac{2\pi}{n}$$  \hspace{1cm} (5.3)

Theta $\theta$ is defined by equation 5.3 as the angle between the centerline for each petal of the reflector, where $n$ is the number of petals or sides of the polygon.

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Figure 5.2: Diagram of a flat 8-sided ESPR and its associated parameters
Figure 5.3: Diagram of a formed 8-sided ESPR and its associated parameters
\[ s = 2R_c \sin \frac{\theta}{2} \quad (5.4) \]

The length of the individual sides of the inner polygon \( s \) is defined by equation 5.4 and depends on \( R_c \) which is the chosen circumcircle radius of the inner polygon.

\[ R_i = R_c \cos \frac{\theta}{2} \quad (5.5) \]

The radius of the incircle of the inner polygon \( R_i \) also depends on \( R_c \), as shown in equation 5.5.

\[ w = 2(R_i + b) \tan \frac{\theta}{2} \quad (5.6) \]

The length of the individual sides of the outer polygon, as formed by the petals when they are brought together, is defined by \( w \) in equation 5.6.

\[ r_c = \frac{w}{2 \sin \frac{\theta}{2}} \quad (5.7) \]

\[ r_i = r_c \cos \frac{\theta}{2} \quad (5.8) \]

\( r_c \) and \( r_i \) in 5.7 and 5.8 respectively are the circumcircle and incircle radii of the outer polygon.

\[ \psi = \arctan \frac{w - s}{2L} \quad (5.9) \]

When flat, the petals are trapezoidal, with the angle \( \psi \) defining the angle from the centerline to the edge of the petal and is defined in equation 5.9.

\[ l = \frac{L}{\cos \psi} \quad (5.10) \]

The parabolic shape formed by the petals when formed has an arc length of \( l \) from the edge of the inner polygon to the edge of the outer polygon.
5.4.1 Folding

Several methods of folding could be used to flatten and stow the ESPR. Two key methods are presented as possible folding options: a sandwich fold and a symmetric bird’s foot fold. Both of these methods require some modification to the already discussed design, but such changes would not necessarily need to be very complex. Figure 5.4 shows the ESPR folded along a crease or hinge down the middle. This is likely the simplest method for folding the ESPR, but requires a symmetric reflector with an even number of petals. Note that the ease of folding is dependent on the size parameters for the reflector and the petals individually, as an increased height of the deployed petals would cause interference during folding. Such issues could be avoided by flattening the system first before folding begins, but this would require external actuators or systems.

Another folding method is the “Symmetric Bird’s Foot” fold and is shown in Figure 5.5. This fold requires four separate fold lines for the folding to occur and allows the system to be reduced in surface area to a smaller profile than the sandwich fold [78]. With thin material like the 0.003” stainless steel shown in the figure, the added thickness of the parts that fold inward is nearly negligible. However, thickness accommodation techniques would need to be implemented if thicker material were used.

5.4.2 Application

The ESPR could be a solution for many applications of parabolic reflectors, especially as solar reflectors or as antennas. Solar reflectors are useful for solar energy collection and even as a method for heating water or food. A flat-foldable parabolic reflector could be a valuable asset if a portable stove-like system were needed for hikers or if power was needed to be generated in remote environments. Antennas are also widely used and can be seen on satellites, vehicles, and communication systems. Having a flat-foldable ESPR on a satellite could allow for simple deployment in outer space and minimal stowed volume. Systems like this could potentially even be manufactured in space given the right laser equipment, and could be useful for in-space fabrication with minimal tooling required. The same could be said for antennas in remote environments on Earth where such an antenna could be transported or fabricated for communication. Naturally, any
Figure 5.4: Sandwich folding configuration of a 24-sided ESPR
of these applications would require further engineering, but the ESPR could provide a structure for such systems.

5.5 Conclusion

The Euler spiral parabolic reflector (ESPR) shows promise as a potential concept for a flat-foldable deployable reflector with geometries that are amenable to folding. Although complex, the geometries for the flat shape and deployed shape are shown to be quantifiable. Several folding methods are proposed as options for stowing the reflector. Being able to fold a curved shape into a flat state is an exciting possibility as more complex systems could be developed that reduce parabolic shapes into drastically condensed forms. Further research would be needed to produce a working model and develop force-deflection relationships for the system. The model for inertia
as it changes along the arc length would be an important to develop to understand how the ESPR would function in reality.
CHAPTER 6. CONCLUSION

The work presented in this thesis expands how we can think about compliant mechanisms and systems fabricated in nontraditional materials. The designs are driven by their fabrication methods. Fabrication can be done in flat sheets and then configured and/or assembled into a transformed shape.

The work presented helps us expand our understanding of creating compliant mechanisms with nontraditional materials or in nontraditional ways. In the cases presented in each of the chapters, the designs of the systems were driven by the fabrication methods used to create them. There are several key learnings obtained by each component of this research.

6.1 Flat-Foldable and Compliant Furniture

Even though furniture is simple in function, the furniture pieces developed show the complexities in creating furniture that is flat-foldable and sturdy. By using novel configurations for LET arrays, stable chairs and stools, they can be fabricated out of a single workpiece and are able to fold into a final shape and back. This improves manufacturability and requires minimal actuation to deploy.

Wood is not often used as a compliant mechanism, but its availability and machinability mean that it is a useful material to create flexible furniture. Origami-inspired shapes can be made out of wood and still be able to retain their shape. Childcare furniture in particular can benefit from such folding techniques as they can save a lot of space while being effective tools for helping children develop.

6.2 Laser-Formed Compliant Mechanisms in Thin Sheet Metal

Thin sheet metal can be difficult to machine into compliant mechanisms because of their complex and delicate features. However, laser forming is shown to be an effective method of
fabricating thin sheet metals into compliant mechanism building blocks. These building blocks can now be used to create systems of mechanisms with more complex behaviors.

Advanced systems like the parabolic reflector demonstrate a novel use of laser forming to fabricate a flat-foldable flexible parabolic reflector. This method and these geometries could be useful in creating a variety of folding Euler spiral mechanisms.

### 6.3 Possible Extensions of Research

The work presented in this thesis focuses on new techniques, concepts, and methods for creating compliant mechanisms with nontraditional materials on a variety of scales. These principles of design can now be implemented in a diverse set of ways to create more refined flat-folding furniture as well as complex, laser-formed metal machines.

The foundational principles presented in this thesis open up a whole new class of ideas. The fact that curved surfaces can be folded flat and condensed into other shapes can be beneficial in creating dish-like shapes, trough-like shapes, wave-like shapes, etc. Also, being able to fabricate complex, flexible systems in the field or in space with simply a laser cutter opens up possibilities for remote machining and creation. The methods of fabrication using laser forming mechanisms by reorienting features could be applied in post-fabrication processes as well. For example, panels of an antenna or other similar systems could be oriented and "tuned" by laser forming. The fact that no unique tooling beyond the laser cutter is required for the demonstrated laser-formed mechanisms is a cost-saving boon that can spare fabrication of the use of complex molds, stamps, and other tools. The field is now wide open and further extensions of the research are sure to be exciting.
REFERENCES


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