System Identification of Postural Tremor in Wrist Flexion-Extension and Radial-Ulnar Deviation

Sydney Bryanna Ward
Brigham Young University

Follow this and additional works at: https://scholarsarchive.byu.edu/etd

Part of the Mechanical Engineering Commons

BYU ScholarsArchive Citation
https://scholarsarchive.byu.edu/etd/9306

This Thesis is brought to you for free and open access by BYU ScholarsArchive. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of BYU ScholarsArchive. For more information, please contact ellen_amatangelo@byu.edu.
ABSTRACT

System Identification of Postural Tremor in Wrist Flexion-Extension and Radial-Ulnar Deviation

Sydney Bryanna Ward
Department of Mechanical Engineering, BYU
Master of Science

Generic simulations of tremor propagation through the upper limb have been achieved using a previously developed postural tremor model, but this model had not yet been compared with experimental data or utilized for subject-specific studies. This work addressed these two issues, which are important for optimizing peripheral tremor suppression techniques.

For tractability, we focused on a subsystem of the upper limb: the isolated wrist, including the four prime wrist muscles (extensor carpi ulnaris, flexor carpi ulnaris, extensor carpi radialis, and flexor carpi radialis) and the two degrees of freedom of the wrist (flexion-extension and radial-ulnar deviation). Muscle excitation and joint displacement signals were collected while subjects with Essential Tremor resisted gravity. System identification was implemented for three subjects who experienced significant tremor using two approaches: 1. Generic linear time-invariant (LTI) models, including autoregressive-exogenous (ARX) and state-space forms, were identified from the experimental data, and characteristics including model order and modal parameters were compared with the previously developed postural tremor model; 2. Subject-specific parameters for the previously developed postural tremor model were directly estimated from experimental data using nonlinear least-squares optimization combined with regularization.

The identified LTI models fit the experimental data well, with coefficients of determination of $0.74 \pm 0.18$ and $0.83 \pm 0.13$ for ARX and state-space forms, respectively. The optimal model orders identified from the experimental data ($4.8 \pm 1.9$ and $6.4 \pm 1.9$) were slightly lower than the orders of the ARX and state-space forms of the previously developed model (6 and 8). For each subject, at least one pair of identified complex poles aligned with the complex poles of the previously developed model, whereas the identified real poles were assumed to represent drift in the data rather than characteristics of the system.

Subject-specific parameter estimates reduced the sum of squared-error (SSE) between the measured and predicted joint displacement signals to be between 10% and 50% of the SSE using generic literature parameters. The predicted joint displacements maintained high coherence at the tremor frequency for flexion-extension ($0.90 \pm 0.10$), which experienced the most tremor.

We successfully applied multiple system identification techniques to identify tremor propagation models using only tremorogenic muscle activity as the input. These techniques identified model order, poles, and subject-specific model parameters, and indicate that tremor propagation at the wrist is well approximated by an LTI model.

Keywords: system identification, parameter estimation, tremor, neuromusculoskeletal, wrist
ACKNOWLEDGEMENTS

I sincerely appreciate all who collaborated with me on this research. First and foremost, I would like to thank my advisor Steven K. Charles who mentored me in a way that always boosted my confidence while also encouraging improvement. I also appreciate Dario Farina of Imperial College London for making time to regularly provide feedback and expertise in neuromechanics and tremor. In addition, I am grateful for my committee members who were involved throughout: Matthew S. Allen provided regular suggestions based on his experience with system identification, Andrew Ning helped significantly regarding optimization, and Spencer P. Magleby graciously participated in data collection.

Finally, I would like to thank my husband Josh for his constant support throughout these two and a half years in this program, as well as my parents Bryan and Sonia Taylor whose examples led me on the path of engineering and graduate education.

My advisor Steven K. Charles and I were supported by the NSF DARE 1806056 grant.
# TABLE OF CONTENTS

TABLE OF CONTENTS ........................................................................................................ iv
LIST OF TABLES .................................................................................................................. vi
LIST OF FIGURES ............................................................................................................... vii
1 Introduction .................................................................................................................... 1
2 Postural Tremor Model ................................................................................................. 4
   2.1 Model Parameters ................................................................................................... 6
   2.2 Inputs and Outputs ............................................................................................... 6
3 Data Collection and Pre-Processing ............................................................................. 8
   3.1 Subjects ................................................................................................................ 8
   3.2 Experimental Setup ............................................................................................. 9
   3.3 Experimental Protocol ......................................................................................... 10
   3.4 Pre-Processing ..................................................................................................... 10
   3.5 Power Spectra of Inputs and Outputs .................................................................. 13
4 System Identification Method 1: Non-Iterative System Identification Techniques and Modal Analysis ........................................................................................................... 15
   4.1 Methods ................................................................................................................ 15
      4.1.1 Theory .......................................................................................................... 15
      4.1.2 Implementation ............................................................................................. 18
   4.2 Results .................................................................................................................. 21
   4.3 Discussion .............................................................................................................. 28
      4.3.1 Performance of LTI Model .......................................................................... 29
      4.3.2 Identified Model Order ................................................................................. 29
      4.3.3 Modal Parameters ......................................................................................... 31
      4.3.4 Limitations ...................................................................................................... 33
      4.3.5 Conclusion ..................................................................................................... 35
5 System Identification Method 2: Parameter Estimation using Nonlinear Least Squares Optimization ..................................................................................................................... 35
   5.1 Methods ................................................................................................................ 37
      5.1.1 Literature Range of Model Parameters ......................................................... 37
      5.1.2 Problem Setup ............................................................................................... 40
      5.1.3 Algorithm ...................................................................................................... 41
      5.1.4 Data Analysis ................................................................................................. 42
LIST OF TABLES

Table 3-1: Subject characteristics, including the measured hand (i.e., the hand with more tremor) and TETRAS score evaluated using only tasks in the performance subset directly related to the upper limb and hand. ................................................................. 8

Table 4-1: Optimal identified model order and corresponding coefficients of determination across data segments, represented as “mean (standard deviation).” ................................................................. 24

Table 4-2: Modal parameters of identified models across data segments, represented as “mean (standard deviation).” ................................................................. 29

Table 5-1: 10th, 50th, and 90th percentile model parameter values from literature for males. .... 38

Table 5-2: 10th, 50th, and 90th percentile model parameter values from literature for females. .. 39

Table 5-3: Median estimated model parameters. ........................................................................ 45
LIST OF FIGURES

Figure 2-1: Postural tremor model, which projects muscle excitation into joint displacements. \( t_1 \) and \( t_2 \) are excitation and contraction muscle time constants; \( C \) is a diagonal matrix of each muscle’s peak isometric force; \( M \) is a matrix of moment arms; and \( I, D, \) and \( K \) are impedance matrices representing coupled joint inertia, damping, and stiffness, respectively. Adapted from [9]. .......................................................... 5

Figure 2-2: Although the neuromusculoskeletal system contains multiple feedback loops, the portion from muscle excitation to joint displacements is purely feedforward. ..................... 5

Figure 3-1: Data were collected of the major wrist muscles and joint DOF while restricting forearm pronation-supination motion. Lasers were utilized in the calibration stage and to visually ensure subjects minimized drift from neutral position during trials. ......................... 9

Figure 3-2: Sample of muscle excitation and joint displacements for each subject, after pre-processing. ........................................................................................................ 12

Figure 3-3: PSDs of muscle excitation and joint acceleration for each subject and each trial. .... 14

Figure 4-1: Flow chart of methods, with the left column representing methods implemented on the theoretical model and the right column representing methods implemented on experimental data. Horizontal arrows represent comparison between the theoretical and experimental models................................................................. 16

Figure 4-2: Representative identified model fits for each subject. The model order and corresponding coefficient of determination (\( R^2 \)) of the models shown are listed in the legend. ........................................................................................................ 16

Figure 4-3: High-pass filtered version of the fit in Figure 4-3 for 3 seconds. .............................. 23

Figure 4-4: Model order versus coefficient of determination (\( R^2 \)); negative \( R^2 \) values are not plotted. ........................................................................................................ 23

Figure 4-5: Subspace poles versus model order and corresponding clusters of physical poles (Subject A segment 1). .................................................................................................................. 25

Figure 4-6: Physical poles across data segment (colored dots) with corresponding mean and standard deviation of each pole (black); major and minor axis of the ellipses represent 1.96 times the real and imaginary standard deviations, capturing 95% of the group.................... 25

Figure 4-7: Theoretical sensitivity of poles to model parameters for males. The results of all poles, grouped by color, are shown on the left, and zoomed-in results are shown on the right. The filled-in markers represent the 50\(^{th}\) percentile results. Labels are shown for results which differed significantly from the 50\(^{th}\) percentile, with nomenclature as follows: stand-alone “10\(^{th}\)” and “90\(^{th}\)” are the cases where all parameters were at the 10\(^{th}\) or 90\(^{th}\) percentile, respectively; parameter elements with subscripts “10\(^{th}\)” and “90\(^{th}\)” are the cases where all parameters except the listed parameter were taken from the 50\(^{th}\) percentile, and the listed parameter was taken from either the 10\(^{th}\) or 90\(^{th}\) percentile. ................................................................. 26

Figure 4-8: Theoretical sensitivity of poles to model parameters for females; see the caption for Figure 4-7 for explanation. ........................................................................................................ 27
Figure 4-9: Identified physical pole mean and standard deviation ellipses (black x’s and ovals) from Figure 4-6 overlaid on the plots of theoretical sensitivity of poles to model parameters (Figure 4-7 and Figure 4-8). ................................................................. 28
Figure 5-1: Objective tradeoff plot for one 20-second segment (Subject A segment 1), where value of $\mu$ is labeled for each scenario. ................................................................. 43
Figure 5-2: The box plot of parameter estimates from all data segments for each subject are shown. The red central mark indicates the median, and the bottom and top edges of the blue box represent the 25th and 75th percentiles. The whiskers extend to the most extreme data points, excluding the outliers which are marked as red plus signs. The green lines indicate literature 10th and 90th percentiles................................................................. 44
Figure 5-3: The SSE between the measured and predicted joint displacement signals are shown using 50th percentile, median estimated, and subject-specific (“Same data”) parameter values. ................................................................................................. 46
Figure 5-4: Example of time-domain fit using parameters generated from the same data and the median estimated parameters (Subject A segment 12)................................................................. 46
Figure 5-5: Example of coherence between measured joint displacements and projected joint displacements, using 50th percentile parameters, the parameters generated from the same data segment, and the median estimated parameters (Subject A segment 12). ................. 47
Figure 5-6: Coherence and amplitude ratio between predicted and measured joint displacements at tremor frequency for 50th percentile values (“FE, 50th” and “RUD, 50th”) and median estimated values (“FE, med.” and “RUD, med.”). The ideal value for both coherence and amplitude ratio is 1. Results of data segments whose optimization did not complete are not shown................................................................. 48
1 INTRODUCTION

Tremor is a common movement disorder that involves involuntary, oscillatory movements of the body, especially the upper limbs. The source of tremor may be an underlying neurological disorder or impairment, a reaction to certain drugs, or due to another medical condition [1]. The most common type of tremor, Essential Tremor (ET), is estimated to affect 0.9% of the global population, including 4.6% of people aged 65 years and older [2]. The two most effective drugs for ET, propranolol and primidone, reduce tremor by 50% [3], but are discontinued by 50% of users due to side effects and inefficacy [4]. Neurosurgical interventions can be very effective but carry the risk and intimidation associated with surgery. For example, deep brain stimulation reduces tremor by up to 90% [5], but due in part to its invasive nature and potential side effects, less than 3% of tremor patients receive this treatment [6]. Consequently, many tremor patients are unsatisfied with their treatment options and desire alternative solutions [7].

Peripheral suppression methods have shown potential to reduce tremor. While developments have been made using electrical stimulation, active and passive orthoses, and other methods [8], the optimal site of intervention (i.e., which joints/muscles) has not been determined because we do not know which muscles/joints contribute most to tremor. Because of neural feedback and mechanical coupling, tremor propagates throughout the upper limb, making it difficult to determine which muscles/joints are most responsible for a subject’s tremor. Ideally, a model of propagation from tremorogenic motoneurons to resulting tremulous movements would
assist in determining which muscle(s) or which joint(s) should be targeted for maximal tremor suppression.

Such a model has recently been developed for the upper limb; taking advantage of known neuromusculoskeletal dynamics, it maps muscle excitation to muscle forces, muscle forces to joint torques, and joint torques to joint displacements [9, 10]. While this model has been useful for generic simulations of postural tremor, it has not been directly compared with subject-specific experimental data. The purpose of this study was to identify postural tremor models directly from experimental data to compare with the previously developed theoretical model and to obtain subject-specific model parameters.

The process of identifying a model from data, or system identification, has been implemented on neuromusculoskeletal systems in many studies. One application has been the identification of models to represent the relationship between muscle excitation and muscle force or joint torque. In some cases, the identified model parameters were physically meaningful (for example, see [11, 12, 13, 14]), whereas others identified generic “black-box” realizations (as in [15, 16]). System identification has also been utilized to identify models of joint dynamics [17, 18], resulting in estimates of stiffness, damping, and inertia. Significantly, these parameters have most commonly been estimated individually (e.g., just stiffness as in [19]) or for a single DOF (such as [20, 21, 22, 23, 24, 25, 26]). Piecing together the full neuromusculoskeletal problem from muscle excitation to joint displacements is more challenging, particularly for multi-input multi-output (MIMO) systems, causing some to only identify a subset of physically meaningful model parameters (e.g., [27]) or to use fully black-box models (including [28, 29]). In relation to tremor, black-box models have been identified in an attempt to predict physiological tremor movements in a single degree of freedom (DOF) from muscle excitation [30] and in one or
multiple DOF from movement history [31, 32]. For pathological tremor, on the other hand, system identification has been applied to the relationship between motor unit excitation and cortical activity [33], whereas the correlation between muscle excitation and joint displacements has been observed [34] but has not yet undergone system identification.

As far as we are aware, this study is the first application of system identification to a MIMO pathological tremor system relating muscle excitation and joint displacements. We approached this problem using two system identification methods, comprising Chapters 4 and 5, for the purpose of both comparing the previously developed postural tremor model with experimental data and obtaining subject-specific model parameters. The previously developed postural tremor model is described in Chapter 2, which, for this study, was reduced to a smaller MIMO subsystem including four wrist muscles and two DOF that tend to experience high amounts of tremorogenic activity and tremor compared to other upper-limb muscles and joints [35]. Chapter 3 describes data collection of postural tremor at the isolated wrist subsystem and data processing preliminary to system identification.
Because the previously developed model [9], which transforms upper-limb muscle excitation into joint tremor, is the starting point for the presented research, a brief overview of the model is included here. This model is designed to approximate postural tremor; since postural tremor involves relatively small displacements about an equilibrium position, a linear time-invariant (LTI) model was chosen, enabling the use of principles and tools from linear systems theory.

The three-stage MIMO model relates muscle excitation to muscle force, muscle force to joint torque, and joint torque to joint displacements (Figure 2-1). More specifically, the first submodel represents excitation-contraction coupling dynamics of muscle, which is approximated by a linear, second-order system; the second submodel represents geometry of the musculoskeletal system from muscle to joint space; and the third submodel represents the coupled, mechanical impedance of the musculoskeletal system in joint space, approximated as a linear, second-order system. Although previous simulations [9, 10] ignored gravity since the fundamental behavior of LTI systems is unaffected by constant inputs, we added gravity to the model because the experimentally measured muscle excitation included muscle activity to overcome gravity. We also reduced the model to include only the wrist subsystem. The ordinary differential equation (ODE) and transfer function representations of the combined submodels are given in Appendix A.
Figure 2-1: Postural tremor model, which projects muscle excitation into joint displacements. $t_1$ and $t_2$ are excitation and contraction muscle time constants; $C$ is a diagonal matrix of each muscle’s peak isometric force; $M$ is a matrix of moment arms; and $I$, $D$, and $K$ are impedance matrices representing coupled joint inertia, damping, and stiffness, respectively. Adapted from [9].

Although the neuromusculoskeletal system includes feedback, the portion of the system that converts muscle excitation into joint displacements is feedforward because muscle excitation contains information from both the descending neural drive and reflex signals (Figure 2-2). This is significant for system identification since closed-loop models usually require more complex identification methods, whereas feedforward models nested in feedback may be treated the same as any other feedforward model [36, p. 435].

Figure 2-2: Although the neuromusculoskeletal system contains multiple feedback loops, the portion from muscle excitation to joint displacements is purely feedforward.
2.1 Model Parameters

The model parameters include the following: muscle excitation and contraction time constants \( t_1 \) and \( t_2 \), which we assumed to be scalars since experimentally estimated time constants are similar across wrist muscles [37]; peak isometric force (PIF) of each muscle, represented by a 4x4 diagonal matrix \( C \); muscle-to-joint moment arms, represented by a 2x4 matrix \( M \); joint impedance matrices, including inertia \( I \), damping constants \( D \), and stiffness \( K \), which are each 2x2 symmetrical matrices; and mass \( m \) and center of mass of the hand \( CoM_{\text{hand}} \), which are both scalars and were used for computation of gravitational torque.

2.2 Inputs and Outputs

The model inputs are muscle excitation in the four prime wrist muscles (extensor carpi ulnaris [ECU], flexor carpi ulnaris [FCU], extensor carpi radialis [ECR], and flexor carpi radialis [FCR]), which are represented by a 4x1 vector at each time step \( u \). The outputs are joint displacements in the two DOF of the wrist (flexion-extension \( \text{[FE]} \) and radial-ulnar deviation \( \text{[RUD]} \)), represented by a 2x1 vector at each time step \( q \).

Of special interest for system identification is whether the inputs to the model are sufficiently excitable (i.e., containing a sufficiently broad range of frequencies) for unique and accurate identification of the model. We chose to use the natural tremorogenic muscle excitation as the system identification input; this has benefits of simplicity and potential clinical application and does not carry the same risk of potentially changing the system as unnatural excitations (e.g., electrical stimulus). Although tremorogenic muscle excitation is often dominated by the tremor frequency, muscle excitation is a broad-band signal, reflecting in part the fact that muscle excitation is generated by a train of action potentials which, similar to impulses, have a broad
frequency spectrum. Note that we used surface electromyography (sEMG) as a proxy for muscle excitation; sEMG is a lowpass filtered, noisy version of muscle excitation due to the tissues that lie between the muscle and the sEMG sensor (e.g., conductivity), resulting in both the attenuation of some of the muscle excitation frequencies and the addition of frequencies not generated by the muscles. Nonetheless, past studies have shown that it can serve as a proxy for muscle excitation [28, 29, 38].
3 DATA COLLECTION AND PRE-PROCESSING

This chapter describes data collection of postural tremor in the isolated wrist subsystem. Data processing that occurred prior to system identification is also described.

3.1 Subjects

To demonstrate that the proposed system identification methods are feasible, we implemented them using data from three subjects with ET (Table 3-1) who exhibited tremor that was both significant and consistent. Each subject had been diagnosed with ET by a neurologist and was evaluated with The Essential Tremor Rating Assessment Scale (TETRAS) [39] as a measurement of tremor severity. Following procedures approved by Brigham Young University’s (BYU) Institutional Review Board, informed consent was obtained from all subjects.

Table 3-1: Subject characteristics, including the measured hand (i.e., the hand with more tremor) and TETRAS score evaluated using only tasks in the performance subset directly related to the upper limb and hand.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Sex</th>
<th>Age</th>
<th>Hand</th>
<th>TETRAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>M</td>
<td>73</td>
<td>R</td>
<td>2.1</td>
</tr>
<tr>
<td>B</td>
<td>F</td>
<td>77</td>
<td>R</td>
<td>1.7</td>
</tr>
<tr>
<td>C</td>
<td>F</td>
<td>59</td>
<td>L</td>
<td>1.5</td>
</tr>
</tbody>
</table>
3.2 Experimental Setup

Subjects sat at a table with the shoulder of the arm with more tremor in approximately 0° of abduction, 30° of flexion, and 0° of internal-external rotation, and with the elbow in approximately 60° of flexion. In order to isolate the wrist, the subject’s forearm was strapped into an apparatus (Figure 3-1) that constrained the forearm in neutral pronation-supination (midway between pronation and supination) while allowing the subject to move freely in FE and RUD. The apparatus minimized interference with muscles by placing pressure mainly on protruding wrist bones rather than the soft tissue. Gravity acted in the direction of ulnar deviation.

![Figure 3-1: Data were collected of the major wrist muscles and joint DOF while restricting forearm pronation-supination motion. Lasers were utilized in the calibration stage and to visually ensure subjects minimized drift from neutral position during trials.](image)

To measure muscle excitation of the four main wrist muscles, sEMG sensors (Trigno IM sensors by Delsys, Natick, MA) were placed in line with and on the belly of the FCR, FCU, ECR (longus and brevis combined), and ECU muscles. To measure wrist rotation in FE and RUD, an
electromagnetic motion capture sensor (trakSTAR 3DGuidance by Ascension Technologies, Shelburne, VT) was placed on the back of the hand, straddling the 3rd and 4th metatarsals. The trakSTAR system was calibrated by marking bony landmarks and aligning the marks in neutral FE and RUD position using the postural calibration method described in [40]; during calibration, the researcher held the subject’s wrist in place to prevent the hand from tremoring. The sEMG and motion capture data were collected at 1111 and 330 samples/s respectively, and both systems were synchronized at the start of each measurement trial.

3.3 Experimental Protocol

Muscle excitation and motion capture data were recorded as subjects held neutral wrist posture (third metacarpal in line with long axis of forearm) with extended fingers while resisting gravity for 3 sets of approximately 2 minutes, with 1 minute of rest in between. Subjects were instructed to avoid co-contracting their muscles and allow free expression of their tremor. Following the postural tremor measurements, we measured the maximum voluntary contraction (MVC) of the four muscles of interest, with three repetitions per muscle.

3.4 Pre-Processing

We processed sEMG data to obtain the muscle excitation envelope using the following steps: 1. High-pass filter (fourth-order Butterworth filter with 20 Hz cutoff frequency), 2. Rectification, 3. Low-pass filter (fourth-order Butterworth filter with 12 Hz cutoff frequency), and 4. Normalization (divided by the peak of the processed sEMG data during the MVC trials of the corresponding muscle). Both filters were created and implemented using MATLAB’s butter and filtfilt functions.
Motion capture data were processed to obtain joint displacement in FE and RUD by implementing inverse kinematics according to [40], using postural calibration and assuming no motion in forearm pronation-supination. We low-pass filtered displacement signals to reduce noise that would amplify during the numerical derivation inherent in the model; we used the same low-pass filter applied on sEMG signals to maintain the linear relationship between the input and output signals [36, p. 466].

Following the processing above, we down-sampled both the muscle excitation and joint displacement signals to a common frequency. The selected frequency was chosen differently for each system identification method. Specifically, to minimize challenges associated with overly small step size in the system identification methods presented in Chapter 4, we down-sampled to 50 samples/s, which is roughly 10 times the frequency of interest [36, pp. 448-452]. Since the system identification method described in Chapter 5 does not have the same risks, we simply down-sampled to the slower of the two sensor sampling rates, or 330 samples/s. In both cases, down-sampling was accomplished using MATLAB’s `resample` function, which applies an anti-aliasing filter before down-sampling.

To gain insight into the variability of results, we increased the number of data segments by dividing the postural tremor measurements into 20-second segments, capturing about 100 cycles of the ~5Hz tremor per segment. Samples of each subject’s muscle excitation and joint displacement signals are shown in Figure 3-2.
Figure 3-2: Sample of muscle excitation and joint displacements for each subject, after pre-processing.
3.5 Power Spectra of Inputs and Outputs

To evaluate the distribution of power in the frequency domain, we calculated the power spectral densities (PSD) of each subject’s muscle excitation and motion data. This was done using the full ~2 minutes of data for each trial (i.e., not segmented into 20-seconds) and using a common sampling frequency of 330 samples/s. We found the PSD of muscle excitation and joint acceleration using the MATLAB `pwelch` function with 18 windows and 50% overlap between windows. Joint acceleration was obtained from joint displacement signals using forward difference numerical differentiation combined with a 10th order low-pass Butterworth filter with 20 Hz cutoff (using MATLAB’s `butter` and `filtfilt` functions) implemented prior to each differentiation to reduce noise amplification. Each subject’s tremor frequency was taken to be the frequency of maximum joint acceleration power; if this frequency was not exactly the same for each trial, we selected the tremor frequency from the trial with the highest power.

The PSDs of muscle excitation and joint acceleration are shown for each subject in Figure 3-3. For Subjects A and B, muscle excitation tremorogenic power dominates other frequencies, resulting in spectrums that are less broad-band than expected. On the other hand, while muscle excitation tremorogenic power is still clearly visible for Subject C, the magnitude is smaller in comparison with other frequencies and thus the spectrum is more broad-band. There are likewise clear peaks at the tremor frequency in joint acceleration power, from which the tremor frequencies were determined to be 4.4, 5.3, and 6.0 Hz for Subjects A, B, and C, respectively. Note that there is more tremor power in FE than RUD for all trials except one.
Figure 3-3: PSDs of muscle excitation and joint acceleration for each subject and each trial.
4 SYSTEM IDENTIFICATION METHOD 1: NON-ITERATIVE SYSTEM IDENTIFICATION TECHNIQUES AND MODAL ANALYSIS

As this was the first application of system identification to this system, we used simple, non-iterative identification methods (i.e., solvable using linear-least squares or other linear algebra techniques) to identify black-box models. Characteristics of the identified models, including model order and modal parameters, were then compared with the theoretical model to ensure consistency.

4.1 Methods

We first present the theoretical foundations of our methods, followed by how we implemented these methods experimentally.

4.1.1 Theory

A flow chart of our methods is shown in Figure 4-1. We started with the theoretical postural tremor model and converted it into two generic forms that can be identified using non-iterative system identification techniques. The theoretical model order and modal parameters were then obtained using these forms of the model. We then used system identification to obtain realizations of the two generic model forms from experimental data, which we used to find experimental model order and modal parameters. Finally, we compared model order and modal parameters between the theoretical and experimental models.
Figure 4-1: Flow chart of methods, with the left column representing methods implemented on the theoretical model and the right column representing methods implemented on experimental data. Horizontal arrows represent comparison between the theoretical and experimental models.

4.1.1.1 Model Representations Identifiable using Non-iterative Techniques

In this study, we implemented non-iterative system identification methods, which utilize linear techniques such as linear least squares and guarantee a global solution. However, basic linear least squares requires that the model be linear in its parameters, and although our model is linear in its variables, it is not linear in its parameters (see Appendix A). Thus, instead of using system identification to estimate the model parameters directly, we used system identification to identify two generic model structures that are solvable non-iteratively, which we then compared with the theoretical model. The generic model structures we selected were autoregressive-exogenous (ARX) and state-space.
In order to guide the system identification procedures and to compare results with the theoretical model, we first transformed the theoretical model into equivalent ARX and state-space forms.

**ARX:** Transforming our model to an equivalent ARX representation (see Appendix B.1) lumped the original model parameters together to form coefficients of two difference equations (one for each output DOF), where the current output is a linear combination of previous outputs and previous and current inputs. The ARX form of the model is sixth order and has the same set of coefficients that multiply previous outputs for both difference equations.

**State-space:** An equivalent state-space form of the model was likewise derived (see Appendix C.1), resulting in an eighth order model. Note that although the transfer function and ARX representations are sixth order, the eighth order state-space form was confirmed to be a minimal representation based on the characteristic polynomial (see Appendix C.2). Thus, in the conversion from state-space to the transfer function matrix, the model order is reduced through pole-zero cancellation. Also note that certain elements of the state-space matrices, which have values of zero and unity, do not depend on the model parameters.

Although we used system identification to identify ARX and state-space models from experimental data (see the Implementation section below for details), comparing these realizations with the theoretical model is not as simple as directly comparing ARX coefficients or state-space matrix elements. The issue is that the theoretical ARX and state-space forms have specific structure and dependencies that cannot be enforced with non-iterative system identification. For example, each of the ARX representation’s coefficients are related to the model parameters, with each model parameter affecting multiple ARX coefficients; this complex dependency of one ARX coefficient on another is not enforced during system identification.
Similarly, the elements of the state-space matrices that are known to be zero or unity cannot be fixed at these values using non-iterative system identification methods. Instead, we selected for comparison other fundamental characteristics that can be compared between the identified and theoretical models, including model order and modal parameters.

4.1.2 Implementation

**ARX:** Based on the theoretical ARX representation, we required the set of coefficients that multiply the previous outputs in both difference equations to be the same, which reduced the number of coefficients to estimate and did not interfere with the ability of non-iterative system identification methods [36]. We solved for ARX parameter values using linear least squares, which minimized the squared error between the measured and predicted joint displacement signals. We hard-coded the linear regression solution (see Appendix B.2) and implemented QR factorization [36, pp. 318-320] using MATLAB `triu` and `qr` functions.

**State-space:** We identified state-space realizations from the data using subspace methods, which are non-iterative system identification techniques that combine linear least squares with other linear algebraic methods (including singular value decomposition) [41]. We used the MATLAB `n4sid` function to implement subspace identification.

4.1.2.1 Model Order

The optimal identified model orders of ARX and state-space models were determined for comparison with the corresponding theoretical representations. We determined the optimal model order by comparing model order versus model fit as follows. Models of orders 1 through 10 were identified for each 20-second data segment, then each identified model was used to
predict joint displacements from the muscle excitation data of that same segment. The projection from muscle excitation to joint displacements was performed using the ARX difference equation starting with zeros for the ARX models and using MATLAB functions *lsim* and *findstates* for state-space models.

As a measure of fit, the coefficient of determination was computed between the experimental and predicted joint displacement signals for each model order. The model order at which the coefficient of determination began to plateau was considered to be the optimal model order for that data segment; this was determined by taking the lowest model order whose coefficient of determination was within 0.02 of the highest coefficient of determination (for model orders ranging from 1 to 10). The value of 0.02 was chosen after review of the data.

### 4.1.2.2 Modal Analysis

We turned to modal analysis (i.e., extracting poles from the identified models [42]) to obtain physically meaningful information about the measured system and to ensure consistency with the theoretical model.

*Poles:* Poles were extracted as roots of the continuous ARX model denominator and as the eigenvalues of the state-space model (both using the MATLAB *pole* function); the continuous ARX model was found by converting the identified discrete-time ARX model into continuous time (using the MATLAB *d2c* function). Poles that were consistent across model order were taken to be the physical poles of the system, whereas other identified poles were assumed to be merely computational (noise based). Physical poles were identified as clusters of poles from different model orders as follows. After compiling poles for even model orders from 2 to 20, clusters of physical poles were found using MATLAB’s density-based clustering algorithm.
dbscan (using the Euclidean distance metric, with a 0.75 Hz neighborhood search radius threshold and a minimum of 5 points per cluster). The average of each cluster of physical poles was recorded for comparison across data segments; these were compiled for all data segments of a given subject, and we again used dbscan (with the same settings as above) to determine which physical poles remained consistent across data segments. From the physical poles, we extracted natural frequencies, damping ratios, and time constants.

Comparison with the theoretical model: The theoretical variability of poles was determined by mapping model parameters into poles for parameter values found in literature. Since we do not know each subject’s model parameter values, we projected not only the 50th percentile values from the literature, but also the variability (as 10th and 90th percentiles). This gave us bounds within which we would expect to find the subject’s poles. In addition, this provided a sort of sensitivity analysis, enabling us to identify the parameters to which the poles are most sensitive. The mapping between the theoretical model parameters and poles is given by the roots of the transfer function denominator (see Appendix A.2), which are given by

\[
p_1 = -\frac{1}{t_1},
\]

(4-1)

\[
p_2 = -\frac{1}{t_2},
\]

(4-2)

and

\[
p_3, p_4, p_5, p_6 = \text{roots}\left[(I_{11}s^2 + D_{11}s + K_{11})(I_{22}s^2 + D_{22}s + K_{22}) - (I_{12}s^2 + D_{12}s + K_{12})^2\right]
\]

(4-3)

where \(p_i\) represents the \(i^{th}\) pole. Poles were determined first with all parameters set to 10th, 50th, then 90th percentile parameter values, followed by setting one parameter at a time to either the 10th or 90th percentile value with the rest of the parameters at 50th percentile (See section 5.1.1
for 10th, 50th, and 90th percentile values). Note that muscle PIF, moment arms, and gravity parameters theoretically do not affect the poles.

4.2 Results

The identified ARX and state-space models fit the measured data well, especially if one overlooks the low-frequency drift and focuses instead on the tremor itself (Figure 4-2, Figure 4-3).

The fit generally improved with model order with a clear plateau, although this was not always the case due to increased ill-conditioning with increased model order [36, pp. 496-497] (Figure 4-4). In some state-space cases, the identified model was unstable due to subspace methods ill-conditioning [43], resulting in extremely poor fits that were excluded from further analysis. The average optimal model order across all subjects was 4.8 for ARX and 6.4 for state-space, which both fall slightly below the corresponding theoretical model orders (Table 4-1). The average coefficient of determination at the optimal model order across all subjects was 0.74 for ARX and 0.83 for state-space.
Figure 4-2: Representative identified model fits for each subject. The model order and corresponding coefficient of determination ($R^2$) of the models shown are listed in the legend.
Figure 4-3: High-pass filtered version of the fit in Figure 4-3 for 3 seconds.
Figure 4-4: Model order versus coefficient of determination ($R^2$); negative $R^2$ values are not plotted.

Table 4-1: Optimal identified model order and corresponding coefficients of determination across data segments, represented as “mean (standard deviation).”

<table>
<thead>
<tr>
<th>Subject</th>
<th><strong>Optimal model order</strong></th>
<th><strong>$R^2$ value at optimal model order</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$ARX$</td>
<td>State-space</td>
</tr>
<tr>
<td></td>
<td><strong>Optimal model order</strong></td>
<td>ARX          State-space</td>
</tr>
<tr>
<td>A</td>
<td>4.1 (1.9)</td>
<td>6.4 (1.9)</td>
</tr>
<tr>
<td>B</td>
<td>5.8 (2)</td>
<td>6.2 (2.1)</td>
</tr>
<tr>
<td>C</td>
<td>4.6 (1.3)</td>
<td>6.5 (1.5)</td>
</tr>
<tr>
<td>Total</td>
<td>4.8 (1.9)</td>
<td>6.4 (1.9)</td>
</tr>
</tbody>
</table>

For each 20-second data segment, the clustering algorithm found between 3 and 7 physical pole clusters (Figure 4-5), whereas 3 or 5 poles were consistently grouped across data segments (Figure 4-6). While the theoretical model has two real poles, the identified models only retrieved one real pole per subject. Subjects A and C had two sets of complex poles like the theoretical model, while Subject B only had one set of complex poles.
Figure 4-5: Subspace poles versus model order and corresponding clusters of physical poles (Subject A segment 1).

Figure 4-6: Physical poles across data segment (colored dots) with corresponding mean and standard deviation of each pole (black); major and minor axis of the ellipses represent 1.96 times the real and imaginary standard deviations, capturing 95% of the group.

The theoretical sensitivity of poles to model parameters is shown for males (Figure 4-7) and females (Figure 4-8). Note that since many of the model parameters are related to body size,
it is more likely that all parameters would be at the 10\textsuperscript{th} or 90\textsuperscript{th} percentile compared to just one parameter being at the 10\textsuperscript{th} or 90\textsuperscript{th} percentile with the rest at the 50\textsuperscript{th} percentile. We can see that one set of complex poles varies more with the FE or “11” element of the impedance matrices (magenta/cyan), while the other set varies more with the RUD or “22” element of the impedance matrices (green/blue). Significantly, the range of these two sets of complex poles overlaps in some cases. The real poles represent those sensitive to muscle time constants.

![Sensitivity of poles to model parameters (M)](image)

**Figure 4-7:** Theoretical sensitivity of poles to model parameters for males. The results of all poles, grouped by color, are shown on the left, and zoomed-in results are shown on the right. The filled-in markers represent the 50\textsuperscript{th} percentile results. Labels are shown for results which differed significantly from the 50\textsuperscript{th} percentile results. Labels are shown for results which differed significantly from the 50\textsuperscript{th} percentile results. Labels are shown for results which differed significantly from the 50\textsuperscript{th} percentile results. Labels are shown for results which differed significantly from the 50\textsuperscript{th} percentile results. Labels are shown for results which differed significantly from the 50\textsuperscript{th} percentile results. Labels are shown for results which differed significantly from the 50\textsuperscript{th} percentile results. Labels are shown for results which differed significantly from the 50\textsuperscript{th} percentile results. Labels are shown for results which differed significantly from the 50\textsuperscript{th} percentile results.

stand-alone “10\textsuperscript{th}” and “90\textsuperscript{th}” are the cases where all parameters were at the 10\textsuperscript{th} or 90\textsuperscript{th} percentile, respectively; parameter elements with subscripts “10\textsuperscript{th}” and “90\textsuperscript{th}” are the cases where all parameters except the listed parameter were taken from the 50\textsuperscript{th} percentile, and the listed parameter was taken from either the 10\textsuperscript{th} or 90\textsuperscript{th} percentile.
Figure 4-8: Theoretical sensitivity of poles to model parameters for females; see the caption for Figure 4-7 for explanation.

We combined plots of the physical poles identified from experimental data with the theoretical range of poles (Figure 4-9). Note that at least one pair of complex poles was within the theoretically expected range, usually falling in the range of the theoretical poles dominated by FE dynamics. In all cases, real poles were significantly different from the theoretically expected range.

The natural frequencies, damping ratios, and time constants were similar between ARX and state-space models, with similar amounts of variance (Table 4-2). Interestingly, the natural frequency of one of the pairs of complex poles was near the tremor frequency for each subject. In addition, for the subjects with two sets of complex poles, the natural frequencies were nearly integer multiples of each other. The damping ratios were clearly underdamped but significantly
different from zero. The identified real pole in each case had a much larger time constant than the theoretical model.

Figure 4-9: Identified physical pole mean and standard deviation ellipses (black x’s and ovals) from Figure 4-6 overlaid on the plots of theoretical sensitivity of poles to model parameters (Figure 4-7 and Figure 4-8).

4.3 Discussion

This study represents the first application of system identification of MIMO pathological tremor system from muscle excitation to joint displacements, which we implemented for purposes of comparison with the previously developed theoretical postural tremor model. We took advantage of identifying linear models because small amplitude movements during postural tremor justify time-invariant parameters and a linear model, and we implemented this using non-iterative system identification methods. We assumed natural tremorogenic muscle activity
provides a stimulating input to the system, thus not requiring artificial stimulation for generating data. In addition, since muscle excitation contains both descending drive and reflex feedback, we treated the system as open-loop.

**Table 4-2: Modal parameters of identified models across data segments, represented as “mean (standard deviation).”**

<table>
<thead>
<tr>
<th></th>
<th>Subject A</th>
<th>Subject B</th>
<th>Subject C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARX</td>
<td>State-space</td>
<td>ARX</td>
</tr>
<tr>
<td>Natural freq. [Hz]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complex 1</td>
<td>4.42 (0.49)</td>
<td>4.56 (0.22)</td>
<td>5.12 (0.38)</td>
</tr>
<tr>
<td>Complex 2</td>
<td>10.49 (0.3)</td>
<td>9.98 (0.21)</td>
<td></td>
</tr>
<tr>
<td>Damping ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complex 1</td>
<td>0.15 (0.06)</td>
<td>0.12 (0.03)</td>
<td>0.29 (0.05)</td>
</tr>
<tr>
<td>Complex 2</td>
<td>0.11 (0.03)</td>
<td>0.06 (0.01)</td>
<td></td>
</tr>
<tr>
<td>Time constant [s]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real 1</td>
<td>0.48 (0.29)</td>
<td>0.61 (0.83)</td>
<td>0.70 (0.76)</td>
</tr>
</tbody>
</table>

### 4.3.1 Performance of LTI Model

Since ARX and state-space models are both LTI, identifying ARX and state-space models from experimental data provided insight on how well a LTI model approximates the true musculoskeletal system. The identified models were able to predict joint displacements similar to measurements, thus confirming the assumption that postural tremor can be approximated well by a LTI model.

### 4.3.2 Identified Model Order

The optimal model orders for ARX and state-space were both slightly below theoretical. This makes sense for the following reason: the identified parameters are independent of each
other, whereas the corresponding theoretical parameters are dependent on the original model parameters; the increased freedom likely led to a lower model order than theoretically expected. On the other hand, one might expect that since the experimental data includes noise, the identified model may fit this noise and result in a higher model order. However, looking at the average optimal model order across many data segments minimized the effects of noise. In addition, one might have expected the experimental model order to be higher than theoretical because the model includes only a subset of muscles that influence the joints of interest. However, since the unmeasured muscles are synergists to the measured muscles, they likely have similar tremorogenic signals [44]; since the same information is being passed to the model, a higher model order is not required.

Since the identified model order is also related to the number of poles, we considered the reason for different number of complex poles between subjects and why some complex poles did not fall into the theoretically expected range. For example, poles that are close to each other may be difficult to identify separately, which may be why we only identified one pair of complex poles for Subject B. In addition, FE had more tremor, so it may be harder to accurately identify poles that are more related to RUD; this may be why the pair of poles with larger natural frequency did not fall within the theoretical range for Subjects A and C. Regarding the real poles, only one real pole was identified per subject, and it was always very close to the imaginary axis. It is possible that the input signals did not have a broad enough spectrum to identify the two theoretically expected first-order real poles; while an input frequency near resonance was likely sufficient to estimate the complex, second-order poles based on the peak in the magnitude ratio bode plot at resonance for underdamped second-order models, the shape of the magnitude ratio bode plot for first-order systems illustrates the need for frequencies both before and after the
cutoff frequency. In addition, it is likely that the single identified real pole, which has a large time constant, represents drift in the data rather than a physical pole of the system.

We compared the optimal model order found by looking at the coefficient of determination with the number of poles. Based on the theoretical ARX and state-space forms of the model, differing model orders between the two forms depends on the real poles, which are repeated for state-space, whereas the complex poles are theoretically the same for both ARX and state-space. Since we do not trust the identified real poles, the number of identified complex poles is thus a lower bound of the model order. In all cases, the optimal model order based on the coefficient of determination was at least as large as the number of identified complex poles, the difference likely being that the coefficient of determination may have fit noise with increased order, whereas the identified poles that were consistent across data segments excluded computational poles. Thus, since the following align with intuition, 1. the number of identified complex poles were slightly below the optimal model order based on the coefficient of determination and 2. the optimal model order based on the coefficient of determination was slightly below the theoretical model order, these suggest that the theoretical model order is reasonable.

### 4.3.3 Modal Parameters

Based on the results of the theoretical sensitivity of model poles to parameters, each pair of complex poles includes coupling between DOF but is dominated by one DOF. For Subjects A and C, the pair of complex poles with smaller natural frequencies fell within the range of expected poles dominated by FE, whereas the other pair of complex poles fell close to but not quite within the expected theoretical range of poles dominated by RUD. Subject B, whose complex poles may be a combination FE and RUD, experienced significantly more tremor power in FE than RUD, so the identified poles are likely biased towards FE. Therefore, we assume each
subject’s pair of complex poles with the smaller natural frequency correlate strongly with FE, and the other pair (for Subjects A and C) correlates somewhat with RUD.

*Comparison of identified natural frequencies with literature:* We found natural frequencies of the poles with the smaller natural frequency to have means ranging from 3.08-5.12 Hz, which fall in the middle of the range for FE seen in literature: [45] found passive natural frequency to range from 1.5-2.9 Hz, [46] measured natural frequency to have a mean of 3.07 Hz (ranging from 2.19 to 4.35 Hz), and [47] found natural frequency to range from 5.6-9.7 Hz. Although no prior studies measured natural frequency of RUD, we expect the natural frequency of RUD to be larger than that of FE because inertia is similar between the two DOF [48] but stiffness is higher for RUD [49]. In our study, the other pairs of complex poles indeed had larger natural frequencies, being about twice the natural frequency of the FE poles and ranging from 6.50-10.49 Hz.

*Comparison of identified damping ratios with literature:* We found damping ratios of the poles with the smaller natural frequency to range from 0.12-0.42, which is well aligned with literature for FE: [45] measured the passive damping ratio to be 0.25, [46] found the damping ratio to be 0.2071 (ranging from 0.1289 to 0.3018), and [47] found the damping ratio to range from 0.10 to 0.39.

Time constants were not compared with literature since, as discussed above, we likely identified drift rather than meaningful system time constants.
4.3.4 Limitations

4.3.4.1 The Use of Tremorogenic Activity as the Input Signal

While the use of tremorogenic activity as the input signal was simple to implement and did not disrupt the system, it also posed some challenges for system identification. For example, based on the subjects’ PSDs of muscle excitation (see Chapter 3), the input signals were not as broad-band as we hoped, thus making it more difficult to accurately identify the system. In addition, we see peaks in the PSDs of both muscle excitation and joint acceleration that are close to the identified natural frequencies, while we do not see peaks unique to joint acceleration (i.e., not also in the muscle excitation). The integer harmonic nature of the PSD peaks and of the identified poles may be due to nonlinearities in pre-processing of muscle excitation, as most natural systems do not have integer harmonic natural frequencies [42]. Each of these insights suggest that it is possible system identification is finding poles of the input rather than of the system.

On the other hand, it is expected that the system natural frequencies are close to the driving tremor frequency [10, 50], so it is also possible that poles of the system physically align with the input signals. Ultimately, distinguishing between physical poles of the system and other computational poles is a challenge that most system identification studies face, and while attempts have been made to mitigate this challenge, these algorithms come with tradeoffs (for example, see [51] which has tradeoffs described in [52]).
4.3.4.2 Treating the Nested Feedback Loop as an Open-Loop System

While treating the system from muscle excitation to joint displacements as open-loop is the simplest approach, the nested feedback loop poses additional risk of not accurately identifying the forward path. Since there is noise in the muscle excitation inputs, we may be identifying a combination of both the forward and feedback paths instead of purely identifying the forward path [17]. In addition, treating the nested feedback loop as an open-loop system does not have the usual benefit of decreased bias with more data points [53, p. 110]. More sophisticated system identification methods approach these challenges of feedback systems, but these are reserved for future studies as the purpose of the current study was to start with the simplest methods and to glean from those what we could.

4.3.4.3 Including Only Subjects with Significant Tremor

Although this study only included subjects who exhibited significant tremor during data collection, subjects with less severe tremor tend to have more broad-band tremorogenic muscle excitation [35], which may actually improve identifiability of the system. Interestingly, we collected measurements of wrist postural tremor for more than the three subjects described in Chapter 3 (for a total of 14 subjects), most of which did not show any visible tremor during the experiment (including many who had high TETRAS scores). Seeing either severe tremor or no tremor is perhaps a limitation of isolating wrist FE and RUD, whereas mild tremor is commonly prevalent in less-restricted systems (e.g., the entire upper limb as in [54]). Thus, implementing these methods on subjects with significant tremor was useful for evaluating feasibility of our methods, whereas future implementations of system identification on larger systems would merit from including subjects with a variety of tremor severity.
4.3.5 Conclusion

By implementing system identification on postural tremor data at the wrist using generic, LTI models, we confirmed that an LTI model fits the data quite will. In addition, the range of the optimal identified model order, based on coefficient of determination and the number of identified poles, falls within the expected range of the theoretical model order. The simple use of tremorogenic muscle excitation as the input signal for system identification also seemed to allow identification of some poles that align with the theoretical poles and modal parameters. Each of these results suggest that the theoretical postural tremor model, including the use of only major superficial muscles as the inputs, is indeed plausible.

Consistent results between the two generic model types, ARX and state-space, as well as between data segments of a given subject, validate the repeatability of these methods. That being said, the biggest concern with this study is whether the identified models reflected modes of the system as opposed to modes of the input signals, as either was possible with the limited input spectrum. This concern could be resolved by producing a broader input perturbation to the system, but this comes with tradeoffs. For example, encouraging the subjects to have voluntary random movements during data collection would increase low-frequency content of the muscle excitation, but movement removes the ability to assume the model is LTI. Another possibility includes using electrodes to create additional frequency content in the muscles, but this could alter the system we are trying to identify. Ultimately, this study showed that simple identification methods, including the use of tremorogenic activity as the system identification stimulation, enabled the identification of two LTI black-box models with characteristics aligning with the theoretical postural tremor model.
5 SYSTEM IDENTIFICATION METHOD 2: PARAMETER ESTIMATION USING NONLINEAR LEAST SQUARES OPTIMIZATION

Since the simple system identification methods described in Chapter 4 showed that the LTI postural tremor model is plausible and that system identification using tremorogenic muscle excitation of the major superficial muscles identified repeatable results, we turned to more sophisticated methods to directly estimate subject-specific parameters of the postural tremor model. With reliable subject-specific parameters, one could, for example, predict the tremor output using muscle excitation from one muscle at a time, which cannot be achieved with black-box models. Thus, the ability to identify subject-specific model parameters would enable the advancement of tremor propagation studies from generic to subject-specific.

The objective of the system identification methods presented in this paper was to estimate subject-specific model parameters that predict the data well and are realistic. To achieve this, we used nonlinear least squares combined with regularization. Note that although nonlinear least squares is an iterative optimization technique, it can directly estimate the postural tremor model parameters, whereas linear least squares would first require the approximate linearization of the model with respect to the model parameters.
5.1 Methods

In this section, we first present a compilation of realistic model parameter ranges from literature, which we used for regularization during and validation following system identification. The parameter estimation problem setup and algorithm are then described.

5.1.1 Literature Range of Model Parameters

We compiled approximate 10th, 50th, and 90th percentile values for each parameter, separately for males and females (Table 5-1, Table 5-2). Details of our methods are included in Appendix D.

In the model, some parameters always appeared as a product with another parameter. To reduce the number of independent model parameters, we consolidated such pairs into a single parameter representing the product. This is the case for elements of $C^*M$ and $m^*CoM_{hand}$. In addition, we assumed parameters with a value of zero to be known, which is the case for $I_{12} = I_{21}$. These reduced the number of independent parameters to 19 scalars.
### Table 5-1: 10th, 50th, and 90th percentile model parameter values from literature for males.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>10th percentile</th>
<th>50th percentile</th>
<th>90th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Muscle Time Constants, $t_1$ &amp; $t_2$ [s]</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Excitation</strong></td>
<td>0.015</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td><strong>Contraction</strong></td>
<td>0.02</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td><strong>Peak Isometric Force, $C$ [N]</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FCR</strong></td>
<td>193.7</td>
<td>407.9</td>
<td>622.1</td>
</tr>
<tr>
<td><strong>FCU</strong></td>
<td>293.7</td>
<td>479.8</td>
<td>665.9</td>
</tr>
<tr>
<td><strong>ECR</strong></td>
<td>314.0</td>
<td>589.8</td>
<td>865.6</td>
</tr>
<tr>
<td><strong>ECU</strong></td>
<td>96.3</td>
<td>192.9</td>
<td>289.5</td>
</tr>
<tr>
<td><strong>Moment Arms, $M$ [m]</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FE</strong></td>
<td>0.0147</td>
<td>0.0149</td>
<td>0.0152</td>
</tr>
<tr>
<td><strong>FCU</strong></td>
<td>0.0147</td>
<td>0.0149</td>
<td>0.0152</td>
</tr>
<tr>
<td><strong>ECR</strong></td>
<td>-0.0114</td>
<td>-0.0115</td>
<td>-0.0119</td>
</tr>
<tr>
<td><strong>ECU</strong></td>
<td>-0.0061</td>
<td>-0.0063</td>
<td>-0.0068</td>
</tr>
<tr>
<td><strong>RUD</strong></td>
<td>-0.0076</td>
<td>0.0214</td>
<td>0.0152</td>
</tr>
<tr>
<td><strong>FE</strong></td>
<td>-0.0114</td>
<td>-0.0169</td>
<td>-0.0075</td>
</tr>
<tr>
<td><strong>FCU</strong></td>
<td>-0.0061</td>
<td>0.0246</td>
<td>0.0221</td>
</tr>
<tr>
<td><strong>ECR</strong></td>
<td>-0.0063</td>
<td>0.0246</td>
<td>0.0221</td>
</tr>
<tr>
<td><strong>ECU</strong></td>
<td>-0.0068</td>
<td>0.0246</td>
<td>0.0221</td>
</tr>
<tr>
<td><strong>Joint Moments of Inertia, $I$ [kg m$^2$]</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FE</strong></td>
<td>0.00242</td>
<td>0.00301</td>
<td>0.00531</td>
</tr>
<tr>
<td><strong>RUD</strong></td>
<td>0.00278</td>
<td>0.00346</td>
<td>0.00609</td>
</tr>
<tr>
<td><strong>Joint Damping Constants, $D$ [Nm s/rad]</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FE</strong></td>
<td>0.021</td>
<td>0.050</td>
<td>0.078</td>
</tr>
<tr>
<td><strong>RUD</strong></td>
<td>0.001</td>
<td>-0.005</td>
<td>-0.008</td>
</tr>
<tr>
<td><strong>Joint Stiffness, $K$ [Nm/rad]</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FE</strong></td>
<td>0.685</td>
<td>3.198</td>
<td>4.459</td>
</tr>
<tr>
<td><strong>RUD</strong></td>
<td>0.042</td>
<td>1.838</td>
<td>6.869</td>
</tr>
<tr>
<td><strong>Hand Center of Gravity, $CoM_{hand}$ [m]</strong></td>
<td>0.065</td>
<td>0.069</td>
<td>0.073</td>
</tr>
<tr>
<td><strong>Hand Mass, $m$ [kg]</strong></td>
<td>0.406</td>
<td>0.526</td>
<td>0.706</td>
</tr>
</tbody>
</table>
Table 5-2: 10th, 50th, and 90th percentile model parameter values from literature for females.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>10th percentile</th>
<th>50th percentile</th>
<th>90th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muscle Time Constants, $t_1$ &amp; $t_2$ [s]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excitation</td>
<td>0.015</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Contraction</td>
<td>0.02</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>Peak Isometric Force, $C$ [N]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FCR</td>
<td>105.2</td>
<td>221.6</td>
<td>337.9</td>
</tr>
<tr>
<td>FCU</td>
<td>159.5</td>
<td>260.6</td>
<td>361.7</td>
</tr>
<tr>
<td>ECR</td>
<td>170.5</td>
<td>320.4</td>
<td>470.2</td>
</tr>
<tr>
<td>ECU</td>
<td>52.3</td>
<td>104.8</td>
<td>157.3</td>
</tr>
<tr>
<td>Moment Arms, $M$ [m]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>0.0140</td>
<td>0.0144</td>
<td>-0.0110</td>
</tr>
<tr>
<td>FCU</td>
<td>0.0141</td>
<td>-0.0107</td>
<td>-0.0052</td>
</tr>
<tr>
<td>ECR</td>
<td>-0.0084</td>
<td>0.0198</td>
<td>-0.0157</td>
</tr>
<tr>
<td>ECU</td>
<td>0.0191</td>
<td>0.0150</td>
<td>0.0225</td>
</tr>
<tr>
<td>Joint Moments of Inertia, $I$ [kg m$^2$]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>0.00109</td>
<td>0.00165</td>
<td>0.00264</td>
</tr>
<tr>
<td>RUD</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Joint Damping Constants, $D$ [Nm s/rad]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>0.015</td>
<td>0.036</td>
<td>0.054</td>
</tr>
<tr>
<td>RUD</td>
<td>0.003</td>
<td>0.036</td>
<td>-0.002</td>
</tr>
<tr>
<td>Joint Stiffness, $K$ [Nm/rad]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>0.570</td>
<td>2.307</td>
<td>3.236</td>
</tr>
<tr>
<td>RUD</td>
<td>0.107</td>
<td>1.776</td>
<td>-0.087</td>
</tr>
<tr>
<td>Hand Center of Gravity, Com$_{hand}$ [m]</td>
<td>0.057</td>
<td>0.0600</td>
<td>0.063</td>
</tr>
<tr>
<td>Hand Mass, $m$ [kg]</td>
<td>0.304</td>
<td>0.4082</td>
<td>0.591</td>
</tr>
</tbody>
</table>
5.1.2 Problem Setup

The two objectives were 1. To find a set of model parameters that predict the output well, and 2. To find model parameters that are realistic. We defined a good prediction as minimal least-squares error between the measured and modeled output (joint displacement). Similarly, we defined realistic model parameters as minimal least-squares difference between the model parameters and 50\textsuperscript{th} percentile literature values. We combined these two objectives with a regularization factor matrix to weigh the tradeoff:

\[
\text{minimize } \|q - \hat{q}(x)\|^2 + W\|x - x_{50}\|^2
\]

by varying \(x\)

(5-1)

where \(q\) is a Nx2 matrix of measured joint displacements (N time steps and 2 DOF), \(\hat{q}\) is a Nx2 matrix of the projected joint displacements using the model and measured inputs, \(W\) is a 19x19 diagonal matrix of regularization factors, \(x\) is a 19x1 vector of model parameters that are varied in the optimization process, and \(x_{50}\) is a 19x1 vector of the 50\textsuperscript{th} percentile parameter values from literature. \(q\) and \(\hat{q}\) were reshaped from matrices to vectors prior to taking the norm.

The purpose of the regularization factor matrix is twofold. First, as previously mentioned, it determines the relative tradeoff between the two objectives. In addition, we selected the regularization factor matrix to allow model parameters with a greater range in the literature to deviate more from the 50\textsuperscript{th} percentile value. These two purposes were achieved by defining the regularization factor matrix as
\[ W = \mu \ast \text{diag}\left( \frac{x_{50}}{|x_{90} - x_{10}|} \right) \] (5-2)

where \( \mu \) is a scalar determining the relative tradeoff between the two objectives and the elements of the diagonal matrix represent a normalized range of the parameters in literature (with \( x_{90} \) and \( x_{10} \) being the 19x1 vectors of 90\(^{th}\) and 10\(^{th}\) percentile values), with a larger range in literature having a smaller regularization factor and therefore less weight; the division represents element-by-element division; and the vertical brackets represent absolute value.

To choose the value for \( \mu \), we implemented the optimization with different values of \( \mu \) and plotted the two objectives for each case. Since increasing \( \mu \) increases one objective and decreases the other, the optimal tradeoff between model fit and realistic parameters is the point that sufficiently minimizes both objectives. We created this plot for values of \( \mu \) between 0.5 and 2.5 in increments of 0.25 for each 20-second data segment, and we determined the optimal tradeoff by inspection. Based on results, the same value for \( \mu \) was chosen for all data segments.

### 5.1.3 Algorithm

Since the model is not linear with respect to the model parameters, we solved the optimization problem using nonlinear least squares. This was implemented using the MATLAB function \( \text{lsqnonlin} \) with the following settings: the algorithm was trust-region-reflective, the function and step tolerance were \( 10^{-11} \), the optimality tolerance was \( 10^{-2} \), the maximum number of iterations was \( 10^3 \), and the maximum number of function evaluations was \( 10^5 \). We specified the objective gradient using complex-step derivative approximation to improve accuracy without increasing computational cost [55]. The initial guess for \( x \) was the 50\(^{th}\) percentile parameter values. At each iteration of the optimization, \( \hat{q}(x) \) was computed using the state-space form of the model (using \( \text{lsim} \)), with initial conditions approximated using forward difference.
derivatives. Note that this approximation of initial conditions is subject to noise and time-shift error, but initial conditions only affect the first few seconds of data.

We scaled the model parameters to have magnitudes near unity to reduce challenges associated with gradient-based optimization in long, narrow valleys in the design space [55]. This was achieved by dividing each model parameter by the 50\textsuperscript{th} percentile value.

5.1.4 Data Analysis

Parameter estimation was performed on each 20-second data segment. Estimates were compared across data segments of a subject to determine variability of estimates and with literature ranges to determine whether estimates were realistic.

The median of all estimated parameters for a given subject were used to predict joint displacement from muscle excitation for each of that subject’s data segments. Parameter estimates from data segments that did not successfully complete the optimization were excluded from computing the median estimated parameters.

The success of the prediction from median estimated parameters was determined with three measures of fit: sum of squared error (SSE), coherence at the tremor frequency, and amplitude ratio at the tremor frequency. SSE between predicted and measured joint displacement measures fit across the entire time domain signal and indicates the success of the optimization since one of the objectives was to minimize SSE. In order to specifically capture the fit of tremor (as opposed to low-frequency drift or higher-frequency phenomena), we observed coherence between the predicted and measured signals at the tremor frequency. Coherence was found using MATLAB’s \textit{mscohere} function with 500-point window length, 50\% overlap, and $2^{11}$ sampling points. Since coherence does not contain information on how well the amplitudes match, we also found the
ratio between the predicted and measured signals’ amplitude at the tremor frequency using MATLAB’s \textit{fft} function.

The prediction performance of each subject’s median parameter estimates was compared with the prediction fit using 50\textsuperscript{th} percentile parameter values. To gage against the best fit, we also observed fits using parameter estimates corresponding with a specific data segment (as opposed to the median of such parameters).

5.2 Results

Based on the plots of objectives with changing values of $\mu$ (Figure 5-1), we selected the value of 1.5 for all subjects. Using this value of $\mu$, all but 4 of 51 data segments optimized successfully.

\textbf{Figure 5-1: Objective tradeoff plot for one 20-second segment (Subject A segment 1), where value of $\mu$ is labeled for each scenario.}
Most parameters were estimated fairly consistently across data segments for a given subject, and most parameters estimates fell within the approximate 10th and 90th percentile range (Figure 5-2). The median estimated model parameters across data segments for each subject are given in (Table 5-3).

Figure 5-2: The box plot of parameter estimates from all data segments for each subject are shown. The red central mark indicates the median, and the bottom and top edges of the blue box represent the 25th and 75th percentiles. The whiskers extend to the most extreme data points, excluding the outliers which are marked as red plus signs. The green lines indicate literature 10th and 90th percentiles.

SSE using the median estimated parameters was on average reduced to 15%, 10% and 50% (for Subjects A, B, and C, respectively) of the SSE using 50th percentile parameters (Figure 5-3). SSE was reduced even further using segment-specific parameters, which on average were 5%, 1%, and 25% (for Subjects A, B, and C, respectively) of the SSE using 50th percentile parameters.
### Table 5-3: Median estimated model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Subject A (M)</th>
<th>Subject B (F)</th>
<th>Subject C (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$ [s]</td>
<td>0.0333</td>
<td>0.0519</td>
<td>0.0572</td>
</tr>
<tr>
<td>$t_2$ [s]</td>
<td>0.0459</td>
<td>0.0728</td>
<td>0.0802</td>
</tr>
<tr>
<td>$C_1M_{11}$ [Nm]</td>
<td>4.0881</td>
<td>2.1664</td>
<td>2.0367</td>
</tr>
<tr>
<td>$C_2M_{12}$ [Nm]</td>
<td>1.3559</td>
<td>2.5772</td>
<td>3.2508</td>
</tr>
<tr>
<td>$C_3M_{13}$ [Nm]</td>
<td>-9.5765</td>
<td>0.2085</td>
<td>-3.8532</td>
</tr>
<tr>
<td>$C_4M_{14}$ [Nm]</td>
<td>-1.2544</td>
<td>-0.5473</td>
<td>-0.6343</td>
</tr>
<tr>
<td>$C_1M_{21}$ [Nm]</td>
<td>-2.9980</td>
<td>-1.4857</td>
<td>-1.8239</td>
</tr>
<tr>
<td>$C_2M_{22}$ [Nm]</td>
<td>8.0089</td>
<td>4.8907</td>
<td>4.7878</td>
</tr>
<tr>
<td>$C_3M_{23}$ [Nm]</td>
<td>-11.0933</td>
<td>1.1155</td>
<td>-4.0677</td>
</tr>
<tr>
<td>$C_4M_{24}$ [Nm]</td>
<td>4.6920</td>
<td>2.7828</td>
<td>2.4660</td>
</tr>
<tr>
<td>$l_{11}$ [kg m$^2$]</td>
<td>0.0018</td>
<td>0.0020</td>
<td>0.0017</td>
</tr>
<tr>
<td>$l_{22}$ [kg m$^2$]</td>
<td>0.0036</td>
<td>0.0018</td>
<td>0.0018</td>
</tr>
<tr>
<td>$K_{11}$ [Nm/rad]</td>
<td>0.8895</td>
<td>1.6257</td>
<td>0.5092</td>
</tr>
<tr>
<td>$K_{12}$ [Nm/rad]</td>
<td>-0.2675</td>
<td>-0.1020</td>
<td>-0.0776</td>
</tr>
<tr>
<td>$K_{22}$ [Nm/rad]</td>
<td>1.9833</td>
<td>3.5045</td>
<td>3.3463</td>
</tr>
<tr>
<td>$D_{11}$ [Nms/rad]</td>
<td>0.0163</td>
<td>0.0458</td>
<td>0.0387</td>
</tr>
<tr>
<td>$D_{12}$ [Nms/rad]</td>
<td>-0.0031</td>
<td>-0.0016</td>
<td>-0.0015</td>
</tr>
<tr>
<td>$D_{22}$ [Nms/rad]</td>
<td>0.1602</td>
<td>0.1034</td>
<td>0.1018</td>
</tr>
<tr>
<td>$m \ast CoM_{hand}$ [kg m]</td>
<td>0.0400</td>
<td>0.0124</td>
<td>0.0265</td>
</tr>
</tbody>
</table>

While the prediction using median-estimated parameters did not fit the measured joint displacement signals as well as using segment-specific parameters, median-estimated parameters still performed well in the tremor portion (i.e., high frequency content) of at least the FE signal (Figure 5-4). Surprisingly, coherence did not necessarily improve using estimated parameters compared to literature 50th percentile values (Figure 5-5). At the tremor frequency in particular, both 50th percentile and subject-specific parameter values had high coherence for FE, with coherence of $0.90 \pm 0.10$ for median estimated parameters (Figure 5-6). Significant improvement was observed using subject specific parameters in the ratio between measured and predicted
amplitudes at tremor frequency, which was brought closer to the value of 1 for most cases (Figure 5-6).

Figure 5-3: The SSE between the measured and predicted joint displacement signals are shown using 50th percentile, median estimated, and subject-specific (“Same data”) parameter values.

Figure 5-4: Example of time-domain fit using parameters generated from the same data and the median estimated parameters (Subject A segment 12).
Figure 5-5: Example of coherence between measured joint displacements and projected joint displacements, using 50th percentile parameters, the parameters generated from the same data segment, and the median estimated parameters (Subject A segment 12).

5.3 Discussion

The purpose of this study was to estimate subject-specific parameters for the postural tremor model that have reasonable values and successfully predict joint displacements from muscle excitation. The achievement of these objectives is significant for more accurately modeling tremor propagation for a given subject, as compared to using generic literature values.

5.3.1 Estimated Parameters

Most parameters were estimated with significantly less variability than the variability in literature, suggesting the general validity of the time-invariant assumption of the model and confirming the reliability of the optimization algorithm. Elements of the stiffness and damping matrices were often among the parameters with more variability. Joint stiffness and damping in multiple DOF are affected by many factors and notoriously difficult to measure; unsurprisingly,
their ranges in the literature are large, resulting in small regularization factors for these parameters and less weighting of the second objective (minimizing the difference between the model parameters and 50\textsuperscript{th} percentile values). On the other hand, stiffness and damping are also susceptible to change over time (especially compared to more constant parameters such as inertia), so perhaps the larger range in values reflects physical variability, deviating from the general time-invariant assumption. Another possibility for larger spread in some parameters is that the model is less sensitive to these parameters, meaning the value of these parameters do not affect the model prediction performance as much as the others.

Figure 5-6: Coherence and amplitude ratio between predicted and measured joint displacements at tremor frequency for 50\textsuperscript{th} percentile values (“FE, 50th” and “RUD, 50th”) and median estimated values (“FE, med.” and “RUD, med.”). The ideal value for both coherence and amplitude ratio is 1. Results of data segments whose optimization did not complete are not shown.
The median estimate for most parameters fell within the 10th and 90th percentile literature approximations, with the exception of three or fewer parameters per subject. Falling outside of the literature range may be due to imperfections of literature 10th and 90th percentile values. For example, in all cases where C*M parameters were outside of the expected literature range, the estimated values fell below the 10th percentile; this may be due to the fact that literature PIF values (C) were based on young adults, whereas the subjects in this study were between the ages of 59 and 77 and may have smaller PIFs than the 10th percentile young adult. It is also possible that the identified C*M parameters represent weighted averages between the measured and unmeasured muscles, whereas the literature values only took into account the measured muscles.

5.3.2 Prediction Performance

As expected, the prediction using the estimated subject-specific parameters greatly reduced the SSE between the measured data and model prediction compared to using 50th percentile model parameters. Note that while segment-specific model parameters resulted in the smallest SSE, the finding that the median-estimated parameters also had small SSE is much more significant; it is no surprise that the optimal model parameters for a given data segment resulted in a strong fit, whereas the strong fit using the median parameters shows that the model consistently predicts the data well using a single set of subject-specific parameters.

Initially surprising, the coherence at tremor frequency did not generally improve with subject-specific parameters compared to 50th percentile values (Figure 5-6). Note, however, the high coherence using 50th percentile values in FE; while subject-specific estimates did not improve this coherence, they did maintain the already adequately high coherence (well above 0.8) while at the same time greatly reducing the SSE of the time-domain data. In addition, the
tremor coherence in RUD was not reduced by much, if any. Tremor was much more prevalent in FE than RUD during all but one of the 2-minute trials of one of the subjects (Figure 3-3), so higher coherence in FE than RUD was not surprising.

Using subject-specific parameters brought the amplitude ratio at the tremor frequency closer to unity with using subject-specific parameters in most cases. In particular, using subject-specific parameters resulted in a significant improvement in RUD of Subject 6, whose default-parameter model vastly overestimated tremor amplitude. While the 50th percentile values had high coherence at the tremor frequency, the subject-specific parameters generally improved the prediction performance of tremor amplitude.

5.3.3 Limitations

5.3.3.1 Obtaining Realistic Parameter Estimates

While parameter estimates were generally realistic and consistent for a given subject, whether these are the subject’s actual physical parameters is not certain. Since we minimized two objectives that accounted for prediction fit and realistic parameters, the values of the parameter estimates depended on the selected regularization factor matrix. More weighting of the prediction fit would have improved the fit for that data segment, but the parameter estimates would have been farther from literature expected values. Significantly, the reduced constraint on parameters would have likely increased the variability of estimates across a subject’s data segments. The median estimates across all data segments would have then been less reliable, perhaps worsening rather than improving the prediction performance of the median estimates. On the other hand, giving more weight to identifying realistic parameters would have worsened
the prediction fit for that data segment but reduced variability of the parameter estimates across data segments. This would have also, however, reduced the subject-specific nature of the parameter estimates, constraining each subject’s estimates to closely match literature generic values. Ultimately, to obtain parameter estimates that sufficiently predict the data and are realistic requires a balance between prediction fit and parameter regularization, as we have done.

The way we defined the diagonal matrix of regularization factors, $W$, assumes the parameters have equal spread below and above the 50th percentile value (similar to a normal distribution). While this is not necessarily true for all parameters, it was simple to implement and likely sufficient. Note that encouraging the model parameter estimates to be realistic could have been achieved by imposing upper and lower constraint bounds instead of the second objective; we chose against this option to avoid the possibility of over-constraining the model parameters, especially since the range of realistic values is not well defined in literature (necessitating the extensive methods for approximating literature ranges described in Appendix D).

We found during the previous implementation of system identification on this data that drift was identified as a component of the model (See section 4.3.2). While the postural tremor model accounts for drift, since imbalanced muscle excitation signals between antagonistic muscles leads to drift in joint displacements, there could be additional drift in the data that the model does not account for. For example, it is possible that some of the drift (or imbalance) in muscle excitation is attenuated in sEMG data processing (see section 3.4) or obscured by noise, thus leading to displacement signals with inaccurate drift. Therefore, it is possible that parameter estimates are compensating for unaccounted-for drift, leading to, for example, muscle time constants estimates with larger values than are present in the physical system. Parameter
estimates and prediction fit may improve by either removing drift in the data or adding a factor to the postural tremor model specifically compensating for otherwise unaccounted-for drift.

5.3.3.2 Sufficient Prediction of Tremor

With the purpose of the postural tremor model being to study tremor propagation, one might suggest that it would be better to find subject-specific model parameters that specifically improve the prediction performance of tremor, with less emphasis on frequencies outside of the tremor band (4-12 Hz). However, we chose to minimize the SSE of the full time-domain joint displacement signals, which emphasizes a good fit for all frequencies. While we could have filtered the joint displacement signals into the tremor band prior to parameter estimation to emphasize the prediction performance of tremor, this likely would have resulted in less realistic parameter values. For example, muscle gains and moment arms significantly affect low frequency content, so neglecting these frequencies would likely result in less accurate muscle PIF and moment arm values. A potential improvement would be to minimize the SSE in the frequency domain and weigh the tremor band more heavily than other frequencies.

Another limitation is that the optimization algorithm minimized the SSE for FE and RUD simultaneously, as opposed to minimizing the DOF separately. While separate objectives for each DOF would likely have improved the fit for RUD, which generally had smaller tremor amplitude, we care most about the prediction performance of the DOF with most tremor. By combining the minimization of the SSE of both DOF, the optimization naturally weighed the DOF with greater amplitude tremor more, which in this study was FE. For the sake of utilizing the model to improve tremor suppression, it is better to predict the DOF with the worst tremor.
5.4 Conclusion

Subject-specific model parameters were estimated that were both acceptably realistic and had strong prediction performance. The variability of parameter estimates between data segments was significantly smaller than the range in literature for most parameters, confirming the reliability of the estimation algorithm. The median parameter estimates from all data segments of the same subject maintained high coherence at the tremor frequency, particularly in FE which expressed more tremor, and significantly reduced the SSE compared to using 50th percentile values. Thus, this method for obtaining subject-specific parameters has the potential to be expanded for implementation on the postural tremor model of the full upper arm to improve the full model’s prediction performance, aid in the study of tremor propagation, and improve developments in tremor suppression.
6 SUMMARY

As far as we are aware, this study was the first to implement system identification methods on a MIMO system of pathological tremor from muscle excitation to joint displacements. Although the ultimate goal is to implement similar methods on the full upper limb for the purpose of tremor propagation studies, we started with a smaller system: the isolated wrist with four muscles and two DOF. We thus reduced the previously developed postural tremor model to the wrist subsystem and collected new data that isolated the muscles and DOF of interest.

Using this dataset, we implemented two system identification methods for purposes of comparing the theoretical model with experimental data and estimating subject-specific model parameters. The two approaches were as follows.

1. Non-iterative techniques and modal analysis: We first converted the postural tremor model to fit the form of two generic, linear model types. We then implemented system identification on the experimental data to identify black-box models of these same two generic forms. Model order and modal characteristics (including poles, natural frequencies, damping ratios, and time constants) were then compared between the theoretical and identified models.

2. Parameter estimation using nonlinear least squares: This second approach directly identified parameters of the postural tremor model. Since the model is not linear with respect to its parameters, this method required an iterative optimization algorithm. In
order to find the best set of subject-specific parameters that are both realistic and predict the experimental data well, regularization was incorporated to balance these two objectives. By taking the median of estimates identified from many data segments, a single set of subject-specific model parameters was found for each subject.

From the first approach, we confirmed that the postural tremor model is indeed plausible based on the high coefficient of determination between the identified LTI models with the experimental data, the identified model order aligning with the theoretical model order, and the most reliable modal parameters falling within the range of expected values. In addition, this approach confirmed that the identified models were consistent across many data segments of a given subject, but also highlighted that the use of tremorogenic muscle excitation as the system stimulus and treating the system as open-loop do not guarantee that the identified models purely represent the feed-forward system. From the second approach, subject-specific parameters were found that were in majority within the realistic range from literature. In addition, the model performance using the subject-specific parameters significantly reduced the SSE of the output compared to using literature 50th percentile values while maintaining the high coherence at tremor frequency.

We anticipate these methods and results will guide future subject-specific tremor propagation studies, particularly of the full upper limb. This work will soon be submitted for publication as two peer-reviewed journal articles.
REFERENCES


A.1 Ordinary Differential Equation

In order to obtain a single ordinary differential equation (ODE) representation of the postural tremor model, we start with the differential equations of the three submodels, where $t_1$ and $t_2$ are scalars and the other parameters are matrices.

\begin{align}
t_1 t_2 \ddot{f} + (t_1 + t_2) \dot{f} + f &= Cu \\
Mf &= \tau \\
I \ddot{q} + D \dot{q} + Kq &= \tau + \tau_g
\end{align}

Substitute (A-2), (A-4), and (A-5) into (A-6).

\begin{align}
t_1 t_2 \dddot{\tau} + (t_1 + t_2) \ddot{\tau} + \tau &= MCu
\end{align}
Take the first and second derivatives of (A-3), assuming gravitational torque is constant.

\[ lq^{(3)} + D\dot{q} + Kq = \dot{\tau} \]  \hspace{1cm} (A-8)

\[ lq^{(4)} + Dq^{(3)} + K\dot{q} = \ddot{\tau} \]  \hspace{1cm} (A-9)

Substitute (A-3), (A-8), and (A-9) into (A-7).

\[ t_1t_2[lq^{(4)} + Dq^{(3)} + K\dot{q}] + (t_1 + t_2)[lq^{(3)} + D\dot{q} + K\dot{q}] + [l\ddot{q} + D\dot{q} + Kq - \tau_g] = MCu \]  \hspace{1cm} (A-10)

Rewrite (A-10) to group into coefficients corresponding with \( q \) and derivatives of \( q \).

\[ t_1t_2lq^{(4)} + [(t_1 + t_2)L + t_1t_2D]q^{(3)} + [I + (t_1 + t_2)D + t_1t_2K]\dot{q} + [D + (t_1 + t_2)K]\ddot{q} + Kq \]

\[ = MCu + \tau_g \]  \hspace{1cm} (A-11)

This is the ODE of the postural tremor model from muscle excitation to joint displacements.

A.2 Transfer Function

Take the Laplace transform of the submodel differential equations (A-1), (A-2), and (A-3), excluding gravity.

\[ [t_1t_2s^2 + (t_1 + t_2)s + 1]F = CU \]  \hspace{1cm} (A-12)

\[ MF = T \]  \hspace{1cm} (A-13)

\[ [Is^2 + Ds + K]Q = T \]  \hspace{1cm} (A-14)

Solve for \( F \) in (A-12).

\[ F = [t_1t_2s^2 + (t_1 + t_2)s + 1]^{-1}CU \]  \hspace{1cm} (A-15)

Substitute (A-14) and (A-15) into (A-13).
\[
M[t_1 t_2 s^2 + (t_1 + t_2)s + 1]^{-1} C U = [I s^2 + D s + K] Q
\]  
(A-16)

Solve for \( Q \).

\[
Q = [I s^2 + D s + K]^{-1} M[t_1 t_2 s^2 + (t_1 + t_2)s + 1]^{-1} C U 
\]  
(A-17)

This is the transfer function from muscle excitation to joint displacements.

Expanding (A-17) for the wrist subsystem results in

\[
H(s) = \frac{1}{(msd_{11}msd_{22} - msd_{12}^2)\text{musc}} \begin{bmatrix}
(msd_{22}M_{11} - msd_{12}M_{21})C_1 & (msd_{22}M_{12} - msd_{12}M_{22})C_2 \\
(-msd_{12}M_{11} + msd_{11}M_{21})C_1 & (-msd_{12}M_{12} + msd_{11}M_{22})C_2 \\
(msd_{22}M_{13} - msd_{12}M_{23})C_3 & (msd_{22}M_{14} - msd_{12}M_{24})C_4 \\
(-msd_{12}M_{13} + msd_{11}M_{23})C_3 & (-msd_{12}M_{14} + msd_{11}M_{24})C_4
\end{bmatrix}
\]  
(A-18)

where

\[
msd_{11} = I_{11}s^2 + D_{11}s + K_{11},
\]

\[
msd_{12} = I_{12}s^2 + D_{12}s + K_{12},
\]

\[
msd_{22} = I_{22}s^2 + D_{22}s + K_{22},
\]

and

\[
musc = t_1 t_2 s^2 + (t_1 + t_2)s + 1.
\]
An autoregressive-exogeneous model (ARX) is a generic model type where the current output is a function of previous values of the output and current and previous values of the input. The order of the model is equal to the number of previous time steps on which the current time step is dependent.

A simple, second order SISO case with the same input and output symbols as the postural tremor model ($u$ and $q$) looks like

$$q(T) = -a_1 q(T - 1) - a_2 q(T - 2) + b_0 u(T) + b_1 u(T - 1) + b_2 u(T - 2) \quad (B-1)$$

where $T$ represents the current time step and $T - n$ represents the $n^{th}$ previous time step. Note that the $a$ coefficients correspond with previous values of the output and the $b$ coefficients correspond with current and previous values of the input. We use this same notation for representing the postural tremor model in an equivalent ARX form.

### B.1 Equivalent ARX Form of the Postural Tremor Model

To convert the postural tremor model into an equivalent ARX form, we start with the postural tremor model ODE, (A-11). To maintain an exact mapping between continuous and discrete frequency domains, we chose to discretize using Tustin’s method (derived from trapezoidal numerical integration) [36, p. 26], resulting in a difference equation for each output,
where the current output is a linear combination of previous outputs, current inputs, and previous inputs. The resulting ARX model has order six.

For output $q_1$, this looks like

$$q_1(T) = [-a_1 \ a_2 \ldots \ a_6] \begin{bmatrix} q_1(T-1) \\ q_1(T-2) \\ \vdots \\ q_1(T-6) \end{bmatrix} + [b_{01}^{11} \ b_{11}^{11} \ldots \ b_{61}^{11}] \begin{bmatrix} u_1(T-0) \\ u_1(T-1) \\ \vdots \\ u_1(T-6) \end{bmatrix} +$$

$$[b_{02}^{12} \ b_{12}^{12} \ldots \ b_{62}^{12}] \begin{bmatrix} u_2(T-0) \\ u_2(T-1) \\ \vdots \\ u_2(T-6) \end{bmatrix} + [b_{03}^{13} \ b_{13}^{13} \ldots \ b_{63}^{13}] \begin{bmatrix} u_3(T-0) \\ u_3(T-1) \\ \vdots \\ u_3(T-6) \end{bmatrix} +$$

$$[b_{04}^{14} \ b_{14}^{14} \ldots \ b_{64}^{14}] \begin{bmatrix} u_4(T-0) \\ u_4(T-1) \\ \vdots \\ u_4(T-6) \end{bmatrix}$$

where each $a$ and $b$ coefficient is directly related to the original model parameters. The relationships between the ARX coefficients and the original model parameters are highly nonlinear.

If we define the following vectors

$$\mathbf{a}^T = [a_1 \ a_2 \ldots \ a_6],$$

$$\mathbf{b}_{ij}^T = [b_{0j}^{ij} \ b_{1j}^{ij} \ldots \ b_{6j}^{ij}],$$

$$\mathbf{q}_i(T) = \begin{bmatrix} q_i(T-1) \\ q_i(T-2) \\ \vdots \\ q_i(T-6) \end{bmatrix},$$

and
For output $i$ and input $j$, we can write the difference equations for both outputs as

$$q_1(T) = -a^T q_1(T) + b_{11}^T u_1(T) + b_{12}^T u_2(T) + b_{13}^T u_3(T) + b_{14}^T u_4(T)$$  \hspace{1cm} (B-3)$$

and

$$q_2(T) = -a^T q_2(T) + b_{21}^T u_1(T) + b_{22}^T u_2(T) + b_{23}^T u_3(T) + b_{24}^T u_4(T).$$  \hspace{1cm} (B-4)$$

Note that the $a$ coefficients for each output’s difference equation are the same; this is consistent with the postural tremor model’s transfer function (see Appendix A.2), where the denominator of each input-output pair are the same. Knowing that the $a$ coefficients are the same for each output’s difference equation and that one output is not dependent on the other output allows us to reduce the number of ARX coefficients to solve for in the system identification process (see the following section).

### B.2 Imposing Structure while Solving for ARX Coefficients

Rearrange the vectors in the difference equations (B-3) and (B-4).

$$q_1(T) = -[q_1(T)]^T a + [u_1(T)]^T b_{11} + [u_2(T)]^T b_{12} + [u_3(T)]^T b_{13} + [u_4(T)]^T b_{14} \hspace{1cm} (B-5)$$

and

$$q_2(T) = -[q_2(T)]^T a + [u_1(T)]^T b_{21} + [u_2(T)]^T b_{22} + [u_3(T)]^T b_{23} + [u_4(T)]^T b_{24} \hspace{1cm} (B-6)$$

Rewrite (B-5) and (B-6) into a single matrix equation.

$$\begin{bmatrix} q_1(T) \\ q_2(T) \end{bmatrix} =$$
If we define the vector of current outputs as \( \mathbf{q}(T) \), the matrix of previous outputs and current and previous inputs as \( \mathbf{\phi}^T(T) \), and the vector of ARX coefficients as \( \mathbf{\theta} \), we can rewrite (B-7) as

\[
\mathbf{q}(T) = \mathbf{\phi}^T(T) \mathbf{\theta}.
\]

We can then solve for the least-squares solution \( \hat{\mathbf{\theta}} \) by left-multiplying both sides of (B-8) by \( \mathbf{\phi}(T) \), taking the summation over all timesteps, and isolating \( \hat{\mathbf{\theta}} \).

\[
\mathbf{\phi}(T) \mathbf{q}(T) = \mathbf{\phi}(T) \mathbf{\phi}^T(T) \hat{\mathbf{\theta}}
\]

\[
\Sigma_{t=1}^{N} [\mathbf{\phi}(T) \mathbf{q}(T)] = [\Sigma_{t=1}^{N} \mathbf{\phi}(T) \mathbf{\phi}^T(T)] \hat{\mathbf{\theta}}
\]

\[
\hat{\mathbf{\theta}} = [\Sigma_{t=1}^{N} \mathbf{\phi}(T) \mathbf{\phi}^T(T)]^{-1} \Sigma_{t=1}^{N} [\mathbf{\phi}(T) \mathbf{q}(T)]
\]
The generic state-space structure has the following form.

\[ \dot{x} = Ax + Bu \]  
\[ y = Cx + Du \]  
\[ (C-1) \]
\[ (C-2) \]

C.1 Equivalent State-Space Tremor Model

Start with the ODE (A-11) and rearrange to isolate \( q^{(4)} \).

\[ q^{(4)} = \]
\[ \frac{1}{t_1 t_2} \left[ \frac{1}{(t_1 + t_2)I + t_1 t_2 D} q^{(3)} - \frac{1}{I + (t_1 + t_2)D + t_1 t_2 K} \dot{q} - \frac{1}{D + (t_1 + t_2)K} \ddot{q} - K q + M C u + \tau_g \right] \]
\[ (C-3) \]

Put (C-2) into state-space form.

\[
\begin{bmatrix}
\dot{q} \\
\ddot{q} \\
q^{(3)} \\
q^{(4)}
\end{bmatrix}
= 
\begin{bmatrix}
0_{2\times2} & 1_{2\times2} & 0_{2\times2} & 0_{2\times2} \\
0_{2\times2} & 0_{2\times2} & 1_{2\times2} & 0_{2\times2} \\
0_{2\times2} & 0_{2\times2} & 0_{2\times2} & 1_{2\times2} \\
-\frac{1}{t_1 t_2} I^{-1} K & -\frac{1}{t_1 t_2} D + (t_1 + t_2) K & -\frac{1}{t_1 t_2} I + (t_1 + t_2) D + t_1 t_2 K & -\frac{1}{t_1 t_2} ((t_1 + t_2) I + t_1 t_2 D)
\end{bmatrix}
\begin{bmatrix}
q \\
\dot{q} \\
q^{(3)}
\end{bmatrix}
\]
(C-4) and (C-5) are the eighth order state-space representation of the postural tremor model.

C.2 Characteristic Equation

It is important to determine whether the eighth order state space model can be reduced to a more minimal form. This can be done by deriving the characteristic polynomial from the transfer function matrix, as the order of the characteristic polynomial represents the order of the minimal state space representation [56]. We do so for a postural tremor model with two inputs and two outputs, which has the same result as with two inputs and four outputs.

We start with a reduced version of the transfer function (A-18) for two inputs and two outputs and add color to make the derivation easier to follow.

\[
H(s) = \frac{1}{(msd_{11}msd_{22} - msd_{12}^2)musc}
\]

\[
\begin{bmatrix}
(msd_{22}M_{11} - msd_{12}M_{21})C_1 & (msd_{22}M_{12} - msd_{12}M_{22})C_2 \\
(-msd_{12}M_{11} + msd_{11}M_{21})C_1 & (-msd_{12}M_{12} + msd_{11}M_{22})C_2
\end{bmatrix}
\]

Since the characteristic equation is equal to the common denominator of all minors of the transfer function, we list all minors for this transfer function. The 1x1 minors are
\[
\frac{(msd_{22}M_{12} - msd_{12}M_{22})C_2}{(msd_{11}msd_{22} - msd_{12}^2)\text{musc}^2}
\]

\[
\frac{(-msd_{12}M_{11} + msd_{11}M_{21})C_1}{(msd_{11}msd_{22} - msd_{12}^2)\text{musc}^2}
\]

and

\[
\frac{(-msd_{12}M_{12} + msd_{11}M_{22})C_2}{(msd_{11}msd_{22} - msd_{12}^2)\text{musc}^2}
\]

and the 2x2 minor is

\[
\frac{(msd_{22}M_{11} - msd_{12}M_{21})C_1}{(msd_{11}msd_{22} - msd_{12}^2)\text{musc}} \ast \frac{(-msd_{12}M_{12} + msd_{11}M_{22})C_2}{(msd_{11}msd_{22} - msd_{12}^2)\text{musc}}
\]

\[
-\frac{(-msd_{12}M_{11} + msd_{11}M_{21})C_1}{(msd_{11}msd_{22} - msd_{12}^2)\text{musc}} \ast \frac{(msd_{22}M_{12} - msd_{12}M_{22})C_2}{(msd_{11}msd_{22} - msd_{12}^2)\text{musc}}
\]

which reduces to

\[
\frac{M_{11}C_1 M_{22}C_2 - M_{21}C_1 M_{12}C_2}{(msd_{11}msd_{22} - msd_{12}^2)\text{musc}^2}
\]

(C-7)

The common denominator of all minors, or the characteristic equation, is thus

\[
(msd_{11}msd_{22} - msd_{12}^2)\text{musc}^2
\]

(C-8)

which has four poles due to the mass-spring-damper and two pairs of repeated poles due to the muscle time constants.

Thus, the minimum realization state-space model is order 8, even though the individual transfer functions are order 6. Thus, in the conversion from state space to the transfer function matrix, the model order is reduced from eight to six through pole-zero cancellation. Note that pole-zero cancellation in the conversion from state space to transfer function representations can mean that the system is either not fully reachable or observable, but not necessarily; the
difference is that the value and direction of the zero has to cancel out with the pole to be unreachable or unobservable, whereas in this case only the value cancels out (otherwise, the eighth order state space representation would not be the minimal representation) [57].
APPENDIX D   10TH, 50TH, AND 90TH PERCENTILE MODEL PARAMETERS

D.1 Muscle Excitation and Contraction Time Constants

We took the muscle time constant 50th, 10th, and 90th percentile values to respectively be the default, lower bound, and upper bound values in [9] for both males and females.

D.2 Peak Isometric Force

D.2.1 Males

The 50th percentile peak isometric force (PIF) values were taken from [58], which were generated from five young adult males. Since PIF is proportional to physiologic cross-sectional area (PCSA), we used the variability of PCSA to estimate the variability of PIF using a constant of proportionality, c:

\[ \sigma_{PIF} = c \sigma_{PCSA} \]

where

\[ c = \frac{\mu_{PIF}}{\mu_{PCSA}} \]

and \( \mu \) is mean and \( \sigma \) is standard deviation. Standard deviations of PCSA were taken from [59], which were generated from 5 males and 5 females.

We found approximate 10th and 90th percentiles of PIF using the standard deviations and
the z-score for ±40% from mean (z = 1.28).

**D.2.2 Females**

In order to approximate female PIF, we first found the average scaling factor between male and female maximum isometric force in wrist flexion and extension [60]. We then divided the male 10th, 50th, and 90th percentile PIF values by this scaling factor to obtain approximate female PIF values.

**D.3 Moment Arms**

We first found 10th, 50th, and 90th percentile heights from the NHANES database (accessed via [61]) for males and females separately. Using these values and length distributions from [48], we generated moment arms using the upper-limb model [62] in OPENSIM [63].

**D.4 Joint Moments of Inertia**

Body-segment inertia values were taken from [48]. Since these values are listed as percentages of height and weight, we used 10th, 50th, and 90th percentile height and weight values from the NHANES database (accessed via [61]).

**D.5 Joint Stiffness**

**D.5.1 50th Percentile Values**

Since few if any measurements of active wrist joint stiffness have been performed in both DOF, a circuitous set of calculations and assumptions was required to obtain estimates. To obtain 50th percentile values, we started with passive stiffness and then added the active stiffness
generated by the muscle contraction needed to overcome gravity in RUD. Passive stiffness values were taken from [49]. To add the effects of gravity, we first found the increase in stiffness due to torque by regressing stiffness in FE vs torque in FE (such measurements do not currently exist for RUD). The value of stiffness at zero torque was taken as an average from [49, 64, 65, 66, 67, 68, 69, 70], and long-range stiffness values (as opposed to short-range stiffness values) at various levels of torque were taken from [66, 67, 71]. Normalizing by passive stiffness allowed us to determine the factor by which passive stiffness increased due to various levels of torque. Assuming torque increases RUD stiffness by the same factor as FE stiffness, we found the factor by which RUD stiffness (the $K_{22}$ element of the stiffness matrix) increased due to gravity. Gravitational torque was estimated as the product of the mass of the hand, the distance from the wrist to the center of mass of the hand, and gravitational acceleration (hand mass and center of mass percentages were taken from [48], and 50th percentile body mass and height from the NHANES database accessed via [61]).

To find the remaining elements of the stiffness matrix ($K_{11}$ and $K_{12} = K_{21}$), we used the relationship between muscle stiffness and joint stiffness with the following assumptions: 1. Only FCR and ECR are activated in the presence of gravity, and their active stiffnesses increase from passive muscle stiffness by the same factor, and 2. Passive muscle stiffness is proportional to PCSA, which is also proportional to PIF. More specifically, the steps we took to find $K_{11}$ and $K_{12} = K_{21}$ were as follows. Assuming negligible change in the Jacobian from muscle to joint space, $J_\mu$, caused by the small changes in posture associated with tremor, the stiffness in joint space, $K$, is related to the stiffness in muscle space, $K_\mu$, as
\[ K = J_{\mu}^T K_{\mu} J_{\mu} \]  

(D-2)

where \( K_{\mu} \) is related to the stiffness of individual muscles, \( K_{\mu,i} \), as

\[
K_{\mu} = \begin{bmatrix}
K_{\mu,1} & 0 & 0 & 0 \\
0 & K_{\mu,2} & 0 & 0 \\
0 & 0 & K_{\mu,3} & 0 \\
0 & 0 & 0 & K_{\mu,4}
\end{bmatrix}
\]

( table 5.1 in [72]).

Expanding (D-2) yields

\[
K_{11} = r_{FE,1}^2 K_{\mu,1} + r_{FE,2}^2 K_{\mu,2} + r_{FE,3}^2 K_{\mu,3} + r_{FE,4}^2 K_{\mu,4}
\]

(D-3a)

\[
K_{12} = r_{FE,1} r_{RUD,1} K_{\mu,1} + r_{FE,2} r_{RUD,2} K_{\mu,2} + r_{FE,3} r_{RUD,3} K_{\mu,3} + r_{FE,4} r_{RUD,4} K_{\mu,4}
\]

(D-3b)

\[
K_{21} = K_{12}
\]

(D-3c)

\[
K_{22} = r_{RUD,1}^2 K_{\mu,1} + r_{RUD,2}^2 K_{\mu,2} + r_{RUD,3}^2 K_{\mu,3} + r_{RUD,4}^2 K_{\mu,4}
\]

(D-3d)

where \( r_{FE,i} \) and \( r_{RUD,i} \) are the moment arms of muscle \( i \) (assuming the wrist has four muscles) about the FE and RUD axes, respectively.

Assuming only FCR and ECR (muscles 1 and 3) are activated in the presence of gravity, we can write the active stiffness for \( K_{22} \) as

\[
K_{22,a} = r_{RUD,1}^2 \alpha_{\mu,1} K_{\mu,1} + r_{RUD,2}^2 K_{\mu,2} + r_{RUD,3}^2 \alpha_{\mu,3} K_{\mu,3} + r_{RUD,4}^2 K_{\mu,4}
\]

(D-4)

where \( \alpha_{\mu,i} \) is the factor between passive and active muscle stiffness for muscle \( i \). Assuming passive muscle stiffness is proportional to its PCSA, which is also proportional to its PIF, we can rewrite (D-4) as
\[ K_{22,a} = r_{FE,1}^2 \alpha \mu_1 C \mu_{max,1} + r_{RUD,2}^2 C \mu_{max,2} + r_{RUD,3}^2 \alpha \mu_3 C \mu_{max,3} + r_{RUD,4}^2 C \mu_{max,4} \]  

(D-5)

where \( \mu_{max,i} \) is the PIF of muscle \( i \) and \( C \) is the constant of proportionality between passive muscle stiffness and PIF.

Further assuming that the factor between passive and active muscle stiffness are the same for FCR and ECR results in

\[ K_{22,a} = r_{FE,1}^2 \alpha \mu C \mu_{max,1} + r_{RUD,2}^2 C \mu_{max,2} + r_{RUD,3}^2 \alpha \mu C \mu_{max,3} + r_{RUD,4}^2 C \mu_{max,4} \]  

(D-6)

\[ K_{22,a} = [r_{RUD,1}^2 \alpha \mu \mu_{max,1} + r_{RUD,2}^2 \mu_{max,2} + r_{RUD,3}^2 \alpha \mu \mu_{max,3} + r_{RUD,4}^2 \mu_{max,4}]C \]  

(D-7)

The passive stiffness of \( K_{22} \) can be similarly written as

\[ K_{22,p} = [r_{RUD,1}^2 \mu_{max,1} + r_{RUD,2}^2 \mu_{max,2} + r_{RUD,3}^2 \mu_{max,3} + r_{RUD,4}^2 \mu_{max,4}]C \]  

(D-8)

If we define the following expressions

\[ A_{22,a} = r_{RUD,1}^2 \alpha \mu \mu_{max,1} + r_{RUD,2}^2 \mu_{max,2} + r_{RUD,3}^2 \alpha \mu \mu_{max,3} + r_{RUD,4}^2 \mu_{max,4} \]

\[ A_{22,p} = r_{RUD,1}^2 \mu_{max,1} + r_{RUD,2}^2 \mu_{max,2} + r_{RUD,3}^2 \mu_{max,3} + r_{RUD,4}^2 \mu_{max,4}, \]

we can use these to write \( K_{22,a} \) as being proportional to \( K_{22,p} \)

\[ K_{22,a} = \frac{A_{22,a}}{A_{22,p}} K_{22,p}. \]  

(D-9)

Since we previously used the regression fit to find active stiffness as a multiple of passive stiffness in the presence of gravity, we set \( \frac{A_{22,a}}{A_{22,p}} \) equal to this value and solved for \( \alpha \mu \); note that all moment arms and PIFs are known.
With a value for $\alpha$, we solved for $K_{11,a}$ and $K_{12,a}$ using expressions derived the same way as above

$$K_{11,a} = \frac{A_{11,a}}{A_{11,p}} K_{12,p}$$  \hspace{1cm} (D-10)

$$K_{12,a} = \frac{A_{12,a}}{A_{12,p}} K_{12,p}$$  \hspace{1cm} (D-11)

where

$$A_{11,a} = r_{FE,1}^2 \alpha \mu_{max,1} + r_{FE,2}^2 \mu_{max,2} + r_{FE,3}^2 \alpha \mu_{max,3} + r_{FE,4}^2 \mu_{max,4}$$

$$A_{11,p} = r_{FE,1}^2 \mu_{max,1} + r_{FE,2}^2 \mu_{max,2} + r_{FE,3}^2 \mu_{max,3} + r_{FE,4}^2 \mu_{max,4}$$

and

$$A_{22,a} = r_{FE,1} r_{RUD,1} \alpha \mu_{max,1} + r_{FE,2} r_{RUD,2} \mu_{max,2} + r_{FE,3} r_{RUD,3} \alpha \mu_{max,3}$$

$$+ r_{FE,4} r_{RUD,4} \mu_{max,4}$$

$$A_{22,p} = r_{FE,1} r_{RUD,1} \mu_{max,1} + r_{FE,2} r_{RUD,2} \mu_{max,2} + r_{FE,3} r_{RUD,3} \mu_{max,3} + r_{FE,4} r_{RUD,2} \mu_{max,4}.$$  \hspace{1cm} (D-12)

### D.5.2 10th and 90th Percentile Values

To find 10th and 90th percentiles, we considered both the range of passive stiffness and active stiffness. Passive stiffness range was taken as ±40% from 50th percentile values using average standard deviations from inbound and outbound values from [49] and a z-score of 1.28. Variability of the active stiffness was taken from variability of the stiffness versus torque regression fit by adding and subtracting 1.28 times the standard deviation of the residuals (or root-mean-square deviation) from the 50th percentile active stiffness; note that since the stiffness values of the regression were normalized by passive stiffness, the standard deviation was first
multiplied by the passive stiffness of the corresponding element (e.g., the standard deviation of regression was multiplied by $K_{11,p}$ to find the variability of $K_{11,a}$). We then selected the smaller of the passive and active lower limits as the 10th percentile values, and the larger of the passive and active upper limits as the 90th percentile values.

D.6 Joint Damping

We used the assumption that the damping ratio remains constant during co-contraction [73] to find the diagonal elements of the damping constant matrix ($D_{11}$ and $D_{22}$) in the presence of gravity, for which we used passive damping constants from [10] and passive and active stiffness values as described above. For example, for $D_{11}$ this looks like

$$
\zeta_{11} = \frac{D_{11,p}}{2\sqrt{K_{11,p}I_{11}}} = \frac{D_{11,a}}{2\sqrt{K_{11,a}I_{11}}}
$$

which rearranges to

$$
D_{11,a} = \frac{\sqrt{K_{11,a}D_{11,p}}}{\sqrt{K_{11,p}}}
$$

where $p$ stands for passive and $a$ stands for active (or in the presence of gravity). We found symmetric off-diagonal elements ($D_{12} = D_{21}$) by assuming the orientation of the damping ellipse, which reflects the geometry of muscles and ligaments, is the same as the stiffness ellipse (i.e., both matrices have the same eigenvectors). Finally, for 10th and 90th percentiles, we assumed the damping ratio is the same as for the 50th percentile case.