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PROPOSITIONAL QUANTIFICATION

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Ramsey defined truth in the following way: x is true if and only if $\exists p(x = [p] \ \& \ p)$. This definition is ill-formed in standard first-order logic, so it is normally interpreted using substitutional or some kind of higher-order quantifier. I argue that these quantifiers fail to provide an adequate reading of the definition, but that, given certain adjustments, standard objectual quantification does provide an adequate reading.

I. INTRODUCTION

In his 1927 manuscript on truth, F. P. Ramsey defined truth like this:

A belief is true if it is a belief that p , and p .

He then expresses it “in Mr Russell’s symbolism” like this:

B is true $:= (\exists p) \cdot B$ is a belief that $p \ \& \ p$. Df.¹

¹ Ramsey 1991, pp. 9, 15n.7. Ramsey proposed the definition first, but it was discussed (independently, I believe) and rejected by Tarski 1930. It has been discussed recently by Künne (2003, Chap. 6.2), where it forms the core of Künne’s “Modest Account” of truth; by van Inwagen 2002, who tentatively rejects the definition on the grounds that the quantification cannot be made sense of; and by briefly by Simmons 1999, who dismisses it on the grounds that neither objectual nor substitutional quantification work.

The definition, obviously, ignores Tarski’s argument that the semantic paradoxes make it impossible to define truth for a formal language. The definition’s supporters may believe that a solution to the paradoxes may be available for a definition of the property of truth or of the concept of truth in natural language. In any case, I will ignore the problem here.

Ramsey's definition has been debated recently in propositional form:

DT x is true iff $\exists p (x = [p] \ \& \ p)$.

This version differs from Ramsey's in that it is about the truth of propositions rather than beliefs. Ramsey didn't want an ontology of propositions, but did want to preserve the categorizing feature of propositions, which allows us to categorize beliefs (and other mental states) in terms of their object. Most philosophers now are willing to discuss these propositions directly, because surely a belief is true or false because of the proposition believed.

In making this move to propositions, Ramsey's "*B* is a belief that *p*" has been replaced by " $x = [p]$ ". These brackets might be read "the proposition that", and apparently function in the definition to distinguish the roles of the two occurrences of *p*—to mark that in the first case the *p* is being used as a name of a proposition, instead of its content as it is in the last clause. These brackets are not, however, standard logical operators.

A problem for DT, as for Ramsey's definition, is the last clause. It is a bare term. Ramsey defends it, saying that "we do not at first realize that "*p*" is a variable *sentence* and so should be regarded as containing a verb" (p. 437). Indeed, Ramsey gives us two versions of the definition, one in a greater, one in a lesser degree of formalization, but he doesn't translate it fully into English. He can't: the definition is untranslatable, for English, apparently, has no way to express the content of an unknown propositional variable.

There are these two questions about the definition: what does the last clause mean, and what do the brackets mean? Central to the answer to these two questions is a third question, the question of the quantifier: how is the quantifier to be understood?

We might understand it (1) in the standard way, as an objectual quantifier, with its variable ranging over the names of objects; or we might understand it (2) substitutionally, with its variable ranging over linguistic expressions; or we might understand the variable (3) as ranging over content, and call the quantifier "higher-order", or "propositional", or "sentential", or "contentual". Substitutional and contentual quantification are not without their controversies, but they appear to be necessary, at least one of them, as standard objectual quantification appears to be obviously unsuited for the task. I will argue, however, that in this case at least the

controversy is justified, as these two readings of the quantifier are unsuited for the definition, but that with some modifications objectual quantification provides for the definition a satisfying and intuitive reading. These modifications are Russellian in nature. I would not be comfortable calling this an exposition of Russell, but my views are inspired by his.

2. SUBSTITUTIONAL QUANTIFICATION

Consider first how substitutional quantification answers the three questions.² A substitutional quantifier depends on the existence of a set of expressions (called “substituends”), each of which may be substituted in for the quantified variable. Normally the substitution class ranges over some semantic class, such as nouns or predicates: the substitution instances for x might include, say, “is tall”, “is angry”, and so on; then “ $\exists x(\text{Xanthippe } x)$ ” is true if and only if Xanthippe is tall or angry. Different quantifiers may range over different substitution classes, so there might be one set of nouns and another set of predicates, each with its own variables. But there is no reason that the substituends must be restricted to a single semantic class, or even to any semantic class at all. They may simply be strings of letters or punctuation marks; the interpretation has to do with the sentence that results when the variables are replaced with one of the substituends. If, for example, “ w is w ” is a permissible substituend for x , “ $\exists x(\text{snow } x\text{white})$ ” is true, because “snow is white” is a substitution instance of the sentence, and snow is white. (Depending on what other expressions are in the substitution class, the sentence may well be nonsense for most substituends.)

On the substitutional reading, the brackets in DT are an abbreviation of “is the proposition that”. The right side of the definition is then “ $\exists p(p$ is the proposition that p , and p)”. The substitution instances of p include every sentence, and so the instances of the right side of the definition will include “ x is the proposition that apricots ripen before peaches, and apricots ripen before peaches”. (Even though the p is, the x need not be a substitutional variable.) Given a substitutional reading, the definition makes sense, the questions have been answered, and it appears that sub-

² Hofweber 2005 defends this reading of the quantifier, and van Inwagen 2002 criticizes it.

stitutional quantification does an admirable job explaining the definition.

As a definition of truth, however, this reading of DT is incoherent. Since we are taking propositions to be the primary bearers of truth and related notions, we cannot take the units of logic to be, instead, orthographically individuated sentences, as would be necessary for substitutional quantification. This motivational inconsistency gives rise to concrete problems. Substitutional quantification works best on a view of sentences as strings of characters, but this approach seems to undercount sentences in any language with ambiguity, homonymy, and other such staples of natural language. Thus not every sentence is a separate substitution instance. And further, it seems that there is not a one-to-one matchup between sentences and propositions. It seems, for example, that “John kissed Mary” and “Mary was kissed by John” are different sentences with different linguistic features, but they express the same proposition, or at least they might. And conversely, “I’m hungry” is a single sentence but expresses several different propositions. This is part of the motivation to define truth in terms of propositions instead. Hence the definition doesn’t get the extension of truth right: there is a single instance “ x is the proposition that I’m hungry, and I’m hungry”, but there is no single proposition that is expressed by the sentence “I’m hungry”, so the definition fails to count every proposition.

I have not argued against substitutional quantification altogether. But even if it works in general, it is insufficient as a reading of the quantifier in DT. Substituting sentences does a poor job of explaining truth for propositions.

3. CONTENTUAL QUANTIFICATION

Another reading of the quantifier involves some kind of higher-order propositional quantification.³ Obviously, we may quantify over propositions using standard first-order quantification. When we say “some propositions are true” we use ordinary quantification limited to the domain of propositions. But in DT we need to quantify over the contents of the propositions. We need a kind of quantification that ranges over the contents of contentful objects, not over the objects themselves. We need an

³ Higher-order contentual quantification has been attacked in van Inwagen 2002 and defended in Grover 1992 and Künné 2003.

intensional higher-order quantification, which I prefer to call “contentual” quantification, to contrast with objectual quantification. If we have this, the final clause of DT is easy to read: it is simply the content of the quantified proposition. The brackets also make sense as a kind of “demonstration” operator, marking that the enclosed content is to be read as an object. In English we have nominalization strategies that make properties function grammatically as nouns. One reason we might do this is to apply a property to a property. We attach “-ness” and “-hood” and “the property of being”, and we form infinitives and gerunds. We do the same thing with propositions, attaching “that” or “the proposition that” or removing the tense on the verb. With these devices, we convert sentences into noun phrases, which can grammatically serve as subjects of predicates. This allows us to attribute truth to propositions—where “peaches ripen after apricots is true” is nonsense, “the proposition that peaches ripen after apricots is true” is not nonsense. We can introduce an operator into a formal language that serves a similar purpose, to convert a sentence into a term. This is the purpose of the brackets in DT. The brackets signal that the propositions function primarily as content, and only secondarily as objects, and hence the appropriate quantification is, apparently, contentual.

But can we make sense of this kind of quantification? What does it mean to quantify over content? One way to make sense of the quantifier is to show it is a translation of a natural-language device that is unproblematic and well-understood. I’m thinking here of Peter van Inwagen’s explanation of the standard objectual existential quantifier—an example of what he says is the only legitimate kind of definition (other than ostentation). He explained the quantifier as a paraphrase of the English locution “there is”, plus a strengthened system of pronouns. He takes this to mean that the quantifier is legitimate, and that it means “there is”. Here we’re in a similar position, trying to explain a quantifier. We want to find a locution in English that we understand, that is logically significant, and that we can paraphrase with the contentual existential quantifier.

If there is a locution in English that underlies our logical intuitions that requires contentual quantification, the quantification will be explained. Some have argued that natural language makes use of “prosentences,” which (think of pronouns) are bits of language taking sentences as antecedents. When we translate a sentence from English into logical

notation, the pronouns turn into the variables: “There is something such that *it* is white” becomes “ $\exists x(x \text{ is white})$.” Prosentences should work in the same way. In natural language they refer back to a sentence, and if we could quantify over them (as we do pronouns) we have quantification over content. But there has been no consensus on whether there are prosentences in natural language. Grover suggests “it is true” as an English-language prosentence; Künne suggests “*es verhält sich so*” as a German-language prosentence. He allows, with reservations, “things are thus” as an English-language prosentence. (He doesn’t care much for Grover’s.) Perhaps German has prosentences and English doesn’t.

No proposed prosentence has gained wide support. But even if we accept that Grover and Künne are right and there are natural-language locutions that can be read as prosentences, we have come only halfway. Not every structure of English (or German) requires an expansion of logical terminology. The English must be logically significant. It would suffice for the prosententialist to display some sentence of English that uses prosentences and cannot be translated using the standard logical devices. But this cannot be done. Any sentence that uses prosentences can be translated with normal quantification over objects and “is true”. For example:

For any whether, either thether or not thether.

(Because I am unpersuaded by the proposed natural-language prosentences, I use Prior’s artificial ones [1971, p. 37]. You may replace these with your favourite real ones, e.g., “For any way things might be said to be, either things are that way or they are not.”) This may be translated, using standard logical operators along with the truth predicate, like this:

$$\forall x(Tx \vee \sim Tx).$$

The quantifier is a standard objectual quantifier with the domain restricted to propositions. Prosentences are supposed to explain truth, but here we’ve used truth to explain prosentences. Which way does the correct analysis run?

For Grover’s proposed prosentence, “it is true”, it is apparently perverse to suggest that the correct linguistic explanation is not in terms of truth. That is, that suggestion could be made only in the service of a very

compelling philosophical theory. It may be the best we can do, but the theory would have to have significant benefits to outweigh that cost. I'll just say that the jury is still out on whether prosententialism offers benefits significant enough to be worthwhile.

This is not the only way to show that a proposed logical device is legitimate, but it is the most common way for this device. It may well be that contentual quantification can be made to work, but there are serious problems with it, just as there are with substitutional quantification, problems serious enough to make it worthwhile to try out objectual quantification again.

4. OBJECTUAL QUANTIFICATION

This is certainly the most familiar kind of quantification. We know how objectual quantification works—the variables range over the names of objects and occupy term position in the quantified sentence. But that doesn't seem to be what's going on here. The quantifier question is answered reassuringly, yes, but the other two questions are not reassuringly answered. Question 1: what are the brackets doing? Brackets add nothing to the name they surround; they mean nothing; they may be dropped without loss. Question 2: what is the final clause, that last p ? Objectual quantification requires its variables to occupy term position, but that last p is not in term position. Hence the definition is ill formed—coming on one side of a conjunction there must be a clause, but there is only a term.

In an attempt to preserve objectual quantification, we could abandon the rule that variables must occupy term position, and allow terms to be atomic formulas (Hofweber 2005). What would a formula mean that consists only of a single term? The interpretation of the language would specify some predicate P as understood as being applied to every bare term; so t by itself would be understood as meaning “ t is P ”. If P is “red”, then “ t is tall & t ” would be interpreted as “ t is tall and t is red”. Does this method make sense of DT? Well, on this reading the brackets are still redundant and meaningless, and may be dispensed with. Hence the identity clause adds nothing to the sense:

$$\text{DT}^* \qquad x \text{ is true iff } \exists p (x = p \ \& \ p)$$

is equivalent to

DT** x is true iff x .

And this, since a bare x is taken to mean “ x is true”, is trivially true under the intended interpretation. While it would be perverse to deny that x is true if and only if x is true, this can scarcely be called a theory of anything, even truth. So again it seems that objectual quantification will not work in this definition.

It is these failures that make substitutional or contentual quantification attractive, and make objectual quantification immediately dismissed. But objectual quantification can be made to work. The secret lies in the brackets. Under the contentual reading of the quantifiers, the brackets serve as a demotion operator, converting a content into an object. To make objectual quantification viable for DT, it may be necessary to find a way to reverse the brackets, to make them serve as a “promotion” operator that converts objects into content—or it may be necessary to find a way to remove them altogether. With objectual quantification, the brackets are meaningless surrounding the first p , but with a promotion operator on the second p , the definition might make sense.

A mathematician might express the T schema for the sentences of arithmetic using Gödel coding:

$$T([\phi]) \leftrightarrow \phi,$$

where “ $[\phi]$ ” is the Gödel number of the sentence ϕ . But this could be put a different way:

$$T(n) \leftrightarrow \{n\},$$

where “ $\{n\}$ ” is the sentence coded with number n . Just as “[]” is a device that “demotes” a sentence to a term, “{ }” is a device that “promotes” a term to a sentence. The device is defined for a subset of the numbers. For each number in that class there is a corresponding sentence ϕ . A promoted number n functions semantically as the sentence ϕ . The second T schema above can be quantified using regular first-order objectual quantification over (a subset of) numbers, without worries of quantifying into quotation marks:

$$\forall n (T(n) \leftrightarrow \{n\}).$$

Similarly with propositions—contentual quantification may be a red herring. Given that propositions exist, there is no problem with objectual quantification with the domain restricted to propositions. So instead of using the brackets as a demotion, or nominalizing, operator, we should make use of a promotion, or denominalizing, operator that can take an object and express its content. Here we may use curly brackets: if p is a proposition, $\{p\}$ is its content. The curly brackets are defined for a subclass of the objects in the domain, and for each object in that class the interpretation assigns a sentence. A bracketed object functions semantically as the associated sentence. Given this, we may restate the definition of truth:

$$\text{DT2} \quad x \text{ is true iff } \exists p (x = p \ \& \ \{p\}).$$

Here the quantification is standard first-order objectual quantification (with a domain restriction), and with this explanation of the brackets, the final clause is unproblematic. The three difficulties with the definition have been dealt with adequately.

It may be objected that the denominalizing brackets are syntactically identical with the truth predicate. What I have called “promotion” is (roughly) what Quine called “semantic ascent”. But these brackets differ from the truth predicate in at least two ways. First, they are semantically different. When applied to the name of a proposition, T (Tarski’s favourite proposition) means “Tarski’s favourite proposition is true”; $\{\text{Tarski’s favourite proposition}\}$ means “snow is white”. Even according to redundancy or prosentential theories of truth, “Tarski’s favourite proposition is true” is not semantically identical with “snow is white”. Second, the brackets are more basic than truth in that truth may be defined in terms of the brackets (as in DT2), but the brackets cannot be defined in terms of truth.

It may also be objected that we can’t understand sentences containing these brackets. This is surely true in general. I don’t know what “ $\{\text{Alice’s last words}\}$ and $\{\text{Bob’s favourite conjecture}\}$ ” means. It may mean “apricots ripen before peaches and there are infinitely many twin primes”; it may mean something else. But even though I don’t know what it means, it still has a meaning. Just as “five times the number of planets in the Milky Way” denotes some number, even if I don’t know what it is, so sentences with the denominalizing brackets have a meaning, even if I

don't know what it is. In some cases, I may know the meaning of a sentence containing brackets. For example, I know what "{Fermat's Last Theorem} $\rightarrow 2 + 2 = 4$ " means. If I know what proposition is named, I can understand the sentence; but the usefulness of the brackets is that I can quantify over the content of propositions I don't understand.

One more step: if this device is legitimate, it is unnecessary. The brackets never disambiguate, and so are not needed. If a term in the class is attached to a property, it functions as an object; when it is not, it functions as the corresponding content. This gives us

DT3 x is true iff $\exists p (x = p \ \& \ p)$.

The quantification is unexceptional, the brackets are gone, and the final clause means just what it ought to. DT3 may provide a compelling reading, since it may be that propositional quantification does not fall neatly into the objectual/contentual distinction. If propositions are structured, DT3 might easily be translated into the structural form, with a single proposition occurring twice within the definition, in two different locations corresponding to its two different roles. There is no metaphysical correlate of the brackets. Without such a correlate, the distinction between objectual quantification with the domain restricted to propositions and contentual quantification begins to disappear. If propositions are structured, one location in the structure might take objects and another take contents; the same proposition appears twice, once as an object and once as a content.

DT3 is reminiscent of the propositional quantification that occurs in the expository sections of *Principia Mathematica*. In Chapter II of the Introduction, Russell quantifies over propositions in the course of an argument that propositions do not, strictly speaking, exist (I: 41–2). Here the quantification is into term position: “(p) . p is false.” In Part I, section B, there is again quantification over propositions, but this time over the content of the propositions:

$$\vdash \cdot (p) \cdot p \vee \sim p. \quad (PM \text{ I: } 129)$$

In neither case is there any indication that there is anything unusual about this kind of quantification. This may be explained, in part, by the fact that these sections are not part of the formal system and that,

officially, propositions don't exist.

It may also be explained, in part, by the familiar charge of Russell's sloppiness about (or confusion of) use and mention. And this charge may very well be true. If we are unclear about the nature of propositions—whether, for examples, propositions are linguistic objects or conceptual objects—this kind of confusion is natural.⁴

But I think that if we understand propositions correctly, this confusion is not only natural, but correct. The use/mention distinction is a distinction of linguistic objects. You cannot really use a proposition as you can use a word. We might loosely speak that way if we want, because we can make a related distinction. But making that distinction is not necessary for propositions.

So we can make sense of DT using standard first-order quantification. The cost of this is a little extra logical machinery—certainly DT3 doesn't look right—but in the case of propositions I think this is just as it should be. The duality of propositions makes them tempting as the foundation for a theory of truth. Propositions are objects, and propositions are also contents. Because they are objects, they can bear properties, such as truth. Because they are contents, they are transparent onto the world. This is to say that propositions are their own truth conditions: under what conditions is it true that apricots ripen before peaches? Precisely if and only if apricots ripen before peaches. Propositions appear to bridge the gap between the mind and the world.

I am not sure that Ramsey's definition will, in the end, explain truth in the way it ought to. But I see no grounds for doubt because of problems with quantification.

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⁴ In *The Principles of Mathematics*, Russell tried without success to distinguish between propositions as asserted and propositions as not asserted. Part of this distinction is that between propositions as contents and propositions as objects. Russell's attempted distinction goes beyond this, however, in that he claims that only true propositions can be asserted.

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