Development and Validation of a Vibration-Based Sound Power Measurement Method

Cameron Bennion Jones

*Brigham Young University*

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Development and Validation of a Vibration-Based

Sound Power Measurement Method

Cameron Bennion Jones

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Science

Jonathan D. Blotter, Chair
Scott D. Sommerfeldt
Mark B. Colton

Department of Mechanical Engineering
Brigham Young University

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ABSTRACT

Development and Validation of a Vibration-Based Sound Power Measurement Method

Cameron Bennion Jones
Department of Mechanical Engineering, BYU
Master of Science

The International Organization for Standardization (ISO) provides no vibration-based sound power measurement standard that provides Precision (Grade 1) results. Current standards that provide Precision (Grade 1) results require known acoustic environments or complex setups. This thesis details the Vibration Based Radiation Mode (VBRM) method as one approach that could potentially be used to develop a Precision (Grade 1) standard. The VBRM method uses measured surface velocities of a structure and combines them with the radiation resistance matrix to calculate sound power.

In this thesis the VBRM method is used to measure the sound power of a single-plate and multiple plate system. The results are compared to sound power measurements using ISO 3741 and good alignment between the 200 Hz and 4 kHz one-third octave band is shown. It also shows that in the case of two plates separated by a distance and driven with uncorrelated sources, the contribution to sound power of each individual plate can be calculated while they are simultaneously excited.

The VBRM method is then extended to account for acoustically radiating cylindrical geometries. The mathematical formulations of the radiation resistance matrix and the accompanying acoustic radiation modes of a baffled cylinder are developed. Numerical sound power calculations using the VBRM method and a boundary element method (BEM) are compared and show good alignment. Experimental surface velocity measurements of a cylinder are taken using a scanning laser Doppler vibrometer (SLDV) and the VBRM method is used to calculate the sound power of a cylinder experimentally. The results are compared to sound power measurements taken using ISO 3741.

Keywords: sound power, acoustic radiation modes, radiation resistance matrix, surface velocity, scanning laser Doppler vibrometer, plate, cylinder
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I would also like to thank fellow students who I worked with during my time here. Monty Anderson, Yin Cao, and Pegah Aslani were all great partners in my learning experience. Caleb Goates became a great friend during my time as a graduate student and his willingness to discuss and brainstorm ideas regarding research was much appreciated and needed.

Finally, to the family and friends who supported me through this process, thank you.
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1 INTRODUCTION

This thesis presents work completed to develop and validate a sound power measurement method based on structural vibration measurements. This was completed by using the Vibration Based Radiation Mode (VBRM) method which combines the use of structural vibration measurements using a scanning laser Doppler vibrometer (SLDV) and known acoustic radiation modes of structures to calculate sound power. This chapter provides background regarding the need for this research. An overview of the fundamental concepts used in the development of the method, including the theory of elementary radiators and the International Organization for Standardization (ISO) 3741 will then be reviewed. The thesis objective is then presented.

1.1 Background

Sound power is the measure of the total energy radiated from a source. Unlike other sound related measurements, it is independent of the distance from the source from which the measurements are taken. It is therefore considered to be a global measurement.

Research has shown that increases in noise levels lead to negative health and economic consequences. A 2010 study funded by the Danish Ministry of the Interior and Health as well as other Danish organizations showed that increases in stress, hypertension, and sleep disturbances related to increased traffic noise increased a person’s chance of having a heart attack.\(^1\) A study conducted by the Center for Disease Control and Prevention showed that as many as 24% of U.S.
adults show signs of noise induced hearing loss\(^2\). Finally, in 2004, a study was conducted on the effects of airport noise on the housing market. Using the Noise Depreciation Index (NDI) it showed that a house exposed to an average noise level 10 dB higher than a comparable house is worth 10 percent less due to that noise exposure.\(^3\)

Due to the economic and health concerns associated with unwanted noise, radiated sound power becomes an important consideration in the design process of consumer products. Many companies have a hard time taking sound power measurements of their products due to the often complex and expensive setups required to take such measurements. The majority of current sound power measurement standards provided by the ISO require the use of a known acoustic environment such as a reverberation chamber or an anechoic chamber.\(^4\) These requirements make sound power measurements cumbersome.

The theory of elementary radiators with its development of the radiation resistance matrix provides an additional method for taking sound power measurements which allows for measurements to be taken with fewer restrictions than current ISO standards and allows for the measurements to be taken in-situ.

1.2 The Theory of Elementary Radiators

In the late 1980s and early 1990s a new theory was developed which allowed for the calculation of sound power based on surface velocity measurements.\(^5\) This theory imagined the surface of a structure as being made up of a collection of individual pistons, each radiating noise independently. Each of these imaginary pistons was referred to as an elementary radiator. The following formulation of the radiation resistance matrix using elementary radiators, and its relationship to sound power, follows the formulation developed by Fahy.\(^5\)
Dividing the surface of a structure into \( R \) imaginary individual radiators of equal size allows the surface of the structure to be described using a vector of velocities measured at each discrete radiator, such that

\[
\{\mathbf{v}_e\} = [\tilde{v}_{e1} \ \tilde{v}_{e2} \ \ldots \ \tilde{v}_{eR}]^T. \tag{1-1}
\]

The sound pressures acting on each elementary radiator can be expressed as a column vector so that

\[
\{\mathbf{p}_e\} = [\tilde{p}_{e1} \ \tilde{p}_{e2} \ \ldots \ \tilde{p}_{eR}]^T. \tag{1-2}
\]

Using these relationships, the power radiated by each of the elementary radiators can be used to find the total sound power radiated from the structure such that

\[
\bar{P}(\omega) = \frac{S}{2R} \text{Re}\{\{\mathbf{v}_e\}^H\{\mathbf{p}_e\}\}\tag{1-3}
\]

where \( S \) is the surface area of the structure. The sound pressures and surface velocities of the structures are related through the impedance matrix \([Z]\) through

\[
\{\mathbf{p}_e\} = [Z]\{\mathbf{v}_e\}. \tag{1-4}
\]

Defining the radiation resistance matrix \([R]\) as \([R] = \frac{S}{2R} \text{Re}[\tilde{Z}]\) and substituting \([R]\) and Eq. (1-4) into Eq. (1-3) results in the sound power being expressed as:

\[
\bar{P}(\omega) = \{\mathbf{v}_e\}^H[R]\{\mathbf{v}_e\} \tag{1-5}
\]

The eigenvectors of the radiation resistance matrix are the acoustic radiation modes and the corresponding eigenvalues are proportional to the radiation efficiencies of the eigenvectors. Acoustic radiation modes are different than structural modes in that structural modes define the vibrations of a structure in terms of orthogonal basis functions whereas acoustic radiation modes
define the acoustic field in terms of orthogonal basis functions. Figure 1-1 shows four structural and acoustic radiation modes. The first structural mode shows bending of the plate while the first acoustic radiation mode has a pure piston form. There is also flexure in the second and third structural modes while there is only rigid body rocking in the second and third acoustic radiation modes.

The radiation resistance matrix and its relationship to sound power will be explored further in Chapters 2 and 3 of this thesis.

![Figure 1-1: (a) The first four structural modes; (b) the first four acoustic radiation modes as presented by Fahy.](image)

1.3 ISO 3741

ISO 3741, titled “Acoustics – Determination of sound power levels and sound energy levels of noise sources using sound pressure – Precision methods for reverberation test rooms”, is a Precision (Grade 1) ISO standard that details the methods for measuring sound power in a reverberation chamber. It details the reverberation time, temperature, air pressure, and humidity requirements to take sound power measurements. The standard requires sound pressure measurements to be taken using an array of at least six microphones. The minimum distance between a given microphone and any surface in the reverberation chamber is 1 meter. The minimum required distance between the noise source and any given microphone is given by
\[ d_{s,m} = 0.08\sqrt{V/T_{rev}} \]

where \( V \) is the volume of the reverberation chamber and \( T_{rev} \) is the reverberation time of any given one-third octave band. Each microphone must be separated from other microphones by a minimum distance of \( d_{m,m} = \lambda/2 \) where \( \lambda \) is the wavelength associated with the centerband frequency of the lowest one-third octave band in question.

The reverberation chamber used in this thesis is located in the Eyring Science Center on the campus of Brigham Young University. The volume of the reverberation chamber is 210 m\(^3\) (5m x 6m x 7m) which allows for measurements down to the 100 Hz one-third octave band so long as the noise floor is not within 10 dB of the sound power measurement for any given one-third octave band. Figure 1-2 shows an example setup in the reverberation chamber in preparation to take ISO 3741 sound power measurements. Table 1-1 details the centerband frequencies of interest with the associated reverberation times as measured in the chamber, as well as the minimum distances required between sources and microphones.

![Figure 1-2: Example setup in preparation to take ISO 3741 measurements.](image-url)
Table 1-1 - Centerband frequencies of the one-third octave bands between 100 Hz and 10 kHz with associated reverberation times, distance between microphones and source and distance between microphones at each one-third octave band in BYU's reverberation chamber.

<table>
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<th>d_{m,m} (m)</th>
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<td>0.02</td>
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1.4 Thesis Objective and Outline

The objective of this thesis is to develop and validate a vibration based sound power measurement method which is accomplished through the VBRM method. The remainder of the thesis will continue as follows: Chapter 2 will present a paper to be submitted to the Journal of the Acoustical Society of America that outlines the research methods and results of using the VBRM method to measure the sound power of a single-baffled plate as well as a multi-plate system. It will be shown in the case of the multi-plate system that the contribution to sound
power of each plate can be derived without having to isolate the acoustic radiation of each plate individually.

Chapter 3 will then present a second paper to be submitted to the Journal of the Acoustical Society of America that expands the use of the VBRM method to cylindrical geometries. The development of the radiation resistance matrix and acoustic radiation modes for baffled cylinders will be presented. The radiation resistance matrix will be used in computational and experimental validation of sound power measurements using the VBRM method. The computational and experimental results using the VBRM method will be compared to results measured using boundary element methods and ISO 3741, respectively.

Finally, Chapter 4 will contain conclusions to this work as well as recommendations for future work to further advance the development of a vibration based sound power measurement standard.
2 EXPERIMENTAL VERIFICATION OF SOUND POWER MEASUREMENTS USING A SCANNING LASER DOPPLER VIBROMETER

This chapter contains a paper to be submitted to the Journal of the Acoustical Society of America. The contents have been reformatted to meet the formatting requirements for this thesis.

2.1 Contributing Authors and Affiliations

Cameron B. Jones and Jonathan D Blotter
Department of Mechanical Engineering, Brigham Young University, Provo, UT 84602, United States

Caleb Goates and Scott D. Sommerfeldt
Department of Physics and Astronomy, Brigham Young University, Provo, UT 84602, United States

2.2 Abstract

The International Organization for Standardization (ISO) publishes no vibration-based sound power measurement standard that provides precision grade results. Current standards that provide precision grade results require known acoustic environments or complex setups. This paper details the Vibration-Based Radiation Mode (VBRM) method as one approach that could be used to develop a precision sound power measurement standard. The VBRM method uses measured surface velocities of an object combined with the acoustic radiation modes to calculate sound power. This paper describes the setup of the VBRM method and compares the results of the method to the precision grade ISO 3741 standard in two scenarios. First, a simply-supported
baffled plate is considered. Next, a system of two simply-supported baffled plates is presented. It is shown that the results of the VBRM method agree with ISO 3741 when vibration measurements obey the Nyquist sampling criterion. It also shows that in the case of two simply supported plates, the contribution to sound power of each individual plate can be calculated while they are simultaneously excited with uncorrelated sources and the resulting acoustic waves do not couple.

2.3 Introduction

The International Organization for Standardization (ISO) publishes sound power measurement standards that are followed worldwide to ensure consistency in the measurements. These published standards are rated based on the reproducibility of results as Precision (Grade 1), Engineering (Grade 2), and Survey (Grade 3). Precision grade gives the most accurate results but generally requires laboratory conditions and the most precise measurement systems. Engineering grade can give accurate results and is commonly used. Survey grade is the least accurate and is used mainly in source comparison tests.

The purpose of the work described in this paper is to explore a vibration-based sound power measurement method that could potentially provide precision grade results. Sound power is the standard measure used to quantify the noise radiated from a source. It is considered a global measurement because it measures the total radiated sound. The development of a vibration-based sound power standard would provide an alternate measurement method which avoids some of the pitfalls of current standards such as the need for complex setups or specific environmental conditions. It would also provide an easier method to measure sound power in-situ or to measure the contribution to sound power of specific incoherent sources of a complex setup.
The ISO currently has seven sound power standards based on pressure measurements. All but two of these require specified acoustic environments such as an anechoic or reverberation chamber.\textsuperscript{6-10} The two standards that do not require a known acoustic environment (ISO 3746 and ISO 3747) only provide survey grade results for narrowband measurements and engineering grade results for broadband.\textsuperscript{11-12}

The ISO has published three standards based on sound intensity measurements.\textsuperscript{13-15} These are ISO 9614-1, ISO 9614-2, ISO 9614-3. ISO 9614-1 and ISO 9614-3 can both provide precision results, while the best results obtainable by ISO 9614-2 are engineering grade. These standards require the measurement to completely surround the noise source, or, if placed on a hard/reflective surface, hemispherically surround the noise source. Due to this requirement, intensity-based sound power measurements lose accuracy in built-up structures. Intensity-based measurements also lose accuracy in windy conditions or conditions with varying background noise.

The ISO provides two technical specifications based on structural vibration methods for computing sound power, namely ISO/TS 7849-1:2009 and ISO/TS 7849-2:2009.\textsuperscript{16-17} Technical specifications are different than standards in that they require only 67\% approval by the committee whereas standards require 75\% approval.\textsuperscript{18} ISO/TS 7849-1:2009 provides survey grade results where ISO/TS 7849-2:2009 provides engineering grade results.

Due to the limitations of current standards, a precision grade standard based on structural vibration data is needed. A vibration-based method would greatly reduce the test setup time for facilities with vibration measurement equipment, thus saving companies money and giving them the ability to efficiently measure the sound power radiated from their products.
In this paper, a method that can potentially give precision results based on structural vibration measurements and acoustic radiation modes is presented. This method, known as the Vibration-Based Radiation Modes (VBRM) method, removes many of the limitations of current methods, such as the need for a specific acoustic environment or limitations on background noise or wind conditions. It will also allow for truly in-situ measurements as well as the measurement of the contribution to sound power of different incoherent sources in a multiple source setup. Current limitations on the VBRM theory are that it requires the surface vibrations of the source to be measurable and the source to have known acoustic radiation modes.

This paper will give a brief overview of the theory behind the VBRM method including acoustic radiation modes and their relationship to sound power measurements. The sound power from a single baffled plate will be computed using the VBRM method and ISO 3741. The results from the two methods will be compared.

The sound power from two radiating plates in the same environment will be computed using both methods. The ability of the VBRM method to determine the individual contributions of the two plates during simultaneous vibration will be demonstrated.

### 2.4 VBRM Method Theory

The VBRM method is based on the measurement of surface velocities and the acoustic radiation mode approach to computing sound power. Acoustic radiation modes provide a mathematical means through which the radiated sound power of a vibrating structure can be calculated. One application area where the modes have been used is the field of active structural acoustic control. Research has shown that if the acoustic radiation modes that radiate power most efficiently are attenuated, total sound power radiated from an object will generally be attenuated as well.
Acoustic radiation modes are derived from the radiation resistance matrix. For a baffled flat plate discretized into \( N \) elements of equal area, the radiation resistance matrix is given by

\[
R = \frac{\omega^2 \rho_0 A_e^2}{4\pi c} \begin{bmatrix}
1 & \frac{\sin(kR_{12})}{kR_{12}} & \cdots & \frac{\sin(kR_{1N})}{kR_{1N}} \\
\frac{\sin(kR_{21})}{kR_{21}} & 1 & \ddots & \\
\vdots & \ddots & \ddots & \\
\frac{\sin(kR_{N1})}{kR_{N1}} & \cdots & 1 & \\
\end{bmatrix},
\tag{2-1}
\]

where \( \omega \) is the angular frequency, \( \rho_0 \) is the density of the surrounding fluid, \( A_e \) is the area of a single discrete element, \( c \) is the speed of sound in the fluid, \( k \) is the acoustic wavenumber, and \( d_{ij} \) is the distance from the \( i \)th to the \( j \)th element. The eigenvectors of the radiation resistance matrix are the acoustic radiation modes, and the corresponding eigenvalues represent the radiation efficiencies of the modes.

Using the radiation resistance matrix, sound power can be expressed as

\[
\bar{P}(\omega) = v_e^H(\omega)R(\omega)v_e(\omega),
\tag{2-2}
\]

where \( v_e \) is a vector containing the velocity of each discrete element on the plate, and \((\cdot)^H\) signifies the Hermitian transpose. Converting Eq. 2-2 to use acoustic radiation modes, \( q_r \), and the eigenvalues, \( \lambda_r \), the expression for sound power is,

\[
\bar{P} = \sum_{r=1}^{N} \lambda_r |\tilde{y}_r|^2
\tag{2-3}
\]

where \( \tilde{y}_r = q_r v_e \). Equations 2-2 and 2-3 are mathematically equivalent when all radiation modes are used.

In 2002, Bai et al. conducted research which used vibration measurements and acoustic radiation modes to calculate the sound power of plates. That research focused on the accuracy of these measurements using only the most efficient radiation modes to calculate sound power,
which can be done to limit the cost of such calculations. In addition, the vibration sampling of the structure was very sparse. Experimental results showed agreement with ISO 3745 only up to 800 Hz. The VBRM method used in this paper uses all radiation modes to calculate the sound power at a given frequency.

2.5 Experimental Setup and Results

This section will compare the sound power measurements of a single baffled plate using ISO 3741 and the VBRM method. The setup and measurements will be described followed by a comparison of the results using the two methods. ISO 3741 provides precision sound power results in one-third octave bands, while the VBRM method is a narrowband calculation. As such, the narrowband VBRM results will be converted to one-third octave band results using standardized one-third octave filter definitions. Comments will also be made regarding the conditions required by each method.

After considering the scenario of a single baffled plate, a multiple plate system will be considered. Two baffled plates driven with uncorrelated random noise will be measured using ISO 3741 and the VBRM method and the results compared.

2.5.1 Single Plate

2.5.1.1 Setup and measurements of the single plate system

A single simply-supported aluminum plate of dimensions 48.5 x 42.0 x 0.16 cm was placed in a reverberation chamber with dimensions of 5 x 6 x 7 m. The plate was placed against one wall of the reverberation chamber, which was used to imitate a baffle (see Fig. 2-1). A piezoelectric transducer was mounted in the upper left quadrant of the back of the plate and was excited with random noise between 0-20 kHz.
A scanning laser Doppler vibrometer (SLDV) was used to measure the surface velocities of the plate on an 11x13 point scan grid as shown in Fig. 2-2. The resulting structural frequency response of the plate can be seen in Fig 2-3. The velocities obtained from the SLDV were expanded into the calculated radiation modes as described in Eq. (2-3) to calculate the sound power using all $R$ radiation modes. These results were compared to sound power calculated from pressure measurements according to ISO 3741.
Figure 2-2: 11x13 point scan grid used to take surface velocity measurements using a SLDV.

Figure 2-3: Structural response of the plate as measured by the SLDV.
2.5.1.2 Results of the single plate system

The measured sound power resulting from ISO 3741 and the VBRM method are reported in one-third octave bands with center band frequencies between 0 and 10 kHz in Table 2-1, along with the differences between the two methods. A plot of these data is shown in Fig. 2-4.

![Graph showing sound power vs. frequency for ISO 3741 and VBRM methods.](image)

Figure 2-4: Sound power of a single baffled plate as measured using ISO 3741 and the VBRM method.

Figure 2-4 shows that between the one-third octave bands with center band frequencies of 200 Hz and 4 kHz there is good alignment between the calculations using the two methods. The maximum difference between the two methods was 2.2 dB at the 4 kHz one-third octave band (see Table I). The mean one-third octave band difference between the 200 Hz band and the 4 kHz band was -0.1 dB and the standard deviation of the errors was 1.1 dB. The total sound power between the 200 Hz and 4 kHz bands is 62.7 dB re $10^{-12}$ W using the VBRM method and
64.4 dB re $10^{-12}$ W using the ISO standard resulting in a total difference of 1.7 dB. Nearly all this difference comes from the 3.15 kHz and 4 kHz one-third octave bands.

At frequencies below 200 Hz there are discrepancies between the ISO 3741 results and the VBRM results. These differences arise due to limitations of the ISO 3741 measurements. The Schroeder frequency of the reverberation chamber is 135 Hz and the noise floor of the chamber was within 10 dB of the measured sound power below 200 Hz. According to ISO 3741, if the noise floor is within 10 dB of the measured sound power the results represent an upper bound on sound power. The combination of measurements below the Schroeder frequency and

Table 2-1: Sound power results from a simply supported plate.

<table>
<thead>
<tr>
<th>Third octave band by centerband frequency (Hz)</th>
<th>Sound Power (dB)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ISO 3741</td>
<td>VBRM</td>
<td>Difference</td>
</tr>
<tr>
<td>100</td>
<td>22.6</td>
<td>2.0</td>
<td>20.6</td>
</tr>
<tr>
<td>125</td>
<td>22.8</td>
<td>2.0</td>
<td>20.8</td>
</tr>
<tr>
<td>160</td>
<td>16.8</td>
<td>15.0</td>
<td>1.8</td>
</tr>
<tr>
<td>200</td>
<td>14.8</td>
<td>16.0</td>
<td>(1.2)</td>
</tr>
<tr>
<td>250</td>
<td>13.7</td>
<td>12.4</td>
<td>1.2</td>
</tr>
<tr>
<td>315</td>
<td>30.1</td>
<td>29.6</td>
<td>0.5</td>
</tr>
<tr>
<td>400</td>
<td>29.1</td>
<td>29.2</td>
<td>(0.1)</td>
</tr>
<tr>
<td>500</td>
<td>31.8</td>
<td>32.3</td>
<td>(0.6)</td>
</tr>
<tr>
<td>630</td>
<td>35.0</td>
<td>35.5</td>
<td>(0.5)</td>
</tr>
<tr>
<td>800</td>
<td>41.6</td>
<td>42.1</td>
<td>(0.5)</td>
</tr>
<tr>
<td>1,000</td>
<td>35.6</td>
<td>36.6</td>
<td>(1.1)</td>
</tr>
<tr>
<td>1,250</td>
<td>43.0</td>
<td>44.0</td>
<td>(1.0)</td>
</tr>
<tr>
<td>1,600</td>
<td>48.8</td>
<td>49.1</td>
<td>(0.3)</td>
</tr>
<tr>
<td>2,000</td>
<td>46.0</td>
<td>47.2</td>
<td>(1.2)</td>
</tr>
<tr>
<td>2,500</td>
<td>50.5</td>
<td>51.4</td>
<td>(0.9)</td>
</tr>
<tr>
<td>3,150</td>
<td>57.9</td>
<td>55.9</td>
<td>2.0</td>
</tr>
<tr>
<td>4,000</td>
<td>62.8</td>
<td>60.6</td>
<td>2.2</td>
</tr>
<tr>
<td>5,000</td>
<td>66.8</td>
<td>67.7</td>
<td>(0.9)</td>
</tr>
<tr>
<td>6,300</td>
<td>67.9</td>
<td>70.1</td>
<td>(2.2)</td>
</tr>
<tr>
<td>8,000</td>
<td>70.5</td>
<td>74.8</td>
<td>(4.3)</td>
</tr>
<tr>
<td>10,000</td>
<td>71.8</td>
<td>75.9</td>
<td>(4.1)</td>
</tr>
</tbody>
</table>
the noise floor introduce errors at low frequencies using the ISO 3741 method. In this low frequency regime, the VBRM method may be more accurate.

Above the 4 kHz one-third octave band, discrepancies between the two methods also appear. The 11x13 measurement grid used in this experiment resulted in a spatial sampling of one scan point every 3.73 cm in the horizontal direction and 3.82 cm in the vertical direction. The Nyquist frequency associated with the plate and given spatial sampling was 5.5 kHz. Due to the 5 kHz one-third octave band extending to 5,623 Hz one would expect to see errors in and above the 5 kHz one-third octave band. Therefore, showing agreement between the two methods at and above the 5 kHz one-third octave band would require a finer spatial sampling mesh and is left for future research.

Using the VBRM method to measure the sound power for a single simply-supported plate requires a less restrictive setup when compared to current ISO standards. The ISO 3741 standard requires a reverberation chamber, while the VBRM does not require a specific acoustic environment; thus, the VBRM method allows for sound power measurements in-situ. The VBRM method would also allow for sound power measurements in windy conditions and conditions with varying background noise. Further advantages are gained when extended to scenarios where multiple incoherent sources contribute to sound power.

2.5.2 Multiple separated plates

2.5.2.1 Setup and measurements of the multiple plate system

Following the same procedures used in the single plate system, a second aluminum plate of dimensions 45.5 x 30.3 x 0.16 cm was added to the reverberation chamber on the opposite wall from the first plate. The second plate was also mounted in-plane with the wall of the reverberation chamber with the wall acting as a baffle. A piezoelectric transducer was mounted
Figure 2-5: 7x9 grid of scan points taken to measure the surface velocity vibrations of Plate 2.

Figure 2-6: Structural response of Plate 2 as measured by the SLDV. When compared to Fig. 2-2 one can see the differences between Plate 1 and Plate 2.
in the upper right quadrant of the second plate and was excited with random noise between 0-20 kHz. Using the SLDV, velocity scans of the second plate were taken using a 9x7 grid (see Fig. 2-5) resulting in a spatial sampling of one scan point every 4.3 cm in the horizontal direction and 5.1 cm in the vertical direction. This combination of plate size and spatial sampling resulted in a Nyquist frequency of 3.6 kHz. The structural response curves of Plate 1 (Fig. 2-2) and Plate 2 (Fig. 2-6) illustrate the two plates have distinctive responses to the random noise inputs.

The sound power from the second plate was measured using the same procedure described in Section 2.5.1.1 and the results can be seen in Fig 2-7. Due to the lower Nyquist frequency associated with Plate 2, discrepancies at higher frequencies begin to be seen in the 4 kHz one-third octave band. Plate 1 sound power results showed discrepancies starting in the 5 kHz one-third octave band.

Figure 2-7: Sound power of Plate 2 as measured using ISO 3741 and the VBRM method.
After calculating the sound power of the second plate using the VBRM method, both plates were simultaneously excited using uncorrelated random noise. While both plates were excited, new pressure measurements were taken and the sound power of the multiple plate system calculated according to ISO 3741.

2.5.2.2 Results of the multiple plate system

Using the additive property of sound power from uncorrelated sources, the total sound power of the two-plate system was calculated by summing the sound powers of the two plates individually calculated using the VBRM method. This summation is shown in Fig. 2-8. The sound power at each individual one-third octave band is most impacted by the plate which radiates the most energy in that band. The larger plate (Plate 1) dominates between 300 Hz and 800 Hz and the smaller plate (Plate 2) contributes more at frequencies between 1.9 kHz and 5 kHz.

![Figure 2-8: Sound powers of plate 1 and plate 2 as well as the total sound power of the two-plate system.](image)
The calculated total sound power using the VBRM method is compared to the ISO 3741 power in Fig 2-9 and in Table 2-2. Between the 250 Hz and 3,150 Hz one-third octave bands there is very good alignment between the two methods with the maximum difference being 1.6 dB at the 250 Hz one-third octave band. The mean one-third octave band difference between 250 Hz and 3,150 Hz bands was -0.3 dB with a standard deviation of 0.7 dB. The total sound power difference between 250 Hz and 3,150 Hz was 0.7 dB.

![Figure 2-9: Comparison of the sound power calculations of the multiple plate system using ISO 3741 and the VBRM method.](image)

At frequencies lower than 200 Hz, the Schroeder frequency and the noise floor again caused discrepancies between the two methods, with the VBRM possibly giving more accurate results in this regime. In the single-plate section there was very good alignment up to 4 kHz due to the Nyquist frequency being 5.5 kHz for the large plate. Due to the spatial sampling and the
size of the smaller plate, the Nyquist frequency was 3.6 kHz and there begin to be errors in the 4 kHz region and above. These results indicate that the VBRM method is accurate up to the Nyquist frequency.

Table 2-2: Sound power results from a multiple plate system.

<table>
<thead>
<tr>
<th>Third octave band by centerband frequency (Hz)</th>
<th>ISO 3741</th>
<th>VBRM</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>22.9</td>
<td>2.7</td>
<td>20.2</td>
</tr>
<tr>
<td>125</td>
<td>23.0</td>
<td>8.8</td>
<td>14.2</td>
</tr>
<tr>
<td>160</td>
<td>21.3</td>
<td>18.8</td>
<td>2.6</td>
</tr>
<tr>
<td>200</td>
<td>17.7</td>
<td>20.0</td>
<td>(2.3)</td>
</tr>
<tr>
<td>250</td>
<td>16.5</td>
<td>18.1</td>
<td>(1.6)</td>
</tr>
<tr>
<td>315</td>
<td>30.4</td>
<td>29.8</td>
<td>0.5</td>
</tr>
<tr>
<td>400</td>
<td>31.1</td>
<td>31.3</td>
<td>(0.2)</td>
</tr>
<tr>
<td>500</td>
<td>31.8</td>
<td>32.9</td>
<td>(1.0)</td>
</tr>
<tr>
<td>630</td>
<td>37.3</td>
<td>37.6</td>
<td>(0.3)</td>
</tr>
<tr>
<td>800</td>
<td>41.7</td>
<td>42.4</td>
<td>(0.7)</td>
</tr>
<tr>
<td>1,000</td>
<td>41.2</td>
<td>41.7</td>
<td>(0.5)</td>
</tr>
<tr>
<td>1,250</td>
<td>45.6</td>
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<td>(0.4)</td>
</tr>
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<td>1,600</td>
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<tr>
<td>2,000</td>
<td>51.6</td>
<td>51.6</td>
<td>0.0</td>
</tr>
<tr>
<td>2,500</td>
<td>55.0</td>
<td>55.2</td>
<td>(0.2)</td>
</tr>
<tr>
<td>3,150</td>
<td>60.3</td>
<td>59.0</td>
<td>1.3</td>
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<tr>
<td>4,000</td>
<td>66.8</td>
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<td>(0.4)</td>
</tr>
<tr>
<td>5,000</td>
<td>69.4</td>
<td>71.3</td>
<td>(1.9)</td>
</tr>
<tr>
<td>6,300</td>
<td>70.3</td>
<td>73.3</td>
<td>(3.1)</td>
</tr>
<tr>
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<tr>
<td>10,000</td>
<td>74.6</td>
<td>80.3</td>
<td>(5.7)</td>
</tr>
</tbody>
</table>

In this multiple plate scenario, the VBRM method exhibited the same advantages over current ISO standards as the single-plate scenario. Additionally, the VBRM method also showed the capability of measuring each plate’s contribution to sound power without having to isolate and measure each plate individually, so long as the radiated waves are not correlated or coupled, as would be required by current ISO standards.
2.6 Conclusions

This paper has focused on developing a method to calculate the sound power of radiating structures using vibration-based measurements. Results for a single- and multi-plate system have been presented, using both ISO 3741 and the VBRM method.

It was shown that for a single plate, sound power calculated using ISO 3741 and the VBRM method had a maximum one-third octave band difference of 2.2 dB between the 200 Hz and 4 kHz one-third octave bands and an overall sound power level difference of 1.7 dB in that frequency range.

It was then shown that for multiple plate systems driven with uncorrelated signals the VBRM method agreed with ISO 3741, with the maximum one-third octave band difference of the multiple plate system being 1.6 dB between the 250 Hz and 3.125 kHz one-third octave bands. The overall difference in sound power level was 0.7 dB in that frequency range.

The VBRM method allows for sound power measurements in a variety of situations made difficult by current ISO standards. These situations include but are not limited to scenarios where anechoic or reverberation chambers are not accessible or cannot fit a specified setup, windy conditions which prohibit the use of ISO standards, or the source of interest is part of a larger system. The VBRM method also allows the measurement of the contribution to total sound power of multiple incoherent sources in the same environment where coupling is negligible without requiring each source to be isolated and tested individually.

The current results are limited to situations where the multiple sources are incoherent, and the frequency range of interest is below the Nyquist frequency as dictated by the spatial density of the surface velocity measurements. Future research will explore the possibility of
extending the VBRM method to coherent sources and built-up structures where there are multiple sources contributing to sound power from one structure.
3 SOUND POWER OF VIBRATING CYLINDERS USING RADIATION RESISTANCE MATRICIES AND THE VBRM METHOD

This chapter contains a paper to be submitted to the Journal of the Acoustical Society of America. The contents have been reformatted to meet the formatting requirements for this thesis. This paper was co-authored by Caleb Goates. The mathematical development of the cylindrical radiation resistance matrix was developed by Caleb. The computational results were the result of a joint effort, with Caleb determining the boundary element methods (BEM) to be used and the author of this thesis calculating results using the VBRM method. The experimental results were compiled by the author of this thesis.

3.1 Contributing Authors and Affiliations

Cameron B. Jones
Department of Mechanical Engineering, Brigham Young University, Provo, UT 84602, United States

Caleb Goates
Department of Physics and Astronomy, Brigham Young University, Provo, UT 84602, United States

Jonathan D. Blotter
Department of Mechanical Engineering, Brigham Young University, Provo, UT 84602, United States

Scott D. Sommerfeldt
Department of Physics and Astronomy, Brigham Young University, Provo, UT 84602, United States

3.2 Abstract

Research has shown that using acoustic radiation modes combined with surface velocity measurements provides an accurate method of measuring the radiated sound power from
vibrating plates. This paper investigates the extension of this method to account for acoustically radiating cylindrical geometries.

The mathematical formulations of the radiation resistance matrix and the accompanying acoustic radiation modes of a baffled cylinder are developed. Computational sound power calculations using the Vibration Based Radiation Modes (VBGM) method and boundary element method (BEM) are then compared and shown to have good agreement. Experimental surface velocity measurements of a cylinder are taken using a scanning laser Doppler vibrometer (SLDV) and the VBGM method is used to calculate sound power. The results are compared to sound power measurements taken using ISO 3741.

3.3 Introduction

Many methods exist for measuring sound power. The International Organization for Standardization (ISO) has published ten standards and two technical specifications detailing how to accurately obtain sound power measurements.4 In the late 1980s and early 1990s a new method was developed to calculate sound power based on a combination of measured surface vibrations of a structure and acoustic radiation modes5.

Structural vibration modes describe the displacement of a structure in terms of orthogonal basis functions. Conversely, acoustic radiation modes describe the acoustic field in terms of orthogonal basis functions and allow the surrounding acoustical field to be calculated based on the vibrations of a structure. Acoustic radiation modes can be derived from the radiation resistance matrix. The radiation resistance matrix relates the surface velocities from discrete elements of the structure to the radiated sound power of the structure through the equation

\[
\bar{P}(\omega) = \mathbf{v}_e^H(\omega)\mathbf{R}(\omega)\mathbf{v}_e(\omega), \tag{3-1}
\]
where $v_e$ is a column vector containing the velocity at each discrete element, $(\cdot)^H$ signifies the Hermitian transpose, $\omega$ is the frequency of interest, and $R$ represents the radiation resistance matrix.\textsuperscript{5} The dependence of various quantities on $\omega$ is implied in expressions throughout the remainder of this paper and will be omitted. The eigenvectors of the radiation resistance matrix are the acoustic radiation modes and the corresponding eigenvalues are proportional to the radiation efficiencies of the eigenvectors. The sound power can be written in terms of the acoustic radiation modes $q_r$ and eigenvalues $\lambda_r$ as

$$P = \sum_{r=1}^{R} \lambda_r |\vec{y}_r|^2$$

where $\vec{y}_r = q_r \cdot v_e$, and $R$ represents the number of elementary radiators over the surface of the structure\textsuperscript{5}.

In 2002, Bai et al. published research showing sound power calculations using the most efficient radiating modes at low frequencies and a modified approach at higher frequencies.\textsuperscript{24} These results showed good agreement between experimental measurements and ISO 3745 at low frequencies, but the results diverged at higher frequencies. Research has also been conducted to show that acoustic radiation modes can be used to calculate the individual contributions to sound power from multiple uncorrelated sources in a system without having to isolate the sources individually.\textsuperscript{4} This research used the Vibration Based Radiation Mode (VBRM) method which will be used throughout this paper. The VBRM method consists of using complex surface velocity measurements with the radiation resistance matrix to compute the sound power.

In addition to sound power calculations, radiation modes have found use in the field of Active Structural Acoustic Control (ASAC).\textsuperscript{21,25-27} In 2010, a new parameter, later labeled the “weighted sum of spatial gradients” (WSSG), was developed to improve ASAC methods.\textsuperscript{23,25}
Research on the effectiveness of WSSG as a parameter in ASAC relied on use of the radiation resistance matrix both computationally and experimentally.\textsuperscript{25,26} Radiation Modes have also been used as a guide for structural design, where certain efficient radiation vibration patterns are suppressed through structural modifications.\textsuperscript{28} Recent work shows that radiation modes may be used as a basis set for acoustical holography source reconstruction.\textsuperscript{29}

Many of the early papers on radiation modes present results for cylinders. These papers that introduced radiation modes included a finite cylinder with hemispherical endcaps\textsuperscript{31} and two finite cylinders with flat endcaps.\textsuperscript{32,33} In each of these cases, only the axisymmetric modes were calculated. These modes were found by an unspecified numerical method, boundary integral methods, and the boundary element method (BEM), respectively. In addition, at least one other paper has treated the hemispherically capped finite cylinder.\textsuperscript{31} Through all these publications there has never been a full development of the radiation modes for a cylinder such that the sound power could be calculated. Boundary element methods can be used to calculate the radiation resistance matrix,\textsuperscript{28} but an analytical formulation has the potential to reduce complexity and computational load. More recently, Aslani \textit{et al.} published a formulation for radiation modes of a finite cylinder sandwiched between two infinite pressure release planes using eigenfunction expansion.\textsuperscript{33} This paper will closely follow the formulation of Aslani \textit{et al.} to develop a full analytical expression for the radiation resistance matrix of vibrating cylinders with infinite cylindrical baffles. These cylindrical radiation modes will provide an additional resource for the calculation of sound power of cylindrical objects.
3.4 Cylinder Radiation Modes

3.4.1 Eigenfunction Formulation of the Cylinder Radiation Resistance Matrix

The radiation resistance matrix is derived from the pressure that a small vibrating element of a structure generates across the structure. Assume that a small portion of a hard-infinite cylinder is vibrating with velocity

\[ u(\theta, z) = \begin{cases} u_0 & \theta_1 \leq \theta \leq \theta_2, z_1 \leq z \leq z_2 \\ 0 & \text{otherwise} \end{cases} \quad (3-3) \]

for some \( \theta_1, \theta_2 \) and \( z_1, z_2 \) such that \( a\Delta \theta \equiv a(\theta_2 - \theta_1) \ll 2\pi/k \) and \( \Delta z \equiv (z_2 - z_1) \ll 2\pi/k \), where \( k \) is the acoustic wavenumber and \( a \) is the radius of the cylinder. This vibration creates a pressure field that can be written in terms of cylindrical eigenfunctions as

\[ p(r, \theta, z) = \sum_{m=0}^{\infty} \int_{0}^{\infty} dk_z \left( A_m \cos m\theta + B_m \sin m\theta \right) D(k_z) \cos k_z z \]

\[ + E(k_z) \sin k_z z H_m^{(2)}(k_r r), \quad (3-4) \]

where \( k_r = \sqrt{k^2 - k_z^2} \), \( k_z \) is the axial acoustic wavenumber, \( m \) is an integer, \( H_m^{(2)}(x) \) is the \( m^{th} \)-order Hankel function of the second kind, and \( A_m, B_m, D(k_z) \), and \( E(k_z) \) are coefficients yet to be determined. The Hankel function of the first kind is omitted as the absence of sources outside \( r = a \) precludes incoming cylindrical waves. The coefficients are determined by the boundary condition at \( r = a \) of

\[ \frac{\partial p}{\partial r} \bigg|_{r=a} = -j\rho_0 \omega u(\theta, z), \quad (3-5) \]
where \( \rho_0 \) is the density of air, and \( \omega \) is the angular frequency. To apply this boundary condition, the velocity is expanded in terms of the \( \theta \) and \( z \) cylindrical eigenfunctions as

\[
u(\theta, z) = \sum_{m=0}^{\infty} (a_m \cos m\theta + b_m \sin m\theta) \int_0^{\infty} (d(k_z) \cos k_z z + e(k_z) \sin k_z z) dk_z. \tag{3-6}\]

Equation 3-6 can be set equal to Eq. 3-3 to find the coefficients \( a_m, b_m, d(k_z) \) and \( e(k_z) \).

Because Eq. 3-6 is a separable expression, the \( \theta \) and \( z \) dependence may be treated separately:

\[
\sum_{m=0}^{\infty} (a_m \cos m\theta + b_m \sin m\theta) = \begin{cases} u_0 & \theta_1 \leq \theta \leq \theta_2, \\ 0 & \text{otherwise}, \end{cases} \tag{3-6a}
\]

\[
\int_0^{\infty} (d(k_z) \cos k_z z + e(k_z) \sin k_z z) dk_z = \begin{cases} 1 & z_1 \leq z \leq z_2, \\ 0 & \text{otherwise}, \end{cases} \tag{3-6b}
\]

where the constant \( u_0 \) has been arbitrarily assigned to the \( \theta \) dependence expression. The coefficients may now be solved for using orthogonality and sine and cosine transforms:

\[
a_m = \frac{u_0}{\pi} \int_{\theta_1}^{\theta_2} \cos m\theta \, d\theta \approx \frac{u_0 \Delta \theta}{\pi} \cos m\theta_0,
\]

\[
b_m = \frac{u_0}{\pi} \int_{\theta_1}^{\theta_2} \sin m\theta \, d\theta \approx \frac{u_0 \Delta \theta}{\pi} \sin m\theta_0,
\]

\[
d(k_z) = \frac{1}{\pi} \int_{z_1}^{z_2} \cos k_z z \, dz \approx \frac{\Delta z}{\pi} \cos k_z z_0,
\]

\[
e(k_z) = \frac{1}{\pi} \int_{z_1}^{z_2} \sin k_z z \, dz \approx \frac{\Delta z}{\pi} \sin k_z z_0,
\]

where \( z_0 = (z_2 + z_1)/2, \Delta z = z_2 - z_1, \theta_0 = (\theta_2 + \theta_1)/2, \Delta \theta = \theta_2 - \theta_1 \). The approximate equalities may be assumed because \( \Delta z \) and \( a \Delta \theta \) are small compared to a wavelength.
Substituting Eq. 3-7 into Eq. 3-6, applying the boundary conditions in Eq. 3-5 and simplifying results in the final pressure expression,

\[ p(r, \theta, z) = -j \frac{u_0 \rho_0 \omega \Delta \theta \Delta z}{\pi^2} \sum_{m=0}^{\infty} \cos[m(\theta - \theta_0)] \int_0^\infty \frac{H_m^{(2)}(k_r r)}{k_r H_m^{(2)'(k_r a)}} \cos[k_z(z - z_0)] \, dk_z. \]  

(3-8)

Dividing Eq.3-8 by the velocity of the vibrating element, i.e., \( u_0 \), and evaluating at a surface point gives the mutual impedance between the source point, point \( i \), and the field point, point \( j \) such that

\[ Z_{ij} = -j \frac{\rho_0 \omega \Delta \theta \Delta z}{\pi^2} \sum_{m=0}^{\infty} \cos[m(\theta_j - \theta_i)] \int_0^\infty \frac{H_m^{(2)}(k_r a)}{k_r H_m^{(2)'(k_r a)}} \cos[k_z(z_j - z_i)] \, dk_z. \]  

(3-9)

The radiation resistance matrix is concerned only with the real part of this expression. Thus, since \( H_m^{(2)}(k_r a)/k_r H_m^{(2)'(k_r a)} \) is purely real for imaginary \( k_r \), the integration need only be carried out from 0 to \( k \). The elements of the radiation resistance matrix are then found as

\[ R_{ij} = \frac{S_e}{2} \text{Re}\{Z_{ij}\} \]

\[ = \frac{S_e^2 \omega \rho_0}{a \pi^2} \int_0^k \left[ \frac{1}{k_r} \cos[k_z(z_j - z_i)] \sum_{m=0}^{\infty} \text{Im}\left\{ \frac{H_m^{(2)}(k_r a)}{H_m^{(2)'(k_r a)}} \right\} \cos[m(\theta_j - \theta_i)] \right] \, dk_z, \]  

(3-10)

where \( S_e = a \Delta \theta \Delta z \) is the area of a single discrete element of the structure, and the summation has been moved inside the integral.
3.4.2 Numerical Evaluation

Equation 3-10 is not closed-form; it involves an infinite sum that must be truncated and an integral that must be numerically evaluated. This section offers guidance on how the expression may be evaluated.

3.4.2.1 Truncating the Infinite Sum

The sum is performed first for each integration point. As \( m \) increases the ratio

\[
\text{Im}\left\{ \frac{H_m^{(2)}(k_r a)}{H_m^{(2)'}(k_r a)} \right\}
\]

decreases with the coefficient approaching zero rapidly after \( m \approx k_r a \). Therefore, this coefficient is used as the test for convergence. For the purposes of this research once the inequality \( \text{Im}\left\{ \frac{H_m^{(2)}(k_r a)}{H_m^{(2)'}(k_r a)} \right\} < 10^{-8} \) is true, the sum is considered to have converged.

3.4.2.2 Numerical Evaluation of the Integral

The truncated sums may be calculated at desired integration points as dictated by a given integration method. This paper uses the midpoint rule, with the integrand evaluated at 80 points over the interval \([0, k]\). Though this is a rather simple method to perform the integration, it has been shown to be sufficiently accurate for the purposes of this research.

It appears there could be a singularity in the integral at \( k_z = k \), where \( k_r \) becomes zero. Use of the limiting forms of the Hankel functions as the argument goes to zero shows that

\[
\lim_{k_r \to 0} \frac{\text{Im}\left\{ \frac{H_m^{(2)}(k_r a)}{H_m^{(2)'}(k_r a)} \right\}}{k_r} = 0,
\]

so the integrand may be replaced with zero at the endpoint if it is needed for the integration.
3.4.3 Radiation Modes

Acoustic radiation modes are computed with an eigendecomposition of the radiation resistance matrix and provide a useful way to characterize \( R \). The eigenvectors represent the acoustic radiation modes while the associated eigenvalues are proportional to the radiation efficiency. The first nine radiation modes, ordered by the radiation efficiency of the mode, according to the formulation above are shown in Fig. 3-1 for a cylinder with \( a/L = 0.2 \) at \( ka = 0.01 \) rad. The first mode resembles a monopole with all parts of the cylinder vibrating in phase and at equal amplitude. The next three modes resemble dipoles and the final five modes resemble quadrupoles. Due to the symmetries associated with a cylinder, all radiation modes with a \( \theta \) dependence come in pairs of degenerate modes.

Figure 3-2 shows the change in the first nine radiation modes when ordered by radiation efficiencies as \( ka \) is increased to \( ka = 1 \) rad. The first four modes follow the same pattern exhibited in Fig 3-1, though the amplitude is tapered toward the ends of the cylinder. The fifth mode at \( ka = 1 \) is of the seventh mode when \( ka = 0.01 \) when ordered by radiation efficiencies. The sixth and seventh modes at \( ka = 1 \) are the fifth and sixth modes when \( ka = 0.01 \). The eighth and ninth modes in Fig 3-2 are new modes which are not seen in Fig. 3-1.

The modal efficiencies with respect to \( ka \) also give insight into the modal behavior. Efficiencies are plotted in Fig. 3-3 for each of the nine modes shown in Fig. 3-1, with degenerate mode efficiencies combined into one line. This plot shows the monopole/dipole/quadrupole radiation characteristics of the modes at low \( ka \): The first mode increases in efficiency, and therefore power, at a rate of 6 dB/octave, the next three modes at 12 dB/octave, and the last five at 18 dB/octave.
Sound power can be calculated using either the radiation resistance matrix (Eq. 3-1) or the acoustic radiation modes (Eq. 3-2). Sound power calculations using the radiation resistance matrix require a matrix-vector multiplication and a dot product to evaluate while using acoustic radiation modes requires an eigenvalue decomposition of a matrix, several dot products to find $\tilde{y}_r$, and a sum. Since the complexity of eigenvalue decomposition is theoretically limited to that of matrix-vector multiplication$^{34-35}$ and is in practice much slower, there is no benefit to using the
acoustic radiation modes for the sound power calculation presented in this work. Therefore, for implementing the VBRM method, the radiation resistance matrix will be used in sound power calculations. It is possible that interpolation of the radiation modes could, in some future work, make radiation modes faster for power computation, but in the simple uses described by Eqns. (3-1) and (3-2) there is no real benefit to using the radiation modes. The power curves in this paper are therefore calculated using the radiation resistance matrix.

![Figure 3-3: Efficiencies of the nine radiation modes that are most efficient at low $ka$. Degenerate mode efficiencies are combined into one line.](image)

### 3.5 Computational Verification of Sound Power Calculations Using the VBRM Method

To verify the methodology above, sound power measurements using the VBRM method were compared to those calculated using the boundary element method (BEM). BEM simulations were performed using VibroAcoustics One (VA One), a commercial package
produced by the ESI Group. The infinite cylindrical baffle assumed in the theory was approximated by a 1-meter baffle connected to each end of the vibrating portion of the cylinder. Simulations were performed with ridged ends on each end of the cylinder instead of a baffle and the results showed the baffle had negligible effect on measured sound power.

Once the cylinder was modeled in VA One, the surface velocities of the shell were computed at each nodal point of the cylindrical mesh using the modal expansion method developed by Bernoulli for a cylinder excited by a point force.\(^3\) For each location on the mesh created in VA One, the complex surface velocities were calculated using

\[
u_3(x, \theta) = \frac{2P}{\rho ahL\pi} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \epsilon_n \omega_{mn}^2 \left[ 1 - \left( \frac{\omega}{\omega_{mn}} \right)^2 \right] \sin(\frac{m\pi z^*}{L}) \sin(\frac{m\pi z}{L}) \cos n(\theta - \theta^*) e^{-j\phi_{mn}}
\]  

(3-11)

where \(P\) is the point load, \(\rho\) is the density of the cylinder’s material, \(h\) is the thickness of the cylindrical wall, \(L\) is the length of the cylinder, \(m\) and \(n\) are the longitudinal and radial mode numbers, respectively, \(z^*\) and \(\theta^*\) are the longitudinal and radial location of the point force, \(z\) and \(\theta\) are the longitudinal and radial locations of the nodal points, \(\omega\) is the angular frequency of interest, \(\omega_{mn}\) is the natural angular frequency of a given mode, \(\zeta\) is a damping coefficient where

\[
\phi_{mn} = \tan^{-1} \left( \frac{2\zeta_{mn}(\omega/\omega_{mn})}{1 - (\omega/\omega_{mn})^2} \right)
\]

(3-12)

and

\[
\epsilon_n = \begin{cases} 
1 & n \neq 0 \\
2 & n = 0 
\end{cases}
\]

(3-13)

Using the velocities described by Eq. 3-11, the VA One BEM simulation was used to calculate the sound power of a 41 cm long cylinder with a 7.6 cm radius.

Equation 3-11 was also used to calculate the surface velocities used as inputs into the VBRM method. Multiple simulations were run using different spatial sampling grids and
different numbers of points to analyze the effect the spatial sampling had on the VBRM method.

Velocities were calculated on a 41 cm cylinder with a 7.6 cm radius with the following spatial sampling patterns (longitudinal points x theta points): 8x9 (72 points), 10x12 (120 points), 16x19 (304 points) and 26x31 (804 points). Figure 3-4 shows the numerical results using the VBRM method for cylinders for each of the previously stated grids.

![Sound power results using different numbers of data points](image)

Figure 3-4: Numerically calculated sound power using the radiation resistance matrix and simulated complex velocities at a different number of data points.

Figure 3-4 shows the 72-point spatial sampling simulation agrees with simulations using a denser spatial sample below 1.5 kHz. Above 1.5 kHz the results begin to diverge. The 120-point spatial sampling simulation agrees with simulations using a denser spatial sample below 3 kHz, after which the results diverge. The reason for the divergence of the 72- and 120-point simulations is due to the low spatial sampling density associated with fewer data points. The
304-point simulation and the 806-point simulation agree through the 6 kHz range shown on the plot.

Using the sound power results from the 304-point simulation shown in Fig 3-4, a comparison was made with the sound power results derived using the BEM method. These results are shown in Fig 3-5.

![Sound power comparison using the VBRM and BEM methods](image)

Figure 3-5: Numerically derived sound power of a 41 cm long cylinder with a 7.6 cm radius using the VBRM and BEM methods.

Figure 3-5 shows good agreement between the results using the BEM method and the VBRM method. The VBRM method calculates sound power to be slightly higher between 1.5 kHz and 2 kHz and again between 3 kHz and 4 kHz but the difference between the two methods is less than 1.5 dB at most frequencies.
The VBRM method and BEM method were also used to calculate the sound power of a 41 cm long cylinder with a 15.2 cm radius. Due to the larger surface area of the 15.2 cm radius cylinder the number of points used in the simulation was increased to 576 to ensure the spatial sampling was dense enough for accurate results. Figure 3-6 shows the comparison between the VBRM method and BEM results for the 15.2 cm radius cylinder. Similar to the results from the 7.6 cm radius, there is good alignment between the results from the two methods for the 15.2 cm radius cylinder with slight discrepancies at frequencies higher than 3 kHz. These discrepancies are less than 1.5 dB.

![Sound power comparison using the VBRM and BEM methods](image)

Figure 3-6: Numerically derived sound power of a 41 cm long cylinder with a 15.2 cm radius using the VBRM and BEM methods.
3.6 Experimental Verification of Sound Power using the VBRM method

This section will review the experimental setup and results of measuring the sound power of a cylinder using the VBRM method. The results calculated using the VBRM method will then be compared to sound power measurements taken using ISO 3741 in a large reverberation chamber with the results being reported in one-third octave bands.

3.6.1 Experimental Setup and Measurement of a Cylindrical Shell

A 41-cm long aluminum cylinder with a radius of 7.6 cm was mounted on a plywood board. A Modal Shop 2007E shaker was supported by the same plywood board with a small piece of foam to isolate the vibrations of the shaker from the plywood. The stinger of the shaker was attached to the cylinder 8.5 cm from its bottom edge. The mounted cylinder and shaker were then attached to an Outline ET250-3D electronic turntable and placed in a reverberation chamber with dimensions 5m x 6m x 7m (see Fig. 3-7). In preparation to make ISO 3741 sound power measurements, six microphones were set up inside the reverberation chamber according to the guidelines of the standard.

It is important to note the experimental setup described above does not perfectly match the theoretical and computational assumptions presented in previous sections. Previous sections assumed a simply-supported cylinder with an infinite cylindrical baffle (approximated by a 1-meter baffle in VA One) extending from the end of each cylinder. While the use of acoustic radiation modes is agnostic to boundary conditions, the lack of a baffle and the inclusion of a turntable and wooden base are departures from previously made assumptions. The case of a cylinder without a baffle was tested using the BEM in VA One and was shown to have a negligible effect on the outcome.
The shaker was excited using pseudo-random noise between 0 and 12.4 kHz. Using a scanning laser Doppler vibrometer (SLDV), line scans measuring the complex surface velocities of the cylinder were taken at 10-degree intervals around the circumference of the cylinder. Each line scan contained 31-points resulting in a total of 1,116 scan points over the surface of the cylinder. This number of experimental points was well above the number of points needed to obtain accurate results up to 6 kHz as shown in Fig 3-4. Fig 3-8 shows an example of the setup of one-line scan.

Figure 3-7: Setup of a mounted cylinder on a turntable with a shaker to excite the cylinder.
One section of the cylinder was blocked by the shaker and surface velocity measurements using the SLDV were unattainable (see Fig. 3-9). Velocity data from surrounding points were used to approximate velocity data of the blocked points. There were a total of 25 out of the 1,116 scan points where the velocity data was approximated using surrounding points. It is notable that due to the proximity of the blocked portions of the cylinder to the point of excitation on the cylinder, the approximated velocity data at those points is expected to be underestimated. The resulting velocity data collected by the SLDV were then used as inputs to the VBRM method to calculate sound power.

After the surface velocity measurements were collected, the SLDV was removed from the reverberation chamber and sound pressure measurements were taken according to the procedures set forth in ISO 3741 and sound power was derived. The calculated sound power measurements using the VBRM method were then compared to the ISO 3741 results reported in one-third octave bands.
3.6.2 Sound Power Results of the Cylinder

Figure 3-10 shows the comparison between the VBRM method and the ISO 3741 sound power results. The results are also summarized in Table 3-1 which shows the difference between the methods at each one-third octave band.

Below 200 Hz the ISO measured sound power results were within 10 dB of the noise floor of the chamber. ISO 3741 states that if the sound power measurements are within 10 dB of the noise floor the results should be considered upper bounds of the radiated sound power. Between the 200 Hz and 10 kHz one-third octave bands there is good alignment between the two methods. In this region the mean difference between the two methods was 0.1 dB with a standard deviation of 1.4 dB.

Figure 3-9: The shaker blocked certain points on the cylinder from being measured. Velocity data from surrounding points were used in place of these points.
The results using the VBRM method were slightly lower than the ISO 3741 results at lower frequencies. This could be due to a combination of factors. The theoretical and computational work assumed an infinitely baffled cylinder, but the experimental setup included endcaps on each end of the cylinder instead of a baffle. The endcaps could have radiated noise contributing to the sound power measured by ISO 3741. The experimental setup of the cylinder also included a plywood mount, a turntable, and a shaker, which could have contributed to sound power when measured by ISO 3741 but would not have contributed to the sound power as measured using the VBRM method.

Figure 3-10: Results of the sound power measurements using the VBRM method compared to the ISO 3741 standard results.
Between the 4 kHz and 8 kHz one-third octave bands there were slight discrepancies between the ISO measured sound power and the VBRM measured sound power with the maximum difference between the two methods being 2.1 dB at the 4 kHz and 5 kHz one-third octave bands. These differences are in line with the differences seen between the BEM and VBRM methods compared numerically in Fig. 3-5.

### 3.7 Conclusions

After a brief review of the concept of radiation resistance matrices and their uses, a derivation of the cylindrical radiation resistance matrix was presented. This produced a full,
analytical expression for the matrix, which can be used in sound power calculations. The radiation modes computed from this matrix were shown to match multipole trends at low frequencies as would be expected from canonical radiation modes for other geometries. Numerical methods to solve the non-closed form equations were presented.

Following the derivation of the cylindrical radiation modes, numerical results were presented and compared using the VBRM method and the BEM method using VA One. A comparison of these results showed very good agreement between the two methods between 0 Hz and 4 kHz with slight discrepancies of less than 1.5 dB appearing between 3 kHz and 4 kHz.

Experimental surface velocity measurements were collected using a SLDV and the sound power was determined using the VBRM method. The sound power was also measured using ISO 3741. These experimental results showed good agreement through the 10 kHz one-third-octave band. Between the 200 Hz and 10 kHz one-third octave bands the mean difference in the sound power obtained using ISO 3741 and the VBRM method was 0.1 dB with a standard deviation of 1.4 dB. The maximum difference between the two methods in any one-third octave band was 2.1 dB which occurring at the 4 kHz one-third octave band.

The results of the numerical simulations and the experimental work presented in this paper have shown that the cylindrical radiation resistance matrix and the accompanying acoustic radiation modes developed in the paper, combined with the use of the VBRM method, are useful tools which allow for the sound power measurement of cylinders.
4 CONCLUSIONS AND RECOMMENDATIONS

This chapter will present the conclusions drawn from the work presented in this thesis and will include recommendations for future work which will continue to refine the VBRM method and will continue the process of developing a vibration based sound power measurement method that could potentially provide Precision (Grade 1) results.

4.1 Conclusions

The work presented in this thesis presents significant progress towards the development of a vibration based sound power measurement. The VBRM method of measuring the sound power was presented. This proved to be an efficient method of measuring the sound power of plates. The results showed good alignment when compared to sound power measurements taken following ISO 3741. In the case of a single plate, the mean one-third octave band difference between the two methods between the 200 Hz and 4 kHz one-third octave bands was -0.1 dB with a standard deviation of 1.1 dB.

In a multiple plate system, the VBRM method was shown to measure the contribution of each plate to total sound power without requiring the acoustic radiation of each plate to be measured in isolation. This capability requires each plate be driven by uncorrelated sources and the plates be separated by a distance large enough that the resulting acoustic waves do not couple. This presents a solution for situations where multiple incoherent sources of radiation
cannot be isolated, and it is desirable to know individual contributions to sound power. Between the 250 Hz and 3.15 kHz one-third octave bands, the multiple plate system had a mean one-third octave band difference of -0.3 dB and a standard deviation of 0.7 dB when comparing the ISO 3741 and VBRM method results.

The VBRM method was then extended to situations with acoustically radiating cylindrical geometries. The formulation of the radiation resistance matrix and acoustic radiation modes were presented. Numerical results using the VBRM method and BEM were compared and the results showed good alignment with slight discrepancies of less than 1.5 dB between 3 kHz and 4 kHz.

An experimental setup of a 41 cm long cylinder with a 7.6 cm radius was developed and an SLDV was used to take surface velocity measurements. The experimental results were used as inputs to the VBRM method to calculate sound power. These results were compared to sound power measurements taken using ISO 3741. The results showed good agreement between the ISO 3741 measurements and the VBRM method up to 10 kHz with a mean difference between the methods of 0.1 dB with a standard deviation of 1.4 dB.

4.2 Recommendations and Future Work

The results of this work have highlighted many areas where future research could contribute to the development of a vibration based sound power measurement method. Recommendations for future work include research into correlated sources and built-up structures, better experimental techniques for measuring non-flat surfaces, and continued research into the radiation resistance matrix for more complex geometries.
4.2.1 Multiple Coupled Sources and Built-Up Structures

This thesis showed that in scenarios where multiple sources are separated by a distance large enough that radiated waves remain uncoupled, that accurate sound power measurements could be taken. As the distance between sources narrows, the resulting waves begin to couple, resulting in inaccurate sound powers measurements if the sound powers from each source are simply added. To test this the two plates described in Section 2.5.2 were mounted side by side in the reverberation chamber (see Fig. 4-1) and the sound power was calculated using ISO 3741 and the VBRM method. The results shown in Figure 4-2 demonstrate the error introduced into sound power measurements as the distance between the plates narrowed and the acoustical waves began to couple.

Figure 4-1: Setup of two plates close together that resulted in wave coupling and poor sound power results.
The problem faced by built up structures is similar to the problem faced by sources that are close together. Vibrations from built up structure are often driven by the same source or coherent sources which lead to coupled waves. Once the acoustic waves are coupled the sound power is no longer additive. Figure 4-3 shows one example of a built-up structure. The small plate could be considered one source of acoustic radiation while the larger box surrounding the plate could be considered another. In practice, cars would be a good example of built up structures where each window could be considered a source of acoustic radiation contributing to the total sound power inside the car. Future research is needed to understand how vibration-based methods could be used to measure the sound power of built up structures.

Figure 4-2: Two plates separated by a small distance begin to have errors in their sound power measurements due to wave coupling.
4.2.2 Experimental Measurement Techniques

Taking accurate surface velocity measurements of a cylinder is a difficult process and could become a limiting factor for future use of the VBRM method in practice. Research into methods to measure the surface velocities of a cylinder in a simple and accurate way is needed if this method is to be used. Such methods could include the use of a 3D SLDV system. Use of a 3D system could limit the time it takes to collect velocity data on a cylinder and would avoid alignment errors as the SLDV alignment would only happen three or four times during the scan of an entire cylinder.
4.2.3 The Radiation Resistance Matrix

The number of structural geometries for which the mathematical description of the radiation resistance matrix has been developed is limited. Future work into understanding the radiation resistance matrix for different geometries should be considered.

Caleb Goates, co-author of the paper presented in Chapter 3 of this work, has started research into calculating the radiation resistance matrix of curved plates and partial cylinders. This work will expand the set of scenarios where the VBRM method may be used. Future research could also look at scenarios for plates and cylinders that are not baffled or do not have endcaps to further expand the situations in which the VBRM method could be used.

Finally, other non-theoretical methods should be researched which would allow the radiation resistance matrix of non-conventional shapes to be calculated. These methods could include computational or experimental derivations of the radiation resistance matrix. Such a method would allow for vibration based sound power measurement methods to be used to calculate the sound power of in a variety of situations other than situations using plates and cylinders.
REFERENCES


