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A Dynamic Optimization Framework with Model Predictive Control Elements for Long Term Planning of Capacity Investments in a District Energy System

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ABSTRACT

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The capacity expansion of a district heating system is studied with the objective of evaluating the investment decision timing and type of capacity expansion. District energy is an energy generation system that provides energy, such as heat and electricity, generated at central locations and distributed to the surrounding area. The study develops an optimization framework to find the optimal investment schedule over a 30 year horizon with the options of investing in traditional heating sources (boilers) or a next-generation combined heat and power (CHP) plant that can provide heat and electricity. In district energy systems, the investment decision on the capacity and type of system is dependent on demand-side requirements, energy prices, and environmental costs. The main contribution of this work is to formulate the capacity planning over a time horizon as a dynamic optimal control problem. In this way, an initial system configuration can be modified by a "controller" that optimally applies control actions that drive the system from an initial state to an optimal state. The optimal control is a model predictive control (MPC) formulation that not only provides the timing and size of the capacity investment, but also guidance on the mode of operation that meets optimal economic objectives with the given capacity.

Keywords: heating, network, capacity, expansion, boilers, energy, controller, optimal, timing, formulation, economic, dependent

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CHAPTER 1. INTRODUCTION

Private and public utilities are confronted with a number of investment options to supply their growing energy demands; investment in new energy systems must compete with other opportunities to improve the financial bottom line of the overall business or institution's goals [1] . Stakeholders want the answers to four fundamental questions for the success of the investment: How much will be earned and when, and how much to spend and when? In the power generation sector, uncertainty on type, timing and stringency of potential air emission regulations coupled with uncertainties on fuel prices, future costs, and energy demand inhibit stakeholders from making robust investment decision early on [2, 3]. Power producers also face the task of balancing the opposite objectives of economic viability, security of supply, and environmental regulations. These uncertainties and conflicting objectives that stakeholders face may cause them to make suboptimal decisions that delay investments. These suboptimal investments are are usually done in small increments that in the long term can cost more than if a less uncertain investment option is taken early on. In like manner, end users of energy like commercial and institutional facilities that have a large energy demand from buildings also face similar challenges as they try to fulfill their growing energy needs and simultaneously comply with pressures to reduce their carbon footprint. One example of those regulations come from EPA Title V of the Clean Air Act, an air operating permitting program that applies to major emitters of air pollution and some other non-major source producers [4]. Title V regulations can affect non industrial institutions like hospitals, universities, or research campuses that produce their own energy for mainly space heating and cooling such as on-site heating plants. In large building campuses, energy demand is usually supplied through a combination of heating plants and electricity from the grid. To face those conflicting objectives, a growing number of institutions are turning to more energy efficient combined heat and power (CHP) plants to meet energy demands. When an on-site CHP system is properly planned, designed, constructed, and operated, it offers a proven method to lower overall facility energy consumption and costs, and reduces total overall utility system fuel consumption [5].

Incorporating a CHP system may be a good alternative in leveraging the uncertainties from the economic and environmental forces to the growing energy demands of the institution. But in considering such investment, the institution must decide on numerous options like whether to continue expanding traditional combustion units for heating plants, installing pollution abatement equipment, reconfiguring the entire space heating/cooling system to an electricity only system, adding energy storage capabilities, or rather to build a new CHP unit that may reduce the costs associated with future CO_2 emissions [3]. With so many options and an uncertain outlook on fuel costs, demand, and regulations; there is a motivation to optimize the investment and operations decisions for these types of problems to ensure reasonable return on the investment [6]. An optimization approach to solve such problems has been attempted in the past decades that mainly focuses on characterizing uncertainty in model parameters and key inputs under a linear model framework. A holistic optimization approach that takes into account multi-objective issues under uncertainty, system dynamics, and intrinsic non-linear constraints is yet to be fully developed for capacity expansion and investment planning problems. This work introduces a dynamic multi-objective optimization approach based on a Model Predictive Control (MPC) framework to find an optimal long-term planning horizon decision outlook for the capacity of a future CHP plant to answer the pressing questions timing and capacity of investment. The optimization framework is applied to an university campus energy system, but the goal of this work is to provide the optimization methods and modeling knowledge to other energy systems with dynamics, system uncertainties, and nonlinear constraints that affect the capacity planning over a time horizon.

CHAPTER 2. BACKGROUND

2.1 District Energy Systems and Combined Heat and Power

District energy systems are energy generation systems that provide any combination of electrical distribution, heating, and cooling, where heating and cooling are generated at central locations and distributed to the surrounding area. District energy systems take advantage of economies of scale to efficiently and cost-effectively provide heating, cooling, or electricity for an immediate surrounding area. Buildings can be supplied by large centrally-located generation equipment, rather than smaller individual units for each building [7]. District energy systems can be composed of various energy conversion technologies such as traditional gas or coal boilers, reciprocating engines, combustion turbines, or industrial processes that generate excess heat. In addition to the energy conversion technology available, the district energy system may include thermal energy storage to offset demand constraints when thermal energy is not sufficiently available or shift production to times of the day when it is more cost effective to produce energy. Regardless of the components that constitute the district energy system, the overall effectiveness of the energy systems heavily relies on how the components of the energy systems interact with each other [8]. The interaction of the system components will depend on the energy demand, system constraints, and system dynamics.

One of the most efficient district energy system arrangements is that of a combined heat and power (CHP) plant, also known as a cogeneration plant. A CHP plant simultaneously produces heat and generates power from a single fuel source [5]. CHP systems can consist of a number of individual components like the prime mover, generator, heat recovery, and electrical interconnection. The prime mover is the type of equipment that drives the overall system and it will typically identify the CHP system. Prime movers for CHP systems include reciprocating engines, combustion or gas turbines, steam turbines, microturbines, and fuel cells. These prime movers are capable

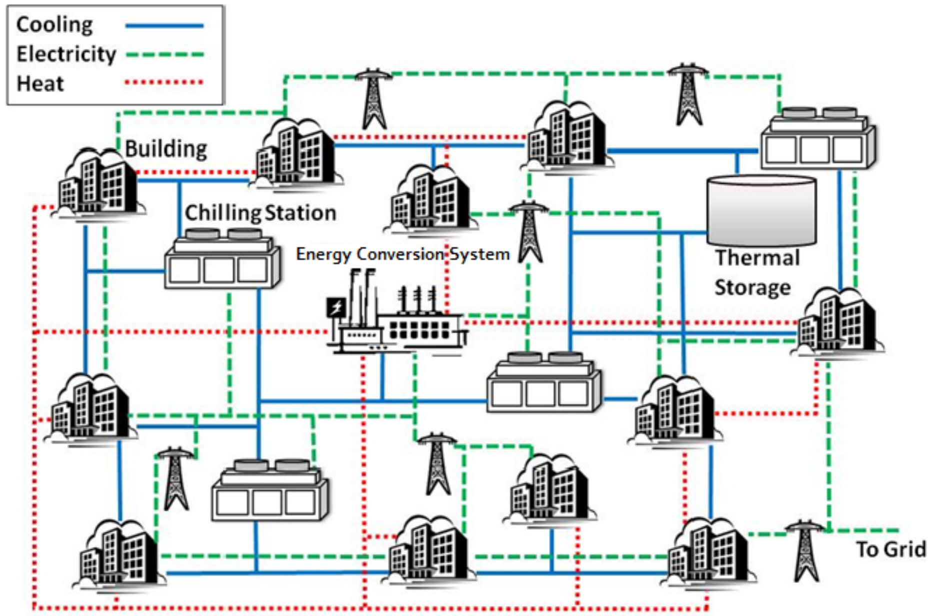

Figure 2.1: Generalized Schematic of District Energy System of an university campus. (courtesy Kody Powell Ph.D., UT Austin).

of burning a variety of fuels, including natural gas, coal, oil, and alternative fuels to produce shaft power or mechanical energy [9].

In building campuses where electric loads are larger than 1 MW, combustion turbine generators (CTG) are popular prime movers for CHP considerations. A combustion turbine system mainly consists of compressor, combustor, and turbine. CTG are commercially available in many capacities varying from small 1 MW to 100 MW utility scale generators [5]. Martens et al. in a survey of CHP efficiencies reports that small gas turbines (<10 MWe) have electric efficiencies lower than 30%, whereas gas turbine based CHP systems between 10 and 40 MW reach electric efficiencies of 30 to 40%. Gas turbines larger than 40 MW have electric efficiencies of about 35% [10]. The remaining energy from the combustion is discarded in the exhaust where it is then recovered by a heat recovery boiler. A heat recovery boiler is similar to a typical fuel fired boiler but instead of using the heat from a separate combustion reaction, the exhaust from the turbine is the source of heat to make hot water or steam depending on the need of the overall district energy system [5]. When the heat recovery boiler produces steam, it is called a heat recovery steam generator (HRSG). If the energy from the exhaust gas from the turbine is not enough to supply the heating needs of the system, supplemental burners and or extra boilers in parallel maybe needed

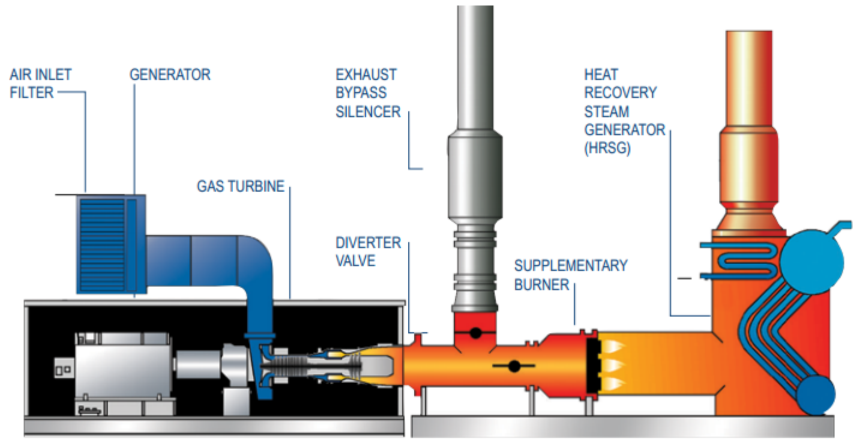

Figure 2.2: Typical Combined Heat and Power System with HRSG [11].

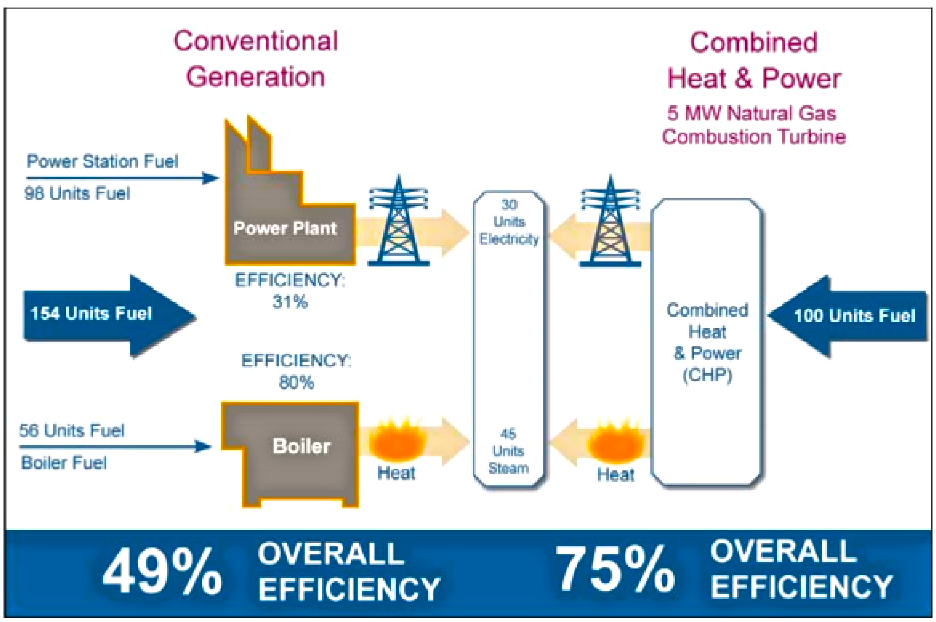

Figure 2.3: CHP versus Separate Heat and Power Production [9].

to supplement the heat recovery boiler. The heat recovery part of the system will have efficiencies of traditional boilers of around 80%. Although operating conditions and capacity size have significant effects on the overall efficiency of a CHP system, the combined processes of power generation and subsequent heat recovery as illustrated in Figure 2.2 have expected efficiencies of around 75 percent [10]. Conventional generation to provide electric power and heat separately will have expected efficiencies of less than 50 percent as illustrated in Figure 2.3 where it compares conventional generation and a CHP option using a natural gas combustion turbine [9].

District energy systems also extend the opportunities for optimization beyond electrical generation and distribution, creating the opportunity for a smart and diverse energy network which provides energy for electrical, heating, and cooling demands [7]. While there is more opportunity for optimization in these systems, the optimization problems themselves are more complex and require models of a diverse range of systems. They also have additional constraints which must be adhered to, including simultaneously meeting other (non-electrical) loads, such as heating and cooling [7]. While optimization methods have been used to exploit savings in energy and reduction of operating costs in operating district CHP systems, the same considerations have not been extended to planning of capacity of district energy systems.

2.1.1 Energy Systems Optimization Under Uncertainty

Decision making in many industries inherently involves consideration of multiple objectives and uncertain outcomes; and in many situations, we must make decisions at different times and at different levels. Those types of problems are generally referred as multi-objective decision processes under uncertainty [12].

Investment and planning decisions in power generation systems fit the above description. As power generation systems become larger and more complex, the number of possible system configurations and technologies that could possibly meet the designer's objectives in an optimal manner increases greatly [13]. The added level of complexity in energy systems usually comes in the form of more efficient and novel equipment because of motivations to conserve energy resources due to economic and geo-political justifications and greater efforts in reducing green house gasses that are increasingly tied to rising global temperatures. In addition, the system may need to be developed taking into account both the dynamic, economic, and environmental effects on system performance. Thus, the difficulty of developing the entire system via the formulation of a single optimization problem is great due to the complexities involved. These complexities are further heightened with the introduction of uncertainty analysis into the problem, transforming the problem from a purely deterministic one into a probabilistic one [13].

Subramanyan et al. [14] describes that in the design of a power system there are three basic types of uncertainties that must be taken into account:

1.) Uncertainty with respect to the model parameters: These parameters are a part of the deterministic model and not actually subject to randomness. Theoretically their value is an exact number. The uncertainty results from the impossibility of exactly modeling the physical behavior of the system.

2.) Uncertainty in the input variables: This kind of uncertainty originates from the random nature and unpredictability of certain process inputs.

3.) Uncertainty in the initial conditions: These types of uncertainties result due to the complications in predicting the initial conditions of the operation.

In a capacity expansion or investment planning optimization problem, such as the one presented here, it may only be necessary to focus on the uncertainty with respect to the model parameters and that of the input variables and disturbances. It can be assumed there is little uncertainty in initial conditions because the status quo of the system is known.

In the past decades the majority of the methods dealing with uncertainties for power generation systems are related to stochastic mathematical programming [15]. More specifically, operations researchers have developed two main types of solution methods: multi-stage stochastic programming (MSSP), and stochastic optimal control (SOC). A notable number of studies that approach the problem of power generation system capacity expansion and investment planning under uncertainty have been developed as MSSP problems [2, 13, 16]. SOC solution approaches are also found for similar problems but in much less frequency [17].

MSSP more specifically deals with problems that involve a sequence of decisions reacting to outcomes that evolve over time. At each stage a decision is made based on currently available information [12]. In many problems where random variables follow multi-dimensional continuous distributions it becomes very difficult to numerically solve those problem because it requires multivariate integration. To avoid this problem, sampling or discrete approximation of the distributions is done to represent the probable space. Those scenarios are many times modeled as a scenario tree that represent discrete scenarios to satisfy specified statistical properties [12]. When a scenario tree is specified, the stochastic program becomes a deterministic equivalent program that is easier to solve [18]. Although this approach makes the problems much more tractable, it stills begs the question of how well the scenario tree actually describes the uncertainty in the variables. Defining a suitable scenario tree is a challenge by itself and still special numerical techniques based on decomposition, aggregation and parallelization are required to solve large-scale problems [12].

SOC can also be referred as Markov decision process, in which the algorithm searches for optimal actions to take at generally discrete points in time in the state being occupied [18]. The actions are taken based on predefined decision rules or policies which are influenced by random outcomes at each specific state and stage [12, 18]. In this fashion, the solution approach is to form a backward recursion that results in an optimal decision associated with each state and each stage [18]. Both MSSP and SOC suffer from the "curse of dimensionality," but in different ways: in MSSP because of the large sample space, and in SOC because of the immense state space [12, 19].

There are certain criteria that are useful in explaining whether a MSSP approach or a SOC approach should be employed. MSSP approaches are reported to be more suitable for solving long-term strategic planning problems, such as capacity planning that have relatively small number of periods and scenarios [12]. SOC problems are reported to work better in problems such as production and inventory control where there are relatively many periods and scenarios but a state space of modest size [12]. This explains the greater use in the literature of MSSP versus SOC for energy systems infrastructure planning under uncertainty. Although the approach to the solution by the two methods is different, Cheng et al. demonstrated that the two methodologies are equivalent in that the decision policy prescribed by SOC is the same as the corresponding optimal decision found by MSSP [12].

In a similar problem of investment and planning for power generation systems under uncertainty Fuss et al. used a real options valuation approach to find a solution [19]. They reported that a MSSP or SOC approach would have resulted in the same outcomes as those obtained in a real options approach. Their work also reports that the main reason for not using stochastic methods was the increased computational intensity due to the dimensionality explosion when there are many periods and scenarios, as well as a modest state space [19].

Other less common, but reported methods in the literature to account for uncertainty in power generation systems expansion and investment planning problems include fuzzy logic [20] and Monte Carlo simulations [21]. Others have used a combination of methods to account for uncertainty. For example, joint probabilistic programming and fuzzy possibility programming was used by Lou et al. in an optimization approach for power generation planning under uncertainty in a mixed integer linear programming (MILP) framework [15]. Another combined method to account for uncertainty was recently reported by Y.F. Li et al. in which a MSSP and fuzzy linear programming is introduced into a MILP framework [22]. Li et al. report that the benefits of such formulations lies in that their approach can tackle uncertainties described in terms of interval values, fuzzy sets, and probability distributions [22]. In energy system planning under uncertainty, the combined method approach can reflect dynamic decisions for facility-capacity expansions and energy supply over a multistage context [22].

2.1.2 Dynamic Optimization and Model Predictive Control for Capacity Planning

Dynamic Optimization constitutes a methodology to optimize systems represented by dynamic models in the form of differential and algebraic equations (DEA). The optimization algorithms for dynamic optimization may handle nonlinear objective functions and constraints with continuous or integer variables. Dynamic optimization is an integral part of some advanced control algorithms such as Model Predictive Control (MPC). MPC is an important advanced control technique that utilizes explicit process models to predict future response of a plant or system [23]. The process models used in MPC are in many cases dynamic and non-linear and capture the dynamic and static interactions between inputs, outputs, and disturbances affecting the system [24]. In control applications of complex chemical and energy processes, MPC technology is extremely beneficial because the algorithms attempt to optimize not only the present optimal control moves, but also optimize future system behavior by computing a sequence of future decision variables adjustments [23]. MPC's ability to predict future variable moves through optimization has similarities to the objectives of capacity planning over a future horizon where economic, environmental, and operational targets must be achieved while the capacity of the system must be optimally planned out under the constraints and uncertainty of the system.

Ricardez-Sandoval et al. [26] reviewed different approaches to simultaneously design and control large systems under process parameter uncertainty. Large systems such as chemical plants are usually designed based on steady state economic calculations, while the control aspects are studied independently. The sequential fashion of the approach from design and control give rise to unforeseen constraints and limitations that can greatly hinder the economic operation of the system once online. The simultaneous optimization of dynamic control variables and design variables

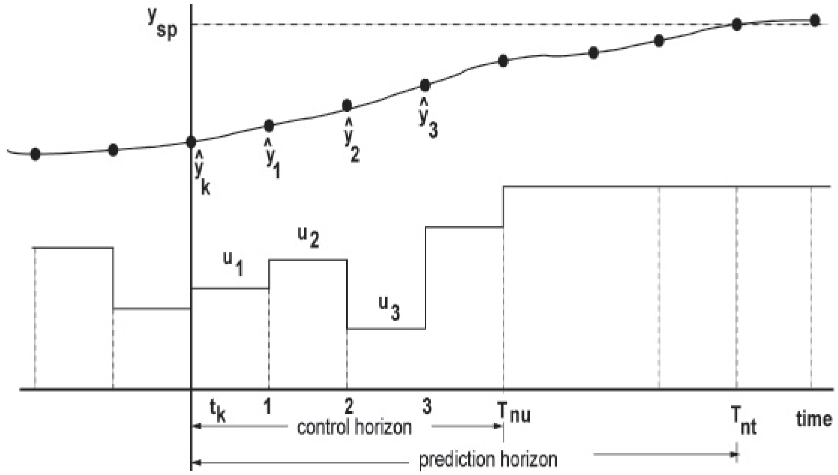

Figure 2.4: MPC approach. y_{SP} is the target set point of the system (i.e. energy demand target or emissions target). T_{nt} number of prediction horizon intervals, and T_{nu} is the number of control horizon intervals. $\hat{y}_i$ are the values of the output controlled variables obtained by applying input manipulated variables u_i [25].

can thus greatly reduce the effect of under sizing or over sizing the capacity of the system and improved profitable operation under different market conditions [26]. From an uncertainty point of view, the dynamic behavior of system parameters and variables is a factor that must addressed in power systems optimization [14], thus the explicit inclusion of system dynamics in the optimization problem as proposed in this work should have a measurable reduction of uncertainty on the optimal size of the system.

From the previous section on stochastic optimization approaches, there is a lack of literature that reports on optimization frameworks that use system dynamics for capacity planning of energy systems. One reason can be attributed to the curse of dimensionality limits found in stochastic programming approaches. System dynamics in the formulation of the problem adds complexity and enlarges the problem, making it even more difficult to solve with current optimization technology. Another reason could be attributed to the recursive features of stochastic programming which do not lend itself to a dynamic model form. The motivation to include system dynamics to optimize capacity planning of energy systems comes from practical experience reported in already planned and constructed plants in which owners find that the planned capacity is not being fully utilized, not enough, or no longer cost effective given changing economic conditions and the load following requirements of the system [27].

2.1.3 Multi-objective Optimization in MINLP Framework

Although the MSSP and SOC approaches historically have been able to provide solutions to planning and scheduling problems under uncertainty, computational expense is one serious drawback. When multi-objective options are incorporated in the MSSP problems, solution times even in the order of days are not uncommon for relatively modest size problems [12]. Science and engineering problems normally feature several and contradictory design and or operation objectives that can be benefited from using new and integrated ways of solving such problems [28]. One innovation is solving these types of problems as a multi-objective, mixed integer programming formulation.

Antunes et al. [29] reports on a multiple-objective mixed integer linear programming model for power generation expansion planning. One important contribution is the consideration of modular expansion capacity values. This approach avoids the need to discretize results in a post-processing phase. In addition their multi-objective MILP approach also focuses on an interactive algorithm that provides decision support in the selection of satisfactory compromise designs (Pareto front designs) [29]. This is an important matter to consider because it can help identify in a systematic way the potential compromised solutions that otherwise can be ignored.

Whether the method to account for uncertainty is a stochastic, fuzzy, Monte-Carlo, or a combination of more than one, the most common optimization framework reported in the literature is the MILP approach. The disadvantage in a MILP approach lies in cases where the system contains non-linear and non-convex constraints in which suboptimal solutions can arise when solved with methods that assume convexity [6]. The application of a mixed integer non-linear programming (MINLP) approach has the potential of providing better results when encountering non-linearity and non-convexity as it is presented in this research.

Gupta and Grossmann in a recent conference proceeding [6] demonstrated the use of MINLP for optimal development planning of offshore oil and gas fields with complex fiscal rules and under nonlinear and non-convex features. Although their approach falls short of a multi-objective application, Gutierrez-Limon et al. expand the application of MINLP alongside a multi-objective optimization approach for scheduling and control for a class of chemical reactors [30].

Historically multi-objective problems in MINLP frameworks have been solved by using a single objective function that is subject to different weights. Those weights are subjective and can give misleading or suboptimal results [25]. On the other hand, multi-objective optimization gives rise to the Pareto front which enables the designer to assess the advantage/disadvantage of a given optimal solution [28]. An advantage from this approach is that the results from the Pareto front allow us to pick up an optimal point containing target behavior [28].

Multi-objective mixed integer non-linear programming (MO MINLP) approaches are the latest advances in optimization techniques for problems suitable for this type of formulation. MO MINLP is starting to be applied to scheduling and control problems in highly nonlinear chemical processes [28]. The possibility exists of online applications in the form of MO MINLP model predictive control as long as the computational times can be feasible for the online process [25]. But as demonstrated by Gupta and Grossman [6], MINLP can be applied to investment and infrastructure planning and a MO MINLP that simultaneously takes into account process uncertainty is the technological frontier that must be further developed.

CHAPTER 3. MODELING DISTRICT HEATING SYSTEM AND CHP

3.1 Modeling for Optimization

A model is a mathematical description of a physical system. One class of mathematical models is referred to as optimization models. Optimization algorithms search for a feasible design according to specified criteria [31]. If the aim is to minimize cost and plan for optimum capacity of a district energy system, the model must be able to calculate cost and represent the capacity with mathematical equations that represent the energy output of the system. The economic and mathematical representation of the dynamics and physical system can be done through first principles or empirical models. Modeling the system through first principles is usually more accurate and intuitively descriptive because the equations will account for thermodynamic, mechanical, chemical, transport phenomena, and any other pertinent fundamental science, but it takes the largest amount of effort and system knowledge to accurately represent all aspects of the system with these fundamentals. On the other hand, pure empirical models are easier to generate but require data from the system being modeled which in many cases might be difficult to obtain at the range of operating conditions desired. A combination of first principles and empirical modeling may be the best option to leverage first principles system knowledge and available data. Parts of the system may be too complex to mathematically represent and require an empirical model. In this work, the mixed approach of first principles and empirical models was used to represent the CHP energy system. This is necessary because the purpose of the optimization program is to find capacity and timing of investment while at the same time responding to diurnal power and heat energy loads of the system. Those goals do not require detailed operating conditions of the CHP system, but do require dynamic response to energy load changes.

3.2 Modeling Boilers and CHP Elements

The motivation in planning for capacity improvements of a district energy system comes from the fact that many district energy systems are district heating systems with an aging fleet of coal and natural gas burning boilers. A district heating system only provides thermal heat from local generation, while power is imported from the grid. The coal or gas boiler based district heating system can be modified to next generation CHP systems that may include a gas turbine and heat recovery boiler elements in the model.

Boiler Modeling

In preparation for this work, considerable efforts were made to understand the system dynamics of the boilers which provide the main heating source for a building campus district energy system. The first step was to generate a model that could represent the cycling time and temperature changes of a coal-fired furnace. The model was created using first principles based on material and energy balances. The energy balance was built around the boiler, with appropriate heat-transfer terms for exchange between the bed, tubes, and high temperature water. A heat transfer term was incorporated into the model to represent the time delay of heating up and cooling down. The heat transfer was based on irradiative heating, as this is the dominant form of heating certain types of coal-fired boilers [32].

A second step in making the boiler model more representative of an operating system was the inclusion of nonlinear dynamic equations that represent the dynamics of the steam drum in a boiler. In operating district energy systems where steam is generated to provide heating and or power for turbines, the main operating goal is to control the steam drum pressure. Since the steam turbines and steam network for heating operate at target pressures, dramatic load changes or sudden upsets in the steam network can upset the pressure and water level of the steam drum with dangerous consequences such as drying or over flowing the drum. Poor drum water level control has been reported to cause up to 30 % of emergency shut downs in utility scale boilers [33].

From the extensive and widely cited work of Alstöm et. al. in modeling boiler dynamics [33], model drum pressure equations were used to simulate boiler load following. Although this dynamic boiler model is not used in the capacity planning problem, the initial effort to model and simulate control of boilers provides fundamental understanding of the interaction and limitations a district energy system has in meeting energy demands. See Appendix A for full dynamic boiler model.

Gas Turbine

The models for the gas turbine generators can be developed using steady-state or dynamic first principles models. The power generated by the gas turbine is a function of the air flow, the fuel flow, the inlet temperature, the temperature at the exit of the compressor, the firing temperature, and the exhaust temperature [7]. Although these factors are extremely important in optimizing online control and interaction of the gas turbine to predict fast dynamics, in a capacity planning problem only the relationship of fuel consumption and power generation are important to establish capacity requirements. To establish a simulated dynamic response to power generation, a first-order differential empirical equation may suffice along with other algebraic relationships for fuel consumption fitted from manufacturer's data.

3.3 From District Heating to CHP System

Figure 3.1 illustrates a typical university campus district energy system. In such arrangement, the buildings and cooling system represent the heating and cooling loads. The heat is provided solely by coal-fired or gas-fired boilers while the electric power supply only comes from the city's electrical grid. The heat produced from the boilers is directly used to provide all the energy for space heating during the winter months as well as any auxiliary uses such as kitchens, showers, laboratories, etc. During the summer months, the boilers continue to operate to provide heat for absorption chillers and auxiliary uses.

A conversion of the current district energy system to one that includes a CHP arrangement is illustrated in Figure 3.2. The new arrangement adds a few important components:

- gas turbine with generator for electricity
- heat recovery boiler
- back-up boiler capacity

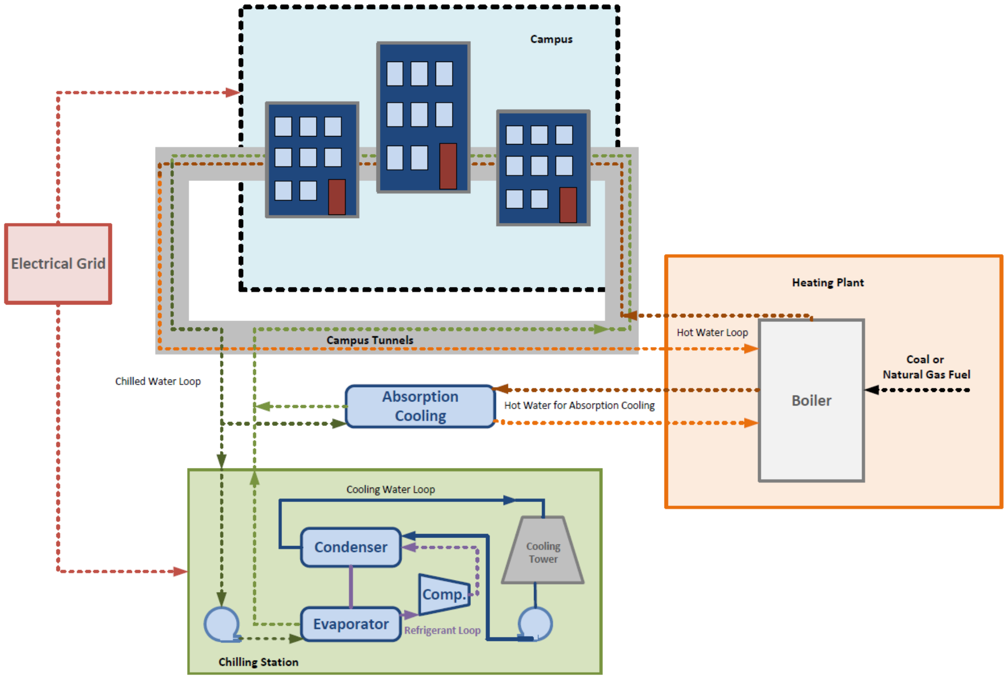

Figure 3.1: Schematic of district heating system with boiler for heating only.

The distribution of heat and electricity to the district energy system remains the same, thus making it only necessary to upgrade the energy generation units and interconnections to the heating and electrical network. This work seeks to answer how large the the CHP plant must be, in terms of capacity, and when it should be installed. The CHP in this analysis will reference a gas turbine and heat recovery boiler combination, while the boiler will refer to a stand-alone combustion chamber that only provides heat.

One of the most important factors in the effectiveness of a CHP plant is how the system can respond to diurnal energy loads. District energy systems can have heating and power loads that match their diurnal and seasonal cycles, but in many cases they don't. In a campus building system during the winter months, heating loads usually peak in the morning times as buildings warm up from the cooling effect of the night, and power peaks in the afternoon when building occupancy is at a maximum and lighting, electronics, etc. are at peak use. During the summer months, heating has a more leveled load, but the electrical load will be much larger. The mismatch between heating and electric load profiles creates tradeoffs that facility owners evaluate and optimize in terms of

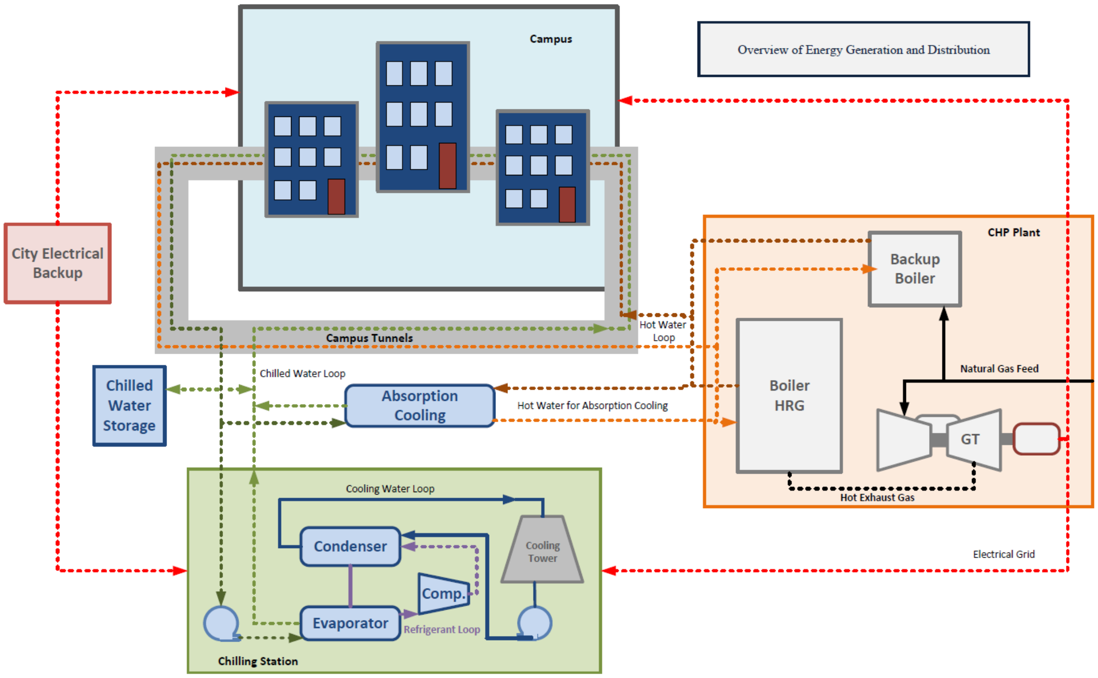

Figure 3.2: Schematic of district energy system with CHP and backup boiler.

the type of energy system that will match energy needs. Figure 3.3 shows energy profiles for the system being studied for a particular summer 24 hour period.

3.4 Traditional Capacity Expansion Models

The design capacity of a power plant is the maximum amount of energy per unit time that it can produce [34]. Typical values in the US are measured in MMBTU/hr (1 million BTU per hour) for heat generating systems and in megawatts for medium-size electric power generation systems. Most systems are oversized for typical load scenarios or may include multiple units that can be staged on or off as demand changes.

Because of growing energy demands and regulations on existing energy production fleets, new investments in design capacity must be considered to meet increases in energy demand and compliance with regulations. At the same time, energy demand is uncertain because of weather and economic factors. Capacity expansions projects are mostly considered irreversible investments because of high capital cost and the plants remain available for an extended period of time. This

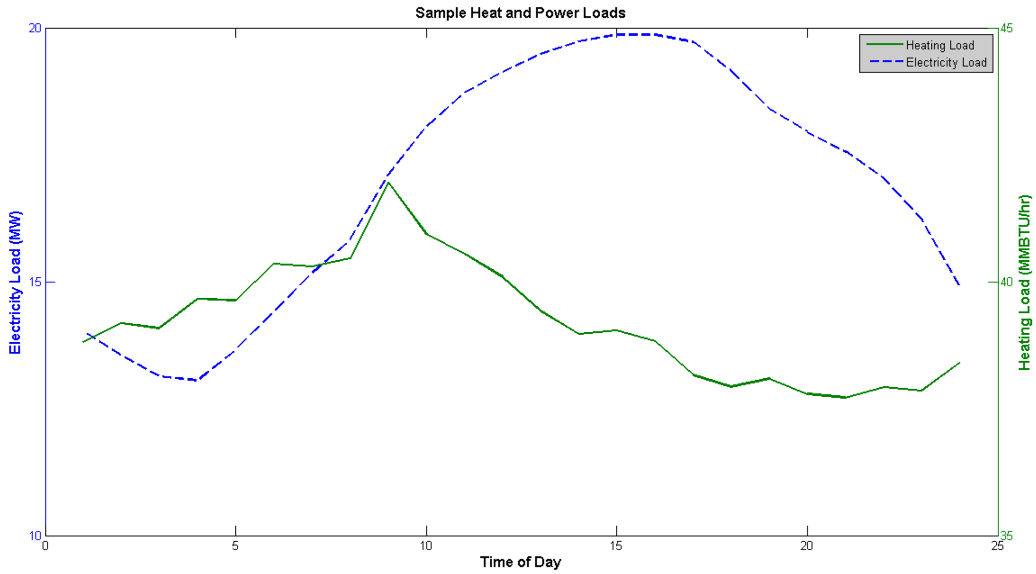

Figure 3.3: Sample heating and electric load profiles for a particular summer 24 hour period showing the mismatch in heating and electric load peaks.

makes the design capacity investment decision a nontrivial one. When the design capacity exceeds demand, the overall capital cost is likely to be too high. Alternatively, when the capacity is insufficient, the plants can be expected to operate at peak capacity and extra supply must be imported from an external grid. Both these events are generally costly in terms of either excess capital cost or higher priced power from external sources.

As previously stated at the beginning of this chapter, the modeling approach of a system will dictate the quality of information obtained from an optimization program. The level of complexity such as the use of linear versus non linear equations, differential versus algebraic only, etc. will also dictate the optimization method and algorithms available to solve those types of problems. To name a few, these include linear programming (LP), quadratic programming (QP), nonlinear programming (NLP), mixed integer linear programming (MILP), mixed integer nonlinear programming (MINLP). Detailed explanations about these optimization methods can be found in many references [31, 35]. The field of operations research continually seeks to improve solution methods to solve ever more complex and larger problems. From a historical perspective, linear methods have been widely used because of their relatively simple modeling complexity and ease of solving with available computer power. Newer non-linear methods that can now solve larger models with larger amounts of data are the new frontier in optimization technology.

In the next two sections, the traditional modeling formulation approach based on linear steady state model and LP optimization method is introduced with an effort to contrast the differences with the proposed dynamic non-linear optimization approach with differential and algebraic equations (DEAs) to solve capacity planning problems. An innovation of this research is the direct application of DEAs to simultaneously optimize operating strategy and long-term planning.

3.5 Simplified Demand Profiles

Electricity demand varies over days, weeks, seasons, and years. Rather than model demand over extended periods of time, traditional power expansion studies only consider a representative single time period of one day. This choice simplifies the model, but still allows for demand scenarios that typify demand throughout a planning horizon of years [34].

Further simplifications of the one day time period is usually done by first discretizing the one day profile into 24 separate periods. Although some dynamic fidelity is lost, it roughly follows the demand curve. Next the 24 separate periods are rearranged to create a cumulative load curve. From the cumulative load curve peak and base load periods can be extracted to approximate the maximum capacity and base capacity needed to supply the demand. Although this approach provides a good approximation of demand, the dynamic interaction between heat and electricity in a CHP system is lost. In Figure 3.4 the energy load profile reduction process is illustrated.

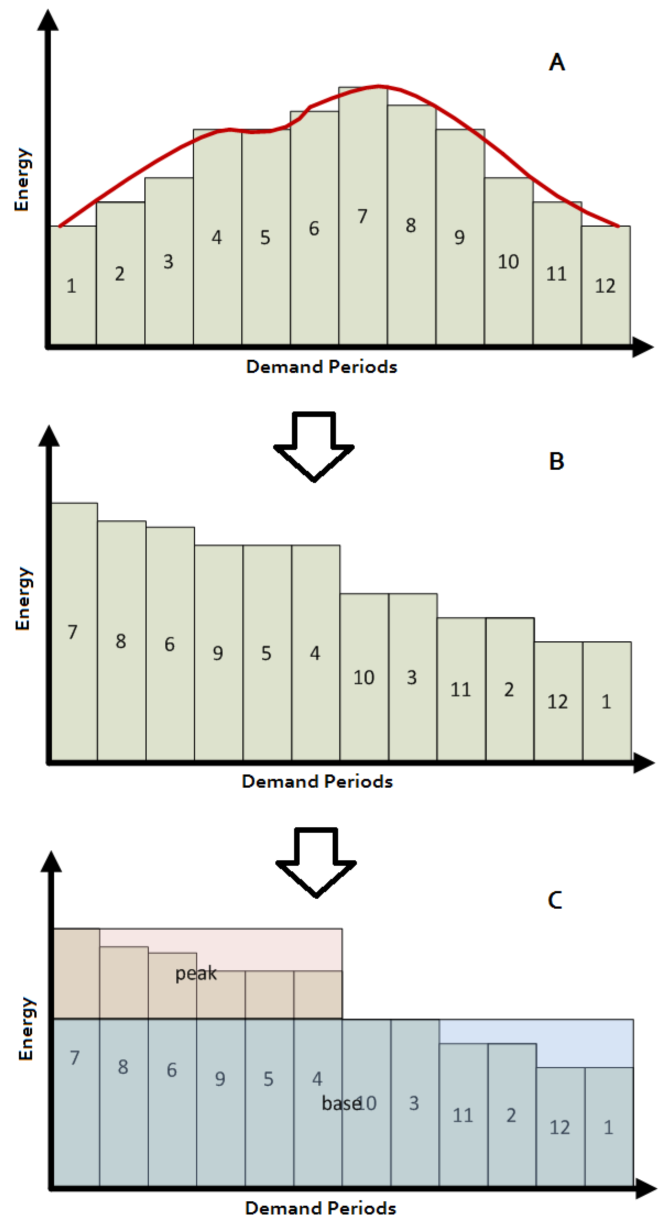

Figure 3.4: Schematic A show the discretization of actual demand profile represented by the solid curve. B shows the rearrangement to make a cumulative load curve. C shows the reduction to base and peak load period.

The simplification of the demand profiles to two periods, base and peak, greatly reduces problem size when optimizing over a long term time horizon.

3.6 Discrete Linear Energy System Model

To contrast the novel contributions of this work, first a traditional linear power plant capacity planning problem from [34] was modified to a campus energy system to optimize capacity planning with CHP and boiler options. This model does not have differential equations or higher than first order algebraic equations. The traditional linear capacity expansion models depend on discretization of time steps and indexation of equations to approximate demand load changes and utilization of the capacity, but cannot model dynamic behavior of the system.

Indices :

p plant type $\{chp, \ boiler\}$

k demand category $\{base\ load, \ peak\ load\}$

s season $\{summer, \ winter\}$

i energy type $\{electric, \ thermal\}$

Parameters :

e_p existing capacity of plant type p $[MW]$

cc_p daily fraction of capital cost of plant p $[MW]$

oc_p daily operating cost of plant p $[USD/MWh]$.

$ic_{k,s}$ electricity import cost $[USD/MWh]$

$d_{k,s,i}$ instantaneous energy demand $[MW]$

$du_{k,s,i}$ duration of demand $[hours]$

$r_{k,s,i}$ required energy $[MWh]$

f_{hrg} recovered heat factor $[unitless]$

where the parameter $r_{k,s,i}$ is defined as $r_{k,s,i} = (d_{k,s,i} - d_{k-1,s,i}) \cdot du_{k,s,i}$

Variables :

x_p new design capacity of plant type p $[MW]$

$y_{p,k,s}$ allocation of capacity to demand $[MW]$

$z_{k,s}$ import of electricity $[MW/h]$

Minimize :

$$\sum_p cc_p(e_p + x_p) + \sum_k \sum_s ic_{k,s} \cdot z_{k,s} + \sum_k \sum_s \sum_i du_{k,s,i} \cdot \left(\sum_p oc_p \cdot y_{p,k,s} \right) \tag{3.1}$$

Subject to :

$$e_p + x_p \geq \sum_k \sum_s y_{p,k,s} \quad for\ all\ p \tag{3.2}$$

$$r_{k,s,electric} = z_{k,s} + du_{k,s,electric} \cdot \left(\sum_p y_{chp,k,s} \right) \quad for\ all\ k,s \tag{3.3}$$

$$r_{k,s,thermal} \leq du_{k,s,electric} \cdot \left(\sum_p y_{boiler,k,s} + y_{chp,k,s} \cdot f_{hrg} \right) \quad for\ all\ k,s \tag{3.4}$$

$$x_p \geq 0, y_{p,k,s} \geq 0, z_{k,s} \geq 0 \tag{3.5}$$

The above linear model in general typifies the traditional optimization approaches for capacity planning expansion. The capacities denoted by x_p are lumped variables that quantify capacity without a realistic physical or dynamic representation of it. Linear models are also the basis for stochastic programming formulations. The stochastic formulations explicitly take into account probabilistic factors as discussed in the background section of this work, but they lack the intrinsic dynamic features. Yet perhaps the greatest deficiency is the inability for the formulation to handle differential and algebraic equations in continuous space to explicitly handle the differential and algebraic relationships.

The next sections illustrate the novel approach of this work in formulating and solving capacity expansion planning problems. This novel approach considers the long-term capacity planning problem with seasonal and dynamic energy demand horizon while subsequent sections treat costs associated with environmental, operational, and capital equipment expansions.

3.7 Dynamic Energy System

3.7.1 Generating a Dynamic Energy Demand Horizon

The diurnal energy profiles, as those illustrated in Figure 3.3, characterize the net energy usage of the system. Over long periods of time the demand profile curves reflect the overall effects weather, building occupancy, and growth have on energy consumption. Energy generation systems must be able to handle these uncertainties and projected growth. Due to increasing energy demand and tougher environmental standards, new capacity must be added or completely new systems installed. Those options must be evaluated against a realistic projected energy demand over the lifetime of the proposed system. From historical data, energy system owners can obtain seasonal and diurnal energy demand data over many years to analyze the cyclical aspects of the demand, and then make year over year growth projections. In institutions such as university campuses, long-term energy consumption growth typically has a well defined projected target due to finite student enrollment goals and building plans. Figure 3.5 shows the campus demand data over an entire year which shows cyclical daily profiles, as well as weekly and seasonal patterns exhibited by the system.

In this work, the entire energy profile of two years is analyzed and broken into two parts, summer and winter. Separately for summer and winter parts, the individual 24 hour periods are isolated and averaged over each hour. This process in return gives an average 24 hour energy demand profile for winter and summer periods. With this approach the extreme demand cases that the system encounters in any one year are lost in the averaging. To counter that deficiency, the most extreme 24 hour episodes for each season are isolated and then averaged with the seasonal daily demand average. This creates a representative 24 hour demand profile that represent most energy demand cases. These procedures are repeated for both heating and electrical energy demands.

Upon finding the average demand curves, these are propagated over a 30 year horizon with a stochastic linear growth of 2.5% to simulate year over year energy demand increase of an university campus. Each 24 hour cycle represents a seasonal average day for that year. At the end, the demand curves for every two years are averaged thus assuming that energy demand stays constant every two years. Although these assumptions are far from actual conditions, it is a method to reduce the number of time points the optimization must consider while at the same time providing the diurnal demand profile that the energy system must adapt to. This method contains more information and dynamic features than the lumped peak and base load periods of traditional modeling methods.

Although diurnal and seasonal energy cycles as well as energy growth projections are uncertain, the level of uncertainty is greatly diminished from the analysis of historical demand data because better projections can be made for the future horizon. This enables stakeholders to fo-

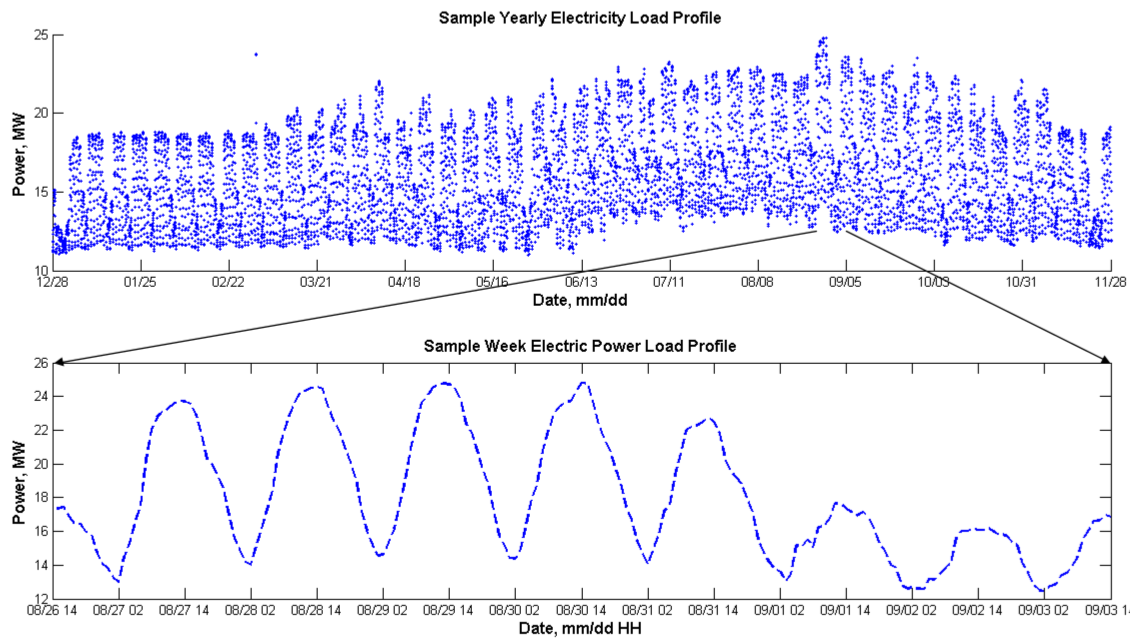

Figure 3.5: Sample electricity load profiles for a representative year (top) and week (bottom).

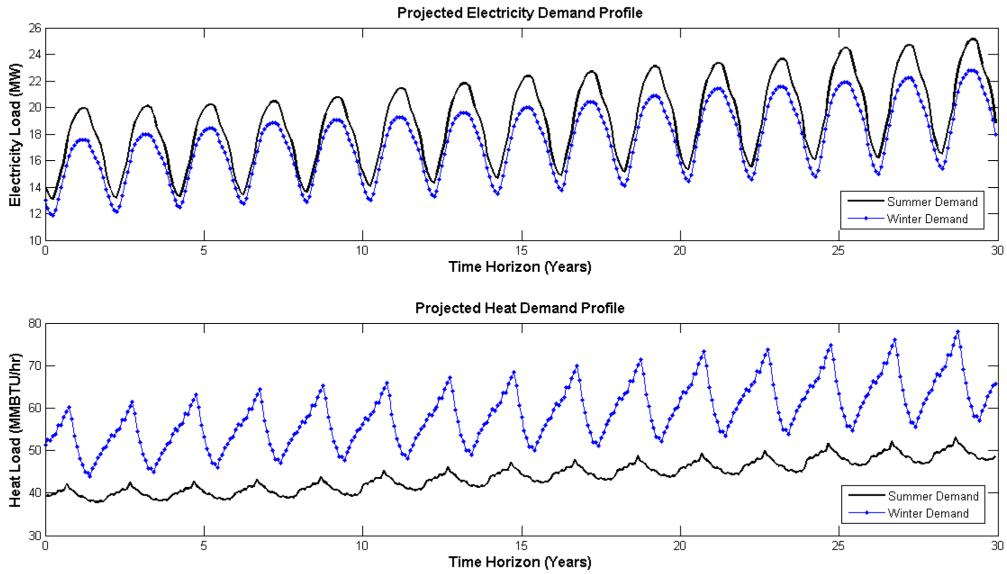

Figure 3.6: Generated electricity and heating demand profiles for dynamic optimization. Each cycle (trough to trough) represents an average day. There are 15 cycles because each represents two years of constant demand for an average winter and summer day.

cus on other uncertain economic forces that have a greater effect on the profitability of energy generation systems such as electricity and natural gas prices and environmental regulation costs.

3.7.2 Economic Horizon and Environmental Costs

Fossil fuel prices and increasing environmental and health-impact concerns have forced decision makers to contemplate and propose comprehensive studies to evaluate energy systems management [22]. A recent report from the European Union indicates that high investment costs, long payback periods of irreversible investments in new power plants, commodity price variability in deregulated power markets, and regulatory uncertainty bear significant risks for the investing stakeholders [36]. Those concerns have already postponed or even canceled planned investments in new fossil power plants [36]. In the United States, similar cost due to environmental based regulations are hampering the growth in coal-based power production. For CHP systems which still depend on fossil fuels, mainly natural gas, the outlook is more favorable due to the efficiency increases and lower carbon footprint. The main driver in the recent economy for natural gas consuming processes comes from falling natural gas prices because of abundant natural gas resources found in shale formations across the United States. There has been a 36% decrease in the average natural gas price paid by manufacturers between 2006 and 2010, and prices are forecasted to remain suppressed for the foreseeable future because of abundant supply, still far below the peaks of the last decade [37].

To evaluate these concerns and the effect on capacity planning over a 30 year horizon, the projected natural gas and electricity rates for commercial level consumptions are gathered from the U.S. Energy Information Administration.

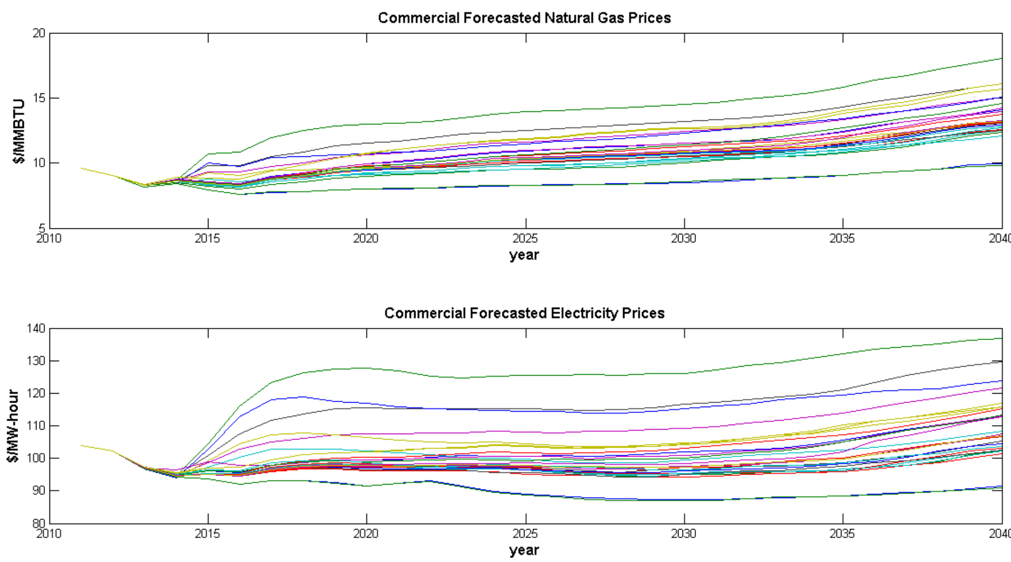

Figure 3.7: 28 U.S. EIA commercial natural gas and electricity price forecasts [38].

As illustrated in Figure 3.7, the spread of these forecasts are a reflection of the price uncertainty over the 30 year horizon. These rates are used as inputs in the optimization program as will be explained in subsequent sections.

In addition to natural gas and electricity prices, environmental regulations can be inhibitors or enablers of investments. These can be very difficult to quantify and understand and are greatly affected by the political environment and public opinion. Title V regulations from the EPA already have great effects on establishing the parameters and rules on whether a major source of Clean Air Act regulated molecules can be emitted by existing or proposed energy generation systems [39]. Title V regulations can be decisive factors on building a new plant or adding capacity regardless of fuel and electricity prices. The assumption for this work is that necessary Title V permitting is available and does not influence capital cost of a potential CHP system. This assumption reduces the uncertainty with potential monetary penalties for non-compliance.

The greatest long-term environmental uncertainty in the U.S. comes from whether CO_2 emissions will be regulated or taxed. To reflect that, some economic EIA forecasts for natural gas and electricity prices already take into account a CO_2 monetary cost starting at \$5/ton, \$15/ton, and \$25/ton and growing 5% each year for the next three decades.

3.7.3 Financial Indicators to Guide Capital Investment

To make valid evaluations of projects that start and end at different times, the time value of money must be considered [1]. This is especially true in capacity planning because the time horizon is typically several decades. To compare different economic cases against each other, the net cash flows that occur in the future are discounted to return to a present value.

The discount rate is used in discounted cash flow analysis to determine the present value of future cash flows. The discount rate takes into account the time value of money and the risk or uncertainty of the anticipated future cash flows [40]. The discount factor (DF) is defined by the following equation

$$DF = (1+r)^{-yr} \tag{3.6}$$

where r is the discount rate and yr is the number of years from present at which the discount factor is being considered.

The discount rate r is based on the weighted average cost of capital (WACC) which is specific to the industry and financial health of the institution or sector. Financial algorithms are available to calculate WACC, but those are beyond the scope of this work. It suffices to state that tables are published with representative discount rates of industrial and commercial sectors. The discount rate for power generation capital projects is around 6% based on a recently published survey [41].

Capital investment costs, if based on current data, must also be modified based on the rate of inflation for investments in a future year. Because the rate of inflation is typically positive, capital investment costs increase in apparent value over time. The rate of inflation will depend on many macroeconomic factors, but in resent history in the US the inflation rate hovers between 3 and 4 percent [42]. It is important to note that the discount rate and inflation rate are not usually the same value and represent different economic indicators. In summary, for this work the discount rate and inflation rate are chosen to be 6% and 3% respectively. Those two important economic values are needed to calculate a representative present cost (*PC*) which is the sum of the discounted costs generated over the project time horizon [1].

3.7.4 Capital Investment Costs

Capital costs are those associated with construction of the CHP system including equipment and installation costs [5]. Installation costs have shown large fluctuations in the recent past in non-monotonic patterns because of fluctuating prices in construction materials [43]. These are uncertainties that influence total capital cost, but are difficult to quantify by continuous mathematical expressions. This work uses gradient based optimization algorithms that require continuous functions to evaluate model variables; therefore, polynomial fits from data for CHP and boiler costs are employed to construct costing functions for capital costs [9]. The capital cost functions are explained along with the dynamic model in subsequent sections. This work assumes that associated uncertainties from construction material cost fluctuations are minimal, although this is something that should be further explored in future work.

3.8 Dynamic and MPC Optimization Framework

The underlying enabler for a dynamic optimization approach to capacity planning comes from novel optimization technologies that allow for the explicit use of differential and non-linear equations that can be solved simultaneously over a time horizon. This work uses APMonitor, a gradient based optimization software for mixed-integer and differential algebraic equations. It is coupled with large-scale solvers for linear, quadratic, nonlinear, and mixed integer programming [44].

A detailed over view of dynamic optimization is given by [45], but this work outlines the important aspects that contribute to solving a capacity planning problem as a dynamic optimization problem.

3.8.1 Non-linear Dynamic Modeling

A general model form for non-linear dynamic problem can be formulated as follows

$$\min J\left(x,y,p,d,u\right) \tag{3.7a}$$

$$0 = f\left(\frac{\partial x}{\partial t}, x, y, p, d, u\right) \tag{3.7b}$$

$$0 = g(x,y,p,d,u) \tag{3.7c}$$

$$0 \leq h(x,y,p,d,u) \tag{3.7d}$$

These equations represent a model that may include differential, algebraic continuous, binary, and integer variables. The solution to the equations can be given by the initial state x_o, parameters p, trajectory of disturbances $d = (d_0, d_1, \ldots, d_{n-1})$, and control moves $u = (u_0, u_1, \ldots, u_{n-1})$ [45]. The solutions for variables x and y are solved for each step n of the time horizon from the differential or algebraic equations in the model respectively. The formulation outlined in Equation 3.7 is especially suitable to handle the complexities of an energy system capacity planning problem. The differential and algebraic equation can model dynamic and nonlinear physical features, while the explicit handling of disturbances as trajectories mimics the dynamic energy demands over the time horizon. The solution method for a DAE model is summarized in the following section.

3.8.2 Numerical Solution of DAE Systems

Usually the time horizon of an energy capacity planning problem is in the order of decades because of the magnitude of the investment associated with capacity increases. Even after simplification of model and reduction of energy demand data, a representative model requires the solution of DAEs over a significant number of time steps. The problem can become very large as the time horizon increases. The problem presented in this work has over 700 degrees of freedom. Degrees of freedom in optimization can be considered as the number of available decision variables the optimizer adjusts to find an optimal solution.

To solve dynamic optimization problems researchers and practitioners have used simultaneous and sequential solution methods [46]. More detail about each approach can be found in the literature [46,47], but the important aspect of simultaneous method is the computational advantage for problems with many decision variables and a moderate number of of state variables [45] as is the case with dynamic capacity planning problems over a large horizon. State variables can be seen as a set of variables used to describe the mathematical state of a dynamic system to determine the future behavior of the system [48].

Simultaneous methods solve the DAE model by converting it to algebraic equations only in a method known as direct transcription [49]. This changes the problem to a nonlinear programming problem that can then be solved by large-scale optimization solvers [45].

3.8.3 MPC Formulation

As discussed in the background section of this work, dynamic DAE models are used in some nonlinear predictive control applications. In a control environment multiple objectives may be desired within a single control application. These objectives can have different priorities or rates at which they must be achieved. Similar multi-objective tradeoffs can exist in capacity planning because although cost minimization is the ultimate goal, certain environmental and operational objectives are also desired over the lifetime of the plant. The additional objective considerations have a significant effect on the capacity of the system. One approach to handle the multi-objective challenge is the use of the $l_1 - norm$ formulation for non-linear dynamic optimization. The $l_1 - norm$ formulation simultaneously optimizes the multiple objectives by selectively manipulating the degrees of freedom that have the highest sensitivity on the most important objective, and then meeting the lower ranking objectives with the remaining degrees of freedom [45]. Priorities are assigned by giving higher weighting to the most important objectives. The dynamic optimization $l_1 - norm$ objective formulation is as follows.

$$\min_{x,y_m,u} \Phi = w_{hi}^T (e_{hi}) + w_{lo}^T (e_{lo}) + (y_m)^T c_y + (u)^T c_u + (\Delta u)^T c_{\Delta u} \tag{3.8a}$$

$$\text{s.t.} \quad 0 = f(\dot{x}, x, u, p, d) \tag{3.8b}$$

$$0 = g(y_x, x, u, d) \tag{3.8c}$$

$$a \geq h(x, u, d) \geq b \tag{3.8d}$$

$$\tau_c \frac{\partial y_{t,hi}}{\partial t} + y_{t,hi} = sp_{hi} \tag{3.8e}$$

$$\tau_c \frac{\partial y_{t,lo}}{\partial t} + y_{t,lo} = sp_{lo} \tag{3.8f}$$

$$e_{hi} \geq \left(y_m - y_{t,hi}\right) \tag{3.8g}$$

$$e_{lo} \geq \left(y_{t,lo} - y_m\right) \tag{3.8h}$$

Nomenclature for Equation 3.8	
Φ	objective function
y_m	model values $(y_{m,0}, \ldots, y_{m,n})^T$
$y_t, y_{t,hi}, y_{t,lo}$	desired trajectory dead-band
w_{hi}, w_{lo}	penalty outside trajectory dead-band
$c_y, c_u, c_{\Delta u}$	cost of y, u,and $\triangle u$, respectively
u, x, d	inputs (u), states (x), and parameters or disturbances (d)
f, g, h	equation residuals (f), output function (g), and inequality constraints (h)
a, b	lower and upper limits
τ_c	time constant of desired controlled variable response
e_{lo}, e_{hi}	slack variable below or above the trajectory dead-band
sp, sp_{lo}, sp_{hi}	target, lower, and upper bounds to final set-point dead-band

The objective function for the $l_1 - norm$ formulation, Equation 3.8a, is drastically different from a dynamic only optimization as noted earlier by Equation 3.7a. The most important distinction beneficial to a capacity planning problem is the ability to include other objectives such as trajectory targets or dead-bands that can be met at different response rates and levels of importance.

Based on the $l_1 - norm$ formulation, the capacity planning problem can be framed as one step of an MPC problem. Manipulated variables such as CHP or boiler set points are moved by the optimizer to meet operating targets like minimizing the error between energy production and demand over the entire planning horizon. At the same time the capital and operating cost are minimized as part of the highest ranking objective function.

3.9 Dynamic Model of CHP System

Formulating the correct problem is perhaps the most important step in optimization [35]. In this work, the objective function represents the sum of operating and capital cost over a 30 year horizon period as functions of key variables and parameters of the system . The rest of the problem is the system model and constraints that describe the interrelationships of key physical and economic variables [35]. To simplify the model, this work utilizes data from Solar Turbines, a Caterpillar company, to construct empirical relationships through first-order and second-order polynomials for the important physical relationships. The data from Solar Turbines is for small-size to medium-size industrial combustion turbines in the range of the capacity requirements for the district energy system considered in this study. The data and polynomial fits are found in Appendix C.

The objective and the equations that make up the deterministic energy system expansion model are first summarized in the following qualitative model formulation.

- Minimize: present value of capital and operating costs
- Subject to:
  - Utilized power production capacity and import supply if needed must be greater than or equal to electricity demand.
  - Utilized heat production capacity must be greater than or equal to heat demand.
  - Utilized CHP and supplemental boiler capacity cannot exceed the maximum capacity of their respective systems.
  - Capacity allocations are irreversible.
  - Other physical and logic constrains that make the model feasible.
  - CO_2 calculations for environmental and cost constraints.

It is important to note that for this proposed optimization framework, there is no need to index or discretize the system by time and allocation of capacity. In the following model, the use of indexes is only used to abbreviate equations and simplify notation. The subscript s is used to denote the season (summer or winter) in which the system operates, but the entire model is solved simultaneously.

The following symbols are used:

Indices :

p plant types (CHP, boiler)

s season (summer, winter)

Parameters :

cn CO_2 content per MMBTU of natural gas $[\frac{lb\,of\,CO_2}{MMBTU\,of\,natural\,gas}]$

dr discount rate

ed_s electric demand $[MW]$

ee electricity price $[\frac{\$}{MWhr}]$

hd_s heat demand $[\frac{MMBTU}{hour}]$

ir inflation rate

n number of years being averaged

ng natural gas price $[\frac{\$}{MMBTU}]$

ηb boiler efficiency assumed constant at 80%

$\triangle t$ time step

yr year in time horizon

Variables :

b_s boiler thermal energy production $[\frac{MMBTU}{hour}]$

CC_p capital costs [\$]

ce_p CO_2emissions from CHP or boiler $[\frac{short\,tons\,CO_2}{hour}]$

ct_s Total CO_2cost in [\$]

fb fuel consumption boiler $[\frac{MMBTU\,of\,natural\,gas}{hour}]$

fc fuel consumption CHP $[\frac{MMBTU\,of\,natural\,gas}{hour}]$

he_s excess heat production $[\frac{MMBTU}{hour}]$

hs_s total heat supply $[\frac{MMBTU}{hour}]$

lf_s load fraction of CHP

mc_p maximum capacity of CHP and boiler $[MW\,or\,\frac{MMBTU}{hour}]$

np_s net power $[MW]$

ηc CHP fuel efficiency

OC_s operating costs [\$]

u_s empirical turbine set point

v_s empirical boiler set point

x_s electricity production from CHP $[MW]$

y_s CHP heat output from heat recovery $[\frac{MMBTU}{hour}]$

The mathematical description of the dynamic model is stated as follows.

Minimize :

The total operating and capital costs is the objective function given by:

$$\sum_s OC_s + \sum_p CC_p \tag{3.9}$$

The operating cost (OC_s) for each season "s" is summed for all steps in the time horizon as well as the capital costs (CC_p) of capacity increases for system type "p" at any step in the time horizon. The operating and capital cost functions are explained later in this section.

Subject to Equations :

Dynamic equations from empirical first order differentials:

$$\tau_{chp} \cdot \frac{\partial x_s}{\partial t} + x_s = u_s \tag{3.10}$$

$$\tau_{boiler} \cdot \frac{\partial b_s}{\partial t} + b_s = v_s \tag{3.11}$$

What differentiates this modeling approach versus traditional linear approaches discussed earlier is the inclusion of DAEs for capacity planning problems. This work uses a first order differential equation model to fit turbine (Equation 3.10) and boiler (Equation 3.11) dynamic responses to power and heat generation respectively. The Greek letter τ or time constant in each equation is a parameter that will manipulate the dynamic response of the system. The differences in the magnitude of τ for CHP and boiler depend from the relative response time each system has when its input variable is changed and the output variable reaches 63.2 of the prescribed change.

The relationship for recoverable heat from the gas turbine is give by

$$y_s = -0.0817 \cdot x_s^2 + 5.6547 \cdot x_s \tag{3.12}$$

Equation 3.12 is a non-linear interaction that describes the recoverable heat y in a heat recovery boiler as a function of power production x from the gas turbine. First principles models of turbines are available to detail the heat versus power response [7]. A polynomial fit is sufficient in this work because heat recovery performance data for different turbine capacities operating at typical operating conditions is available from CHP manufacturers (see Appendix C for data and polynomial fit). The polynomial fit for this data if extrapolated beyond the available data range gives unrealistic properties because of the negative value coefficient; nevertheless, the optimization program is constrained to search only within the acceptable range. Exponential fits of the data give better fits than the polynomial fits especially at smaller turbine capacities, but those are more difficult to solve by optimization solvers. When exponential or logarithmic equations are coupled with other non-linear equations in relatively large optimization problems, optimization solvers may not be able to find a solution.

The total available heat supply to the district system is given by Equation 3.13 which sums the boiler and CHP heat generation.

$$hs_s = y_s + b_s \tag{3.13}$$

Equation 3.14 indicates the excess heat generation which is the difference between total heat production and heat demand. Because heat production can only come from the on-site system, the energy system is constrained to have a difference equal or grater than zero as indicated by Equation 3.15.

$$he_s = hs_s - hd_s \tag{3.14}$$

$$he_s \geq 0 \tag{3.15}$$

Usable heat production from a CHP system is limited by turbine capacity utilization, which means that at maximum load the maximum amount of heat is generated. Excessive heat production beyond the heat demand is discarded to the atmosphere through the flue combustion gases at higher temperatures. Although usually there are design and regulatory limitations on the flue gas temperatures, generally if too much excess heat is being discarded to the atmosphere, then heat integration and efficiency goals of a CHP system are not being achieved. In the results and discussion section, optimization formulations to minimize excess heat he_s are explored.

The net power relationship is given by

$$np_s = x_s - ed_s \tag{3.16}$$

Many CHP systems have the convenience of interconnection with the city power grid. Interconnection with the city grid provides backup power, or if the CHP capacity is designed to partially fulfill the electric load, then the city grid can meet the remaining load. In certain markets net metering measures the difference between the electricity from the utility used by the customer and the electricity generated and provided back to the utility [50]. Depending on the market where the CHP is located, net metering rates and incentives may make the CHP investment more financially attractive even after fuel and efficiency savings. Equation 3.16 defines the net power usage of the system to simulate the net metering options of certain markets. When the CHP power production x_s is greater than the electric demand ed_s then the excess electricity is sold back to the grid. On the other hand, if electric demand is greater than the current CHP power production, the resulting negative np_s denotes that power is being imported from the external electrical grid. This flexibility makes for even greater optimization trade offs that can affect capacity planning of CHP systems. In this work, both net metering and electricity import only options are explored in different optimization cases. For simplicity when net metering is allowed, the same electric rates were considered for either import or export of electricity. Although this is a simplification of more complicated net metering rules, it provides a good basis to explore the realistic effects of such trade offs in energy system capacity planning.

Turbine load fraction and efficiency are given by:

$$lf_s = \frac{x_s}{mc_{chp}} \tag{3.17}$$

$$\eta c_s = 0.9044 \cdot lf_s + 0.0956 \tag{3.18}$$

The natural gas consumption for CHP and boiler are given by:

$$fc_s = \frac{-0.1361 \cdot x_s^2 + 13.347 \cdot x_s}{\eta c_s} \tag{3.19}$$

$$fb_s = \frac{b_s}{\eta b} \tag{3.20}$$

When less than full power is required from a gas turbine, the output is reduced by lowering the turbine inlet temperature which in addition to reducing power this change also reduces efficiency [9]. The load fraction lf_s is the ratio of instantaneous CHP output and CHP capacity. When the turbine is operating at it's maximum capacity load fraction is 1. Although actual turbine efficiency generally reaches a maximum around 30%, in this work the turbine load versus efficiency relationship is normalized to 1 (100%) for its maximum efficiency and a linear relationship is approximated by Equation 3.18. This is done because fuel consumption from manufacturer data for different turbine capacities is rated at the maximum load, most efficient state. The CHP fuel consumption Equation 3.19 is also represented by a polynomial fit from manufacturer data for gas turbines running at their optimal full capacity. Operation below the turbine's rated capacity will consume more fuel, thus the polynomial for fuel consumption is divided by the normalized CHP efficiency ηc_s to estimate the effects of part load operation on fuel consumption. When the normalized CHP efficiency is less than one, fuel consumption increases thus quantifying the effects of operating the gas turbine at less than full capacity. Fuel consumption by the gas boiler (Equation 3.20) is estimated by directly dividing the boiler energy output by the expected boiler efficiency of 80%.

Carbon dioxide calculations are given by:

$$ce_{chp,s} = cn/2000 \tag{3.21}$$

$$ce_{boiler,s} = cn/2000 \tag{3.22}$$

The CO_2 emissions rate from fuel consumption estimated by Equations 3.21 and 3.22 based on a stoicheometric relationship of natural gas combustion.

When there is a cost on CO_2 emissions, the cost is calculated by:

$$ct_s = (ce_{chp,s} + ce_{boiler,s}) \cdot cp \cdot (907.185/1000) \tag{3.23}$$

The capital and operating cost functions are given by:

$$CC_{chp} = (-0.0069 \cdot (\triangle mc_{chp})^2 + 1.3351 \cdot (\triangle mc_{chp})) \cdot (10^6) \cdot (1+ir)^{yr} \cdot (1+dr)^{-yr} \tag{3.24}$$

$$CC_{boiler} = 2 \cdot (-39.504 \cdot (\triangle mc_{boiler})^2 + 29900 \cdot (\triangle mc_{boiler})) \cdot (1+ir)^{yr} \cdot (1+dr)^{-yr} \tag{3.25}$$

Correlating the capital costs is important for an accurate objective function. Polynomial approximation of manufacturer data on capital costs for a range of CHP capacities is used for the CHP capital cost function. The data comes from a survey of estimated total capital costs for typical gas turbine-based CHP systems (see Appendix C). One of the innovations of this work is in using differential equations that explicitly consider system changes with respect to time for capacity planning problems. Over the entire time horizon the energy generation capacities may increase when the energy demands requires expansion, but those changes are also limited by the economic objective of minimizing costs. In Equations 3.24 and 3.25, $\triangle mc_p$ denotes the change in maximum capacity of system p. To describe this in the optimization program, the multiplication operation $\triangle t \cdot \frac{\partial mc_p}{\partial t}$ yields an approximation to $\triangle mc_p$, the magnitude of the maximum capacity change at a particular time period. The differential time change ∂t is approximately equal to the time discretization $\triangle t$ interval used in the model. The optimization software used in this work explicitly handles differential equations, which allows for the calculation of capital cost at any time period of the horizon when the optimizer finds optimal to have a change in capacity in the energy system. The capital cost equations are multiplied by $(1+ir)^{yr}$ to account for inflation and and by $(1+dr)^{-yr}$ to account for the time value of money. The capital costs are summed together and included as minimization terms in the objective function Equation 3.9.

The operating cost is evaluated by:

$$OC_s = (ng \cdot fc_s + ng \cdot fb_s - ee \cdot np_s + ct_s) \cdot (0.5 \cdot 365) \cdot (1+dr)^{-yr} \tag{3.26}$$

The operating expenses CHP fuel, boiler fuel, net electric power , and CO_2 emissions are summed and multiplied by $(0.5 \cdot 365 \cdot n)$ to approximate the number of days (half a year) in each season S. The letter n is the number of years each demand cycle represents, and is used to reduce the number of individual cycles the optimization program must solve. The operating cost equation also contains the term $ee \cdot np_s$ which quantifies the cost of electricity. This term has a negative sign because net power (np_s) is positive when excess electricity is produced. Excess electricity production reduces operating cost because it is sold to the grid. When electricity is imported np_s is negative which makes the $-ee \cdot np_s$ term positive and the operating cost increases. Operating costs are also discounted to the present value by $(1+dr)^{-yr}$. There is no inflation factor in Equation 3.26 because the price data already considers inflation. It is important to note that CO_2emissions cost (ct_s) is only considered in economic scenarios that give CO_2 a cost associated per ton emitted. The baseline scenario in this work does not consider CO_2 emission cost.

To specify the irreversible nature of change in capacity, the differential terms are directly constrained by:

$$\frac{\partial mc_{chp}}{\partial t} \geq 0 \tag{3.27}$$

$$\frac{\partial mc_{boiler}}{\partial t} \geq 0 \tag{3.28}$$

Directly constraining differential terms is a feature that few optimization software packages have, but is extensively exploited in this work. Limiting $\frac{\partial mc_p}{\partial t}$ to be ≥ 0 allows for the change in maximum capacity of system p to remain positive meaning there is no decrease in capacity once a decision is made.

The maximum capacity always remains greater than or equal to the instantaneous generation rates of the CHP and boiler system respectively through the following constraints:

$$mc_{chp} \geq x_s \tag{3.29}$$

$$mc_{boiler} \geq b_s \tag{3.30}$$

In summary, the dynamic model includes all the potential energy generation systems to fulfill the heating and electric demands. The optimizer decides which arrangement, and capacity is needed. The optimizer can either decide to have a CHP only system, a boiler only, or a combination of the two along with the choice of import capacity from the grid for electricity. The optimizer uses the model to find timing and potential overall capacity in a 30 year horizon by minimizing total present value costs as stated in the objective function Equation 3.9.

3.10 Uncertainty in Natural Gas and Electricity Prices

With uncertainty in natural gas and electricity prices, there is motivation to optimize the problem to the expected value of the total capital and operating cost over the time horizon. Natural gas and electricity prices are stochastic inputs to the model represented by the spread of prices over the optimization horizon as illustrated earlier in Figure 3.7. Based on the likelihood of each economic model, there is a specific probability associated with each value of natural gas and electricity price. At any particular point in the time horizon the probability of all the possible values of a random variable can be summarized by a probability mass function (PMF) if the random variable is discrete [51]. It is often desirable to summarize the probability information in a single representative number. That is accomplished by the expected value, also called the expectation or the mean, of a random variable, which is a weighted average of the possible values of the random variable [51].

The expectation of a random variable X is mathematically defined as

$$\mathbf{E}[X] = \sum_{x} x \cdot p_X(x) \tag{3.31}$$

where $p_X(x)$ is the probability associated with each variable x. For linear function of random variables it can be verified that the expected value of a function of the form $a \cdot X + b \cdot Y + c$ is given by $a \cdot \mathbf{E}[X] + b \cdot \mathbf{E}[Y] + c$ where X and Y are random variables and a, b, and c are scalars. This property is extended to find the expected value of the total cost. If the total present cost (PC) function is of the form

$$\begin{aligned} PC = {} & f_1\left(\frac{\partial x}{\partial t}, x, y, p, d, u\right) \cdot A + f_2\left(\frac{\partial x}{\partial t}, x, y, p, d, u\right) \cdot A \\ & + f_3\left(\frac{\partial x}{\partial t}, x, y, p, d, u\right) \cdot B + f_4\left(\frac{\partial x}{\partial t}, x, y, p, d, u\right) \end{aligned} \tag{3.32}$$

with non-linear functions for CHP fuel consumption (f_1), boiler fuel consumption (f_2), net power production (f_3), and capital costs (f_4) respectively. The parameters A and B are the stochastic natural gas price and electricity price respectively.

Although the *PC* is the result of a highly non-linear system, each function f does not depend on the stochastic A and B parameters. Because this dynamic optimization simultaneously solves each time step and does not depend on feedback from previous solutions, the expected value of the *PC* at each time step reduces to

$$\begin{aligned}\mathbf{E}[PC] = f_1\left(\frac{\partial x}{\partial t}, x, y, p, d, u\right)\cdot\mathbf{E}[A] + f_2\left(\frac{\partial x}{\partial t}, x, y, p, d, u\right)\cdot\mathbf{E}[A] \\ + f_3\left(\frac{\partial x}{\partial t}, x, y, p, d, u\right)\cdot\mathbf{E}[B] + f_4\left(\frac{\partial x}{\partial t}, x, y, p, d, u\right)\end{aligned} \tag{3.33}$$

Equation 3.33 shows that the uncertainty associated with natural gas and electricity prices on the *PC* at each time step is captured only by the expected value of each random variable as defined in equation 3.31. This is true after the assumptions established earlier that the energy demand is considered a deterministic input and there is negligible uncertainty on model parameters and initial conditions. Otherwise all functions f would be random functions whose PMFs would be very difficult to obtain because of the non-linear nature.

To find the expected natural gas and electricity price, the probabilities of each price data point must be known. Statistical analysis using MatLab statistical toolbox constructs PDFs and cumulative density functions (CDFs) of the price data for each year in the horizon to estimate the probability of each data point. Matlab statistical toolbox uses a kernel distribution to estimate PDFs. A kernel distribution is a nonparametric representation of the PDF of a random variable. This method is employed when a parametric distribution cannot describe the data or to avoid making assumptions about the distribution of the data [52]. Equation 3.31 is invoked to find the expected value of natural gas price and electricity price. If each economic model is assumed to have equal probability, the expected value of natural gas and electricity prices at each year is the arithmetic mean, a very simple computation. The calculated expected values for both natural gas and electricity prices at each year are used as the price inputs in Equation 3.26 to calculate the operating cost. This makes the objective function Equation 3.9 like Equation 3.33, the expected value of the total net present value costs. The stochastic information is encapsulated in the expected natural gas and electricity prices and incorporated in the optimization directly through the objective function.

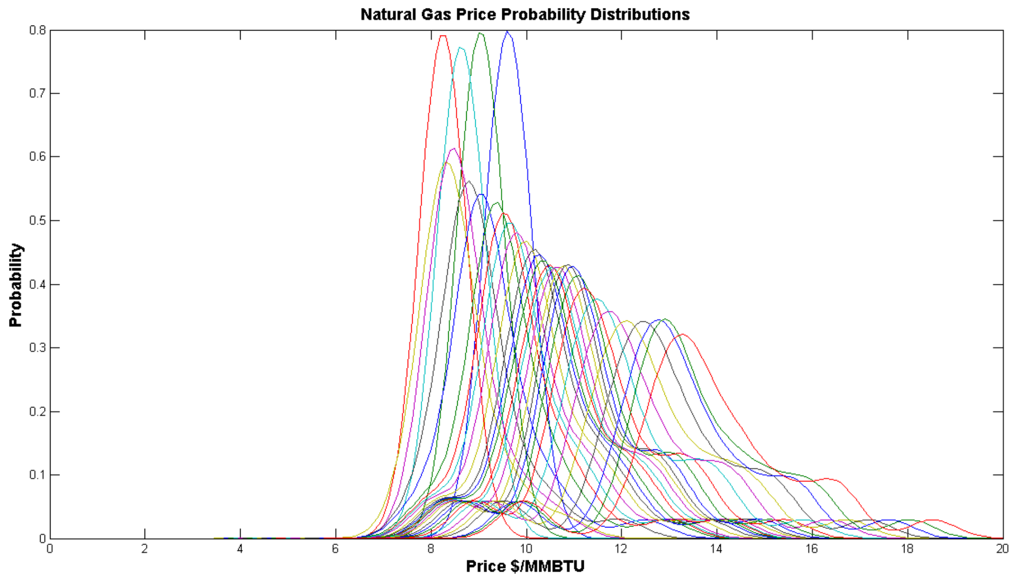

Figure 3.8: Probability distributions for natural gas prices. Each PDF curve denotes a different price distribution for a particular year in the time horizon. The distributions are used to calculate the CDF and then the expected natural gas prices at each point of the horizon through Equation 3.31. The same approach is used to find the expected electricity prices.

CHAPTER 4. OPTIMIZATION RESULTS AND DISCUSSION

The following is a list of the main results and observations further explained in this section:

- MPC formulation enables simultaneous control with multiple objectives in addition to an economic objective while finding optimum capacity.
- Capacity and timing of investment are coupled without the need of discretization, and multi-scaling of short term dynamics with long term planning horizon which are possible because of the explicit use of time derivative terms in the model.
- In addition to capacity and timing, optimization reveals utilization of capacity based on economic and dynamic demand profiles.

The observations listed above are important contributions of this optimization framework; and highlights the ability to give stake holders another tool to evaluate and observe the combined effects of system dynamics and economic scenarios on capacity planning. It is also important to note that the results are not a product of an exhaustive economic evaluation, but rather an optimization formulation that can be further refined with more comprehensive economic metrics and detailed system models.

4.1 MPC Formulations Results

Results for three different MPC formulations of the problem are summarized in Table 4.1. The capital cost and operating cost associated with each case is outlined in its respective column in millions of dollars. All three cases assume the expected energy prices economic case. Evaluators may quickly point out that the lowest overall cost is the best option for the capacity expansion plan, but the dynamic optimization formulation of this problem provides additional information that is important to consider beyond the purely economic results. All three MPC cases have the objective of minimizing capital and operating costs over the entire horizon. MPC Case 1 however has the additional target of reducing excess heat through the $l_1 - norm$ objective formulation by making the heat difference, Equation 3.14 in the model, a target set point. MPC Case 2 does not consider excess heat reduction but the net power demand which is constrained to follow the electric power demand. MPC Case 3 has the economic objective of minimizing costs over the entire horizon and is not constrained to track electric or heat production.

Figure 4.1 shows the optimized capacity planning schedule for the three different MPC formulations discussed above. The dashed line shows the investment schedule for MPC Case 1. This case gives the lowest CHP capacity while the boiler capacity is the highest. For this case, boiler capacity is the highest from the beginning with small increases planned along the horizon. The next highest line denotes MPC Case 2. This optimization case does not attempt to control the excess heat to a minimum, but instead is constrained to meet the electric power demand of the system only. This constraint is realistic if the ability to sell excess power to the grid is not available or the stakeholders do not want to participate in complex net metering rules. Additional boiler capacity investments for MPC Case 2 are delayed to latter years in the planning horizon. This is attributed to the lower cost of capital for conventional boiler capacity versus the higher cost of a CHP system. The blue dotted line outlines MPC Case 3. This case does not have artificial system limitations nor controls, but is free to drive the capacity planning based solely on the economic objective. This includes the ability to sell to the grid as part of the optimization option. This is shown in Figure 4.1 as the dotted blue line for the CHP maximum capacity plot reaches 30 MW, which is the upper bound established for CHP capacity in the model. This indicates that economically, the district energy system behaves as a power utility to sell electric power to the grid. This is obviously not part of a reasonable strategy that a company or institution would pursue if the main goal was to fulfill their electric power and heating needs. Nevertheless, the optimization program finds that selling power to the grid can reduce costs by generating revenue from excess electricity.

The last column of Table 4.1 is the levelized cost of energy. This is an important parameter used to compare, through a single number, the cost of energy over the lifetime of the system. It is defined as

$$\frac{\sum_{t=1}^{n} CC_t + OC_t}{\sum_{t=1}^{n} E_t} \tag{4.1}$$

Table 4.1: Results for three different MPC formulations for the expected energy prices economic case.

MPC Case	Economic Objective	Electricity Target	Heat Target	Max CHP Capacity	Max Boiler Capacity	Capital Cost	Operating Cost	Levelized Cost of Usable Energy
1	✓	✗	✓	9 MW	70 MMBTU/hr	\$14.8MM	\$290.5MM	\$29.90/MWh
2	✓	✓	✗	13 MW	20 MMBTU/hr	\$16.1MM	\$230.8MM	\$26.20/MWh
3	✓	✗	✗	30 MW	5 MMBTU/hr	\$32.2MM	\$323.8MM	\$30.10/MWh

Where

CC_t is the discounted capital cost at time t

OC_t is the discounted operating cost at time t

E_t is the usable energy generation at time t

n is the life of the system

The sum of usable energy (E_t) is the sum of both electric and usable thermal energy in megawatt-hours, but does not account for thermal energy that is wasted through the flue gas. The levelized cost of energy is a useful tool to evaluate which generation system option produces energy in the most cost effective way. For the different MPC optimization cases, MPC Case 2 produces the lowest cost in dollars per megawatt-hour of usable energy generated.

Figure 4.2 shows the effects of the MPC cases on surplus heat. It is important to note that excess heat production from a district heating system is discarded to the atmosphere, and electricity is exported to the grid if generation is above the system needs. Production of thermal energy beyond the heat demand of the district energy system reduces the efficiency gains of heat and power integration, one of the main goals of a CHP system. The $l_1 - norm$ formulation of MPC Case 1 has the lowest excess heat release because the excess heat term of the model is a variable targeted for minimization. The control objective is effective in that it fully utilizes all available heat in the winter season. MPC Case 2 does not reduce the excess heat generation as aggressively as MPC Case 1, but gradually diminishes the excess heat over the 30 year horizon for both summer and winter. MPC Case 3, as discussed earlier, has only the economic objective and thus produces both excess electricity and likewise excess heat.

MPC formulations offer a direct way of expressing operational strategies into a capacity planning problem and observe the effect on size and timing of capacity investment.

4.2 Utilization of Capacity

The results in the previous section reveal that although the economic objective is the main driver for the optimization, operational objectives can be formulated into the problem. Providing the lowest capital and operational cost also depends on how the capacity is utilized given the additional operating objectives such as reducing excess heat production. Reducing excess heat has a direct effect on CO_2 emissions which in the long term can become an important operational ob-

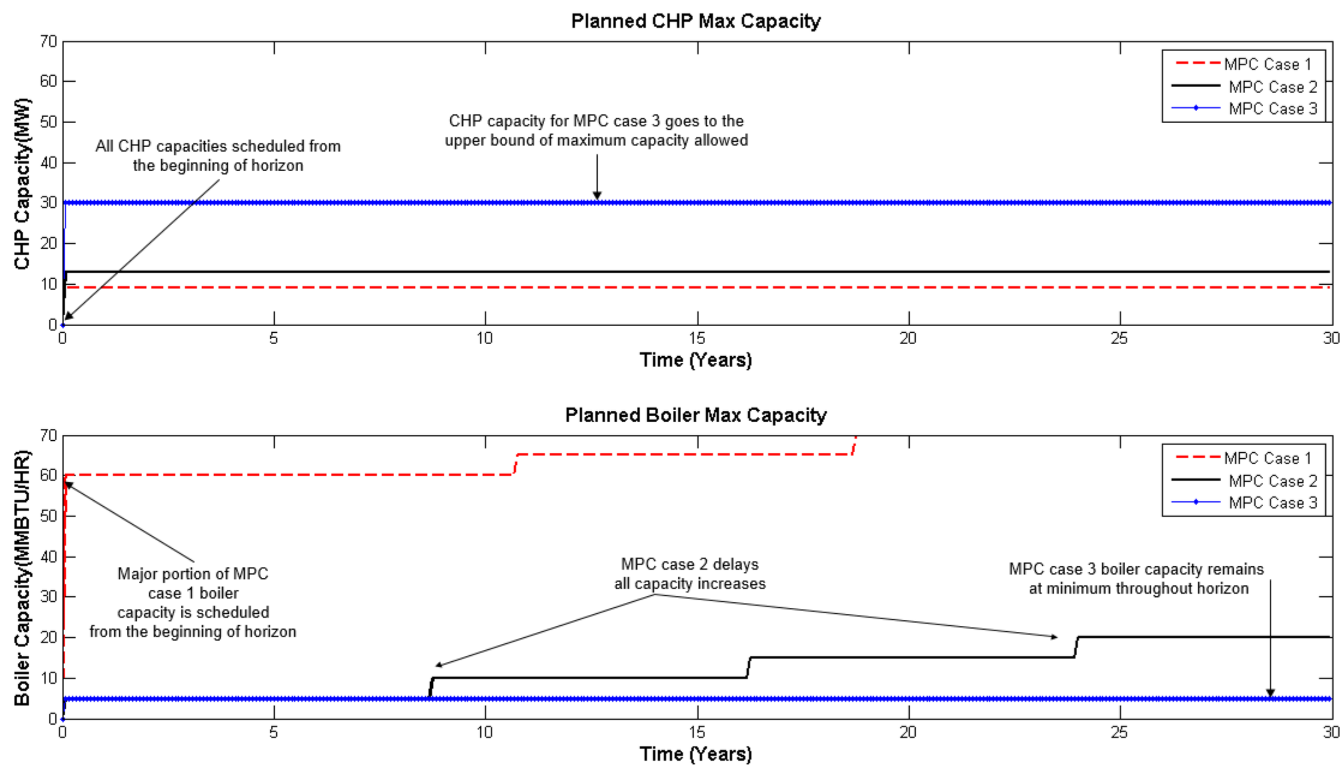

Figure 4.1: CHP and boiler capacity investment plan from MPC formulation based on three different control objectives for the expected energy prices economic case.

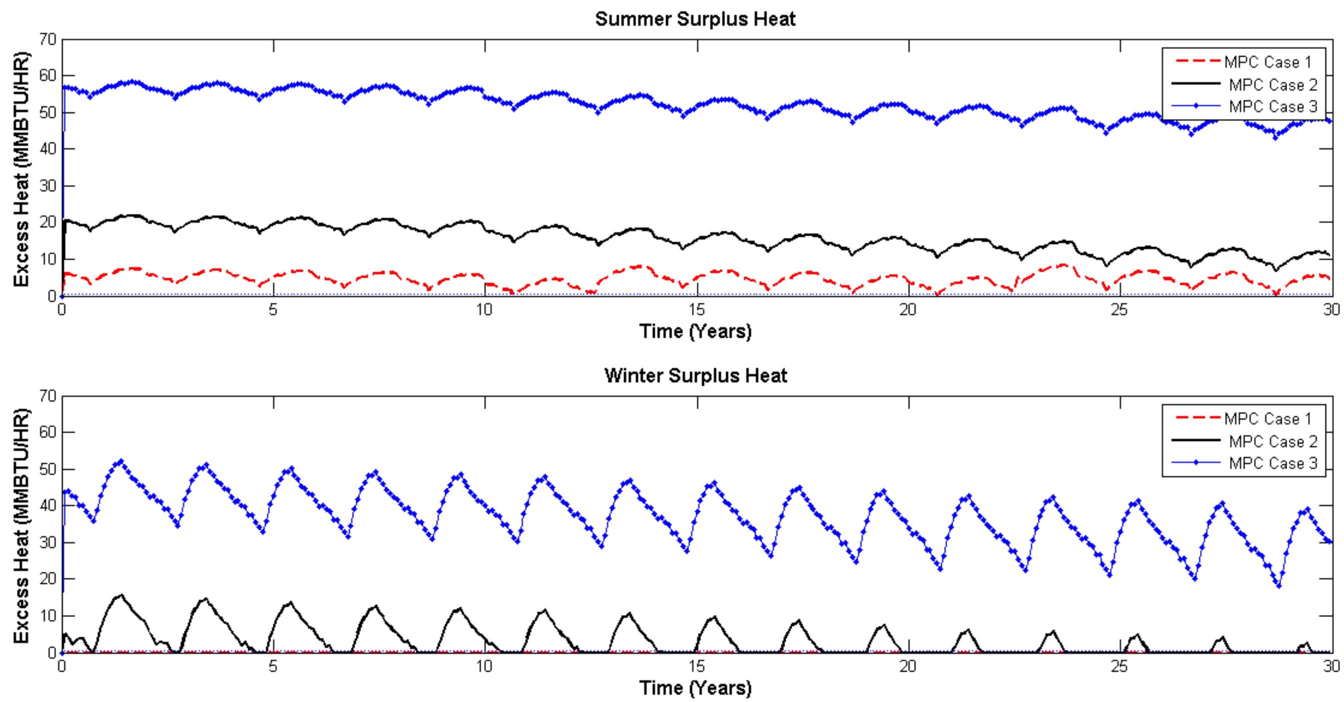

Figure 4.2: Surplus heat profiles for the different MPC cases. MPC Case 3 has the lowest excess heat for both winter and summer. For winter profile it goes to zero for MPC Case 3.

jective if carbon cutting regulations are implemented. Capacity planning has a multi-decade time scale while operational strategy has an hourly time scale. This work merges the differences in time scales by directly using representative diurnal load profiles for heat and electricity of winter and summer seasons and projecting them over a 30 year horizon as illustrated earlier in Figure 3.4. The differential terms in the model directly respond to the short term dynamics for energy generation while the differential of maximum capacity ($\frac{\partial mc_p}{\partial t}$) takes effect only when extra capacity is needed. The multi-scale features provide additional flexibility to the optimizer to move manipulated variables that affect short scale economic and control objectives while simultaneously scheduling capacity investments over the long term horizon. Figure 4.2 shows the heating profiles for both summer and winter seasons along with the utilization of heat supply for MPC Case 1. As mentioned earlier, MPC Case 1, has the control objective of reducing excess heat. The optimizer achieves this by using the extra boiler capacity during the winter season to follow the heat load with only a small fraction supplied by the heat recovery boiler. The total heat supply denoted by the "x" marker is a perfect match with the heat demand of the winter profile. For the summer heating profile, the heat is provided by the heat recovered from the CHP while the extra boiler capacity set point is set at zero (boiler off) denoted by the diamond marker. The CHP system does not follow the heat load but rather stays atop the peak demands throughout the summer horizon. This happens because the heat generated from the turbine to meet the higher summer electric load is enough to meet the heat demand. This strategy is found by the optimizer in order to reduce the excess heat control variable.

In summary, the excess heat minimization objective in MPC Case 1 is achieved by changing the capacity utilization of both winter and summer seasons. During the summer, CHP heat output is maximized and in the winter boiler load following is recommended. This is a good strategy in places where air quality issues are more sensitive during the winter months, and reducing excess heat production to minimum limits is desirable to reduce combustion emissions. Traditional optimization and modeling approaches for capacity planning problems cannot give short-term guidance on utilization of capacity because the models do not detail dynamic response of the system as this work presents.

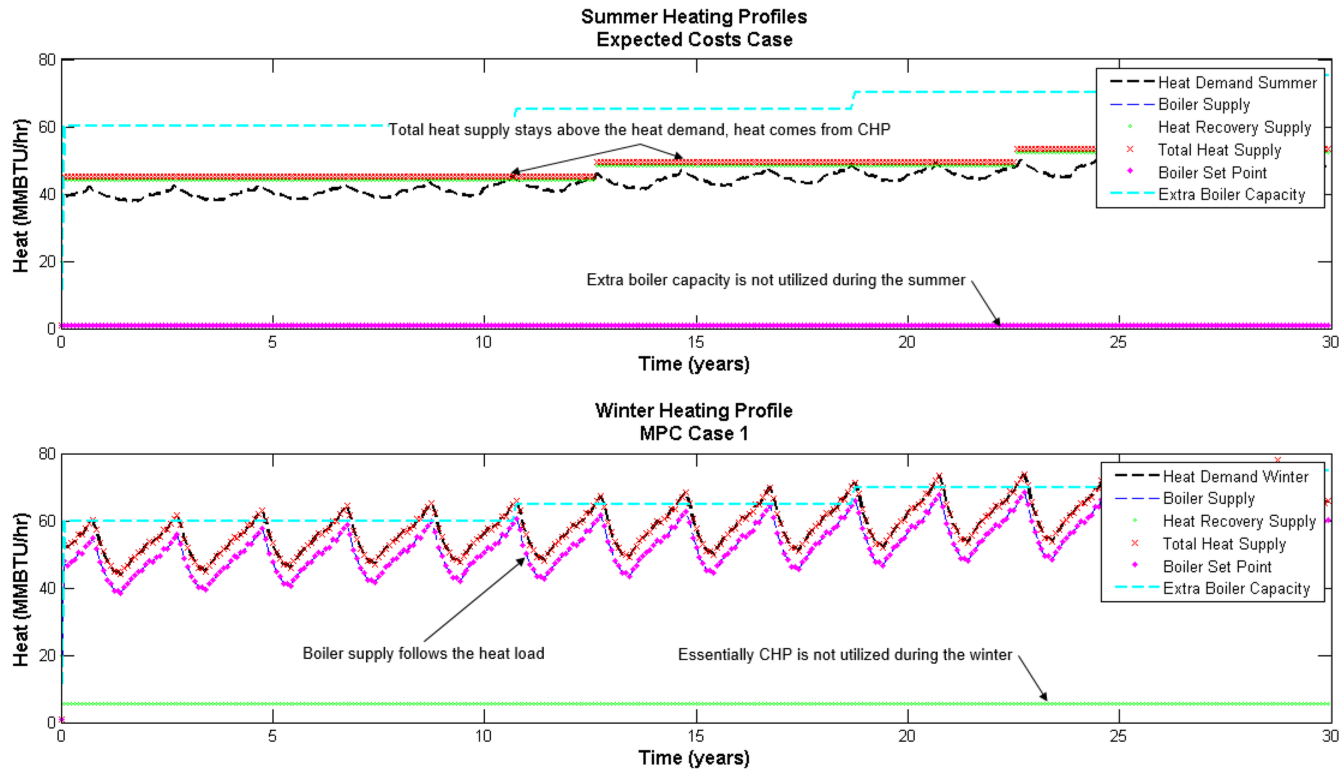

Figure 4.3: Heating supply profiles for MPC case 1.

4.3 Optimization Under Different Economic Scenarios

This section contains optimization results based on different economic scenarios. The results directly reflect the effect of input economic data and not the MPC formulation. The MPC formulation is kept constant and is based on MPC Case 2 formulation above. MPC Case 2 is chosen because computation times are slightly lower than the other cases. As discussed in the previous section, MPC Case 2 formulation maintains the economic objective and restricts electric power production to meet system load. MPC Case 2 does not allow the transfer of power to the grid.

4.3.1 Economic Scenarios Relaxed Problem Results

Optimizing capacity planning under different economic scenarios beyond the expected natural gas and electricity prices generate results that give insights on the sensitivity of the capacity and timing of investment on energy prices. Figure 4.4 shows the CHP capacity plan for seven different economic scenarios. The CHP capacity is lower for all cases where there is monetary cost per short ton of CO_2 emission. The higher the cost per ton, the lower the CHP capacity is calculated to be optimal while the low economic growth case which contains the lowest energy prices and no CO_2 cost has the highest capacity. The reference case, the high economic growth, and no greenhouse gas (GHG) concern cases converge to the same capacity. For this reason only the plot for high economic growth case appears, the other two are hidden behind the same line. Regardless of the economic case and magnitude of capacity, the optimizer indicates that CHP investment should be made from the beginning of the planning horizon. These results are from a relaxed problem formulation on the CHP capacity variable (mc_{chp}). In optimization, a relaxed formulation means that there is no requirement for integer solution of designated variables, but instead the entire continuous feasible space is available. Integer formulations are computationally harder to solve, but give more realistic results. For example, turbine capacities are usually rated by integer number (e.g. 10 MW) instead of a mixed number (e.g. 10.14 MW).

In conjunction with the CHP capacity, the boiler capacity investment schedule is give by Figure 4.5 which shows different timings for additional boiler capacity depending on the economic case. Additional boiler capacity is delayed for those cases that have the lowest energy prices while

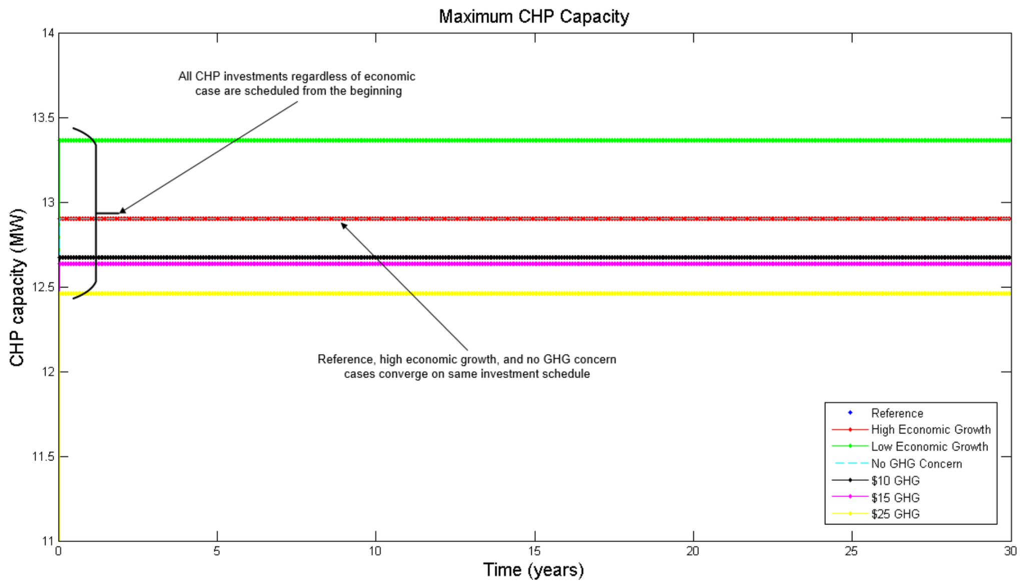

Figure 4.4: CHP capacity schedule based on different economic cases for relaxed problem formulation.

those that consider CO_2 emission costs request additional boiler capacity earlier in the horizon. The optimizer finds a better objective by reducing CHP capacity and increasing investment on the lower cost boiler capacity to fulfill the energy demands when operating costs are higher. The reference case, high economic growth, and no GHG concern cases over lap with the same timing and capacity plotted by the blue and cyan line in Figure 4.5.

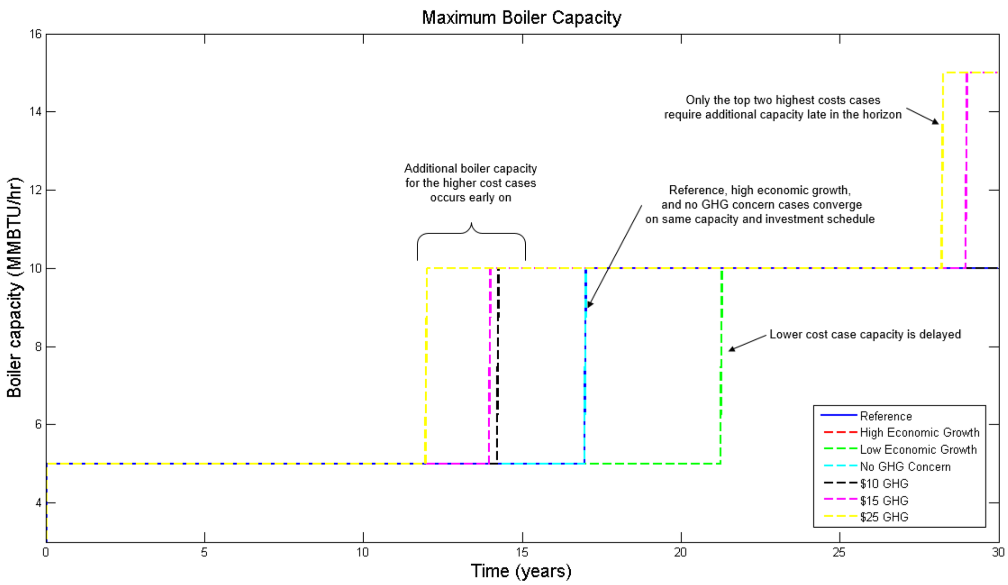

Figure 4.5: Boiler capacity schedule based on different economic cases for relaxed problem formulation.

4.3.2 Economic Scenarios MINLP Results

Results for the mixed integer problem (MINLP) formulation are summarized in Figure 4.6 for a larger set of economic scenarios. Optimization of a wider range of economic scenarios gives further insights about the feasibility of a CHP system. In 15 out of 20 different economic cases that account for natural gas and electricity price fluctuations over the 30 year horizon, the optimization results find feasible to invest in a CHP system, while only five cases suggest to make no CHP investment and keep a boiler only system.

A histogram with the frequency of the integer solutions for CHP and boiler capacities is given in Figure 4.7. For most of the economic scenarios, the CHP maximum capacity recommendations are closely aggregated between 9 MW and 13 MW capacity, a difference of just 4 MW in capacity. Although a 4 MW spread translates to differences in several million dollars in capital investment, the optimization solution provides less uncertain capacity options because more information about the system is encapsulated and solved simultaneously. It is important to mention that for all cases where CHP capacity is cost effective, the capacity investment starts from the beginning of the horizon, mirroring the relaxed problem results of Figure 4.4. Figure 4.7 also shows the

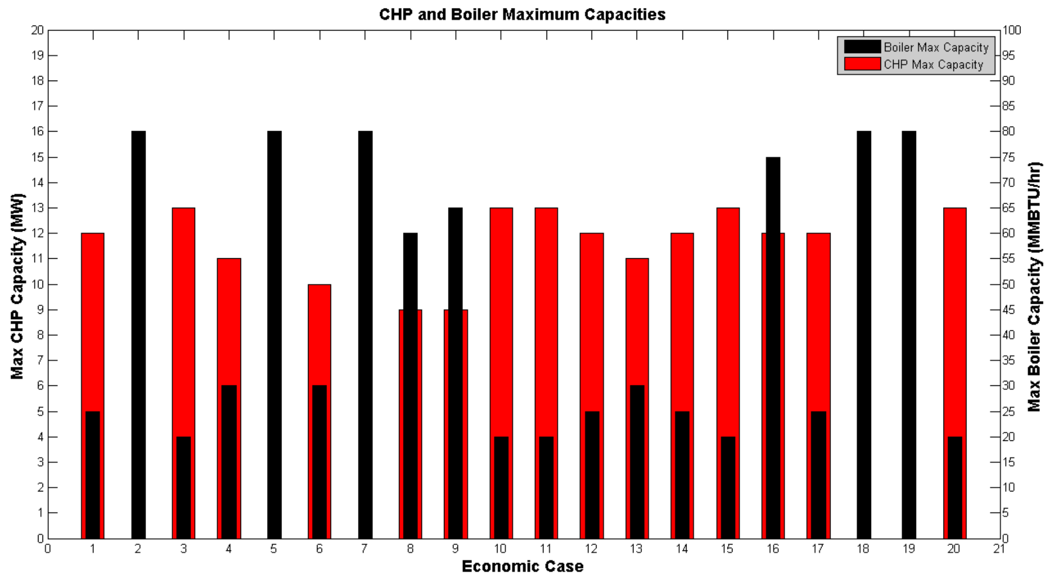

Figure 4.6: CHP and boiler maximum capacities by economic scenario.

frequencies of the maximum boiler capacities. Similar to the relaxed problem solution of Figure 4.5, the boiler capacity increases are distributed at different points of the horizon.

There are five cases where the optimization finds a lower objective function without CHP investment. These results are less intuitive to understand, but underscore the benefits of large scale dynamic optimization formulations that utilize large sets of data and solve the problem simultaneously for the entire horizon. The economic evaluations are performed at every step of the horizon, thus the hidden trade offs from the economic data inputs at different time points are exploited to find an overall lower cost objective. Discrete, steady state evaluations would never be able to compare the dynamic interactions of price and demand fluctuations to differentiate the capacity size decision over a large time horizon.

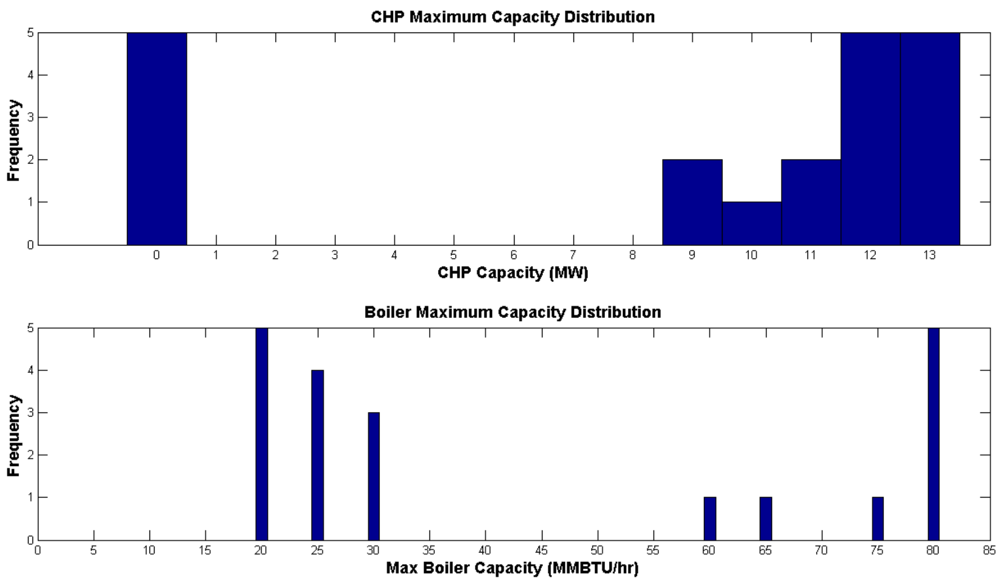

Figure 4.7: CHP capacity histogram (top) and boiler total capacity histogram (bottom).

CHAPTER 5. CONCLUSION AND FUTURE CONSIDERATIONS

This work presents a dynamic optimization framework with MPC elements for capacity planning of an energy district system. Key conclusions from this work are summarized next.

Non-linear terms are essential to capture cost effects such as economies of scale as the optimizer simultaneously captures the economic and physical relationships that describe the system. For example, although the optimal CHP capacity size varies depending on the economic scenario, the optimizer finds that a sizable CHP capacity from the beginning of the horizon has a steeper reduction in overall costs than a delayed CHP investment regardless of how the capacity may be utilized.

The modeling of the district energy system along with the formulation to find the capacity investment over a thirty year horizon is facilitated by the novel use of differential and algebraic equations. The differential elements explicitly account for changes in capacity over time without the need to explicitly discretize the model to account for time as is done in traditional linear programming formulations. For this application, the differential terms tie together energy demand dynamics with response time constraints, and relate the production of energy in the short time scale to the longer term maximum capacity. This cannot be done in models that do not treat time as a continuous, explicit variable.

This work also shows that within the dynamic optimization framework, a model predictive control formulation of the objective function can add other control objectives beside the economic objective. The MPC formulation inserts operational control objectives to the optimization formulation to observe the effects on long term capacity planning. Different economic cases also affect capacity and timing of the investment. This optimization framework is a tool to guide timing of capacity investments that reflects operational strategies and multiple objectives.

For a building campus district energy system, proposing a CHP investment to meet energy needs is a major change. From the results of this optimization study, the question of whether to invest in a CHP system is answered by the optimization recommendations to make some type of CHP investment for most economic cases from the beginning of the horizon. The other half of the question is in regards to the size of the capacity investment. The size of the capacity investment will depend closely on the financial incentive that the optimizer finds based on the uncertain fuel cost and electricity price data. In contrast, without a simultaneous optimization with uncertain parameters, CHP builders have recommend capacity options that are over fifty percent larger than the largest optimized CHP capacity result for a system of similar size. Oversized CHP capacity options not only increase already expensive capital expenditures, but also CHP utilization will be more expensive because there is a greater opportunity for the gas turbine to operate at lower, less efficient loads over the lifetime of the system.

Difficulty in solving large and highly non-linear dynamic optimization problems is something that is still problematic and subject of continuous research. This limitation is visible in this work in using second degree polynomials instead of better fitting exponential functions for capacity vs. cost relationship. The optimizer in certain cases recommends smaller increases in boiler capacity throughout the horizon, which may be an artifact of the slightly non-linear polynomial functions. Close to the origin, the polynomial fits are almost linear; therefore, smaller capacities give smaller cost and potentially an artificially lower objective. Limitations also exist in this work when formulating the problem to explicitly account for uncertainty by introducing larger amounts of data into the problem. The optimizer handles $20,463$ state variables, $18,309$ total equations, and $2,872$ slack variables, with over 700 degrees of freedom when considering a single economic case. When different economic cases are simultaneously solved, the optimization solver is unable to solve the problem. Future work should explore improving optimization solver start up procedures as well as consideration of alternative large scale optimization solvers to diminish both the highly non-linear and large problem limitations. Consideration of other large scale solvers will require formulating this problem in different mathematical modeling languages like GAMS or AMPL.

To further understand and differentiate the effects of important economic parameters, future work should explore global optimization techniques to verify global optimality from given economic inputs. So far, this work does not include comprehensive study on the global optimality of the solutions, but mainly focuses on a gradient based dynamic optimization strategy for energy system capacity expansion problems.

This work does not attempt to show an exhaustive capacity investment schedule for a district energy system, but rather demonstrate that a dynamic optimization framework with MPC elements is a feasible approach to guide capacity planning of energy systems. This approach can be expanded to larger systems such as smart grid applications where a number of interactive energy systems with unique dynamics such as coal power plants, wind and solar renewable power sources, energy storage, and carbon capturing systems interact to meet demands. The intrinsic dynamics, economic, and operational targets affect capacity and timing of investments of system components. This optimization framework brings a holistic approach to handle multiple targets and system data within a single optimization formulation.

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APPENDIX A. DYNAMIC BOILER MODEL

```
Model
  Parameters
    % boiler physical Parameters
    mt = 300000                    % (kg)total metal mass of boiler
         Cp_boiler = 0.49          % (kJ/kg*K) heat capacity of
            metal
         Vt = 88                             %(m^3) total volume of
            the drum, downcomer, and risers
         gashhv = 39.5             % (MJ/m3) natural gas high
            heating value

         % steam/water physical Parameters
    MW_water = 18         % Molecular weight of water(kg/kmol)
         treff = 273.15                              % deg K,
            reference temperature for enthalphy calculations
         tcrit = 647.096                             % (deg K)critical
             temperature of water

         % next are the empirical constants for the formulas from
            steam table data
         %tempsat constants
         Atsat= 6E-11
         Btsat= -2E-6
         Ctsat= 0.0277
```

```
    Dtsat= 168.3
    Arwe = -0.0192
    Brwe = 876.27

    % rhos constants
    Ars = 1E-7
    Brs = 0.0039
    Crs = 2.4107

    % hw constants
    Ahwc=3E-10
    Bhwc=-8E-6
    Chwc=0.126
    Dhwc=705.14

    % hs constants
    Ahsc= 7E-11
    Bhsc= -3E-6
    Chsc= 0.0127
    Dhsc= 2787.2
    qf = 55 % (kg/s) feed water mass flow rate
    tfw = 200 % (deg C)feed water temperature
  Q = 17%  in (MW) fuel power input
    Vwt = 55 % (m3) total water volume inside boiler
End Parameters

Variables
  % Differential States

 qs = 55 % (kg/s) steam mass flow rate
```

```
p  = 8000 % (kPa) drum pressure

temp_sat

gas_flow

V_steam_inside

V_water
End Variables

Intermediates

    tempsat = 35.6784*p^(0.2350)
    tf = tfw +273.15 % (deg K)feed water temperature
    rhow = (3.3659E-8*p + 0.001117)^(-1) % in kg/m^3

    rhos = (211.075/p -0.00294)^(-1) % in kg/m^3

    Vst = Vt - Vwt % total steam volume inside boiler

    hw = 103.749*p^0.28302 % in kJ/kg

    hs = 2802.4 - 1.6038E-5*(p-3124.9)^1.73808 % in kJ/kg

    hf = (2.763E5*(tf -treff) -2.0901E3*(tf^2-treff^2)/2 +
       8.125*(tf^3 - treff^3)/3 -1.14116E-2*(tf^4-treff^4)/4
       + 9.3701E-6*(tf^5-treff^5)/5)*1/(MW_water*1000) % (kJ/
       kg)
```

```
% DERIVATIVE PARTS

drhow = -3.3659E-8/(3.3659E-8*p+0.001117)^2 % in ((kg/m3)
    /kPa)

drhos = 211.075/((p^2)*(211.075/p - 0.00294)^2) %in ((kg/
    m3)/kPa)

dhw = 29.36304198/p^0.71698% in (kJ/kg)/(kPa)

dhs = -0.00002787532704*(p-3124.9)^0.73808 % in (kJ/kg)/(
    kPa)

dts = 8.384424/p^0.765 % (deg C)/(kPa)

gasflow = Q/gashhv + (0.3/0.7)*(Q/gashhv) % (m3/s) total
    natural gas flow rate,
%assuming that 70% of the heating value goes to produce
    steam
%this has to be calculated more closely to carefully
    quantify the gas usage, which ultimately is what we
    want.

e11 = rhow - rhos
e12 = Vwt*drhow + Vst*drhos
e21 = rhow*hw - rhos*hs
e22 = Vwt*(hw*drhow + rhow*dhw) + Vst*(hs*drhos + rhos*
    dhs) - Vt + mt*Cp_boiler*dts
```

```
    % for parameter reporting and comparison to Astrom's
      values
    parameter1 = (hs-hw)*Vst*drhos
    parameter2 = rhos*Vst*dhs
    parameter3 = rhow*Vwt*dhw
    parameter4 = mt*Cp_boiler*dts

  End Intermediates

  Equations
    temp_sat = tempsat
    gas_flow = gasflow
    V_steam_inside = Vst
    V_water = Vwt
    % overall mass balance
    e11*$Vwt+e12*$p = qf - qs
  % energy balance
    e21*$Vwt + e22*$p = Q*(1E3) +qf*hf - qs*hs

  End Equations

End Model
```

APPENDIX B. ENERGY SYSTEM DYNAMIC MODEL

```
model
constants
        I = 1
        chp_cases = 1 % if greater than 1, more than one economic
            case is simultaneously being solved
end constants

parameters
        tau1              = .001
        tau2              = .001
        elecprice[1:I]    = 0 % $/MWhr
        ngprice[1:I]      = 0 % $/MMBTU/hr
        demand1           = 10
        demand2           = 10
        heat1             = 10
        heat2             = 10
        itime             = 0
        naverage          = 2 % this is the number of years being
           averaged for model reduction,
                %if = 1, then the full model is being solved
        co2price[1:I]     = 0     %$/metric ton
        timestep          = 0.041667
        inflation         = 0.03
        discount          = 0.06
```

```
        boilereff       = 0.8
        capdiff         = 0.2
        co2ng           = 124.8 %lb CO2/MMBTU
        boiler_min_cap = 5 % MMBTU/hr

end parameters

variables
% for all variable names in model: 1 is for summer, 2 is for
   winter
% if variable name starts with "int," then it is an integer
   variable
        x_s[1:I]                              = 0 ,>= 0, <=30 %
            x is chp electricity production in MW
        x_w[1:I]                              = 0 ,>= 0, <=30 %
            x is chp electricity production in MW
        y_s[1:I]                              = 20 ,>=0 % y is
           the chp heat output as steam or hotwater
        y_w[1:I]                              = 20 ,>=0 % y is
           the chp heat output as steam or hotwater
        b_s[1:I]                              = 1 ,>=0, <=100
        b_w[1:I]                              = 1 ,>=0, <=100
        u_s[1:I]                              = 1 ,>=0, <=100
                      % u is the setpoint for turbine
        u_w[1:I]                              = 1 ,>=0, <=100
                      % u is the setpoint for turbine
        v_s[1:I]                              = 1 ,>=0, <=100
                      % v is the setpoint for boiler
        v_w[1:I]                              = 1 ,>=0, <=100
                      % v is the setpoint for boiler
```

```
%import1                    = 1  ,>=  0,  <=100
%import2                    = 1  ,>=  0,  <=100
%energy1                            = 1  %,>=  1
%energy2                    = 1         %,>=  1
oper_costs_s[1:I]           = 1  %,>=  0
oper_costs_w[1:I]           = 1         %,>=  0
heatsupply_s[1:I]           = 1
heatsupply_w[1:I]           = 1
fuelchp_s[1:I]              = 0
fuelchp_w[1:I]              = 0
fuelboiler_s[1:I]                   = 0
fuelboiler_w[1:I]                   = 0
loadfrac_s[1:I]                     = 1 % part load fraction
   summer operation
loadfrac_w[1:I]                     = 1 % partl load fraction
co2chp_s[1:I]                       = 0% shortton CO2/hour
co2chp_w[1:I]                       = 0 % shortton CO2/hour
co2boiler_s[1:I]                    = 0 % shortton CO2/hour
co2boiler_w[1:I]                    = 0 % shortton CO2/hour
co2cost_s[1:I]              = 0 % in $
co2cost_w[1:I]              = 0 % in $
%error_energy1   = 0
%error_energy2   = 0
error_heat_s[1:I]                   = 0
error_heat_w[1:I]                   = 0
int_cap_max_chp             = 0
totalcost[1:I]              = 0
dcap_max_chp     = 0
icosts_cap_chp   = 0
cap_npv                     = 0
```

```
        capital_costs    = 0
       %excess_power1    = 0
       %excess_power2    = 0
        netpower_s[1:I]          = 0,  <= 0 % add the <= 0
           constraint if you want to remove ability
                %to sell electricity to grid ( No net positive
                   power production )
        netpower_w[1:I]          = 0,  <= 0 % add the <= 0
           constraint if you want to remove ability
                %to sell electricity to grid ( No net positive
                   power production )
        fueleff_s[1:I]           = 1
        fueleff_w[1:I]           = 1
        int_boiler_step = 0
        cap_max_b                = 0
        dcap_max_b               = 0
        icosts_cap_b     = 0
        netpower_co2     = 0
end variables

equations
%*************** DEMAND PROFILES ***********
       %error_energy1   = energy1 - demand1
       %error_energy2   = energy2 - demand2
        error_heat_s[1:I]               = heatsupply_s[1:I] -
           heat1
        error_heat_w[1:I]               = heatsupply_w[1:I] -
           heat2

       %error_energy1   >= 0
```

```
%error_energy2  >= 0
 error_heat_s[1:I]                 >= 0
 error_heat_w[1:I]                 >= 0
%*************** DYNAMIC SYSTEM ***********
 tau1*$x_s[1:I] + x_s[1:I] = u_s[1:I] % dynamic equation
    for chp summer
 tau2*$x_w[1:I] + x_w[1:I] = u_w[1:I] % dynamic equation
    for chp winter
 tau1*$b_s[1:I] + b_s[1:I] = v_s[1:I] % dynamic equation
    for boiler summer
 tau2*$b_w[1:I] + b_w[1:I] = v_w[1:I] % dynamic equation
    for boiler  winter
 y_s[1:I] = -0.0817*x_s[1:I]^2+5.6547*x_s[1:I] % heat
    output in MMBTU/hr as steam or hot water from HRSG
 y_w[1:I] = -0.0817*x_w[1:I]^2+5.6547*x_w[1:I] % heat
    output in MMBTU/hr as steam or hot water from HRSG
 heatsupply_s[1:I] = y_s[1:I] + b_s[1:I] %total heat
    supply in MMBTU/hr summer
 heatsupply_w[1:I] = y_w[1:I] + b_w[1:I] %total heat
    supply in MMBTU/hr winter
 netpower_s[1:I] =  x_s[1:I] - demand1 % if positive, then
     exporting power, negative is importing power
 netpower_w[1:I] =  x_w[1:I] - demand2

 int_cap_max_chp >= x_s[1:I]
 int_cap_max_chp >= x_w[1:I]
 $int_cap_max_chp >= 0
 dcap_max_chp = $int_cap_max_chp

 cap_max_b = boiler_min_cap*int_boiler_step
```

```
        cap_max_b >= b_s[1:I]
        cap_max_b >= b_w[1:I]
        $cap_max_b >= 0
        dcap_max_b = $cap_max_b
%*************** FUEL CONSUMPTIONS EVALUATIONS ***********
        loadfrac_s[1:I] = x_s[1:I]/(int_cap_max_chp + 10^(-6)) %
           part load fraction summer operation
        loadfrac_w[1:I] = x_w[1:I]/(int_cap_max_chp + 10^(-6)) %
           part load fraction winter operation
        fueleff_s[1:I]          =  0.9044*loadfrac_s[1:I] +
           0.0956 % simplified equation
        fueleff_w[1:I]          =  0.9044*loadfrac_w[1:I] +
           0.0956
        fuelchp_s[1:I]          = (-0.1361*x_s[1:I]^2 + 13.347*
           x_s[1:I])/fueleff_s[1:I] % summer fuel consumptions of
            gas turbine in MMBTU/hr
        fuelchp_w[1:I]          = (-0.1361*x_w[1:I]^2 + 13.347*
           x_w[1:I])/fueleff_w[1:I] % winter fuel consumptions of
            gas turbine in MMBTU/hr
        fuelboiler_s[1:I]       =  b_s[1:I]/boilereff % fuel
           consumptions of boiler in MMBTU/hr
        fuelboiler_w[1:I]       =  b_w[1:I]/boilereff % fuel
           consumptions of boiler in MMBTU/hr
%*************** CO2 EVALUATIONS *********************
        co2chp_s[1:I]                  = ((fuelchp_s[1:I]*co2ng)
           /2000) % shortton CO2/hour
        co2chp_w[1:I]                  = ((fuelchp_w[1:I]*co2ng)
           /2000) % shortton CO2/hour
        co2boiler_s[1:I]               = ((fuelboiler_s[1:I]*
           co2ng)/2000) % shortton CO2/hour
```

```
co2boiler_w[1:I]                    = ((fuelboiler_w[1:I]*
   co2ng)/2000) % shortton CO2/hour
netpower_co2                        = (co2chp_s[1:I] +
   co2chp_w[1:I] + co2boiler_s[1:I] + co2boiler_s[1:I] +
   co2boiler_w[1:I])*(0.5*365*naverage)
%*************** COST EVALUATIONS ********************
icosts_cap_chp = (-0.0069*(timestep*dcap_max_chp)^2 +
   1.3351*(timestep*dcap_max_chp))*(10^6)*(1 + inflation)
   ^itime % function to calculate investment cost in $
icosts_cap_b = 2*(-39.504*(timestep*dcap_max_b)^2 +
   29900*(timestep*dcap_max_b))*(1 + inflation)^itime %%
   function to calculate investment cost in $
cap_npv                   = icosts_cap_chp/((1+ discount)^
   itime) + icosts_cap_b/((1+ discount)^itime) % NPV of
   capital investment in $
co2cost_s[1:I]            = (co2chp_s[1:I] + co2boiler_s[1:
   I])*co2price[1:I]*(907.185/1000) % in $,
   (907.185/1000) is the conversion factor from short ton
    to metric ton
co2cost_w[1:I]            = (co2chp_w[1:I] + co2boiler_w[1:
   I])*co2price[1:I]*(907.185/1000) % in $,
   (907.185/1000) is the conversion factor from short ton
    to metric ton
oper_costs_s[1:I]         = ((ngprice[1:I]*fuelchp_s[1:I] +
    ngprice[1:I]*fuelboiler_s[1:I]  - (1)*elecprice[1:I]*
   netpower_s[1:I] + co2cost_s[1:I])*0.5*365)/((1+
   discount)^itime)*naverage % in $ (summer)
oper_costs_w[1:I]         = ((ngprice[1:I]*fuelchp_w[1:I] +
    ngprice[1:I]*fuelboiler_w[1:I]  - (1)*elecprice[1:I]*
```

```
      netpower_w[1:I] + co2cost_w[1:I])*0.5*365)/((1+
      discount)^itime)*naverage % in $ (winter)
    capital_costs    = cap_npv % in $
    totalcost[1:I] = oper_costs_s[1:I] + oper_costs_w[1:I]
      % in $
    %*************** objective Function
      *********************
    minimize totalcost[1:I]
    minimize capital_costs*chp_cases

end equations
end model
```

APPENDIX C. CHP CORRELATIONS

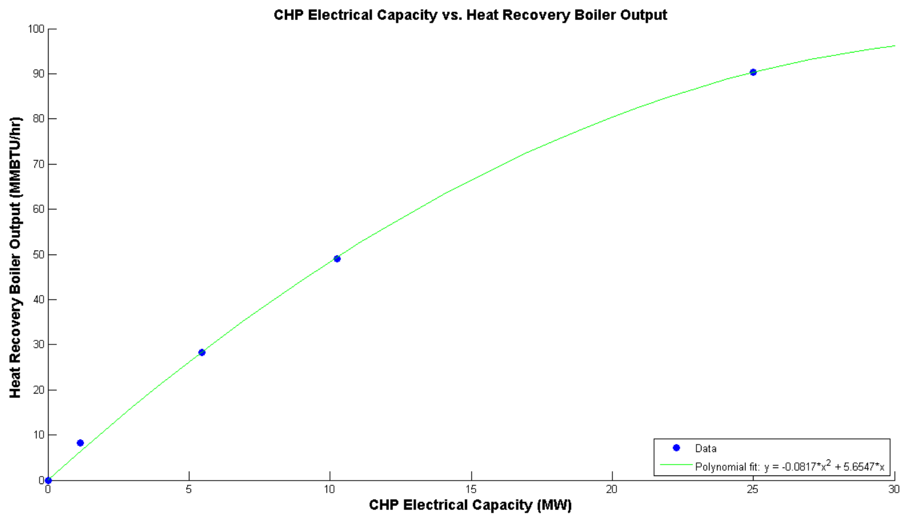

Figure C.1: Data and polynomial from [11] for CHP heat supply Equation 3.13.

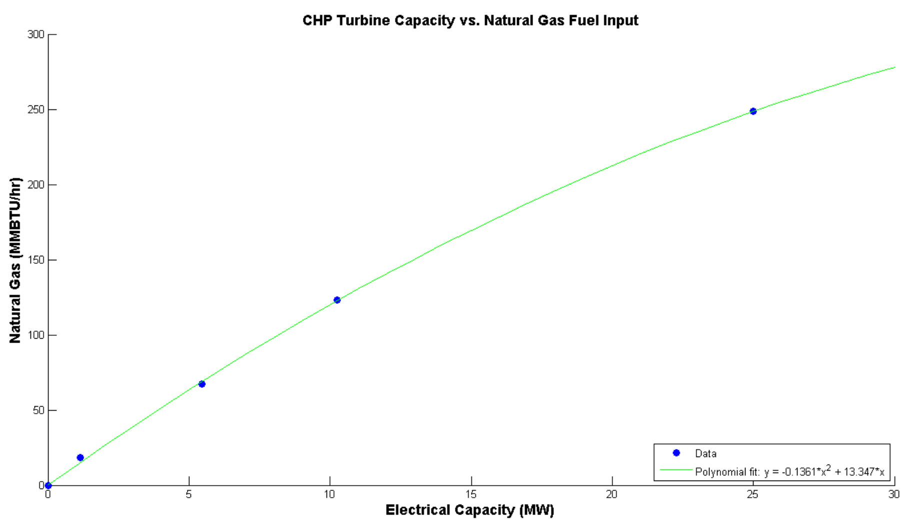

Figure C.2: Data and polynomial from [11] for CHP natural gas consumption Equation 3.19.

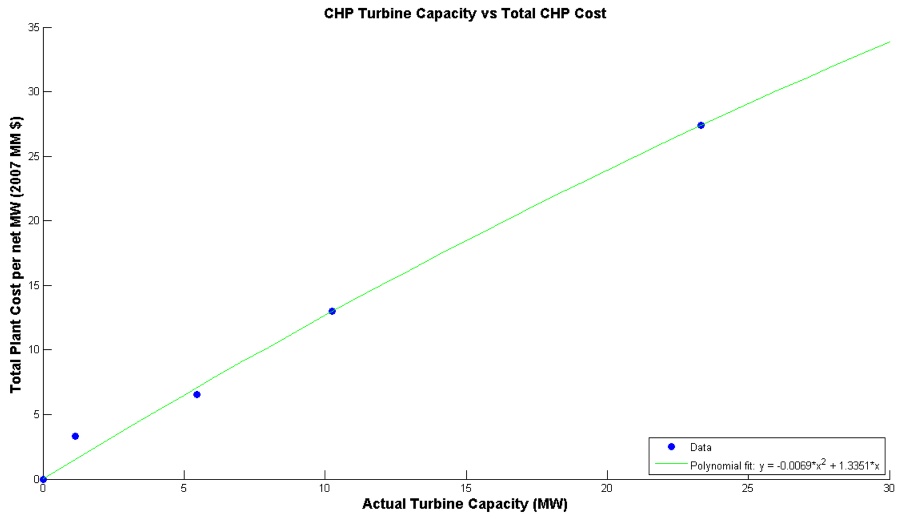

Figure C.3: Data and polynomial from [9] for CHP capital cost Equation 3.24

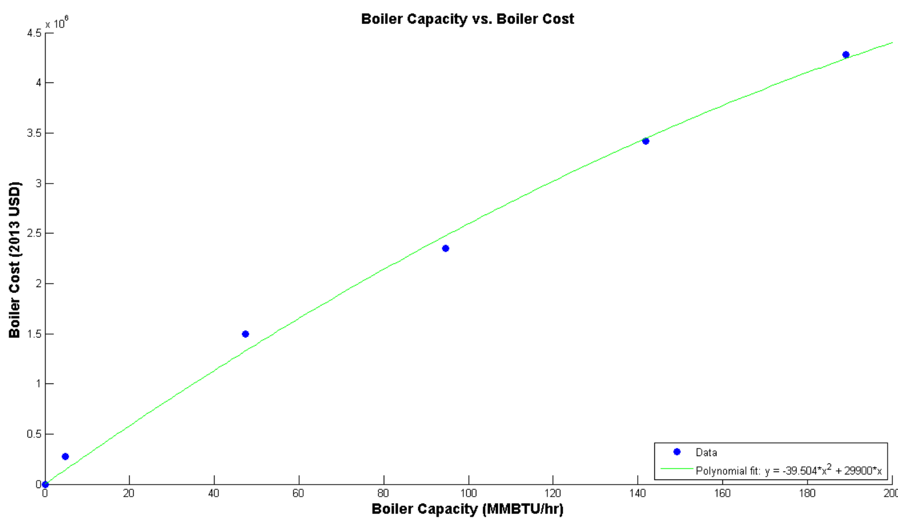

Figure C.4: Data and polynomial from [53] for boiler capital cost Equation 3.25.